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The Effect of Magnetisation and Lorentz Forces in a Two-Dimensional Biomagnetic Channel Flow

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Abstract. The present paper studies the fundamental problem of a Magnetohydrodynamics (MHD) behaviour of the biomagnetic fluid flow in a channel under the influence of a spatially varying magnetic field. The governing equation involves the presence of magnetisation and Lorentz forces for the magnetic field description. Solution of the problem is obtained using an improved finite difference method that employs staggered grid in its discretisation. This approach has successfully handled the pressure of the flow which is the main problem in the standard finite difference method. Results concerning the velocity and skin friction indicate that the presence of magnetic field appreciably influence the flow field. In particular, a distortion in terms of asymmetric flow profile is observed near the magnetic source. Also, vortices have been observed near the lower plate where the magnetic source is placed.

Keywords: Biomagnetic, Magnetisation force, Lorentz force, Two-dimensional channel flow
PACS: 47.11.Bc

INTRODUCTION

Biomagnetic fluid dynamics (BFD) is the study of the interaction of biological fluids with an applied magnetic field. The study of the BFD flow has been investigated by several researchers for numerous applications in bioengineering and medicine. Among them are the development of magnetic devices for cell separation, targeted transport of drugs i.e. using magnetic particles as drug carriers, magnetic wound treatment and cancer tumor treatment causing magnetic hyperthermia (Haik et al. [1], Ruuge and Rusetski [2], Plavins and Lauva [3]). On this regard, encouraging solution couples with supportive findings have been evident in the clinical application in terms of magnetic drug targeting (MDT) in loco-regional cancer treatment (Alexiou et al. [4-6]).

In order to examine the flow of a BFD, mathematical models have been developed by Haik et al. [1]). The BFD model is based on the Ferrohydrodynamics (FHD) [7], which deals with non-inducing electric current and considers that the flow is affected by the magnetisation of the fluid in the magnetic field [8]. For a full description of a blood flow, the contribution of the Lorentz force due to the induced electric current of Magnetohydrodynamic (MHD) [9] should be taken into account. Therefore, an extended BFD mathematical model, which includes the Lorentz force was developed by Tzirtzilakis [8].

From literature there are several methods considered in solving biomagnetic fluid flow using numerical method. The stream function–vorticity formulation was adopted by Loukopoulos and Tzirtzilakis [10] for the numerical investigation of the steady fluid flow in a channel under the influence of a spatially varying magnetic field. The development of a more stable and simpler technique in the application for BFD problems was presented by Tzirtzilakis [11]. The technique was used to investigate the flow in a channel with symmetric stenosis [12]. Finite analytic method was used by Haik et al. [13] for the simulation of flow in a channel with thrombus while the finite volume method was used in Khashan and Haik [14] to simulate a biomagnetic fluid in a stenotic artery.

In the present study, the Finite Difference Method (FDM) will be adopted. One of the main challenges of using the FDM is in the handling of the pressure of the flow. To overcome this problem, a new scheme that is implemented with the SIMPLE-type algorithm for the pressure field calculation is proposed by Zogheib [15]. The nonlinear partial differential governing equations will be discretised using the finite difference approximation which is carried out in a staggered grid. The advantages of using the staggered grid over a non-staggered grid are twofold. First, the continuity equation can be written at node *i,j* with the central difference up to a second order accuracy

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without the interpolation of the relevant velocity components. Second, it prevents the odd-even coupling or what is known as the checkerboarding between the pressure and the velocity fields [16].

However, the formulation by Zogheib [15] is devised for the Navier-Stokes equations only. Therefore, this paper aims to provide a modified scheme that handles the additional magnetic part. The simulation of a flow behaviours using biomagnetic equation for MHD in a channel under the influence of a spatially varying magnetic field based on the new model is explored and discussed. In the followings, the governing equations as well as its boundary conditions are presented in Section 2. Section 3 discusses the numerical scheme used while the validation of the benchmark problem based on the improved model is shown in Section 4. Finally, Section 5 presents the numerical simulation of a MHD flow under the influence of a spatially varying magnetic field.

THE GOVERNING EQUATIONS

In the present study, the steady, two-dimensional, incompressible and laminar flow is considered. The energy equation is not considered i.e. isothermal case. The Lorentz force due to MHD interaction and the magnetisation force due to FHD interaction will be taken into account associated with Navier-Stokes equations. Hence, the governing equations of the fluid flow, under the influence of a spatially varying magnetic field, are given by Tzirtzilakis and Loukopoulos [17] as the followings:

Continuity equation

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

x-momentum equation

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho^*}\frac{\partial p^*}{\partial x^*}+\nu^*\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)+\frac{\mu_0{}^* M^*}{\rho^*}\frac{\partial H^*}{\partial x^*}-\sigma^* B^{*2} u^* \tag{2}$$

y-momentum equation

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho^*}\frac{\partial p^*}{\partial y^*}+\nu^*\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)+\frac{\mu_0{}^* M^*}{\rho^*}\frac{\partial H^*}{\partial y^*} \tag{3}$$

The boundary conditions are

$$x^*=0 \;\text{ or }\; x^*=L^* \;\text{ and }\; 0\le y^*\le L^*: \quad u^*=v^*=0, \tag{4}$$

$$y^*=0 \;\text{ or }\; y^*=L^* \;\text{ and }\; 0\le x^*\le L^*: \quad u^*=v^*=0, \tag{5}$$

where u and v are the velocity components in the x and y directions respectively, p is the pressure, ρ is the constant density, $\nu^*=\mu^*/\rho^*$ is the kinematic viscosity, μ_0 is magnetic permeability in vacuum, M^* is the magnetisation and H^* is the magnetic field intensity. The variation of the magnetic field is at the x^*-y^* plane, so the magnetic terms on the right hand side in equations (2) and (3) represent the magnetisation force per unit mass in the x^* and y^* directions, respectively.

Using the dimensionless definitions given by Tzirtzilakis and Loukopoulos [17]:

$$x=\frac{x^*}{h},\; y=\frac{y^*}{h},\; u=\frac{u^*}{u_r},\; v=\frac{v^*}{u_r},\; p=\frac{p^*}{\rho u_r{}^2},\; H=\frac{H^*}{H_r}. \tag{6}$$

the governing equations (1) to (3) become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{Mn_F}{\mathrm{Re}}H\frac{\partial H}{\partial x}-{Mn_M}^2 u \tag{8}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{Mn_F}{\mathrm{Re}}H\frac{\partial H}{\partial y} \tag{9}$$

and the boundary conditions (4) and (5) become

$$x=0 \text{ or } x=1 \text{ and } 0\le y\le 1: \quad u=v=0, \tag{10}$$

$$y=0 \text{ or } y=1 \text{ and } 0\le x\le 1: \quad u=v=0. \tag{11}$$

Here, $\mathrm{Re}=\frac{h^*\rho^*u_r{}^*}{\mu^*}$ is the Reynolds number, $Mn_F=\frac{B_0^*M^*h^{*2}\rho^*}{\mu_0{}^{*2}}$ is the magnetic number and $Mn_M=B_0^*h^*\sqrt{\frac{\sigma^*}{\mu^*}}$ is the magnetic number.

NUMERICAL METHOD

The solution of the governing equations presented in the equations (7) to (9) subject to the boundary conditions in the equations (10) and (11) are solved numerically using the improved finite difference method as described in our previous work [18]. A first-order upwind differencing scheme is used to approximate the convective terms in the momentum equations, while a second-order central differencing is used for the diffusion terms. The pressure gradients are approximated by a second order central difference scheme.

VERIFICATION

In this section, verification is presented to check the degree of matchness of the simulation with respect to the analytical solutions. Therefore, the developed finite difference formulation is applied to a two-dimensional MHD channel flow similar to that studied by Xu et al. [19]. Note that the magnetic number, $Mn_F=0$ here. This problem is considered merely for the validation of our present model. The set of equations (7) to (9) will be used for this purpose. The geometry for 2D Hartman flow and the boundary conditions are illustrated in Figure 1. The flow is driven by pressure gradient.

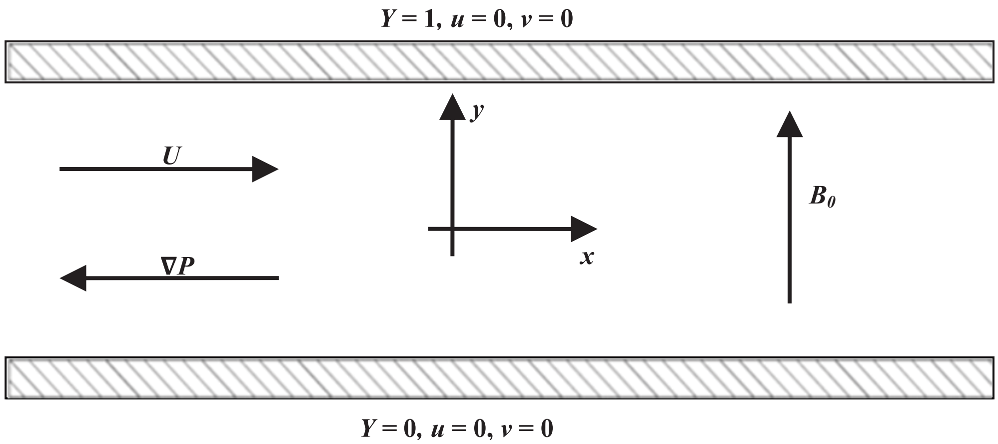


FIGURE 1. 2D Hartman flow and its boundary conditions

The analytical solution of MHD channel flow investigated by Xu et al. [19] is given by

$$U(y) = A\cosh\left(\sqrt{\mathrm{Re}\,N}B_0 y\right) + B\sinh\left(\sqrt{\mathrm{Re}\,N}B_0 y\right) - \frac{1}{NB_0^2}\frac{dP}{dx} \tag{12}$$

where N is the Stuart number, $A = \frac{1}{NB_0^2}\frac{dP}{dx}$, and $B = \frac{1}{NB_0^2}\frac{dP}{dx}\frac{1-\cosh\left(\sqrt{\mathrm{Re}\,N}B_0\right)}{\sinh\left(\sqrt{\mathrm{Re}\,N}B_0\right)}$.

Figure 2 shows the comparison of axial velocity component u along vertical line through geometric centre of cavity for $\mathrm{Re}=1$, $N=1$ and $B_0=2$. In this particular reference, the considered mesh size is 200×200, which means the mesh size for dx and dy are 0.05. It is obvious that the result computed using current formulation has a very good agreement with that obtained by Xu et al. [19].

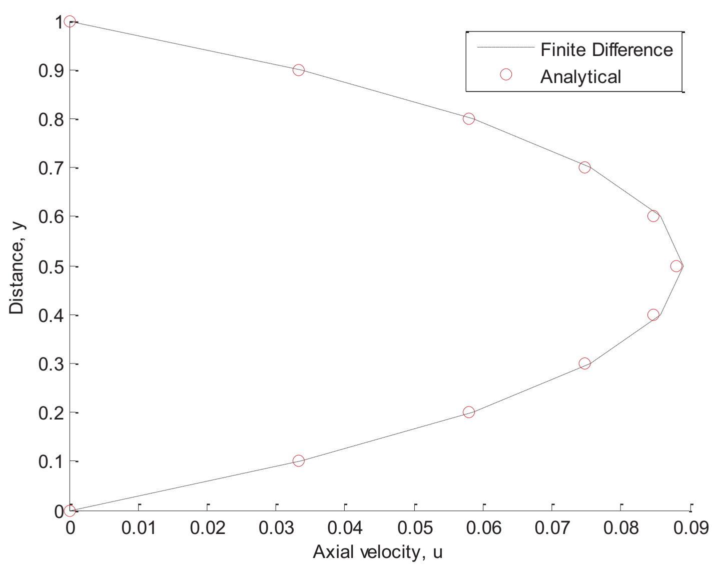


FIGURE 2. Validation with analytical solution by Xu et al. [19]

Figure 3 shows the velocity contour of the flow. It is therefore evident that our improved numerical algorithm also can handle a problem which is driven by pressure gradient. It is constructive to note that the flow previously studied in Rusli et al. [18] is driven by velocity gradient. Our original reference which is based on a numerical algorithm proposed by Zogheib [15] can only handle for flow which is driven by velocity gradient.

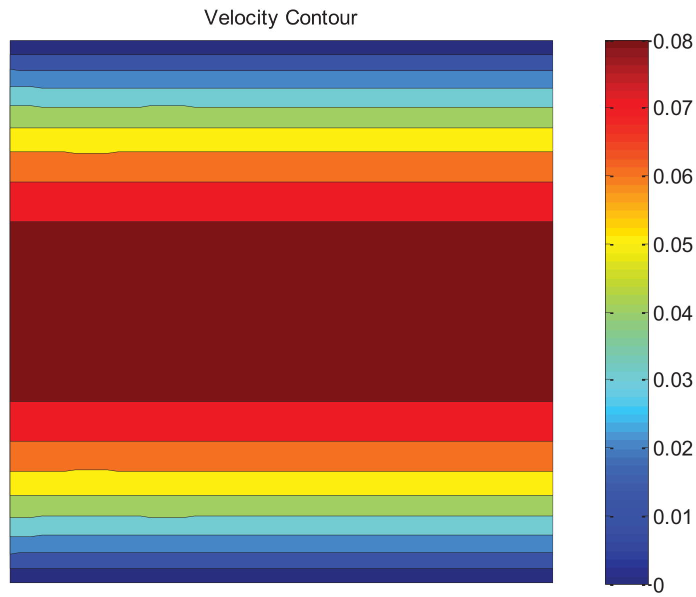
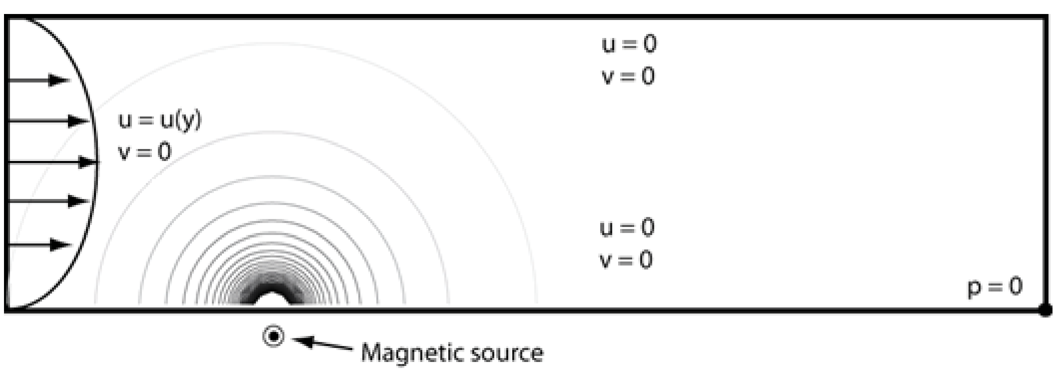


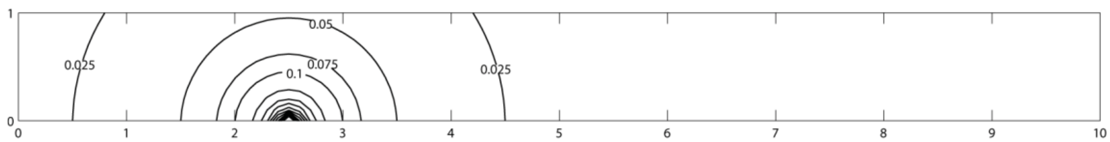
FIGURE 3. Velocity contour of the flow

MHD FLOW IN A CHANNEL

Having just established an agreement with the benchmark problem, we consider here a MHD flow in a channel that is prescribed with a spatially varying magnetic field. The biomagnetic fluid flow is considered taking place between two parallel plates (channel) same as the setting described in Figure 4. In that figure, the length of the plates is L and the distance between them is D. The channel is modeled such that $L/D = 10$. The flow at the entrance is assumed to be fully developed. The flow is subjected to a magnetic source, which is placed at $(a,b) = (2.5,-0.05)$.



(a)



(b)

FIGURE 4. Mesh and boundary condition of biomagnetic flow subjected to a magnetic field (a) Problem domain and boundary condition (b) Magnetic field intensity

Tzirtzilakis and Loukopoulos [17] solved Navier Stokes equation allowing magnetisation and Lorentz forces with inclusion of energy equation. The results concerning the velocity and temperature field, skin friction and rate of heat transfer, show that the flow is influenced by the magnetic field. The major effect is the formation of two vortices which arise at the areas where the magnetic field starts and stops to apply. They considered magnetisation force just for x momentum equation, because the magnetic field was assumed varying in x direction only. Extending this, we will include magnetisation force for both x and y momentum equations, because the current focus is placed on variations both in x and y directions.

Figure 5 shows U velocity profiles for various magnetic effect, i.e. $3T$, $5T$ and $8T$. An increase in the magnetic strength B causes perturbation in the flow pattern and an elevation of maximum velocity along the flow direction. A vortex is arising near the lower plate where the magnetic source is placed. Therefore, blood flow recirculation may occur in this region. Observe also that the waviness of the velocity profile especially that near the magnetic source region has been increased. This is consistent with the enhancement of the flow vortex exerted by an increment of magnetic intensity.

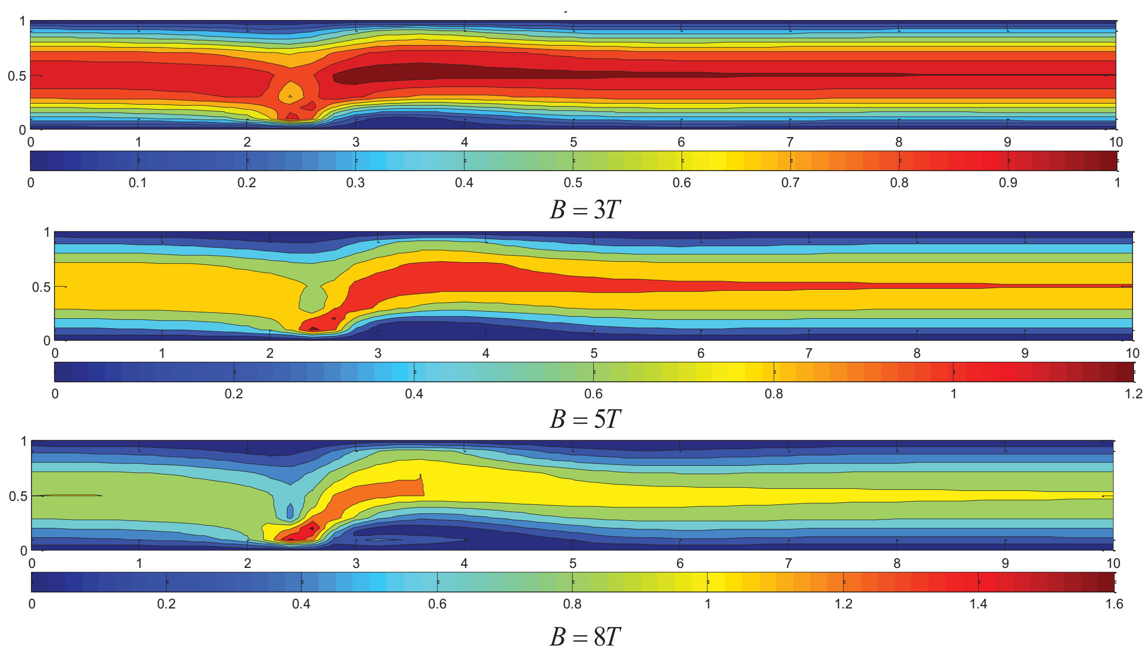


FIGURE 5. U velocity profiles for various magnetic effect

Figure 6 shows the local skin friction coefficient, C_f, for the upper and lower walls for B of $0T$, $3T$, $5T$ and $8T$. Considering lower wall as shown in Figure 6(a), C_f increases rapidly in the region $x = 1.5$ to $x = 2.48$ where it reaches its maximum values. Very close to the region where the magnetic source is located $(x = 2.5)$, a decreasing pattern occurs drastically and at $x = 3.36$, C_f reaches its minimum value. Different to upper wall in Figure 6(b), C_f increases near the area of the magnetic source. Furthermore, with the increase in magnetic strength, the minumum values are enhanced and extended but the shape of the skin friction coefficient remains qualitatively the same. The maximum skin friction coefficient occurs at $x = 2.3$ while the minumum skin friction coefficient occurs at $x = 3.56$

for $B = 8T$. It is also worthy to observe that while the lower wall has the variation of skin friction in the positive and negative range, the upper wall only has negative value of skin friction.

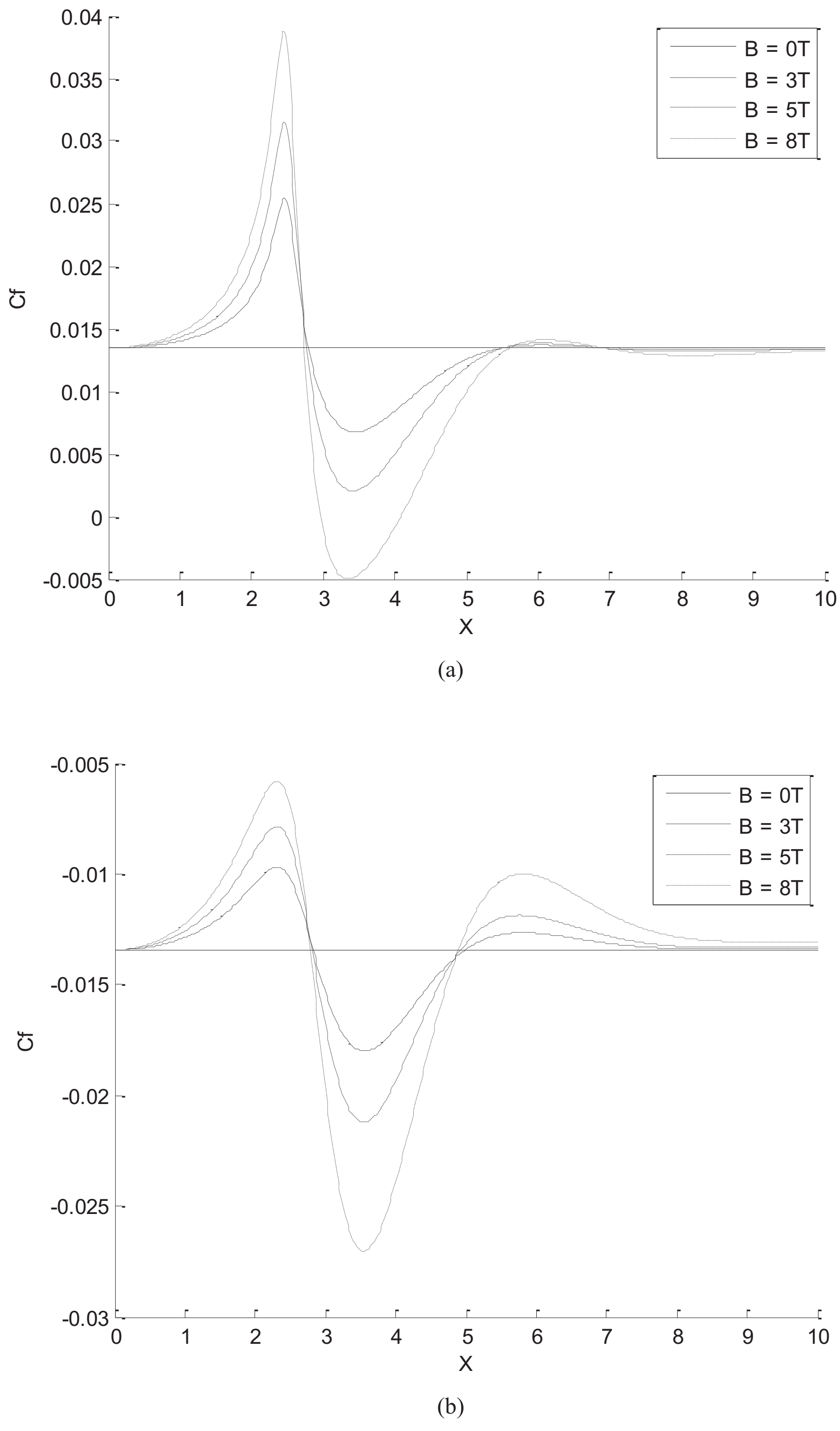


FIGURE 6. Skin friction coefficient along the channel at (a) lower wall, and (b) upper wall subjected to various magnetic field intensities at $\mathrm{Re} = 250$

CONCLUSIONS

MHD flow behaviours in a channel under the influence of a spatially varying magnetic field are presented. The solution of the problem is obtained using a numerical technique based on an improved finite difference method. For validation purpose, the result computed using current formulation has a very good agreement with those obtained from benchmark test. Results concerning the velocity indicates that the presence of magnetic field appreciably influence the flow field. In general, the existence of a magnetic field alters the behaviours blood flow considerably. On practical ground, the findings can offer interesting correlation in assisting the optimization of the magnetically targeted drug delivery, nowadays commonly exercised but laboriously performed clinical applications.

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