CHAPTER 5

Direct-Current Bridges

5-1 Instructional Objectives

Chapter 5 discusses the basic theory and applications of direct-current bridges. Primary attention is given to the principle of operation as well as to measurement and control applications and to the recent use of digital circuitry in bridges. After completing the chapter you should be able to

1. List and discuss the principal applications of Wheatstone bridges.
2. Describe the operation of the Wheatstone bridge.
3. List and discuss the principal applications of Kelvin bridges.
4. Describe the operation of the Kelvin bridge.
5. Solve for Thévenin's equivalent circuit for an unbalanced Wheatstone bridge.
6. Describe how a Wheatstone bridge may be used to control various physical parameters.
7. Define the term *null* as it applies to bridge measurements.
8. List an advantage of comparison-type measurements over deflection-type measurements.
9. Describe the difference between using minicomputers and microprocessors in test equipment.
10. Describe how microprocessors are being used in test equipment.

INTRODUCTION

Bridge circuits, which are instruments for making **comparison measurements**, are widely used to measure resistance, inductance, capacitance, and impedance.

Bridge circuits operate on a **null-indication** principle. This means the indication is *independent* of the calibration of the indicating device or any characteristics of it. For this reason, very high degrees of accuracy can be achieved using the bridges.

Bridge circuits are also frequently used in *control* circuits. When used in such applications, one arm of the bridge contains a resistive element that is sensitive to the physical parameter (temperature, pressure, etc.) being controlled.

This chapter discusses basic bridge circuits and their applications in measurement and control. It also introduces some very recent concepts regarding the use of digital principles in bridges.

5-3 THE WHEATSTONE BRIDGE

The **Wheatstone bridge** consists of two parallel resistance branches with each branch containing two series elements, usually resistors. A dc voltage source is connected across this resistance network to provide a source of current through the resistance network. A *null detector*, usually a **galvanometer**, is connected between the parallel branches to detect a condition of **balance**. This circuit, shown in Fig. 5-1, was first devised by S. H. Christie in 1833. However, it was little used until 1847 when Sir Charles Wheatstone, for whom the circuit is named, recognized its possibilities as a very accurate means of measuring resistance.

The Wheatstone bridge has been in use longer than almost any other electrical measuring instrument. It is still an accurate and reliable instrument and is heavily used in industry. Accuracy of 0.1% is quite common with the Wheatstone bridge as opposed to 3% to 5% error with the ordinary ohmmeter for resistance measurement.

In using the bridge to determine the value of an unknown resistor, say R_4, we vary one of the remaining resistors until the current through the null detector decreases to zero. The bridge is then in a balanced condition, which means the voltage across resistor R_3 is equal to the voltage drop across R_4. Therefore, we can say that

$$I_3R_3 = I_4R_4 \tag{5-1}$$

At balance the voltage drops across R_1 and R_2 must also be equal; therefore,

$$I_1R_1 = I_2R_2 \tag{5-2}$$

Since no current flows through the galvanometer G when the bridge is

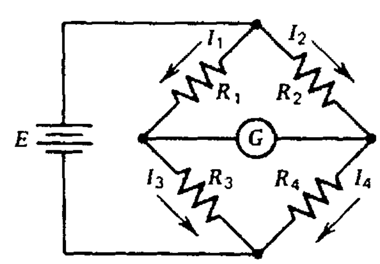

FIGURE 5-1 Wheatstone bridge circuit.

balanced, we can say that

$$I_1 = I_3$$

and

$$I_2 = I_4$$

Substituting I_1 for I_3 and I_2 for I_4 in Eq. 5-1 yields the following:

$$I_1 R_3 = I_2 R_4 \qquad (5\text{-}3)$$

Now, if we divide Eq. 5-2 by Eq. 5-3, we obtain

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

This can be rewritten as

$$R_1 R_4 = R_2 R_3 \qquad (5\text{-}4)$$

Equation 5-4 states the conditions for balance of a Wheatstone bridge and is useful for computing the value of an unknown resistor once balance has been achieved.

AMPLE 5-1

Determine the value of the unknown resistor, R_x, in the circuit of Fig. 5-2 assuming a null exists (current through the galvanometer is zero).

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We see in Eq. 5-4 that the products of the resistance in opposite arms of the bridge are equal at balance. Therefore,

$$R_x R_1 = R_2 R_3$$

Solving for R_x yields

$$R_x = \frac{R_2 R_3}{R_1}$$

$$= \frac{15\ \text{k}\Omega \times 32\ \text{k}\Omega}{12\ \text{k}\Omega} = 40\ \text{k}\Omega$$

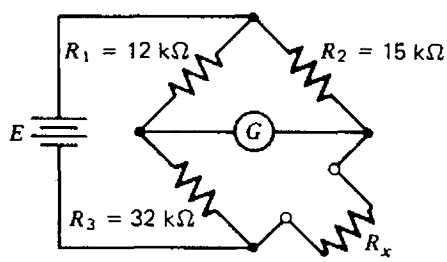

FIGURE 5-2 Circuit for Example 5-1.

SENSITIVITY OF THE WHEATSTONE BRIDGE

When the bridge is in an unbalanced condition, current flows through the galvanometer, causing a deflection of its pointer. The amount of deflection is a function of the sensitivity of the galvanometer. We might think of **sensitivity** as *deflection per unit current*. This means that a more sensitive galvanometer deflects a greater amount for the same current. Deflection may be expressed in linear or angular units of measure. Sensitivity S can be expressed in units of

$$S = \frac{\text{millimeters}}{\mu\text{A}} \text{ or } \frac{\text{degrees}}{\mu\text{A}} \text{ or } \frac{\text{radians}}{\mu\text{A}} \tag{5-5}$$

Therefore, it follows that total deflection D is

$$D = S \times I$$

where S is as defined and I is the current in microamperes (μA). We might naturally question how to determine the amount of deflection that will result from a particular degree of unbalance.

Although general circuit analysis techniques can be applied to this kind of problem, in our approach we will make frequent use of **Thévenin's theorem**. Since our interest is in finding the current through the galvanometer, we want to find Thévenin's equivalent circuit for the bridge as seen by the galvanometer. Thévenin's equivalent voltage is found by removing the galvanometer from the bridge circuit as shown in Fig. 5-3 and computing the "open-circuit" voltage between terminals a and b. Applying the voltage divider equation permits us to express the voltage at point a as

$$V_a = E\frac{R_3}{R_1 + R_3}$$

and the voltage at point b as

$$V_b = E\frac{R_4}{R_2 + R_4}$$

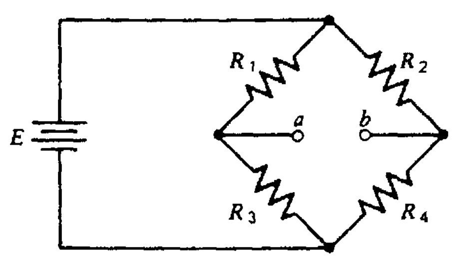

FIGURE 5-3 Wheatstone bridge with the galvanometer removed to facilitate computation of Thévenin's equivalent voltage.

The difference in V_a and V_b represents Thévenin's equivalent voltage. That is,

$$V_{Th} = V_a - V_b = E\frac{R_3}{R_1+R_3} - E\frac{R_4}{R_2+R_4}$$

$$= E\left(\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right) \qquad (5\text{-}6)$$

Thévenin's equivalent resistance is found by replacing the voltage source with its internal resistance and computing the resistance seen looking back into the bridge at the terminals from which the galvanometer was removed. Since the internal resistance of the voltage source E is assumed to be very low, we will treat it as 0 Ω and redraw the bridge as shown in Fig. 5-4 to facilitate computation of the equivalent resistance. The equivalent resistance of the circuit in Fig. 5-4 is calculated as $R_1 \| R_3 + R_2 \| R_4$ or

$$R_{Th} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4} \qquad (5\text{-}7)$$

Thévenin's equivalent circuit for the bridge, as seen looking back into the bridge from terminals a and b in Fig. 5-3, is shown in Fig. 5-5. A galvanometer connected between the output terminals a and b of a Wheatstone bridge (Fig. 5-3) or its Thévenin's equivalent circuit (Fig. 5-5) will experience the same deflection. The magnitude of the current is limited by both Thévenin's equivalent resistance and any resistance connected between terminals a and b. The resistance between terminals a and b generally consists of only the resistance of the galvanometer R_g. The deflection current in the galvanometer is

$$I_g = \frac{V_{Th}}{R_{Th} + R_g} \qquad (5\text{-}8)$$

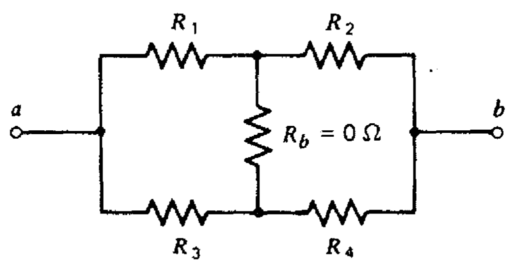

FIGURE 5-4 Circuit for finding Thévenin's equivalent resistance.

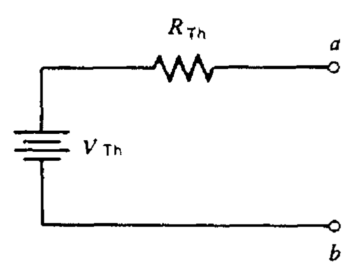

FIGURE 5-5 Thévenin's equivalent circuit for an unbalanced Wheatstone bridge.

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Calculate the current through the galvanometer in the circuit of Fig. 5-6.

The easiest way to solve for the current is to find Thévenin's equivalent circuit for the bridge as seen by the galvanometer. Thévenin's equivalent voltage is calculated as follows:

$$V_{Th} = E\left(\frac{R_3}{R_3+R_1} - \frac{R_4}{R_4+R_2}\right)$$
$$= 6\text{ V} \times \left(\frac{3.5\text{ k}\Omega}{3.5\text{ k}\Omega + 1\text{ k}\Omega} - \frac{7.5\text{ k}\Omega}{7.5\text{ k}\Omega + 1.6\text{ k}\Omega}\right)$$
$$= (6\text{ V})(0.778 - 0.824) = 0.276\text{ V}$$

Thévenin's equivalent resistance is computed as

$$R_{Th} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$$
$$= \frac{1\text{ k}\Omega \times 3.5\text{ k}\Omega}{1\text{ k}\Omega + 3.5\text{ k}\Omega} + \frac{1.6\text{ k}\Omega \times 7.5\text{ k}\Omega}{1.6\text{ k}\Omega + 7.5\text{ k}\Omega} = 2.097\text{ k}\Omega \qquad (5\text{-}7)$$

Thévenin's equivalent circuit can now be connected to the galvanometer as shown in Fig. 5-7. The current through the galvanometer is now calculated as

$$I_g = \frac{V_{Th}}{R_{Th} + R_g}$$
$$= \frac{0.276\text{ V}}{2.097\text{ k}\Omega + 200\ \Omega} = 120\ \mu\text{A}$$

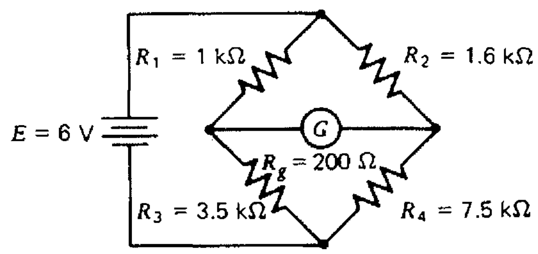


FIGURE 5-6 Unbalanced Wheatstone bridge.

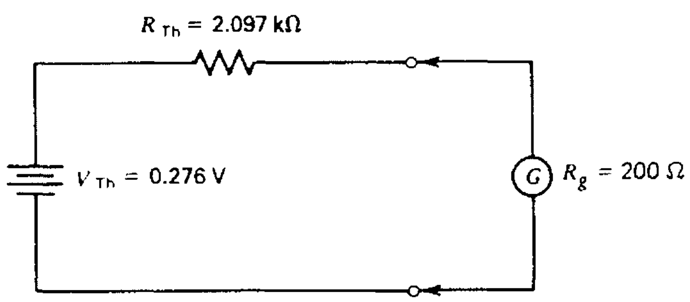


FIGURE 5-7 Thévenin's equivalent circuit for the unbalanced bridge of Fig. 5-6, connected to a galvanometer.

5-4.1 Slightly Unbalanced Wheatstone Bridge

If three of the four resistors in a bridge are equal in value to R and the fourth differs from R by 5% or less, we can develop an approximate but accurate expression for Thévenin's equivalent voltage and resistance.[1] Consider the circuit in Fig. 5-8. The voltage at point a is given as

$$V_a = E\frac{R}{R+R} = E\left(\frac{R}{2R}\right) = \frac{E}{2}$$

The voltage at point b is expressed as

$$V_b = E\frac{R+\Delta r}{R+R+\Delta r}$$

Thévenin's equivalent voltage is the difference in these voltages, or

$$V_{\text{Th}} = V_b - V_a = E\left(\frac{R+\Delta r}{R+R+\Delta r} - \frac{1}{2}\right) = E\left(\frac{\Delta r}{4R+2\Delta r}\right)$$

If Δr is 5% of R or less, then the Δr term in the denominator may be dropped without introducing appreciable error. If this is done, the expression for Thévenin's equivalent voltage simplifies to

$$V_{\text{Th}} \approx E\left(\frac{\Delta r}{4R}\right) \tag{5-9}$$

Thévenin's equivalent resistance can be calculated by replacing the voltage source with its internal resistance (for all practical purposes a short circuit) and redrawing the circuit as seen from terminals a and b. The bridge, as it appears looking from the output terminals, is shown in Fig. 5-9. Thévenin's equivalent resistance is now calculated as

$$R_{\text{Th}} = \frac{R}{2} + \frac{(R)(R+\Delta r)}{R+R+\Delta r}$$

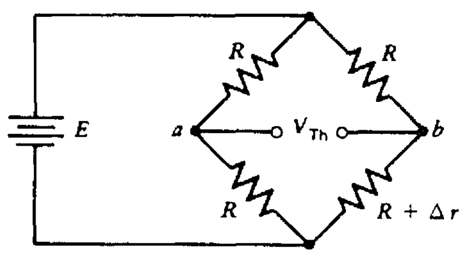


FIGURE 5-8 A Wheatstone bridge with three equal arms.

[1]This is an extremely practical circuit. It is commonly used in strain-gauge measurements. It is also used in transducers of control systems as an error detector. It is therefore important to develop equivalent voltage and resistance relations for it.

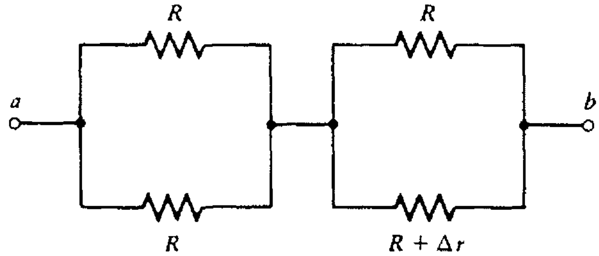


FIGURE 5-9 Resistance of a Wheatstone bridge as seen from the output terminals and *b*.

Again, if Δr is small compared to R, the equation simplifies to

$$R_{Th} \approx \frac{R}{2} + \frac{R}{2} \quad \text{or } R_{Th} \approx R \qquad (5\text{-}10)$$

Using these approximations, we have Thévenin's equivalent circuit as shown in Fig. 5-10. These approximations are about 98% accurate if $\Delta r \leq 0.05R$.

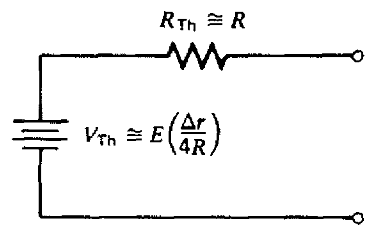


FIGURE 5-10 An approximate Thévenin's equivalent circuit for a Wheatstone bridge containing three equal resistors and a fourth resistor differing by 5% or less.

EXAMPLE 5-3

Use the approximation given in Eqs. 5-9 and 5-10 to calculate the current through the galvanometer in Fig. 5-11. The galvanometer resistance, R_g is 125 Ω and is a center-zero 200-0-200-μA movement.

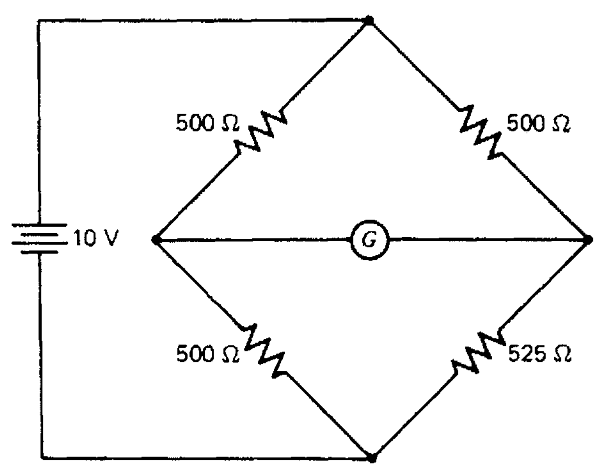


FIGURE 5-11 Slightly unbalanced Wheatstone bridge.

Thévenin's equivalent voltage is

$$V_{Th} = E\left(\frac{\Delta r}{4R}\right) = 10\ \text{V} \times \frac{25\ \Omega}{2000\ \Omega} = 0.125\ \text{V} \qquad (5\text{-}9)$$

Thévenin's equivalent resistance is

$$R_{Th} \approx R \approx 500\ \Omega \qquad (5\text{-}10)$$

The current through the galvanometer is

$$I_g = \frac{V_{Th}}{R_{Th} + R_g} + \frac{0.125\ \text{V}}{500\ \Omega + 125\ \Omega} = 200\ \mu\text{A}$$

If the detector is a 200-0-200-μA galvanometer, we see that the pointer deflected full scale for a 5% change in resistance.

ELVIN BRIDGE

The **Kelvin bridge** (Fig. 5-12) is a modified version of the Wheatstone bridge. The purpose of the modification is to eliminate the effects of contact and lead resistance when measuring unknown low resistances. Resistors in the range of 1 Ω to approximately 1 $\mu\Omega$ may be measured with a high degree of accuracy using the Kelvin bridge. Since the Kelvin bridge uses a second set

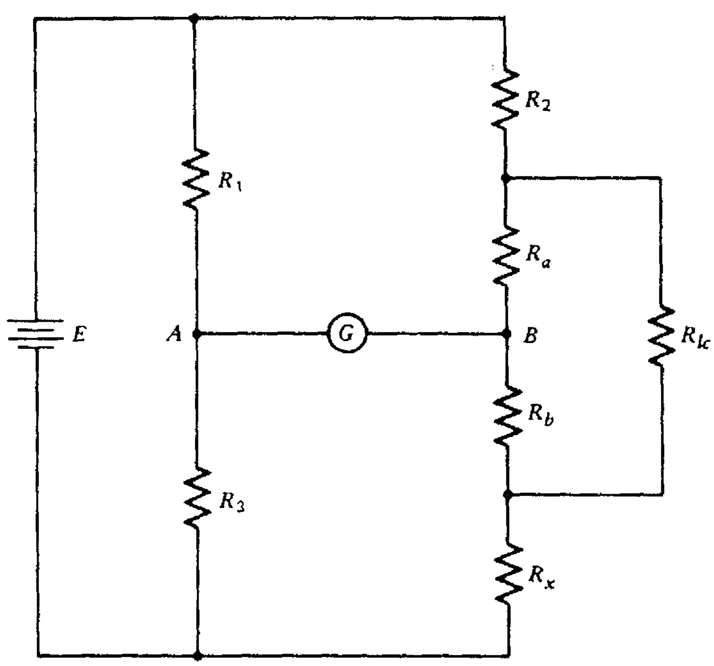


FIGURE 5-12 Basic Kelvin bridge showing a second set of ratio arms.

of ratio arms as shown in Fig. 5-12, it is sometimes referred to as the Kelvin double bridge.

The resistor R_{lc} shown in Fig. 5-12 represents the lead and contact resistance present in the Wheatstone bridge. The second set of ratio arms (R_a and R_b in Fig. 5-12) compensates for this relatively low lead–contact resistance. At balance the ratio of R_a to R_b must be equal to the ratio of R_1 to R_3. It can be shown (see Appendix A) that, when a null exists, the value for R_x is the same as that for the Wheatstone bridge, which is

$$R_x = \frac{R_2 R_3}{R_1}$$

This can be written as

$$\frac{R_x}{R_2} = \frac{R_3}{R_1}$$

Therefore, when a Kelvin bridge is balanced, we can say

$$\frac{R_x}{R_2} = \frac{R_3}{R_1} = \frac{R_b}{R_a} \tag{5-11}$$

EXAMPLE 5-4

If, in Fig. 5-12, the ratio of R_a to R_b is 1000, R_1 is 5 Ω, and $R_1 = 0.5R_2$, what is the value of R_x?

Solution

The resistance of R_x can be calculated using Eq. 5-11 as

$$\frac{R_x}{R_2} = \frac{R_a}{5\ \Omega} = \frac{1}{1000}$$

Since $R_1 = 0.5R_2$, the value of R_2 is calculated as

$$R_2 = \frac{R_1}{0.5} = \frac{5\ \Omega}{0.5} = 10\ \Omega$$

Now, we can calculate the value of R_x as

$$R_x = R_2\left(\frac{1}{1000}\right) = 10\ \Omega \times \frac{1}{1000} = 0.01\ \Omega$$

5-6 DIGITAL READOUT BRIDGES

The tremendous increase in the use of digital circuitry has had a marked effect on electronic test instruments. Early use of digital circuits in bridges was to provide digital readout. The actual measuring circuitry of the bridge remained the same. But operator error in observing the reading was eliminated by incorporating digital readout capabilities. The block diagram for a Wheatstone bridge with digital readout is shown in Fig. 5-13. Note that a logic circuit is used to provide a signal to R_3, sense the null, and provide a digital readout representing the value of R_x.

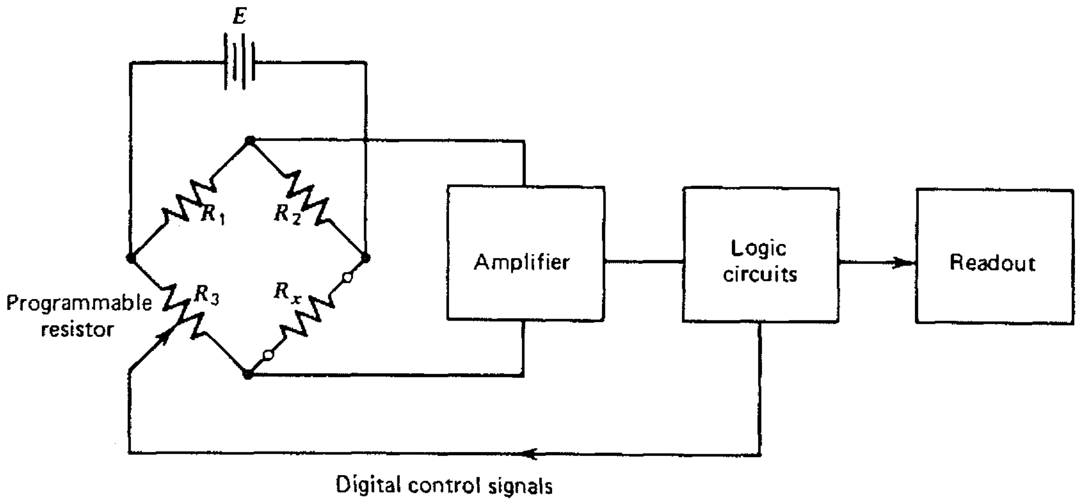


FIGURE 5-13 Block diagram for a Wheatstone bridge with a digital readout.

IICROPROCESSOR-CONTROLLED BRIDGES

Digital computers have been used in conjunction with test systems, bridges, process controllers, and in other applications for several years. In these applications computers are used to give instructions and perform operations on the measurement data. When the **microprocessor** was first developed, it was used in much the same way. However, real improvements in performance did not occur until microprocessors were truly integrated into the instrument. With this accomplished, not only do microprocessors give instructions about measurements, but they can also change the way in which measurements are made. This innovation has brought about a whole new class of instruments called *intelligent instruments*.

The complexity and cost of making analog measurements can be reduced using a microprocessor. This reduction of analog circuitry is important, even if additional digital circuitry must be added, because precision analog components are expensive. In addition, adjusting, testing, and trouble-shooting analog circuits is time-consuming and costly. Often, digital circuits can replace analog circuits because various functions can be done either way.

The following are some of the ways in which microprocessors are reducing the cost and complexity of analog measurements.

- Replacing sequential control logic with stored control programs.
- Eliminating some auxiliary equipment by handling interfacing, programming, and other system functions.
- Giving wider latitude in the selection of measurement circuits, thereby making it possible to measure one parameter and calculate another parameter of interest.
- Reducing accuracy requirements by storing and applying correction factors.

Instruments in which microprocessors are an integral part can take th results of a measurement that is easiest to make in a circuit and then calcula and display the desired parameter, which may be much more difficult t measure directly. For example, conventional counters can measure the perio of a low-frequency waveform. The frequency is then calculated by hand, o extensive circuitry is required to perform the required division. Such calcula tions are done very easily by the microprocessor.

Resistance and conductance are also reciprocals of each other. Som hybrid digital–analog bridges are designed to measure conductance by current measurement. This measurement is then converted to a resistanc value by rather elaborate circuitry. With a microprocessor-based instrument resistance value is easily obtained from the conductance measurement.

Many other similar examples could be presented. However, the importa thing to remember is that the microprocessor is an integral part of th measuring instrument. As such, it produces an intelligent instrument th allows the choice of the easiest method of measurement and requires on one measurement circuit to obtain various results. Specifically, one quanti can be measured in terms of another, or several others, with complete different dimensions, and the desired results can be calculated with th microprocessor.

The General Radio, Model 1658 RLC Digibridge, shown in Fig- 5-14 is microprocessor-based instrument. Such intelligent instruments represent new era in impedance-measuring instruments. The following are some of th features of the instrument.

FIGURE 5-14 General Radio, Model 1658 RLC Digibridge. (Courtesy GenRad In Concord, Mass.)

- Automatic measurement of resistance, *R*, inductance, *L*, capacitance, *C*, dissipation factor for capacitors, *D*, and storage factor for inductors, *Q*.
- 0.1% basic accuracy.
- Series or parallel measurement mode.
- Autoranging.
- No calibration ever required.
- Ten bins for component sorting and binning.
- Three test speeds.
- Three types of display-programmed bin limits, measured values, or bin number.

Most of these features are available as a result of the microprocessor. For example, the component sorting and binning feature is achieved by programming the microprocessor. When the instrument is used in this mode, bins are assigned a tolerance range. As a component is measured, a digital readout (labeled Bin No.) indicating the proper bin for the component is displayed on the keyboard control panel of Fig. 5-15. The theory of how the bridge circuit operates will be discussed in Chapter 13.

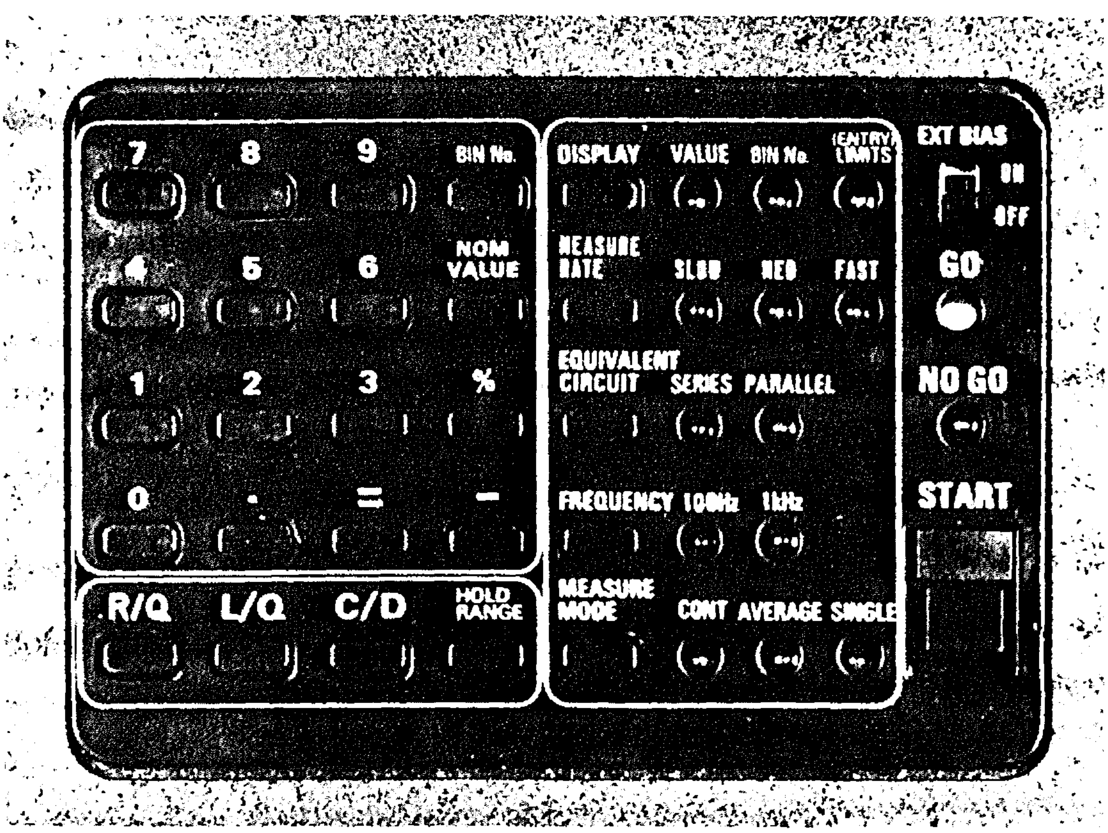


FIGURE 5-15 Control panel for Model 1658 Digibridge. (Courtesy GenRad Inc., Concord, Mass.)

-? BRIDGE-CONTROLLED CIRCUITS

We have seen that whenever a bridge is unbalanced, a potential difference exists at its output terminals. This potential difference causes current through a detector, such as a galvanometer, when the bridge is used as part of a measuring instrument. When a bridge is used as an error detector in a control circuit, the potential difference at the output of the bridge is called an error signal (see Fig. 5-16).

Passive circuit elements such as strain gauges, temperature-sensitive resistors (thermistors), or light-sensitive resistors (photoresistors) produce no output voltage. However, when they are used as one arm of a Wheatstone bridge, a change in their sensitive parameter (heat, light, pressure) produces a change in their resistance. This causes the bridge to be unbalanced, thereby producing an output voltage or an error signal.

Resistor R_v in Fig. 5-16 may be sensitive to one of many different physical parameters such as heat or light. If the particular parameter to which the resistor is sensitive is of such a magnitude that the ratio of R_2 to R_v equals the ratio of R_1 to R_3, then the error signal is zero. If the physical parameter changes, then R_v also changes. The bridge then becomes unbalanced and an error signal exists. In most control applications, the measured and controlled parameter is corrected, restoring R_v to the value that creates a null condition at the output of the bridge. Since R_v varies by only a small amount, Eqs. 5-9 and 5-10 generally apply. Because R_v is corrected rapidly back to the null, the amplitude of the error signal is normally quite low. Therefore, it is usually amplified before being used for control purposes.

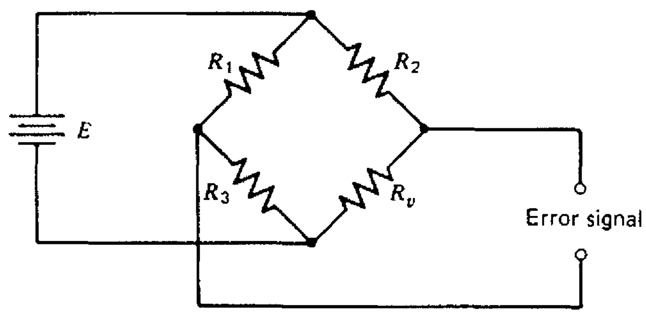


FIGURE 5-16 Wheatstone bridge error detector with R_v sensitive to some physical parameter.

EXAMPLE 5-5

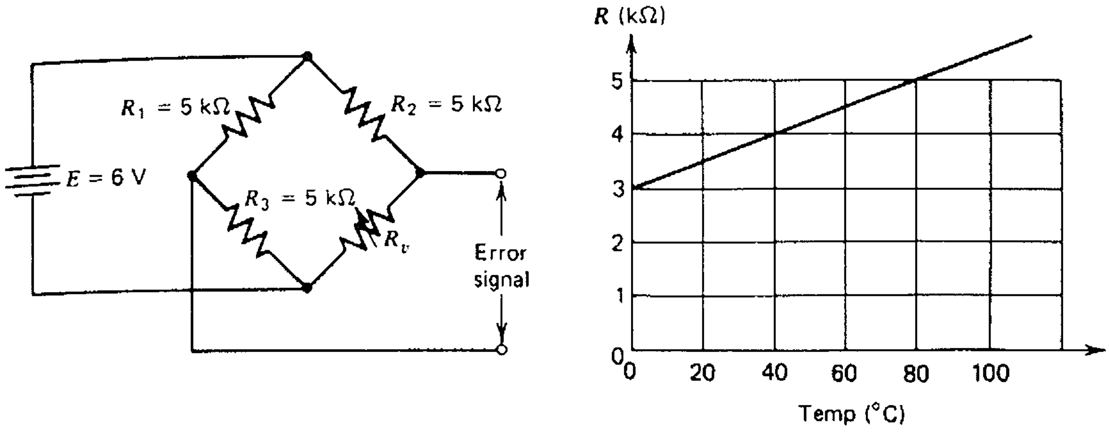
Resistor R_v in Fig. 5-17*a* is temperature-sensitive, with the relation between its resistance and temperature as shown in Fig. 5-17*b*. Calculate

(a) At what temperature the bridge is balanced.
(b) The amplitude of the error signal at 60°C.

Solution

(a) The value of R_v when the bridge is balanced is calculated as

$$R_v = \frac{R_2 R_3}{R_1} = \frac{5\ \text{k}\Omega \times 5\ \text{k}\Omega}{5\ \text{k}\Omega} = 5\ \text{k}\Omega \qquad (5\text{-}4)$$



(a) Circuit (b) Variation of R_v with temperature

FIGURE 5-17 Wheatstone bridge in which one arm (R_v) is temperature-sensitive.

The bridge is balanced when the temperature is 80°C. This is read directly from the graph of Fig. 5-17*b*.

(b) We can also determine, by reading directly from the graph, the resistance of R_v when its temperature is 60°C. This resistance value is 4.5 kΩ; therefore, the error signal, e_g, is

$$e_g = E\frac{R_3}{R_1 + R_3} - E\frac{R_v}{R_2 + R_v}$$

$$= 6\text{ V} \times \frac{5\text{ k}\Omega}{5\text{ k}\Omega + 5\text{ k}\Omega} - 6\text{ V} \times \frac{4.5\text{ k}\Omega}{5\text{ k}\Omega + 4.5\text{ k}\Omega} = 0.158\text{ V}$$

The error signal can also be determined using Eq. 5-9 as

$$e_g = V_{\text{Th}} = E\left(\frac{\Delta r}{4R}\right)$$

where Δr is 5 kΩ − 4.5 kΩ or 500 Ω; therefore,

$$e_g = 6\text{ V} \times \frac{500\ \Omega}{20\text{ k}\Omega} = 0.150\text{ V}$$

APPLICATIONS

There are many industrial applications for bridge circuits in the areas of measurement and control. A few of these applications are discussed here.

A Wheatstone bridge may be used to measure the dc resistance of various types of wire for the purpose of quality control either of the wire itself or of some assembly in which a quantity of wire is used. For example, the resistance of motor windings, transformers, solenoids, or relay coils may be measured.

Telephone companies and others use the Wheatstone bridge extensivel locate faults in cables. The fault may be two lines "shorted" together o single line shorted to ground.

A portable **Murray loop** test method is one of the best known a simplest of loop tests and is used principally to locate ground faults sheathed cables. Figure 5-18 shows a test setup. The defective conducto length L_b is connected at its cable terminals to a healthy conductor of len L_a. The loop formed by these two conductors is connected to the test set shown, and the bridge is balanced with the adjustable resistor R_2. The ra of R_2 to R_1 is generally known as the ratio arms. At balance,

$$\frac{R_2}{R_1} = \frac{R_a + (R_b - R_x)}{R_x}$$

where R_a, R_b, and R_x are the resistances of L_a, L_b, and L_x, respectively.

$$R_2 R_x = R_1 R_a + R_1 R_b - R_1 R_x$$

$$R_2 R_x + R_1 R_x = R_1 R_a + R_1 R_b$$

$$R_x (R_1 + R_2) = R_1 (R_a + R_b)$$

$$R_x = \frac{R_1}{R_1 + R_2}(R_a + R_b)$$

But

$$R = \frac{pl}{A}$$

where

p is resistivity
l is length
A is cross-sectional area

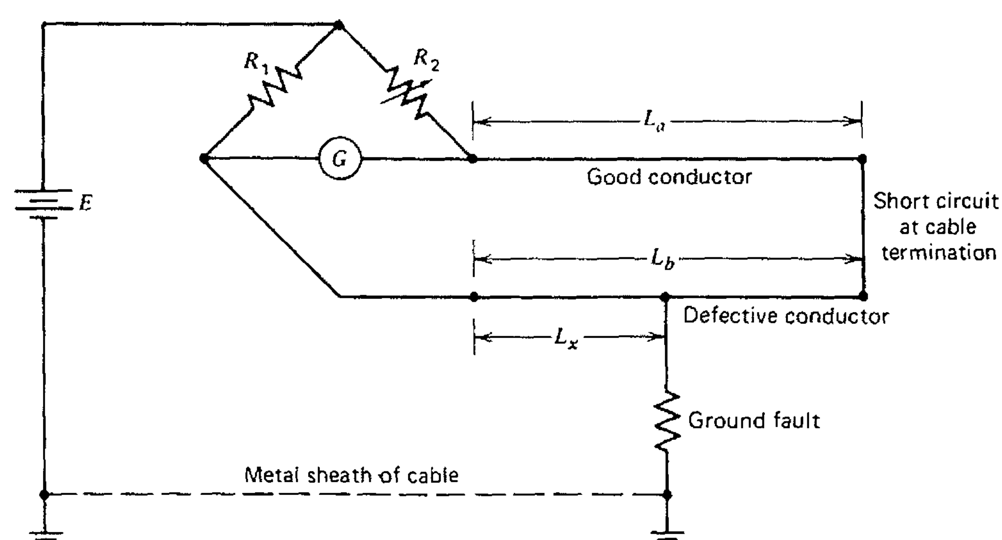


FIGURE 5-18 The Murray loop test to locate a ground fault (short circuit).

Therefore

$$\frac{P_x L_x}{A_x} = \frac{R_1}{R_1 + R_2}\left(\frac{P_a L_a}{A_a} + \frac{P_b L_b}{A_b}\right) \tag{5-12}$$

If both conductors consist of the same material and cross-sectional area, then

$$L_x = \frac{R_1}{R_1 + R_2}(L_a + L_b) \tag{5-13}$$

In a multicore cable the healthy conductor has the same length and same cross section as the faulty cable, so that $L_a = L_b$. Therefore,

$$L_x = \frac{R_1}{R_1 + R_2}(2_L) = \frac{2R_1 L}{R_1 + R_2} \tag{5-14}$$

The portable Wheatstone bridge can reasonably measure low-resistance ground faults. If, however, the fault resistance is high, the battery-operated test set is not adequate, and a high-voltage measurement must be made.

PLE 5-6

The Murray loop test set of Fig. 5-18 consists of two conductors of the same material and the same cross-sectional area. Both cables are connected 5280 feet from the test setup at the cable terminal. The bridge is balanced, when R_1 is 100 Ω and R_2 is 300 Ω. Find the distance from the ground fault to the test set.

$$L_x = \frac{2R_1 L}{R_1 + R_2} = \frac{2 \times 100\,\Omega \times 5280\text{ ft}}{100\,\Omega + 300\,\Omega} = 240\text{ ft}$$

The **Varley loop** test is one of the most accurate methods of locating ground faults and short circuits in a multiconductor cable. It is essentially a modification of the Murray loop test. This method uses a Wheatstone bridge, with two fixed ratio arms R_2 and R_1 and a rheostat R_3 in the third arm. Figure 5-19 shows a commonly used test method.

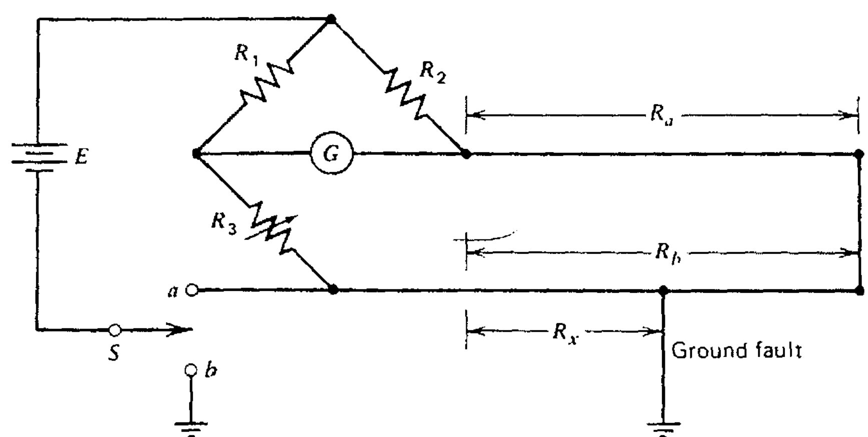


FIGURE 5-19 Wheatstone bridge connected for a Varley loop test.

Suppose a "short to ground" has occurred in the conductor represented by resistance R_b. A good conductor, in the multicore cable, is connected to the defective cable at the cable termination. The healthy conductor is represented by resistance R_a. To locate the fault, first set switch S to position a. Balance the bridge by adjusting R_3. When the bridge is balanced,

$$\frac{R_2}{R_1} = \frac{R_a + R_b}{R_3}$$

$$R_2R_3 = R_1R_a + R_1R_b$$

$$R_1(R_a + R_b) = R_2R_3$$

and

$$R_a + R_b = \frac{R_2R_3}{R_1} \tag{5-15}$$

Now set the switch to position b and balance the bridge again. The equation for balance is now

$$\frac{R_2}{R_1} = \frac{R_a + (R_b - R_x)}{R_x + R_3}$$

$$R_2R_x + R_2R_3 = R_1R_a + R_aR_b - R_1R_x$$

$$R_2R_x + R_1R_x = R_1R_a + R_1R_b - R_2R_x$$

$$R_x(R_1 + R_2) = R_1(R_a + R_b) - R_2R_3$$

Solving for R_x yields

$$R_x = \frac{R_1(R_a + R_b) - R_2R_3}{R_1 + R_2}$$

The value of $R_a + R_b$ can now be obtained from Eq. 5-15.

EXAMPLE 5-7

The Varley loop test set of Fig. 5-20 consists of a defective conductor and a healthy conductor connected at the cable terminal located 10 miles from the test set. The cables have a resistance of 0.05 ohm per 1000 ft. When the switch is in position a and the circuit is balanced, the balancing resistance is

$$R_3 = 100\ \Omega$$

When the switch is in position b and the circuit is rebalanced, the balancing resistor becomes

$$R_3 = 99\ \Omega$$

Find the distance from the ground fault to the test set.

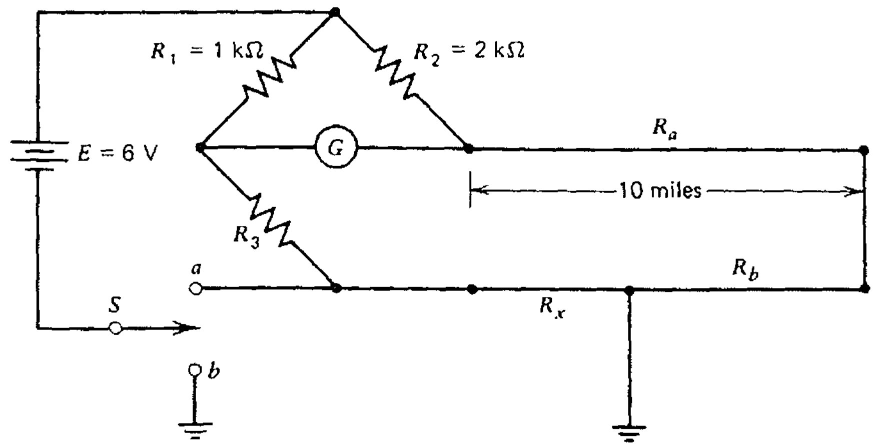


FIGURE 5-20 Varley loop tests to locate grounds or short circuits.

ution

With the switch in position a,

$$R_a + R_b = \frac{R_2 R_3}{R_1} \times \frac{2000 \times 100}{1000} = 200\ \Omega$$

With the switch in position b,

$$R_x = \frac{R_1(R_a + R_b) - R_2 R_3}{R_1 + R_2} = \frac{1000 \times 200 - 2000 \times 99}{1000 + 2000}$$

$$= 0.67\ \Omega$$

A cable resistance of 0.05 Ω represents 1000 feet. Therefore, a cable resistance of 0.67 Ω represents

$$\frac{0.67 \times 1000}{0.05} = 13{,}333\ \text{ft}$$

The Wheatstone bridge is also widely used in control circuits such as those shown in Fig. 5-21.

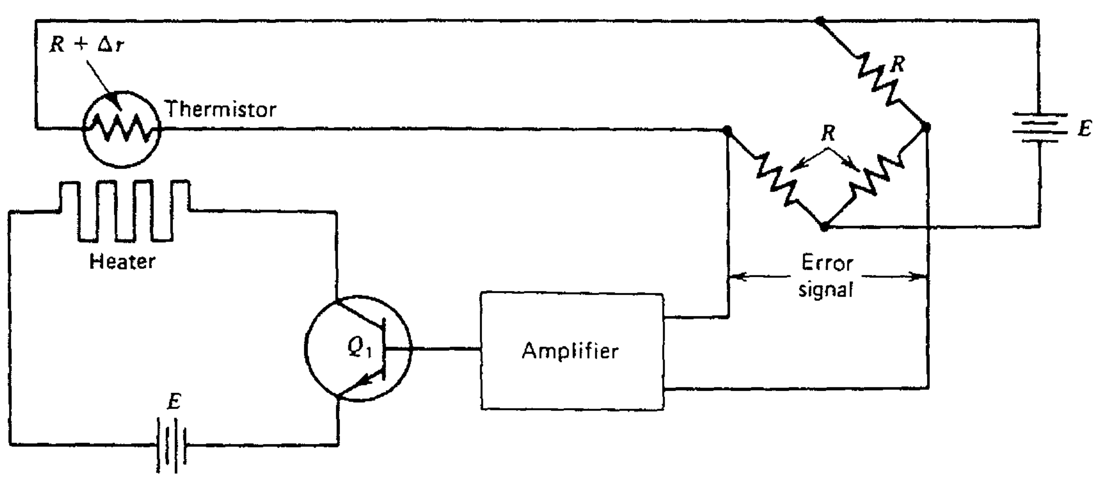


FIGURE 5-21 Basic bridge-controlled heater circuit.

The circuit is a basic heater circuit. At the desired temperature the re tance of the thermistor equals R. This causes the bridge to be balanced, a therefore, there is no error voltage. Since transistor Q_1 is biased off, current flows through the heating element. If the temperature decreases, thermistor resistance decreases, the bridge is unbalanced, and the error sig is amplified, which forward-biases transistor Q_1. This allows current to fl through the heating element until the thermistor resistance increases to value of R, which balances the bridge and turns off the circuit. Such circu are of use in many industrial applications in which the temperature must maintained with close tolerance.

5-10 SUMMARY

The Wheatstone bridge is the most basic bridge circuit. It is widely used measuring instruments and control circuits. Bridge circuits have a high deg of accuracy, limited only by the accuracy of the components used in circuit. The Kelvin bridge is a modification of the Wheatstone bridge and widely used to measure very low resistances.

Recent innovations in bridge-type instruments include digital readout a microprocessor-controlled bridges. Whenever the microprocessor becomes integral part of the measuring circuitry, the instrument becomes hig sophisticated (intelligent).

The most frequently used analytical tool for analyzing an unbalanc Wheatstone bridge is Thévenin's theorem.

5-11 GLOSSARY

Balance: The condition of a bridge when no current flows through t detector (usually a galvanometer).

Comparison measurement: A measurement made with an instrument which a standard against which an unknown is being compared is physica present within the instrument.

Galvanometer: A laboratory instrument using a d'Arsonval meter movem but with zero at center scale. Used to measure very small currents of eith positive or negative polarity.

Kelvin bridge: A modification of the Wheatstone bridge. Contains additional set of ratio arms to compensate for lead and contact resistors 1 Ω or less.

Microprocessor: A "data processor" or "controller" contained on a sin integrated-circuit chip.

Murray loop: A special Wheatstone bridge used to measure shorts betwe lines or to ground.

Null indication: A term used to indicate that no current is flowing throu a galvanometer; hence, the pointer is resting at center scale zero, indicati the bridge is balanced.

Sensitivity: Deflection per unit of current.

Thévenin's theorem: An analytical tool used extensively to analyze an unbalanced bridge. The theorem states that a complex circuit may be replaced with a single equivalent voltage source and a single equivalent series resistance as seen looking back into the circuit from the output terminals with no load connected.

Varley loop: A special Wheatstone bridge configuration used to locate shorts between conductors or faults to ground in conductors.

Wheatstone bridge: A basic circuit configuration used in measuring instruments or control instruments. The bridge is balanced when the products of the resistors in opposite arms are equal.

REVIEW QUESTIONS

The following questions should be answered after a comprehensive study of the chapter. The purpose of the questions is to determine the reader's comprehension of the material.

1. How does the measuring accuracy of a Wheatstone bridge compare with that of an ordinary ohmmeter?
2. What are the criteria for balance of a Wheatstone bridge?
3. In what two types of circuits do Wheatstone bridges find most of their applications?
4. What are the criteria for balance of a Kelvin bridge?
5. What is the primary use of the Kelvin bridge?
6. How does the basic circuit for the Kelvin bridge differ from that for the Wheatstone bridge?
7. How does the use of microprocessors in bridge circuits differ from the use of minicomputers with bridges?
8. What are some ways in which microprocessors are reducing the cost and complexity of analog measurements?
9. What technique lends itself well to analyzing the unbalanced Wheatstone bridge?

PROBLEMS

5-1 Calculate the value of R_x in the circuit of Fig. 5-22 if $R_1 = 400\ \Omega$, $R_2 = 5\ k\Omega$, and $R_3 = 2\ k\Omega$.

5-2 Calculate the value of R_x in Fig. 5-22 if $R_1 = 10\ k\Omega$, $R_2 = 60\ k\Omega$, and $R_3 = 18.5\ k\Omega$.

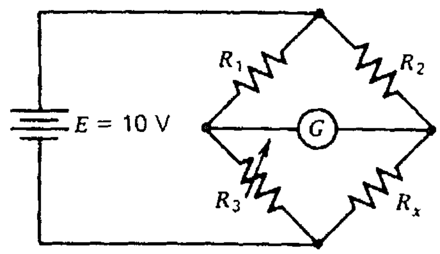


FIGURE 5-22 Circuit for Problem 5-1

5-3 Calculate the value of R_x in Fig. 5-22 if $R_1 = 5\ \text{k}\Omega$, $R_2 = 40\ \text{k}\Omega$, and $R_3 = 10\ \Omega$.

5-4 What resistance range must resistor R_3 of Fig. 5-23 have in order to measure unknown resistors in the range of 1 to 100 kΩ?

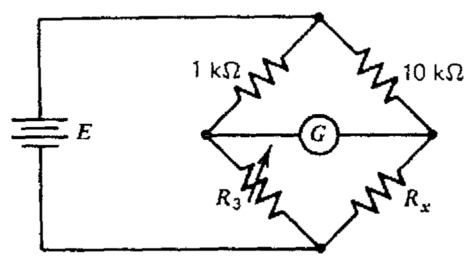


FIGURE 5-23 Circuit for Problem 5-4.

5-5 Calculate the value of R_x in the circuit of Fig. 5-24 if $R_a = 1200\ \Omega$, $R_a = 1600R_b$, $R_1 = 800R_b$, and $R_1 = 1.25R_2$.

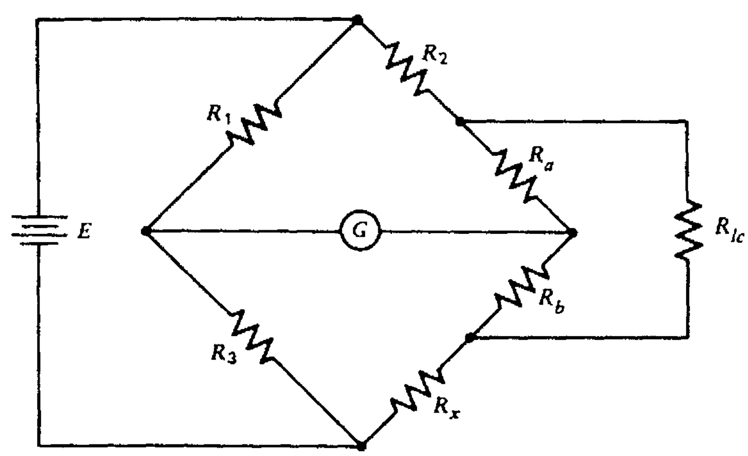


FIGURE 5-24 Circuit for Problem 5-5.

5-6 Calculate the current through the galvanometer in the circuit of Fig. 5-25.

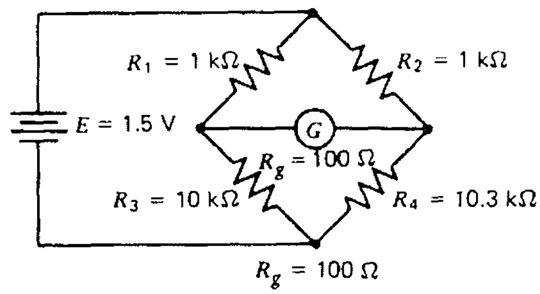


FIGURE 5-25 Circuit for Problem 5-6.

5-7 Three arms of a Wheatstone bridge contain resistors of known value that have a limiting error of $\pm 0.2\%$. Calculate the limiting error of an unknown resistor when measured with this instrument.

5-8 Calculate the percentage of error in the value of the current through the galvanometer in Fig. 5-26 when the approximate Eqs. 5-9 and 5-10 are used to find Thévenin's equivalent circuit.

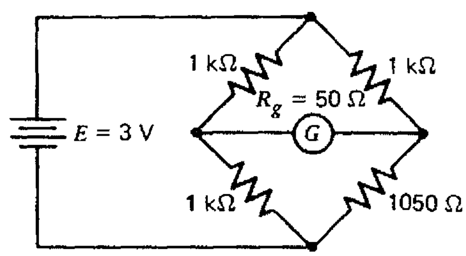


FIGURE 5-26 Circuit for Problem 5-8.

5-9 Calculate the value of R_x in the circuit of Fig. 5-27 if $V_{Th} = 24$ mV and $I_g = 13.6\ \mu A$.

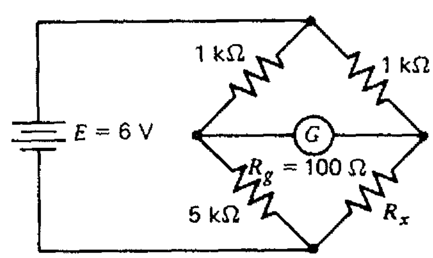


FIGURE 5-27 Circuit for Problem 5-9.

5-10 If the sensitivity of the galvanometer in the circuit of Fig. 5-28 is 10 mm/μA, determine its deflection.

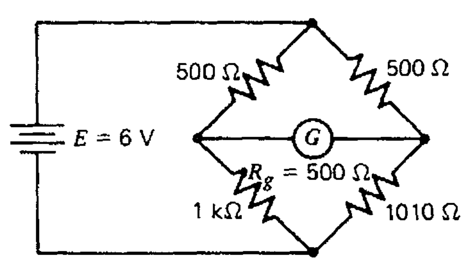


FIGURE 5-28 Circuit for Problem 5-10.

5-11 If the light beam that is directed on the photocell R in the circuit of Fig. 5-29 is interrupted, the resistance of the photocell increases from 10 to 40 kΩ. Calculate the current I_{in} when the beam is interrupted.

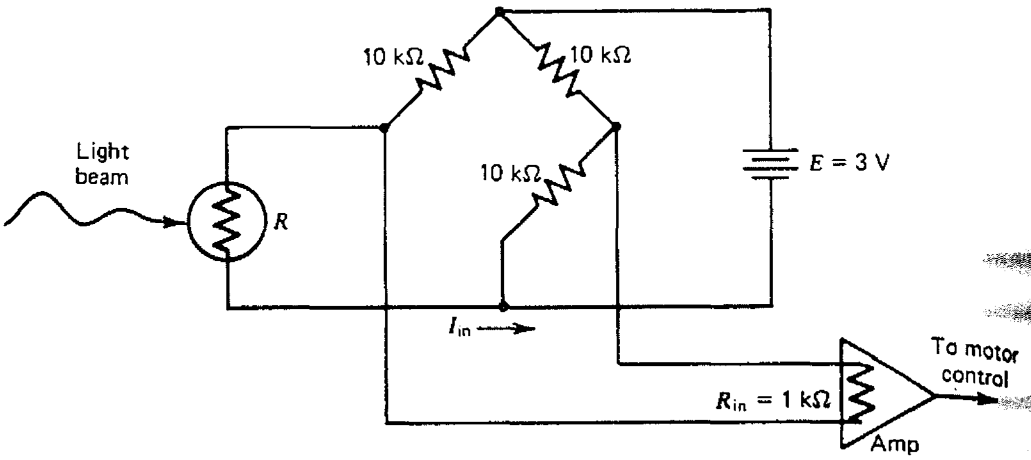


FIGURE 5-29 Circuit for Problem 5-11.

5-12 Calculate the current I_{in} in the circuit of Fig. 5-30 when the temperature is 50°C if $R = 500\ \Omega$ at 25°C and its resistance increases 0.7 Ω per degree.

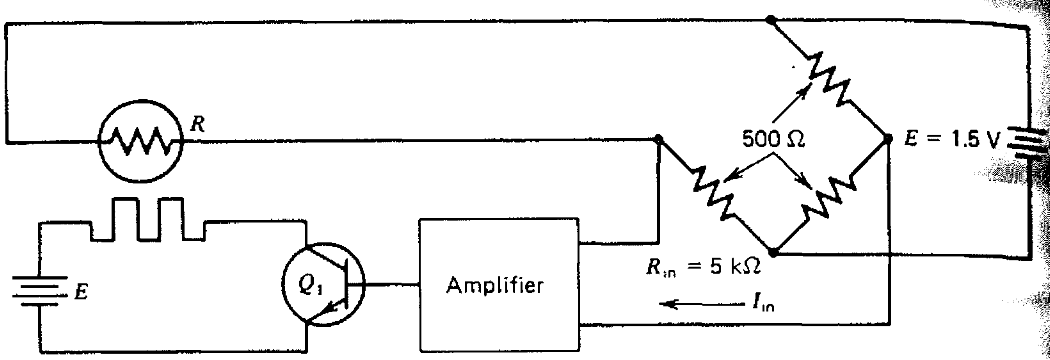


FIGURE 5-30 Circuit for Problem 5-12.

5-13 A Wheatstone bridge is connected for a Varley loop test as shown in Fig. 5-31. When the switch S is in position a, the bridge is balanced

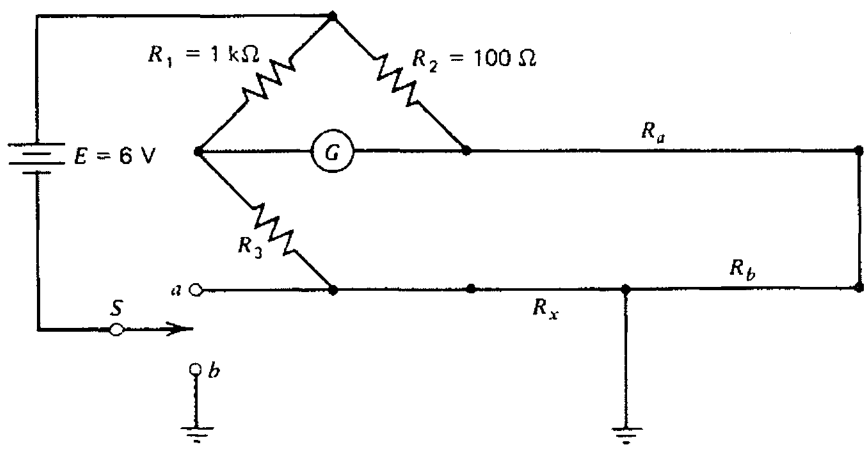


FIGURE 5-31 Circuit for Problem 5-13.

with $R_1 = 1000\ \Omega$, $R_2 = 100\ \Omega$, and $R_3 = 53\ \Omega$. When S is in position b, the bridge is balanced with $R_1 = 1000\ \Omega$, $R_2 = 100\ \Omega$, and $R_3 = 52.9\ \Omega$. If the resistance of the shorted wire is $0.015\ \Omega/\text{m}$, how many meters from the bridge has a short to ground occurred?

5-14 A Wheatstone bridge is connected for a Murray loop test as shown in Fig. 5-32 and balanced. Cable a is an aerial cable with a resistance of 0.1 ohm per 1000 ft. Cable b is an underground cable with a resistance of 0.005 ohm per 1000 ft. Neglecting temperature differences, calculate the distance L_x from the ground fault to the test set if $L_a = L_b$.

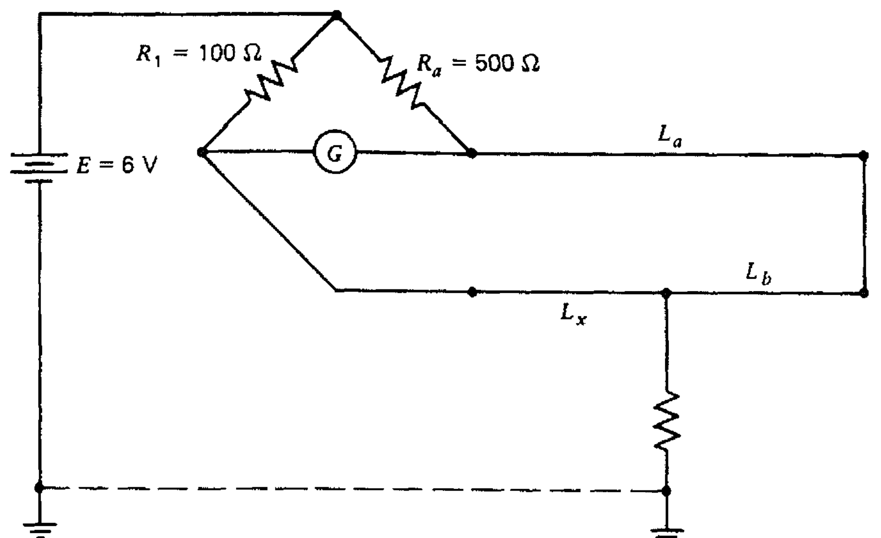


FIGURE 5-32 Circuit for Problem 5-14.

14 LABORATORY EXPERIMENTS

Experiments E9, E10, and E11, pertain to the theory presented in Chapter 5. The purpose of the experiments is to provide hands-on experience to reinforce the theory.

The experiments make use of components and equipment that are found in any well-equipped electronics laboratory. The contents of the laboratory report to be submitted by each student are included at the end of each experimental procedure.