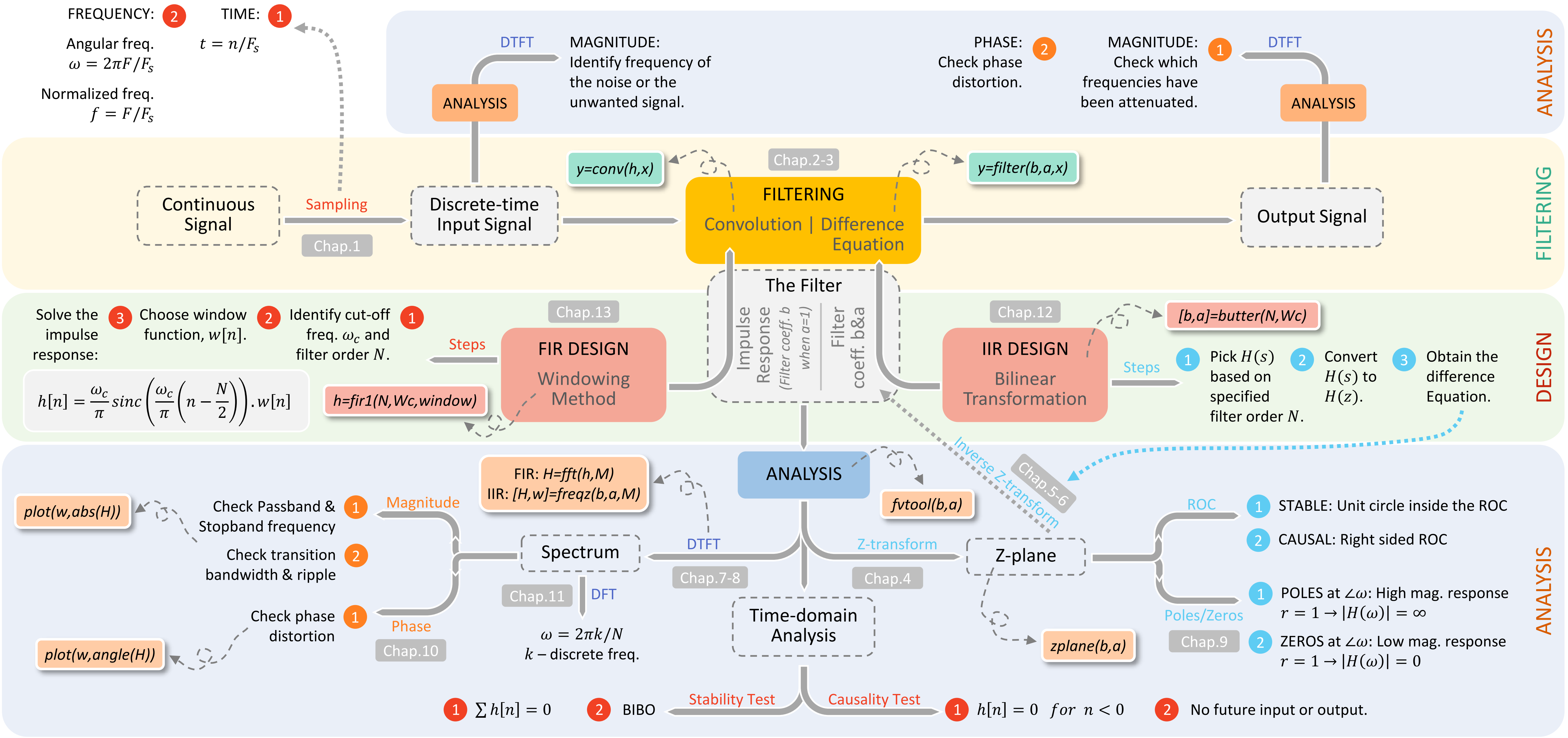
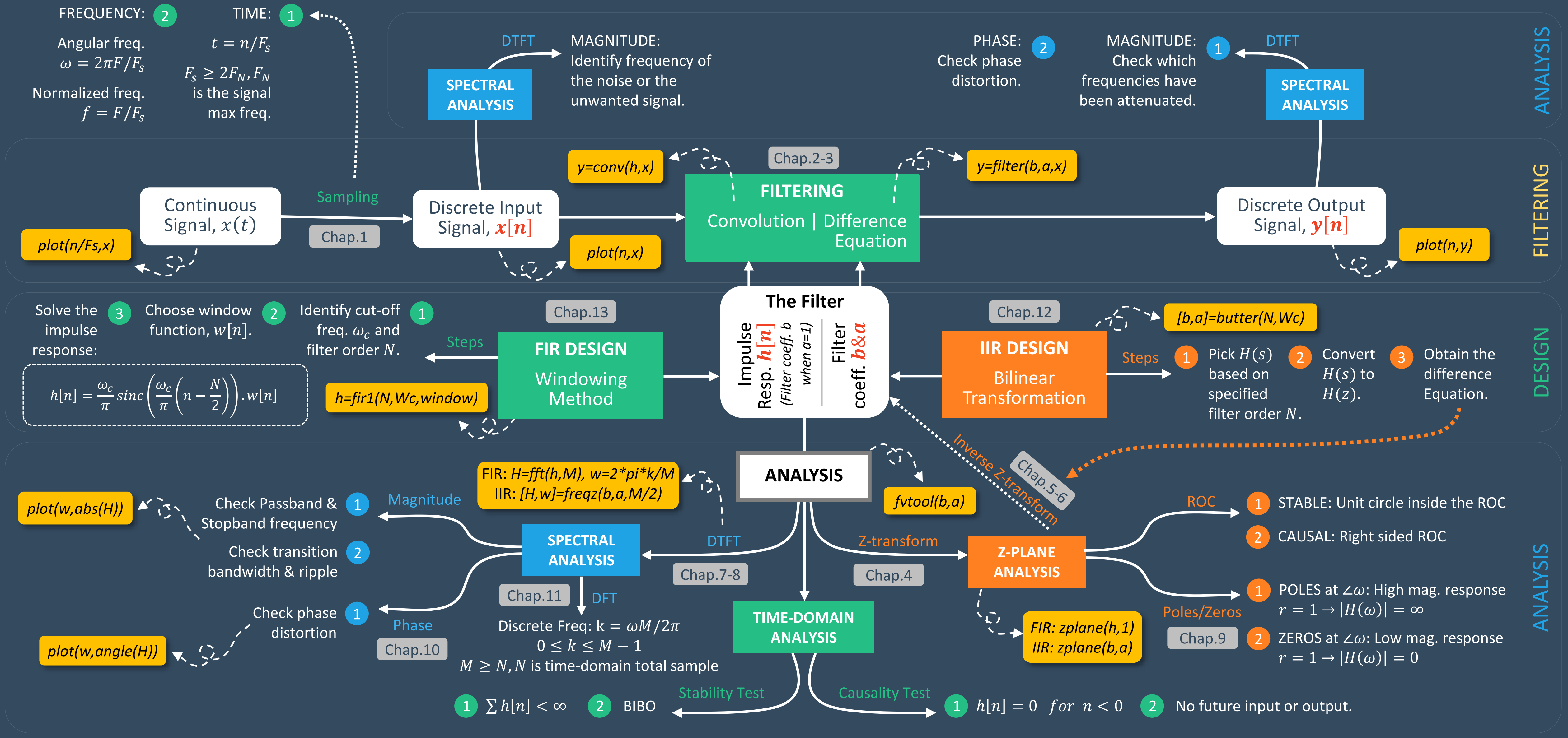
DSP Concept Mapping



DSP Concept Mapping



Discrete-Time Signal

OPERATION ON SIGNAL VALUES
Arithmetic operation is done on each similar sample index.

SIGNAL LENGTH
No of samples counted from the first to the last non-zero samples.

OPERATION ON INDEX
Reordering the samples;
Folding, Shifting, Selection.

Signal Values

$$x[n] = [\ 2.5 \quad 3.1 \quad 4 \quad 2.7 \quad 1.4\ ]$$

Math term: vector

n	0	1	2	3	4
t	0	T_s	$2T_s$	$3T_s$	$4T_s$

$t = nT_s$

Frequency sampling, $F_s = 1/T_s$

$F_s \geq 2F_N$, F_N is the signal max freq.

LTI System

1. LTI system is represented by *DIFFERENCE EQUATION* restricted to operations:

   i. Delay

   ii. Summation to all samples

   iii. Weight multiplication

2. The weights are the filter, called:

   i. Coefficient a – output samples weights

   ii. Coefficient b – input samples weights

3. LTI system can also be represented by *IMPULSE RESPONSE*, h[n]:

   FIR – equals to coefficient b

   IIR – Need z-transform to convert the coefficient a & b to h[n]

System representation based on it's process

DIFFERENCE EQUATION → FIR: Convolution, $y[n] = x[n]*h[n]$

$$1y[n] + 5y[n-1] + 3y[n-2] = 2x[n] + 3x[n-1] + 2x[n-2]$$

$a = [1 \quad 5 \quad 3]$ — Coefficient a

$b = [2 \quad 3 \quad 2]$ — Coefficient b

System representation based on it's filter

IMPULSE RESPONSE

$h[n]$

FIR: $h[n] = b$

IIR: $h[n] \longleftrightarrow a, b$ (z-transform)

Convolution

$x[n] = [\underline{3}\ \ 3\ \ 1\ \ 3]$ $\qquad$ $h[n] = [1\ \ \underline{2}\ \ 1]$

n →		-1	0	1	2	3	4
x[n] →		[	$\underline{3}$	3	1	3	]
h[n] →	x	[1	$\underline{2}$	1			]
		3	3	1	3		
			6	6	2	6	
	+			3	3	1	3
y[n] →		[3	$\underline{9}$	10	8	7	3]

Stability and Causality Check

DIFFERENCE EQUATION

CAUSALITY: NO FUTURE INPUT/OUTPUT
STABILITY: BIBO

IMPULSE RESPONSE

CAUSALITY: $h[n] = 0 \; for \; n < 0$
STABILITY: $\sum|h[n]| < \infty$

Z-DOMAIN ROC

CAUSALITY: ROC OUTWARD
STABILITY: ROC INCLUDE UNIT CIRCLE

$1y[n] = 2x[n] + 4x[n-1]$ ----→ $h[n] = [2 \quad 4]$ ----→ (Z-TRANSFORM)

$$H(z) = 2 + 4z^{-1}$$

ROC: $|z| > 0$

$1y[n] - 0.25y[n-2] = 1x[n] + 2x[n-1]$

$$h[n] = 2.5(0.5)^n u[n] - 1.5(-0.5)^n u[n]$$

←---- INVERSE Z-TRANSFORM

$$H(z) = \frac{1z^0 + 2z^{-1}}{1z^0 + 0z^{-1} - 0.25z^{-2}}$$

Poles (Denominator Roots):

$$1z^2 + 0z^1 - 0.25z^0 = 0$$

$$(z - 0.5)(z + 0.5) = 0$$

ROC: $|z| > 0.5$

CHECKING BIBO FOR STABILITY IS NOT ALWAYS EASY SINCE WE NEED TO TAKE INTO ACCOUNT ALL POSSIBLE INPUTS. AS AN ALTERNATIVE, IT IS EASIER TO CHECK STABILITY ON THE IMPULSE RESPONSE.

CONVERTING DIFFERENCE EQUATION TO $h[n]$ IS NOT STRAIGHT FORWARD FOR IIR. THUS, FOR IIR, CHECKING THE Z-DOMAIN ROC WILL BE THE EASIEST WAY TO CHECK STABILITY.

Z-Transform

TABLE 1 : Z-TRANSFORM PAIR

TABLE 2 : Z-TRANSFORM PROPERTIES

TIME-DOMAIN	Z-DOMAIN
CONVOLUTION	MULTIPLICATION
DELAY n_d	MULTIPLY WITH z^{-n_d}

IMPULSE RESPONSE

$$h[n] = 0.5u[n] + 0.5(0.2)^{n-1}u[n-1]$$

x[n]	X(z)
y[n]	Y(z)
h[n]	H(z)

SYSTEM FUNCTION

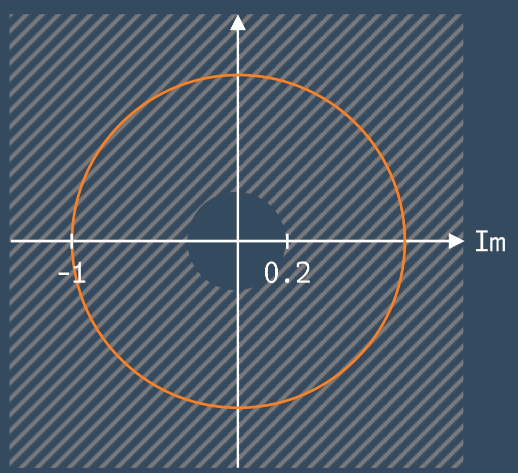
$$\frac{Y(z)}{X(z)} = \frac{0.5 + 0.5z^{-1}}{1 - 0.2z^{-1}}$$

$ROC, |z| > 0.2$

$$\frac{Y(z)}{X(z)} = 0.5\left(\frac{z+1}{z-0.2}\right)$$

ZEROS : $z = -1$

POLES : $z = 0.2$



1y[n] = 0.5x[n] + 0.5x[n-1] + 0.2y[n-1]

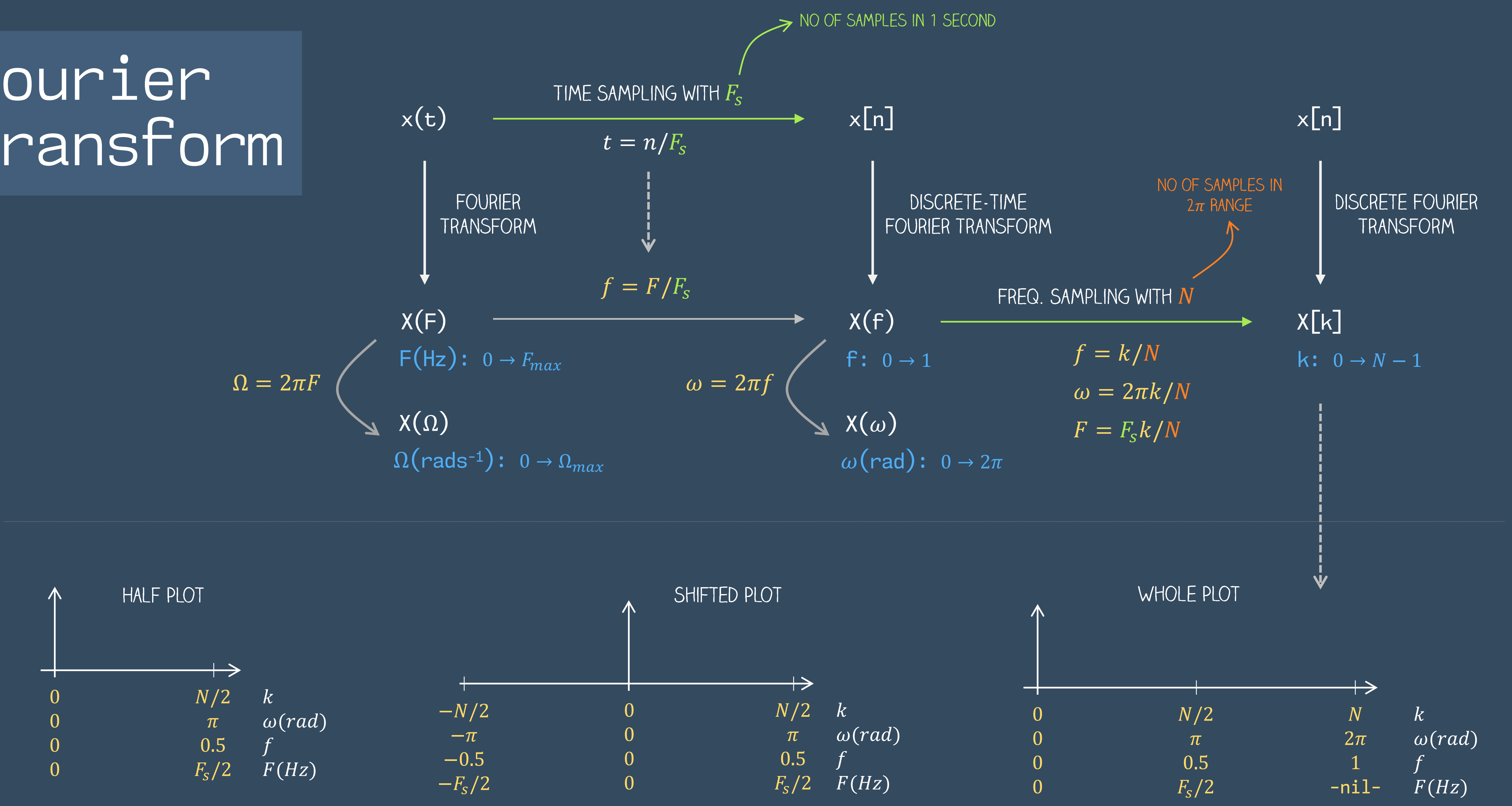
Y(z)(1 - 0.2z^{-1}) = X(z)(0.5 + 0.5z^{-1})

DIFFERENCE EQUATION

SYSTEM REALIZATION

ADDITION MULTIPLICATION DELAY

Fourier Transform



Frequency Response of a System

$h[n] = [1\ 2\ 1]$

IMPULSE RESPONSE

$H(z) = \sum h[n] z^{-n}$

$H(z) = 1 + 2z^{-1} + z^{-2}$

SYSTEM FUNCTION

$H(\omega) = \sum h[n] e^{-j\omega n}$

FIR: $h[n] = b$

$H(z) = Y(z)/X(z)$

$z = e^{j\omega}$

DIFFERENCE EQUATION

$y[n] = x[n] + 2x[n-1] + x[n-2]$

SYSTEM COEFFICIENT

a=[1] b=[1 2 1]

FREQUENCY RESPONSE

$H(\omega) = 1 + 2e^{-j\omega} + e^{-j2\omega}$

$H(\omega)_{real} = 1 + 2\cos(\omega) + \cos(2\omega)$

$H(\omega)_{imag} = -2\sin(\omega) - \sin(2\omega)$

$|H(\omega)| = \sqrt[2]{H(\omega)_{real}^2 + H(\omega)_{imag}^2}$

$\measuredangle H(\omega) = \tan^{-1} \frac{H(\omega)_{imag}}{H(\omega)_{real}}$

MAGNITUDE RESPONSE VARIATIONS

MAGNITUDE SQUARE:

$|H(\omega)|^2 = H(\omega) \times H(\omega)^*$

MAGNITUDE DB:

$|H(\omega)|_{dB} = 20Log_{10}|H(\omega)|$

Filtering

Time Domain

Implementing the filtering process between input $x[n]$ and impulse response $h[n]$ or filter coefficient a & b

Convolution → FIR
$y[n] = x[n] * h[n]$

Difference Equation → FIR & IIR
$y[n] = \sum b_i x[n-i] + \sum a_j y[n-j]$

Z-Domain

Analysing & Designing the filter in terms of the system function $H(z)$

ROC → Analyse Stability & Causality.

Poles & Zeros → Analyse and design the frequency response.

Frequency Domain

Analysing the freq. response of the input $X(\omega)$, the output $Y(\omega)$ and the filter $H(\omega)$

MAGNITUDE RESPONSE

Input, $|X(\omega)|$ - To identify the unwanted frequencies.
Filter, $|H(\omega)|$ - To understand which frequency will be attenuated, preserved or amplified.
Output, $|Y(\omega)|$ - To check the effect of the filtering on the frequency components.

PHASE RESPONSE

Filter, $\angle H(\omega)$ - To understand the type and amount of delay the filter will cause.
Output, $\angle Y(\omega)$ - To check the delay effect.

Filtering

Time Domain

1. Solve Difference Equation

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] + \sum_{k=1}^{N} a_k x[n-k]$$

2. Convolution

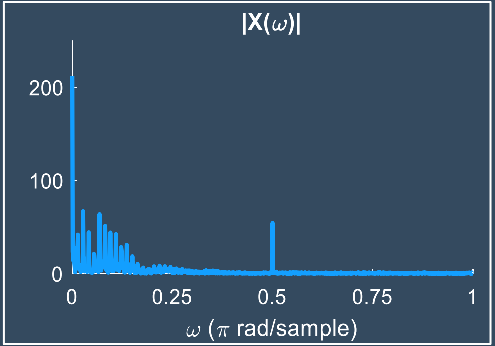
n →		-1	0	1	2	3	4
x[n] →		[	3	3	1	3	]
h[n] →	x	[1	2	1			]
		3	3	1	3		
			6	6	2	6	
	+			3	3	1	3
y[n] →		[3	9	10	8	7	3]

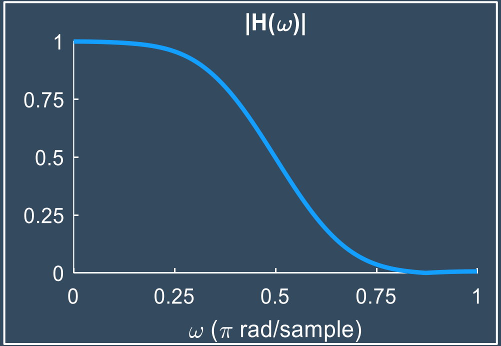
Frequency Domain

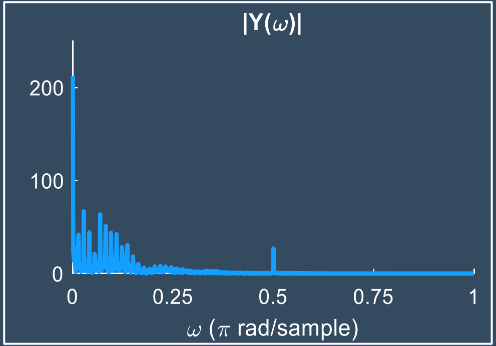
$$Y(\omega) = X(\omega).H(\omega)$$

$$|Y(\omega)| = |X(\omega|.|H(\omega)|$$

$$\measuredangle Y(\omega) = \measuredangle X(\omega) + \measuredangle H(\omega)$$



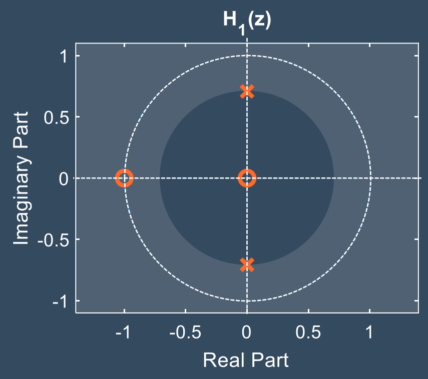


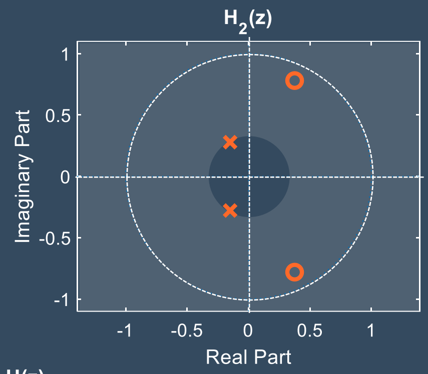


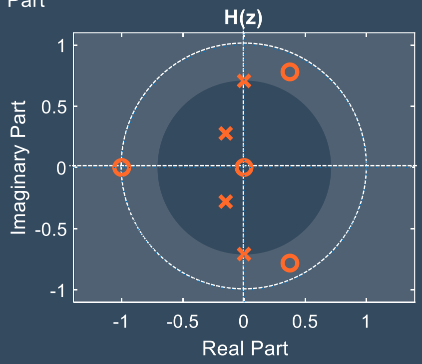
Z-Domain

Represent the filter with system function, $H(z) = \frac{\mathrm{Y(z)}}{\mathrm{X(z)}}$

For two cascaded filters, $H(z) = H_1(z).H_2(z)$:



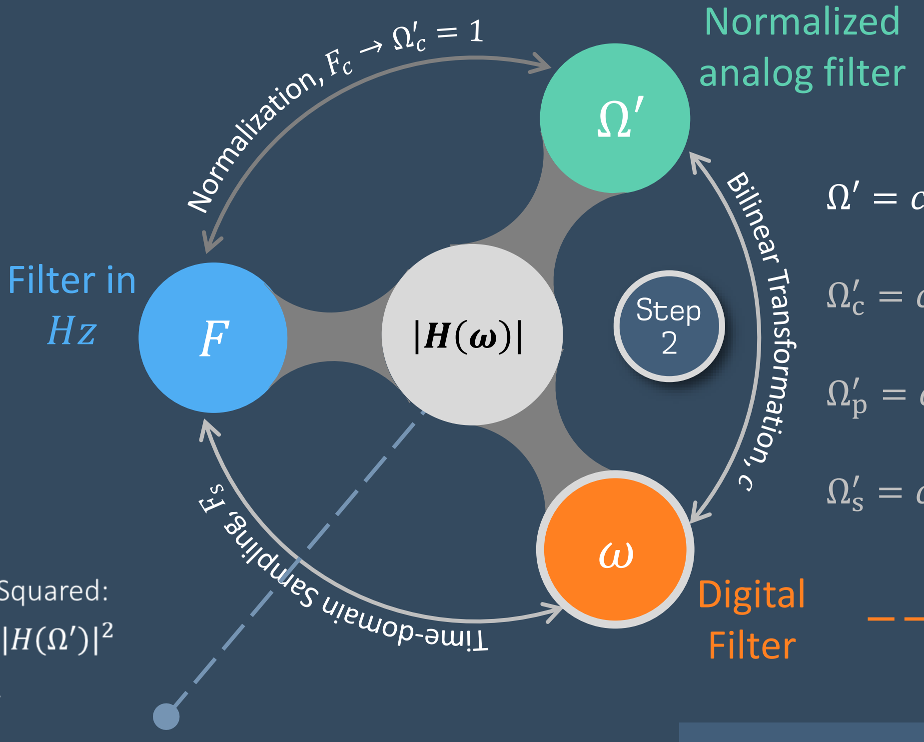




*Combine both poles and zeros

$$*ROC_{H(z)} = ROC_{H_1(z)} \cap ROC_{H_2(z)}$$

IIR Butterworth Filter Design



$$\Omega' = c.\tan\left(\frac{\omega}{2}\right)$$

$$\Omega'_c = c.\tan\left(\frac{\omega_c}{2}\right) = 1$$

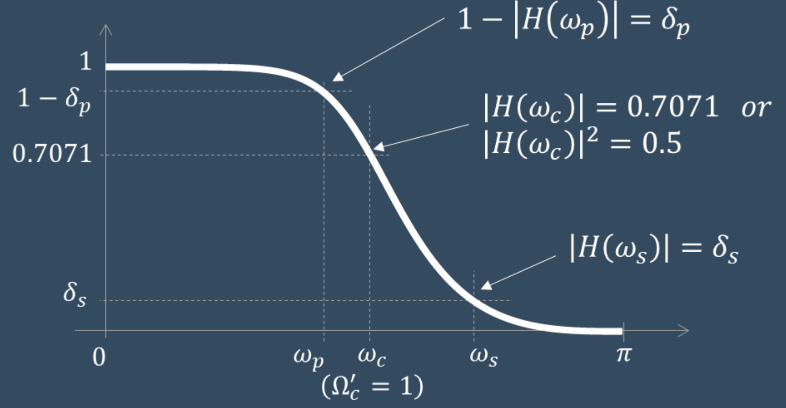
$$\Omega'_p = c.\tan\left(\frac{\omega_p}{2}\right)$$

$$\Omega'_s = c.\tan\left(\frac{\omega_s}{2}\right)$$

Step 1

Magnitude Squared:

$$|H(\omega)|^2 = |H(\Omega')|^2 = \frac{1}{1 + \Omega'^{2N}}$$



Normalized analog filter system function, $H(s)$

1st Order	$\frac{1}{s+1}$
2nd Order	$\frac{1}{s^2 + 1.4142s + 1}$
3rd Order	$\frac{1}{(s+1)(s^2+s+1)}$

a_i – normalized filter coefficient

$$d_1 = 1 + c$$
$$d_2 = 1 - c$$
$$b_{i1} = c^2 + a_i c + 1$$
$$b_{i2} = -2c^2 + 2$$
$$b_{i3} = c^2 - a_i c + 1$$

Step 3

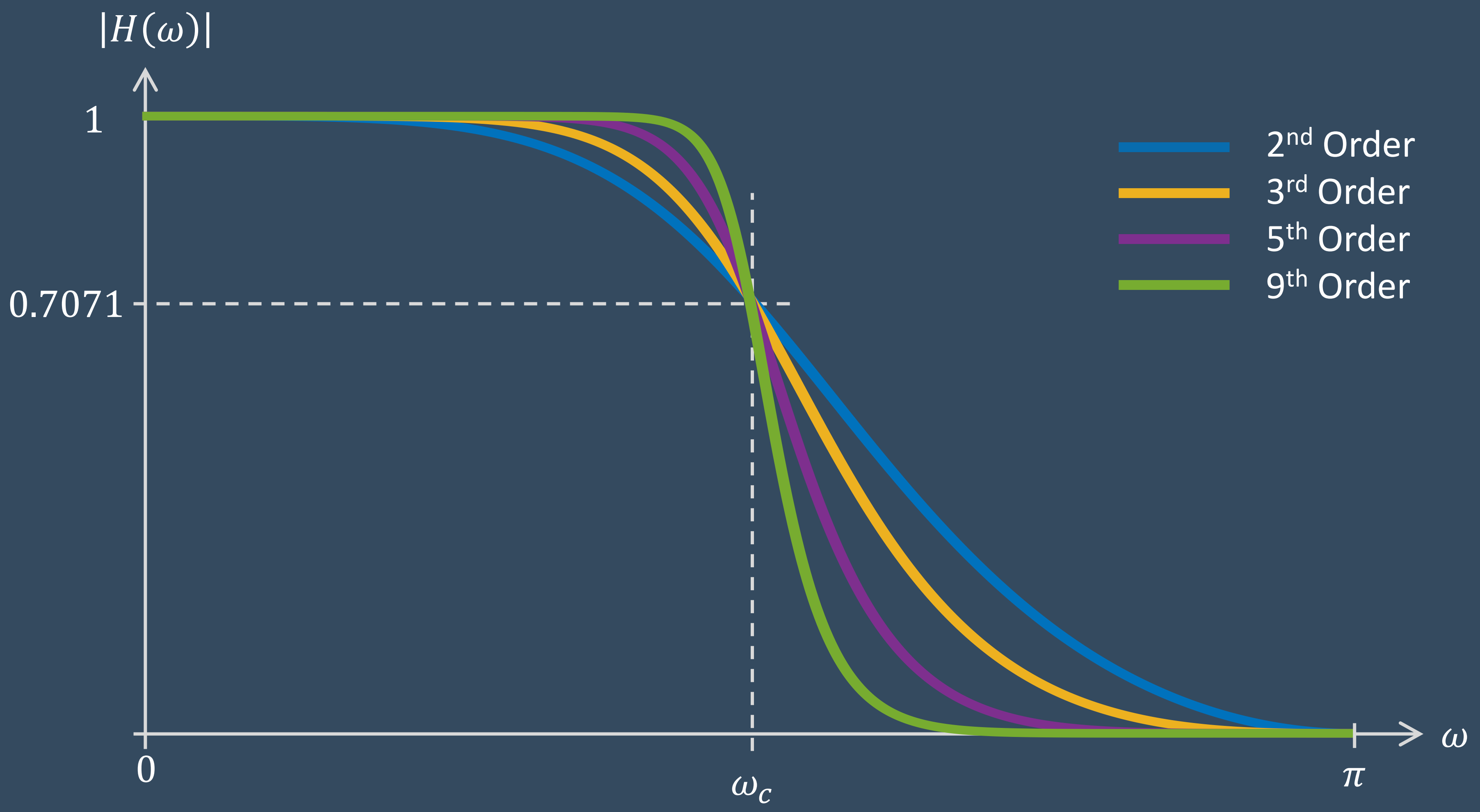
Digital filter system function, $H(z)$

1st Order	$\frac{1 + z^{-1}}{d_1 + d_2 z^{-1}}$
2nd Order	$\frac{(1 + z^{-1})^2}{b_{11} + b_{12}z^{-1} + b_{13}z^{-2}}$
3rd Order	$\frac{(1 + z^{-1})^3}{(d_1 + d_2 z^{-1})(b_{11} + b_{12}z^{-1} + b_{13}z^{-2})}$

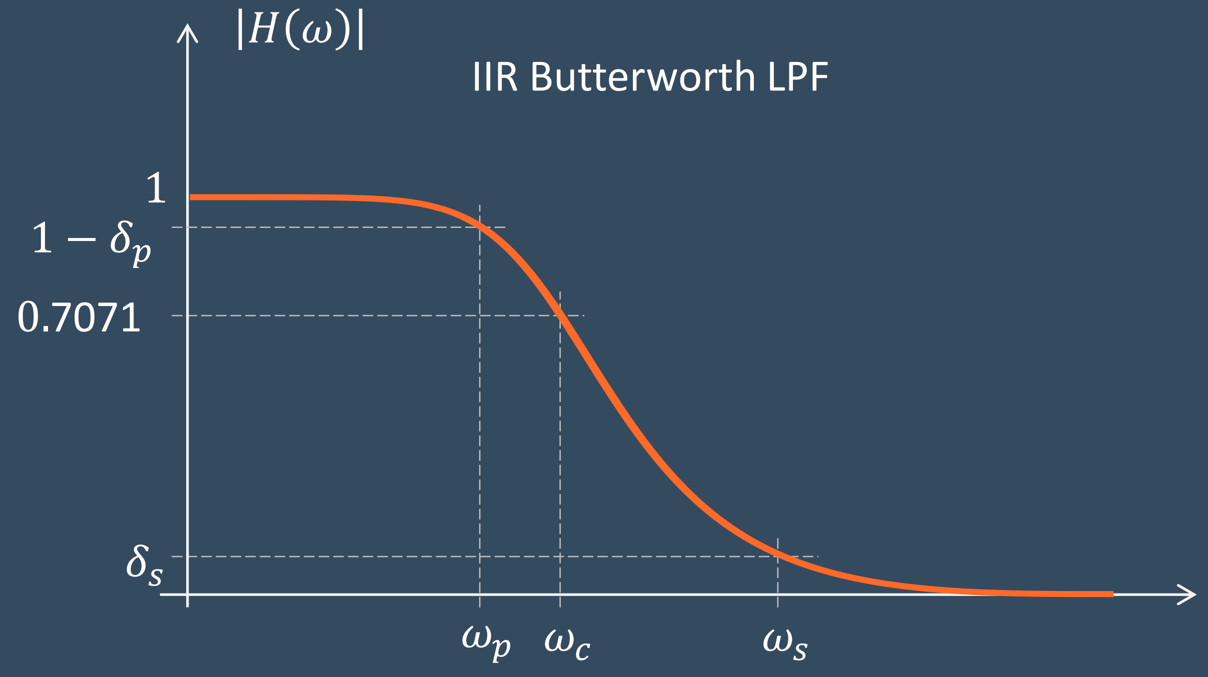
Design Step

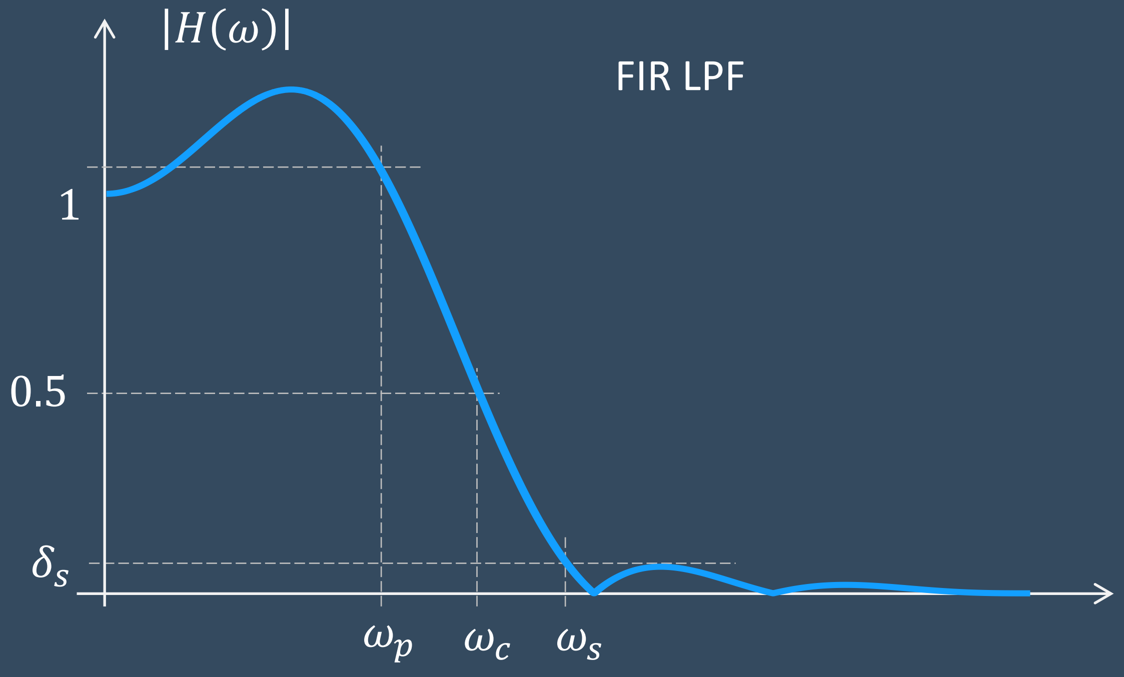
1. Specify filter characteristic: $\omega_c, N, \delta_p, \delta_s, \omega_p, \omega_s$.
2. Find filter order N (if not specified in step 1) and parameter c.
3. Based on the N and c, convert $H(s)$ to $H(z)$.
4. Transform $H(z)$ to the time-domain difference equation

IIR Butterworth Lowpass Filter



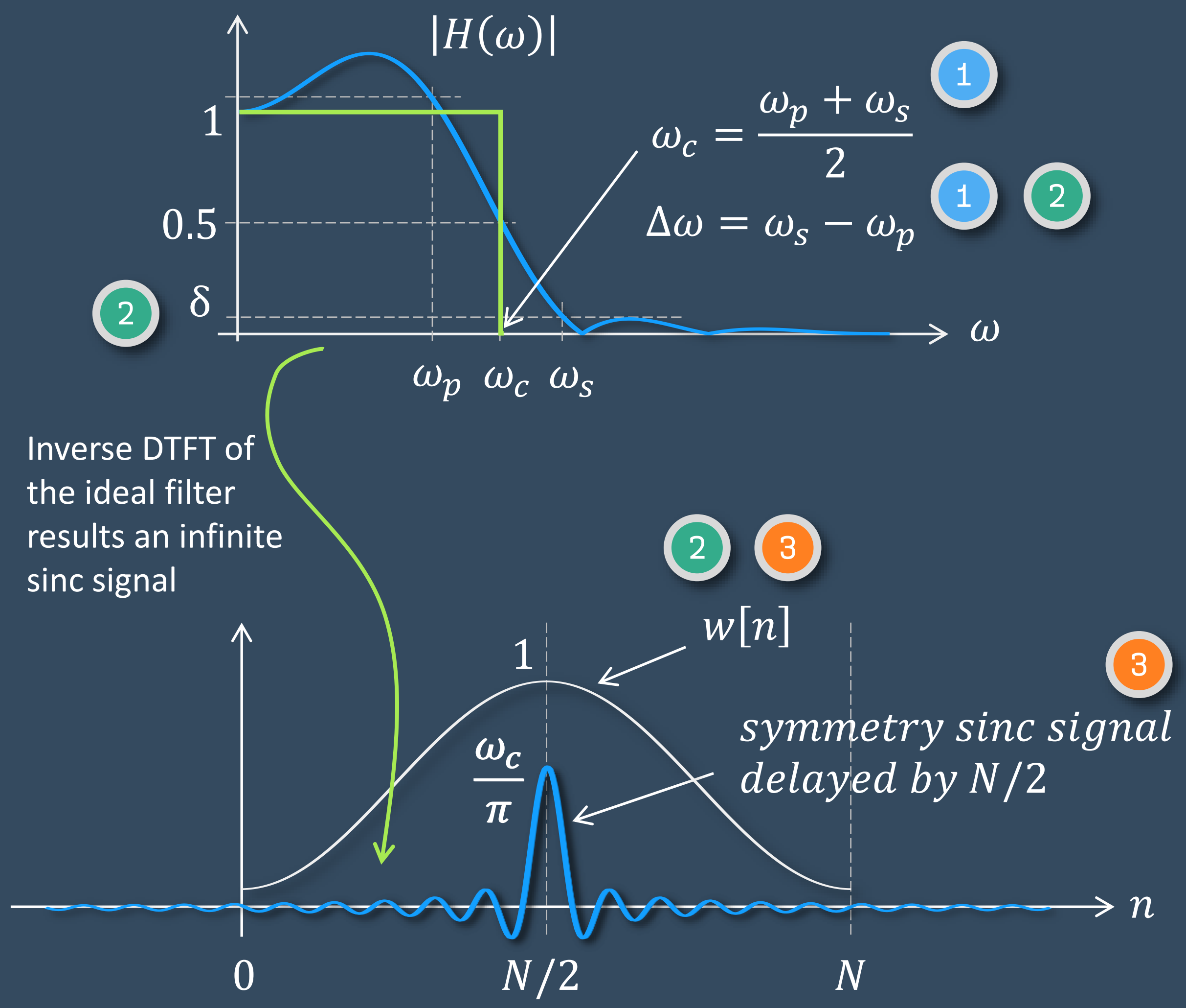
IIR Butterwort Filter vs FIR Filter





Characteristic	IIR Butterworth filter	FIR Filter
Cut-off frequency, ω_C	Closer to ω_p	At center between ω_p and ω_s
$\lvert H(\omega_c)\rvert$	Half power → 0.7071	Half magnitude → 0.5
Ripple	Gradually decreasing	Fluctuated

FIR Filter Design



Step 1

DETERMINE CUT-OFF FREQUENCY, ω_c

Either empirically specified or computed from the transition bandwidth, ω_p and ω_s values.

Step 2

DETERMINE FILTER ORDER, N AND WINDOW FUNCTION, $w[n]$

Either empirically specified or computed from the transition bandwidth and ripple, δ values.

window	$\boldsymbol{\delta}$, dB; (Magnitude)	$\boldsymbol{\Delta\omega}$
Rectangular	-20.9 dB; (0.0902)	$1.8\pi/N$
Hanning	-43.9 dB; (0.0064)	$6.2\pi/N$
Hamming	-54.5 dB; (0.0019)	$6.6\pi/N$
Blackman	-75.3 dB; (0.0002)	$11.1\pi/N$

Step 3

SOLVE THE IMPULSE RESPONSE EQUATION

$$h[n] = \frac{\omega_c}{\pi} sinc\left(\frac{\omega_c}{\pi}(n - N/2)\right).w[n]$$

Tangent Inverse