Fundamentals of Electrical Engineering

(SMJP 2092)

Section 02

Dr. Nelidya Md Yusoff (nelidya.kl@utm.my)

Schedule

Chapt.1. Concept and Function of Electric Circuit

Chapt.2 Power sources and Measurement

Chapt.3 Circuit Analysis

Test 1

Chapt.4 AC Circuit

Chapt.5 Review of Mathematics for Complex Number

Semester Break

Chapt.6 AC Circuit Analysis with Complex Numbers

Chapt.7 Operational Amplifier

Chapt.8 Filter Circuit

Test 2

Laboratory Work (or Group Presentation)

Revision

Final Examination

Assessments

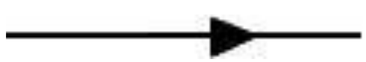
Test 1	:	15%
Test 2	:	20%
Assignment 1	:	5%
Assignment 2	:	5%
Lab's Report (or Group Presentation)	:	5%
Final Exam	:	50%
Total	:	100%

Chapter 1

Concept and Function of Electronic Circuit

Sec.1.1 Electric Current

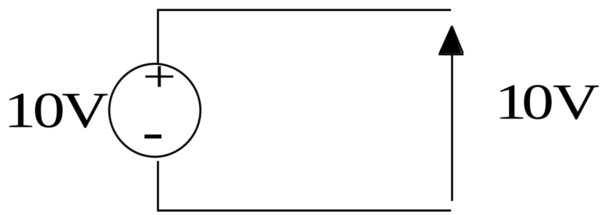
- Electric current is a flow of electric charge.
- The unit for electric current is the ampere [A].
- The abbreviation for the quantity is *I* .
- One may compare the electric current with water flowing in a pipe (hydraulic analogy).
- The current always circulates in a loop: current does not compress nor vanish because charge must be conserved (a basic law of physics).
- The current in a wire is denoted like this:



I = 3mA

Sec.1.2 Voltage

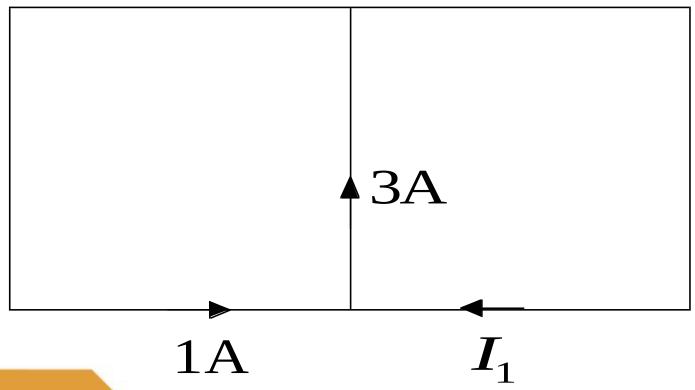
- The potential difference between two points is called voltage.
- The abbreviation for the quantity is *V*.
- The unit of voltage is the volt [V].
- One may compare the voltage with a pressure difference in hydraulic system, or to a difference in altitude.
- Voltage is denoted with an arrow between two points.



Sec.1.3 Kirchhoff's Current Law (or: Kirchhoff's First Law)

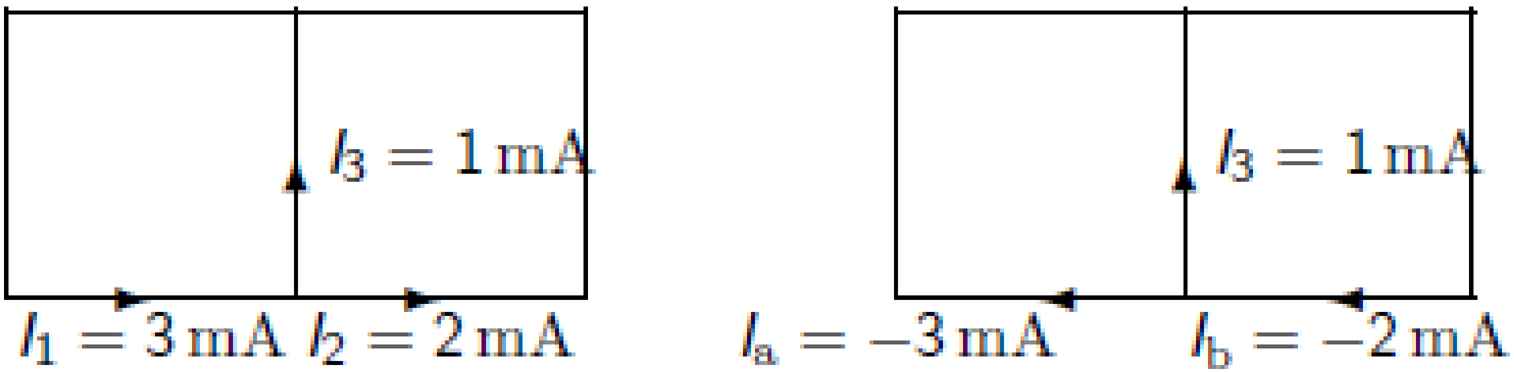
Remember Current Can Not Vanish

- At any area in an electrical circuit, the sum of currents flowing into that area is equal to the sum of currents flowing out of that area.



What is value I_1 ?

Are these the same?

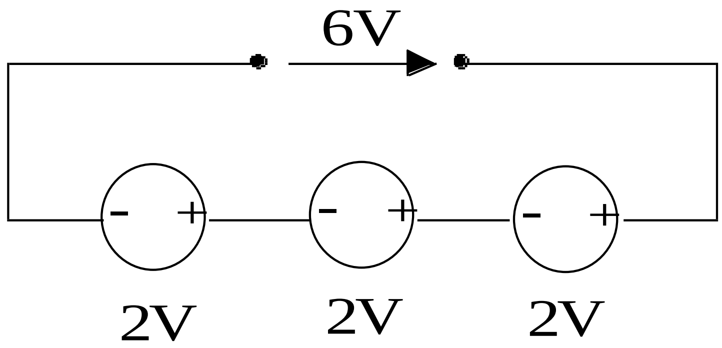


Be careful with signs !

- The sign of the current shows the orientation of the current flow.
- The two circuits on previous slide are exactly identical.
- If you measure a current with an ammeter and it reads −15mA, by reversing the wires of the ammeter it will show 15mA.

Sec.1.4 Kirchhoff's Voltage Law (or: Kirchhoff's Second Law)

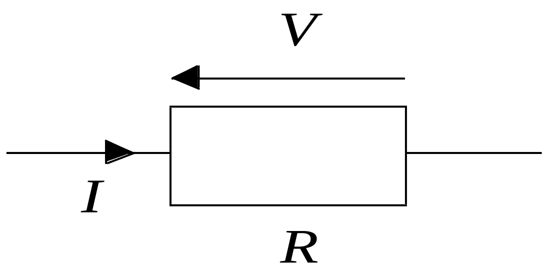
- The voltage between two points is the same, regardless of the path chosen.
- (This is easy to understand by using the analogy of differences in altitude. If you leave your home, go somewhere and return to your home, you have travelled uphill as much as you have travelled downhill.)
- The directed sum of the voltages around any closed circuit is zero.



Sec.1.5 Ohm's Law

- Resistance is a measure of the degree to which an object opposes an electric current flowing through it.
- The larger the current, the larger the voltage
- The abbreviation of the quantity is *R* and the unit is Ω (Ohm).
- The definition of resistance is the ratio of the voltage over the element divided by the current through the element.

$$R = \frac{V}{I} \quad [\Omega]$$



Conductance

- Resistance is a measure of the degree to which an object opposes an electric current through it.
- The inverse of resistance is conductance. The symbol for conductance is *G* and the unit is Siemens [S].
- Conductance is a measure of how easily electric current flows.

$$G = \frac{1}{R} = \frac{I}{V} \quad [\mathrm{S}]$$

Sec.1.6 DC Circuit

- Electric circuit: A system consisting of components, in which electric current flows.
- Direct current (DC): The electrical quantities (voltage and current) are constant (or nearly constant) over time.
- Direct current circuit: An electric circuit, where voltages and currents are constant over time.

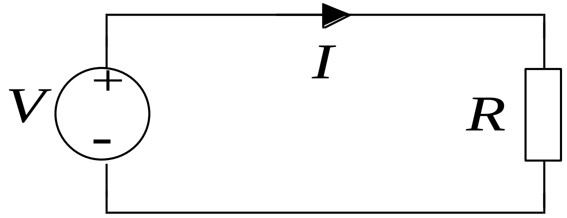
Sec.1.7 Electrical Power

- In physics, power is the rate at which work is performed.
- The symbol for power is *P* and the unit is the Watt [W].
- The DC power consumed by an electric element is P = VI.
- If the formula outputs a positive power, the element is consuming power from the circuit.
- If the formula outputs a negative power, the element is delivering power to the circuit.

Remember:

Energy can not be created nor destroyed.

This means that the power consumed by the elements in the circuit = the power delivered by the elements in the circuit.



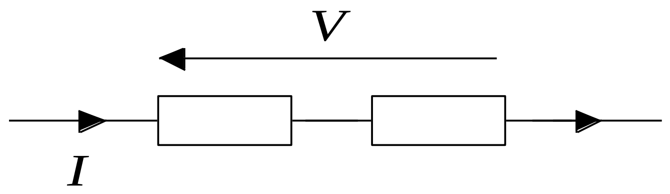
$$I = \frac{V}{R}$$

$$P_R = V\,I = V\frac{V}{R} = \frac{V^2}{R}$$

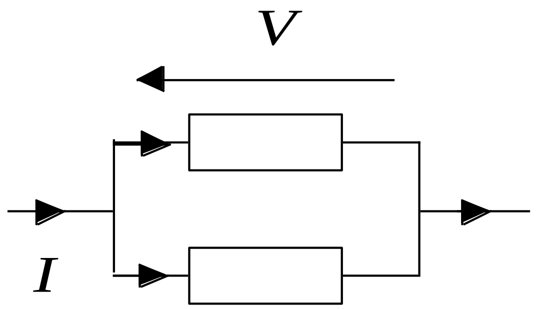
$$P_V = V\cdot(-I) = V\frac{-V}{R} = -\frac{V^2}{R}$$

Sec.1.8 Series and Parallel Circuits

Series Circuit

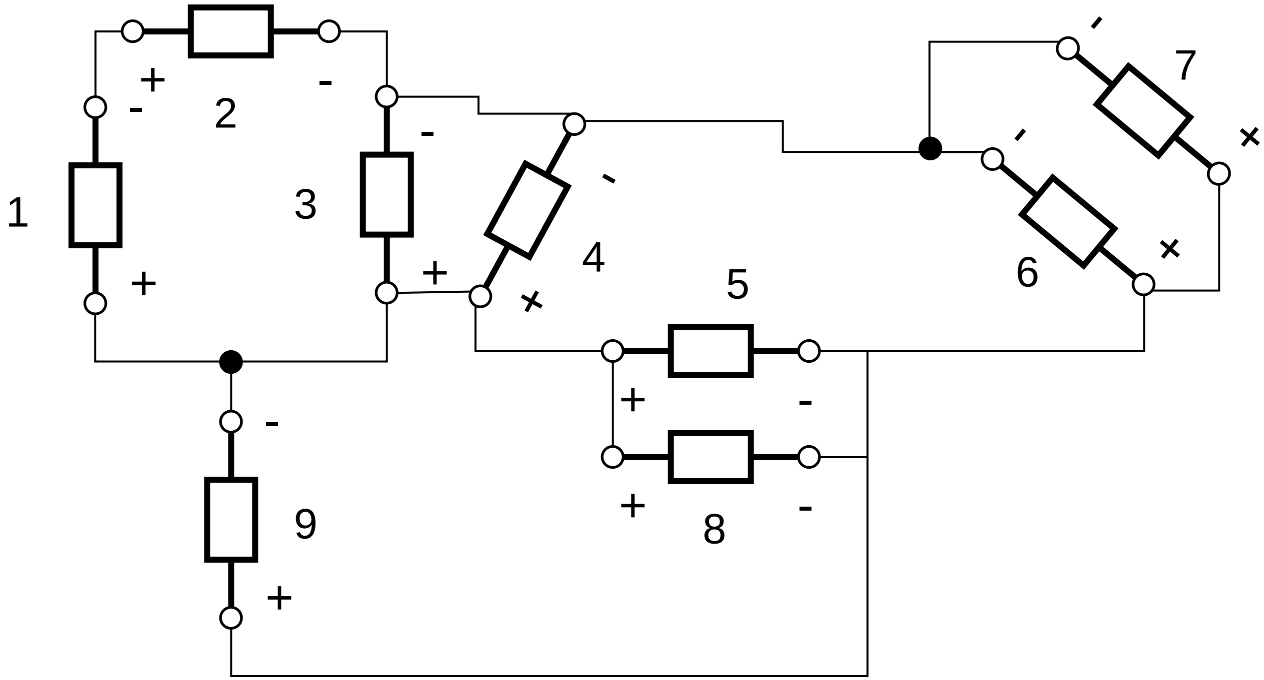


Parallel Circuit



An Example Circuit

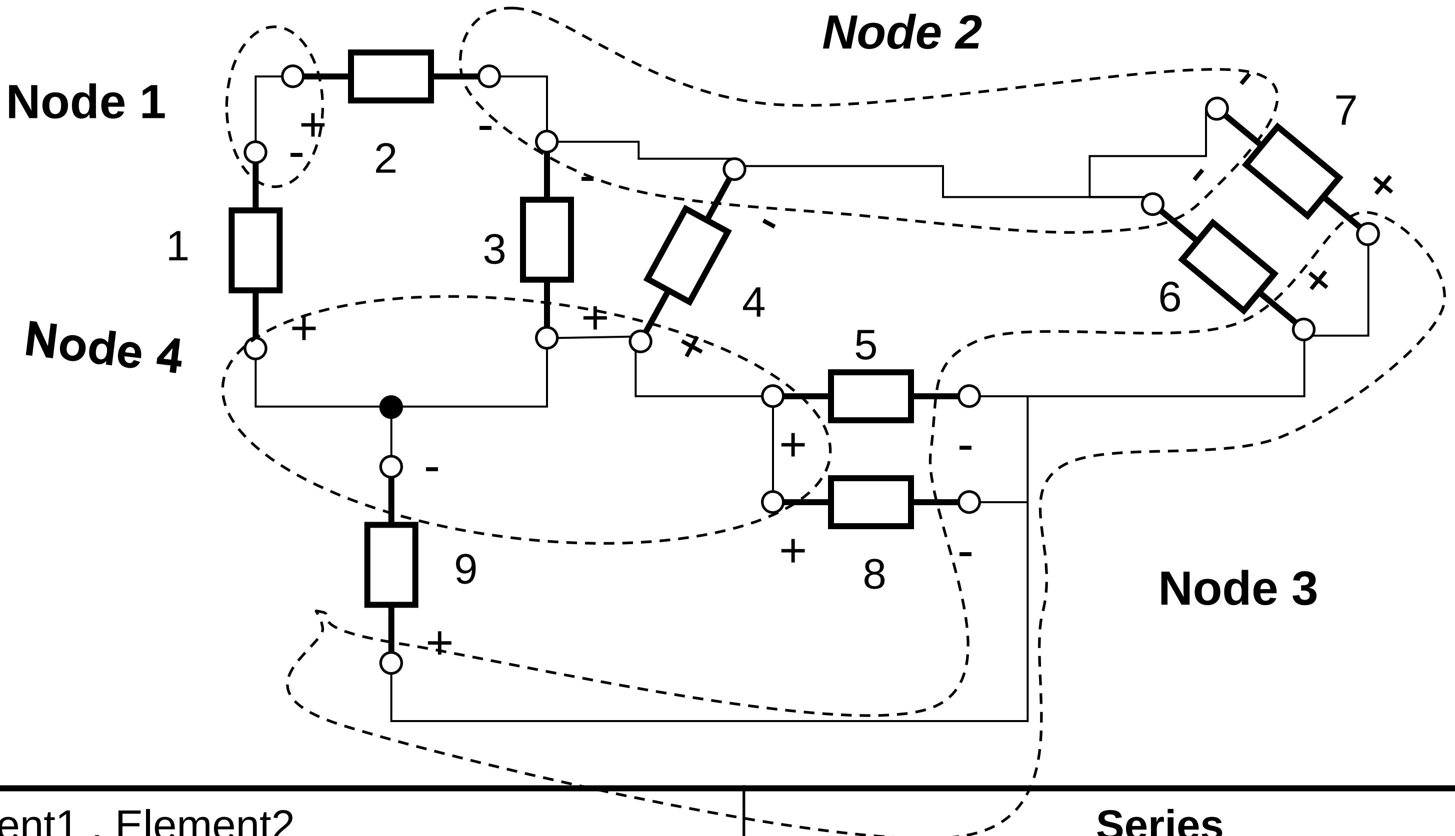
Find the nodes in the circuit shown here



Determine the type of connection between the following elements:-

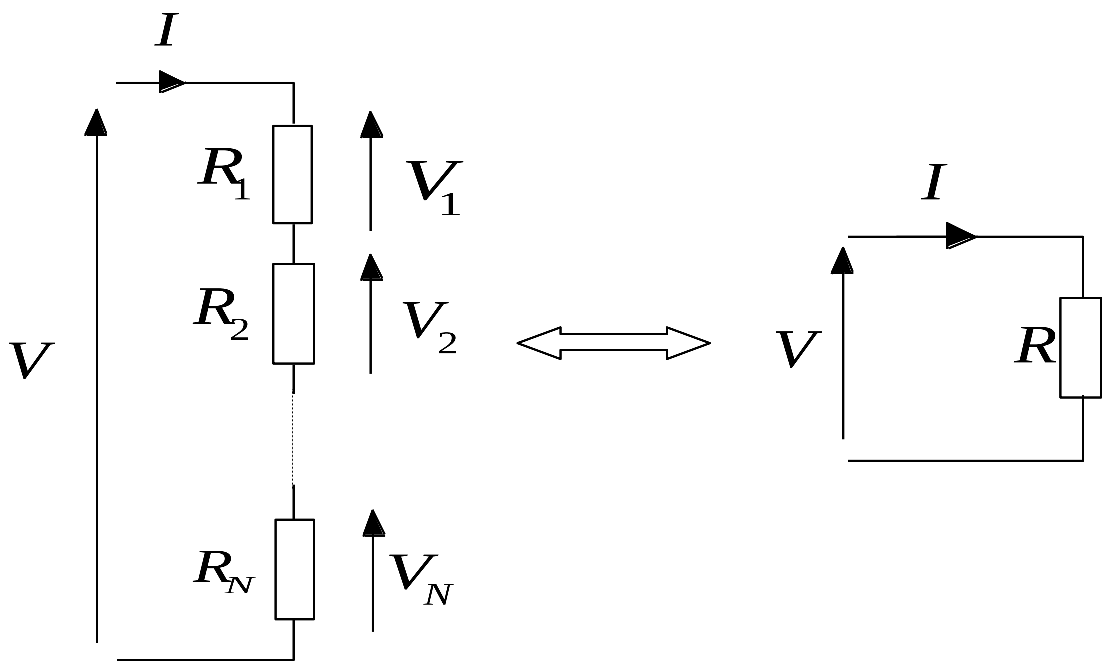
Element1 , Element2	
Element 3 , Element 4	
Element 6 , Element 7	
Element 5 , Element 8 , Element 9	
Element 2 , Element 3	

An Example Circuit



Element1 , Element2	**Series**
Element 3 , Element 4	**Parallel**
Element 6 , Element 7	**Parallel**
Element 5 , Element 8 , Element 9	**Parallel**
Element 2 , Element 3	**Neither in Series Nor in Parallel**

Series connection of resistors



I is common.

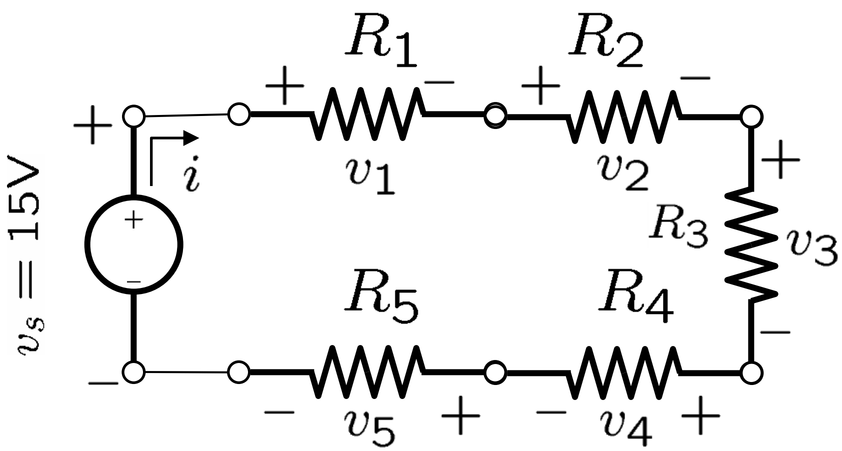
$$V = V_1 + V_2 + \cdots + V_N = \sum_{i=1}^{N} V_i$$

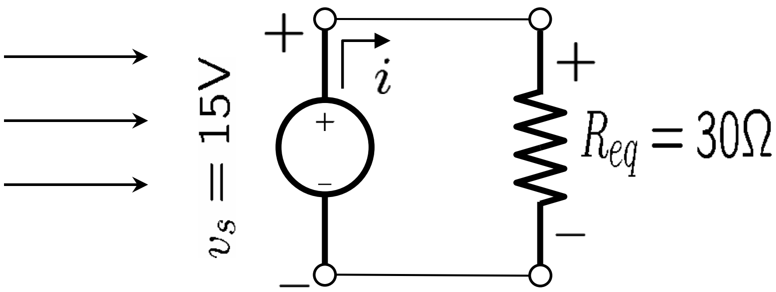
$$R = \frac{V}{I} = R_1 + R_2 + \cdots + R_N = \sum_{i=1}^{N} R_i$$

Resistive Circuits

Resistors in Series

Example: Draw an equivalent circuit. Use the equivalent circuit to find the current ***i***





$R_1 = 2\Omega$

$R_2 = 4\Omega$

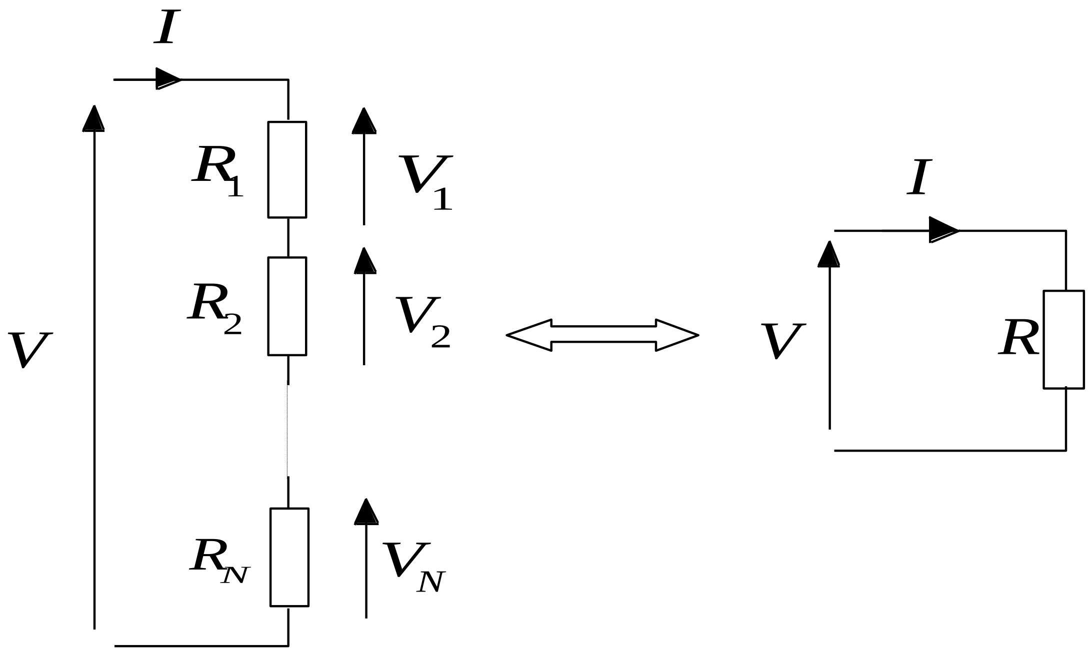
$R_3 = 6\Omega$

$R_4 = 8\Omega$

$R_5 = 10\Omega$

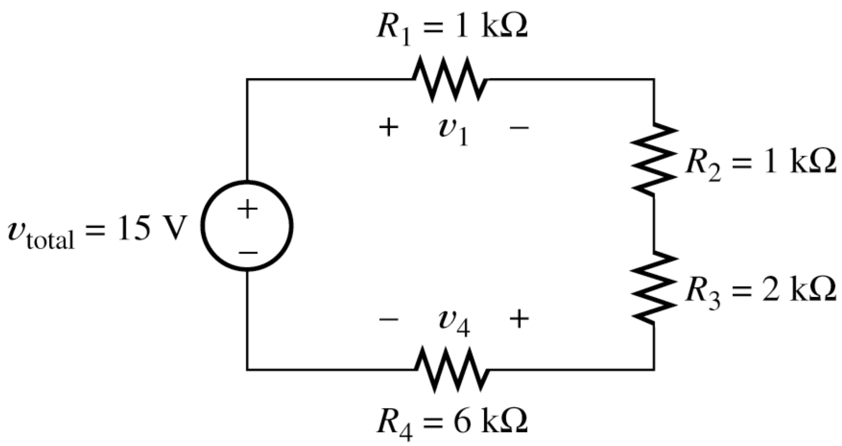
$$i = \frac{v_s}{R_{eq}} = \frac{15}{30} = 0.5\text{A}$$

Voltage Divider



$$V_i = R_i\, I = R_i \frac{V}{R} = \frac{R_i}{\sum_{i=1}^{N} R_i} V$$

Application of the Voltage-Division Principle

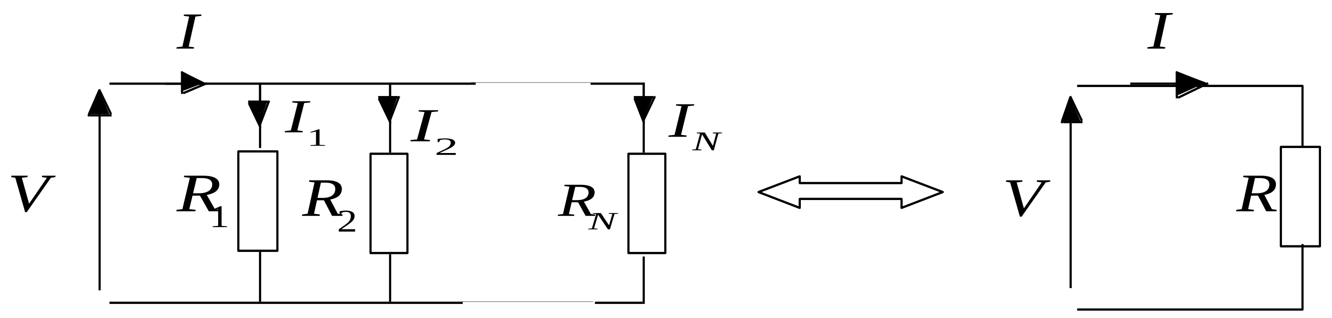


$$v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} v_{total}$$

$$= \frac{1000}{1000 + 1000 + 2000 + 6000} \times 15$$

$$= 1.5\text{V}$$

Parallel connection of resistors



V is common.

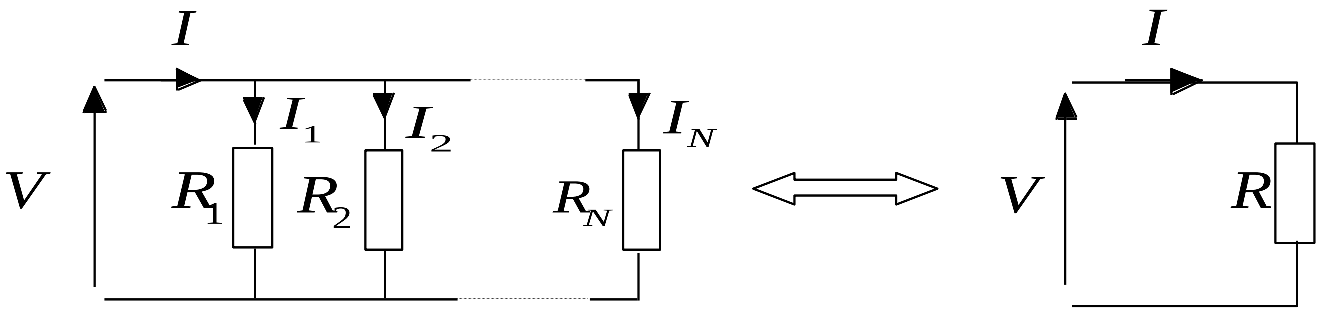
$$I = I_1 + I_2 + \cdots + I_N = \sum_{i=1}^{N} I_i$$

$$\frac{1}{R} = \frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i}$$

$$G = G_1 + G_2 + \cdots + G_N = \sum_{i=1}^{N} G_i$$

Abbreviated writing : $R = R_1 // R_2 // \cdots // R_N$

Current Divider



$$I_i = \frac{V}{R_i} = \frac{R}{R_i} I$$

Applications of Current Division Principle

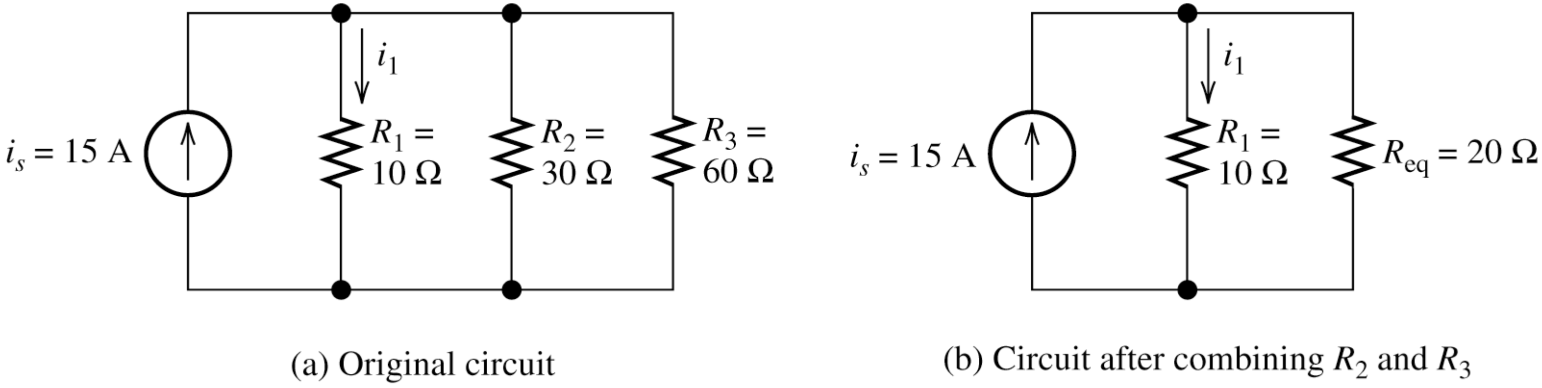


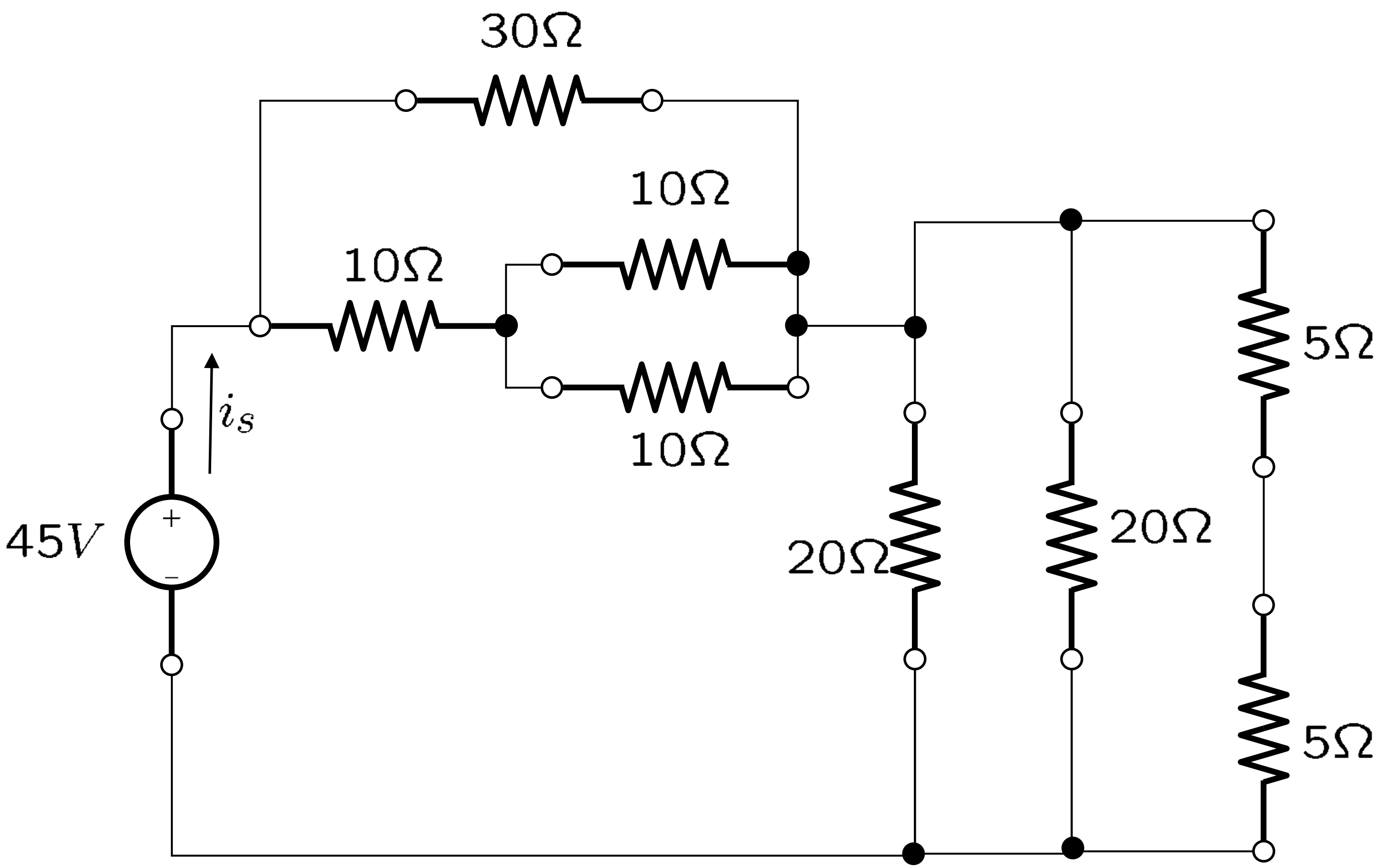
Figure 2.12 Circuit for Example 2.5.

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\Omega$$

$$i_1 = \frac{R_{eq}}{R_1 + R_{eq}} i_s = \frac{20}{10 + 20} 15 = 10\text{A}$$

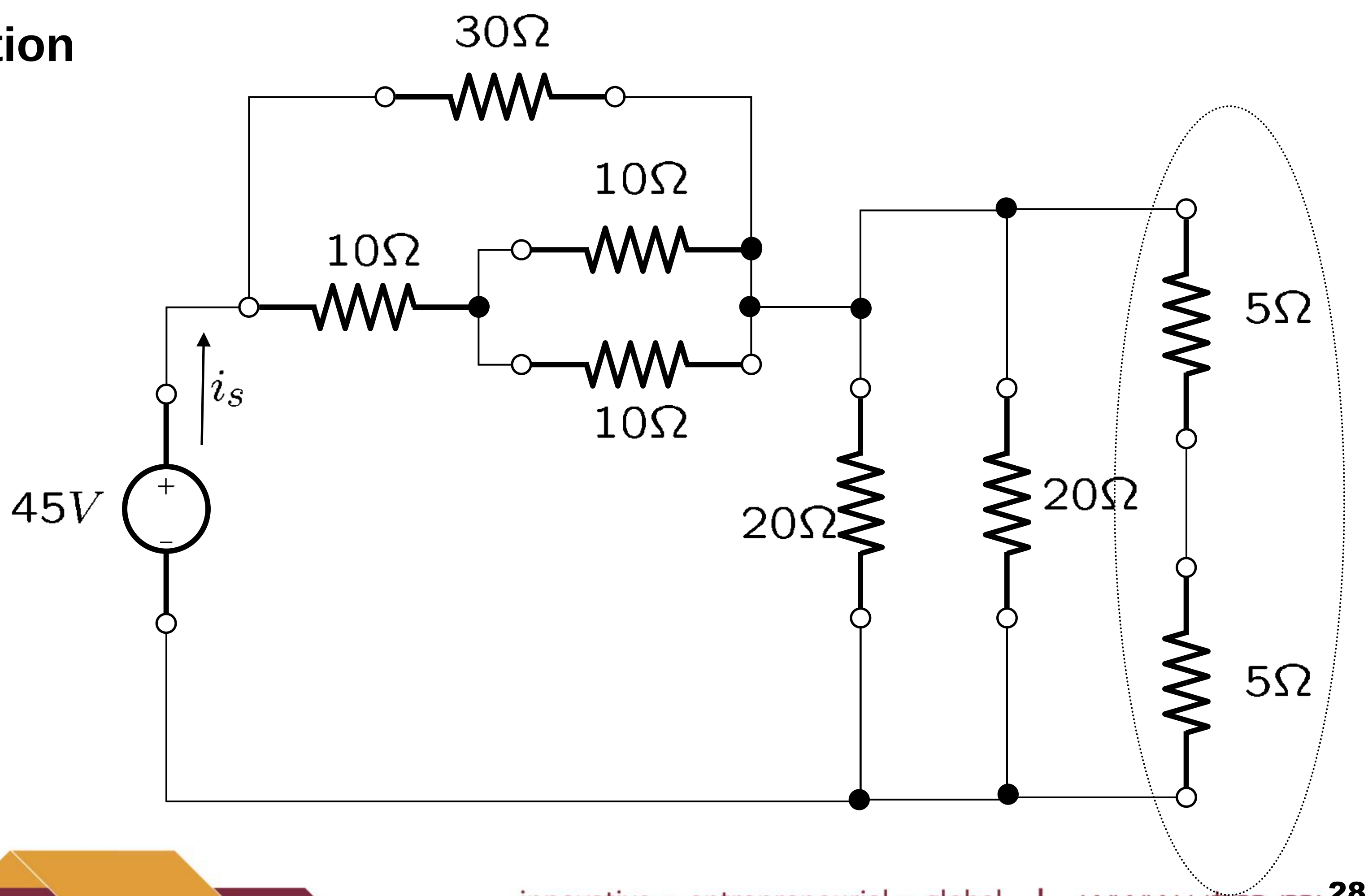
Equivalent Resistance : Example 1

For the circuit shown below, draw a simplified equivalent circuit.
Use the equivalent circuit to calculate the current i_s flowing in the source.



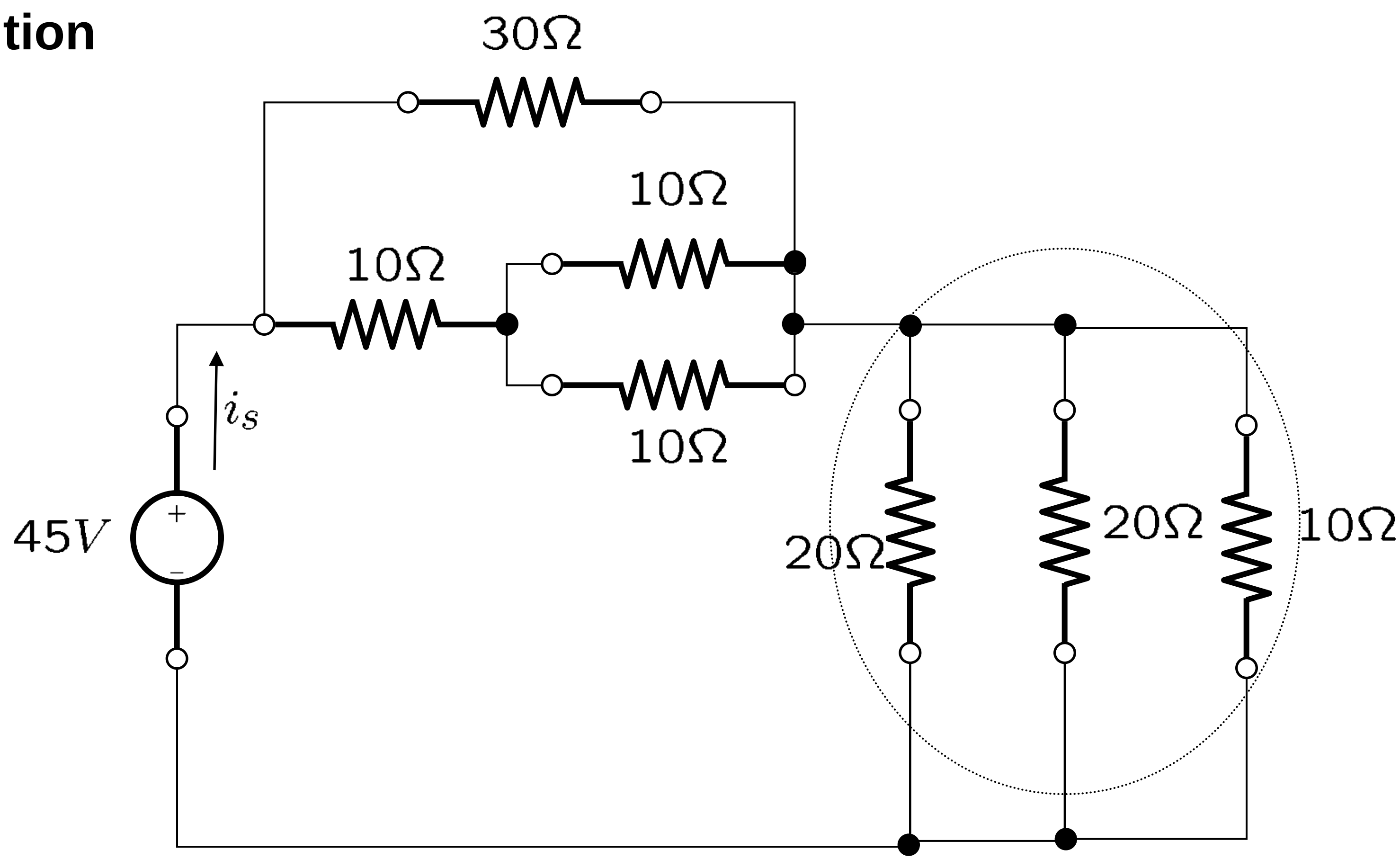
Equivalent Resistance : Example 1

Solution



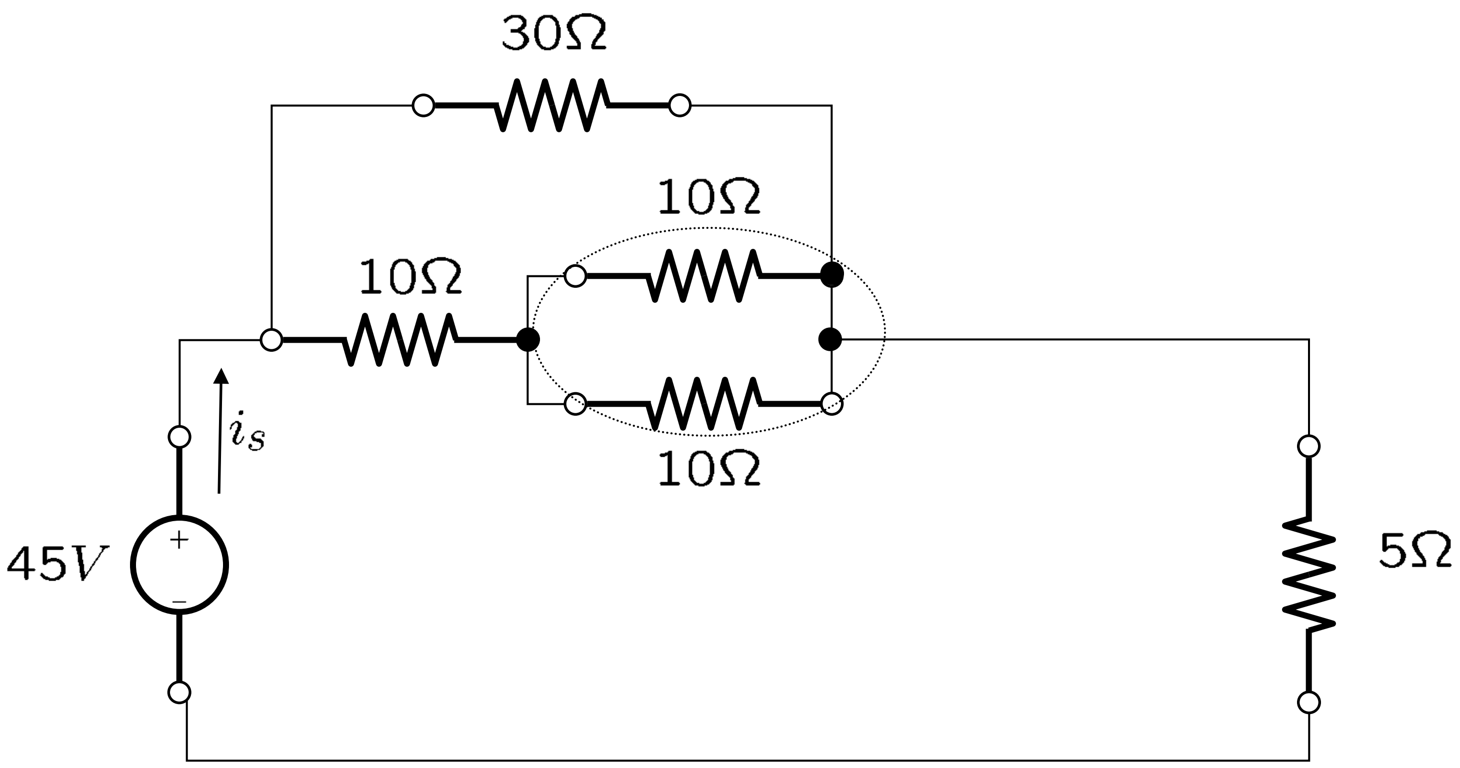
Equivalent Resistance : Example 1

Solution



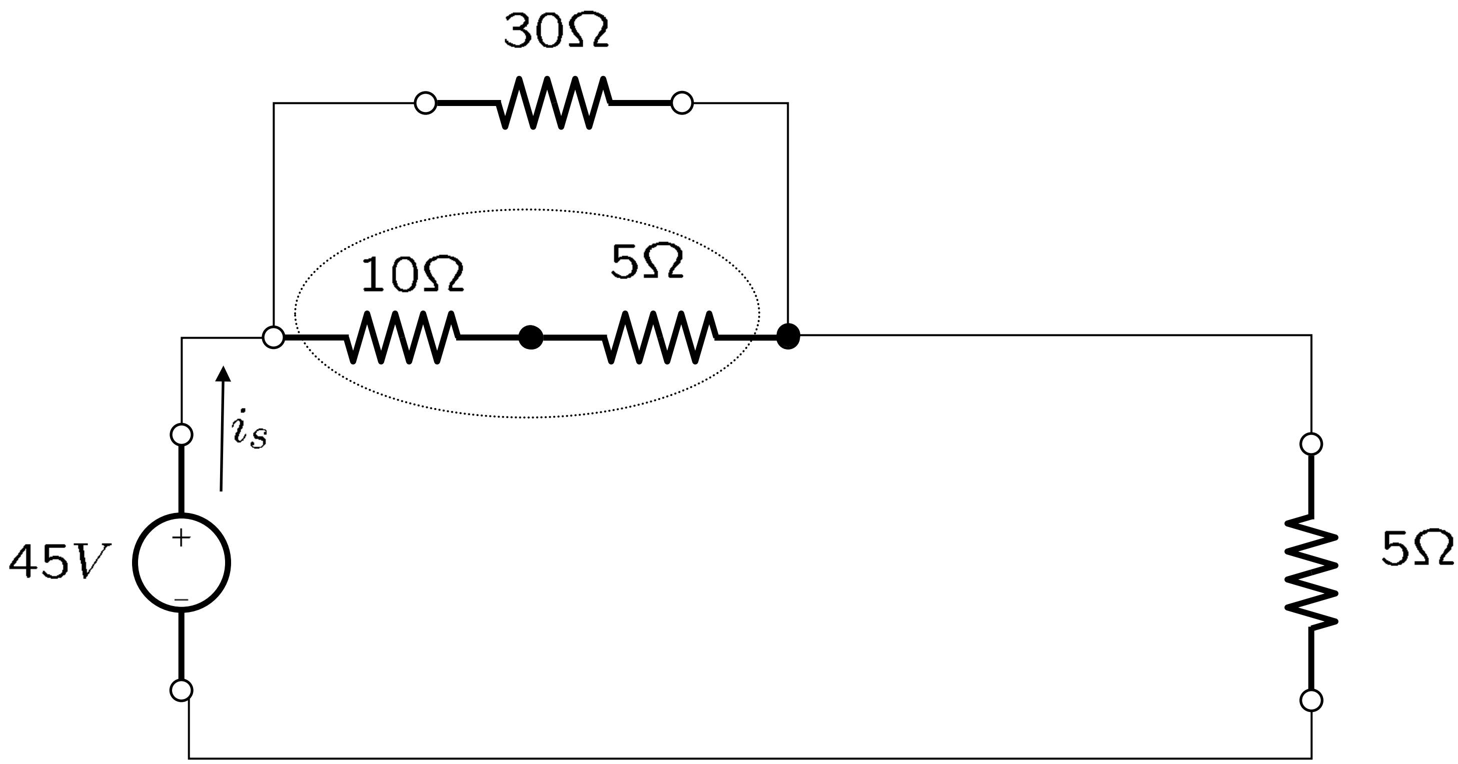
Equivalent Resistance : Example

Solution



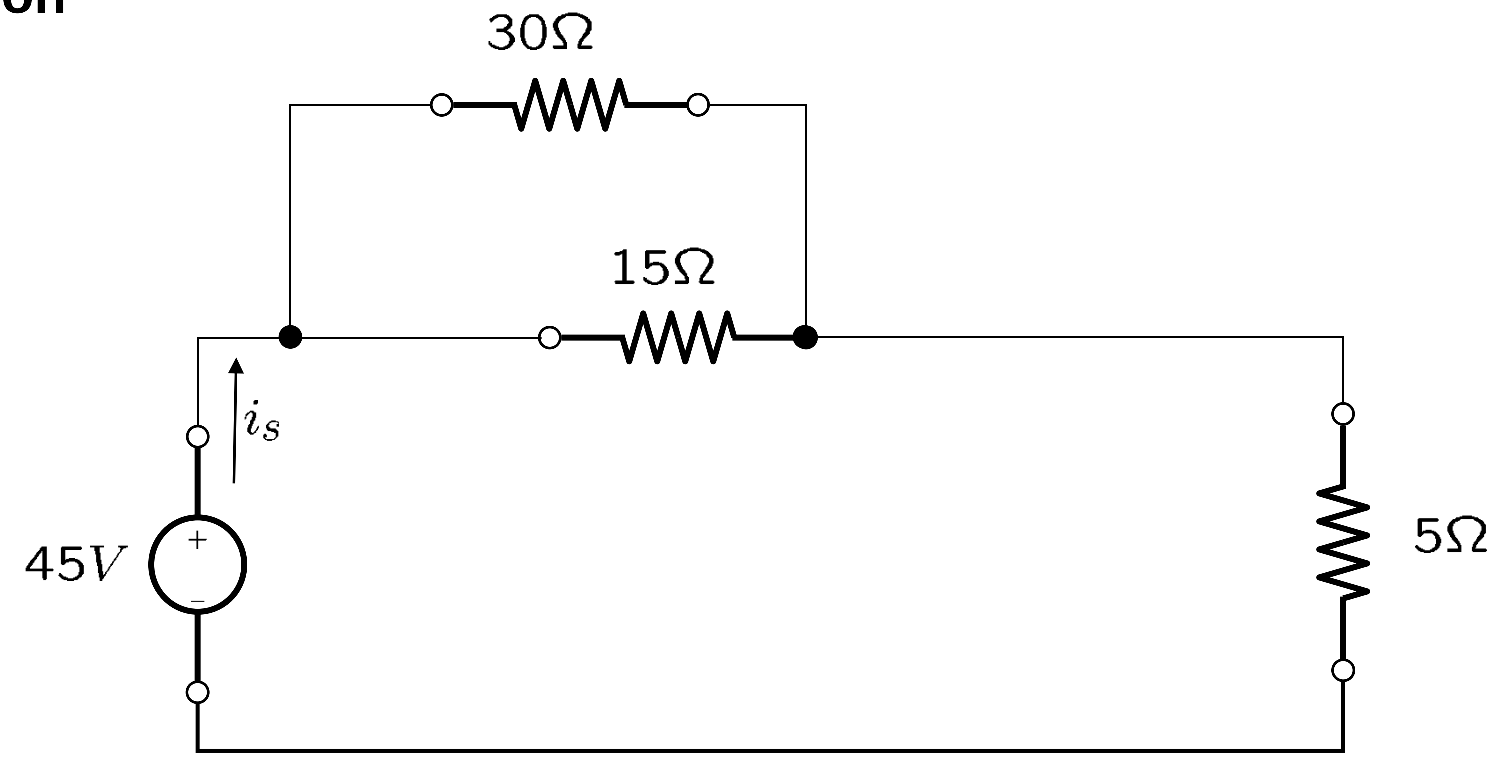
Equivalent Resistance : Example

Solution



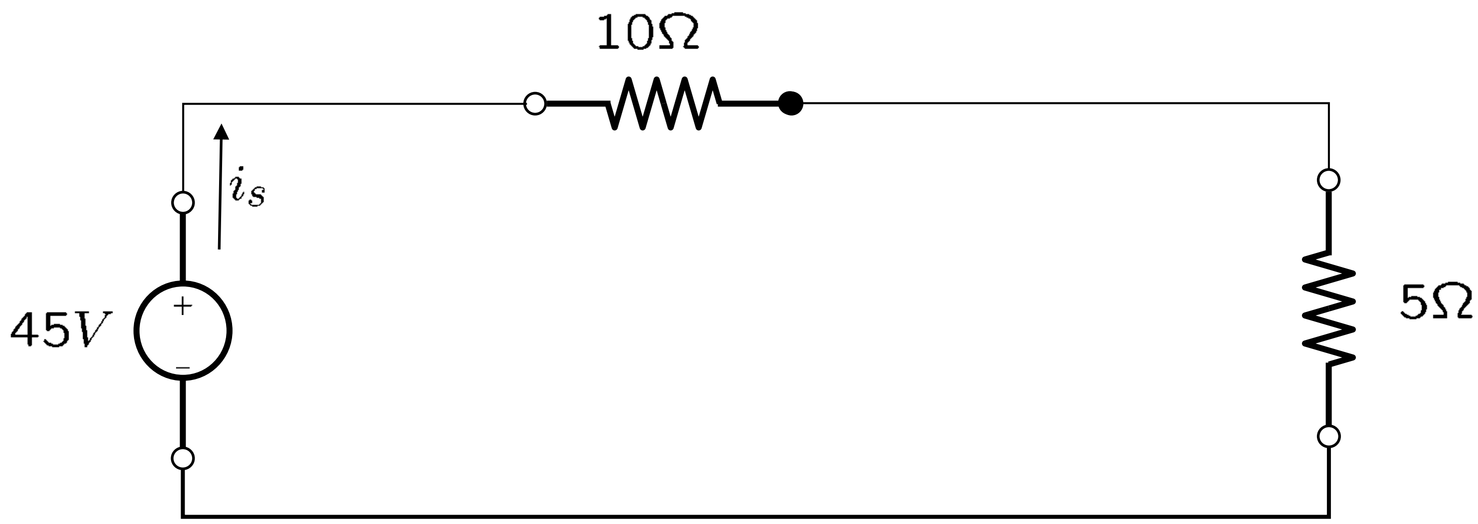
Equivalent Resistance : Example

Solution



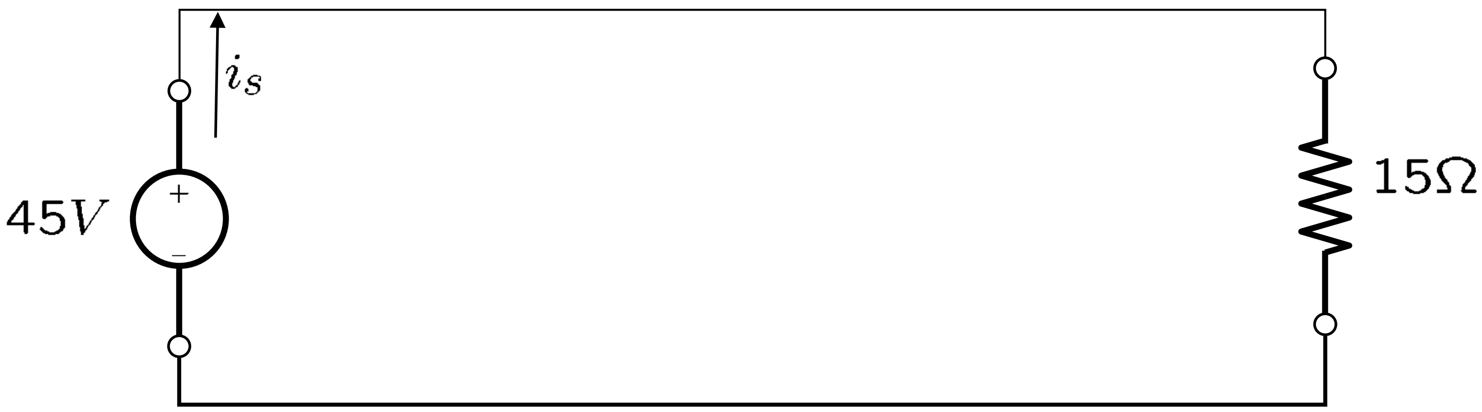
Equivalent Resistance : Example

Solution



Equivalent Resistance : Example

Solution



$$i_s = \frac{45}{15} = 3\text{A}$$

Equivalent Resistance: Example 2

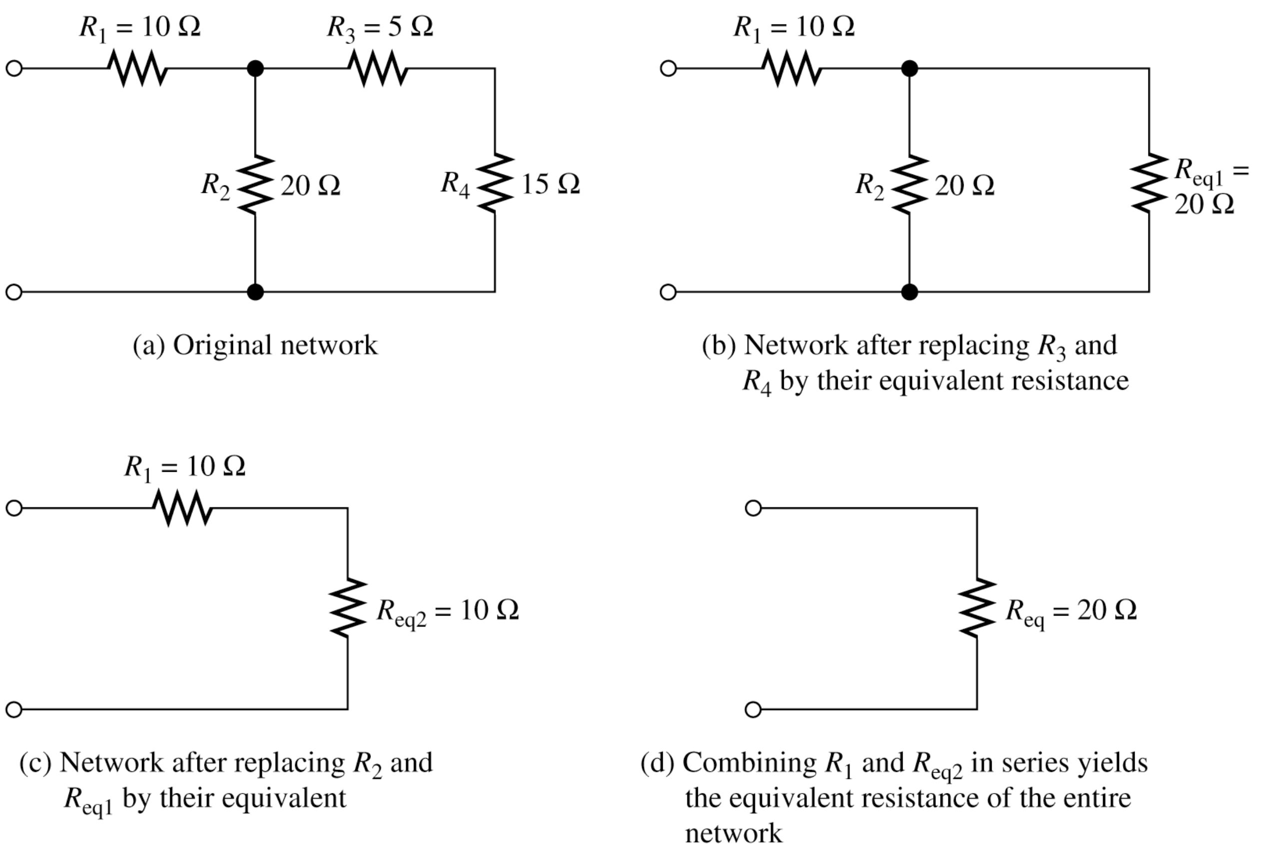


Figure 2.3 Resistive network for Example 2.1.

Equivalent Resistance Exercises

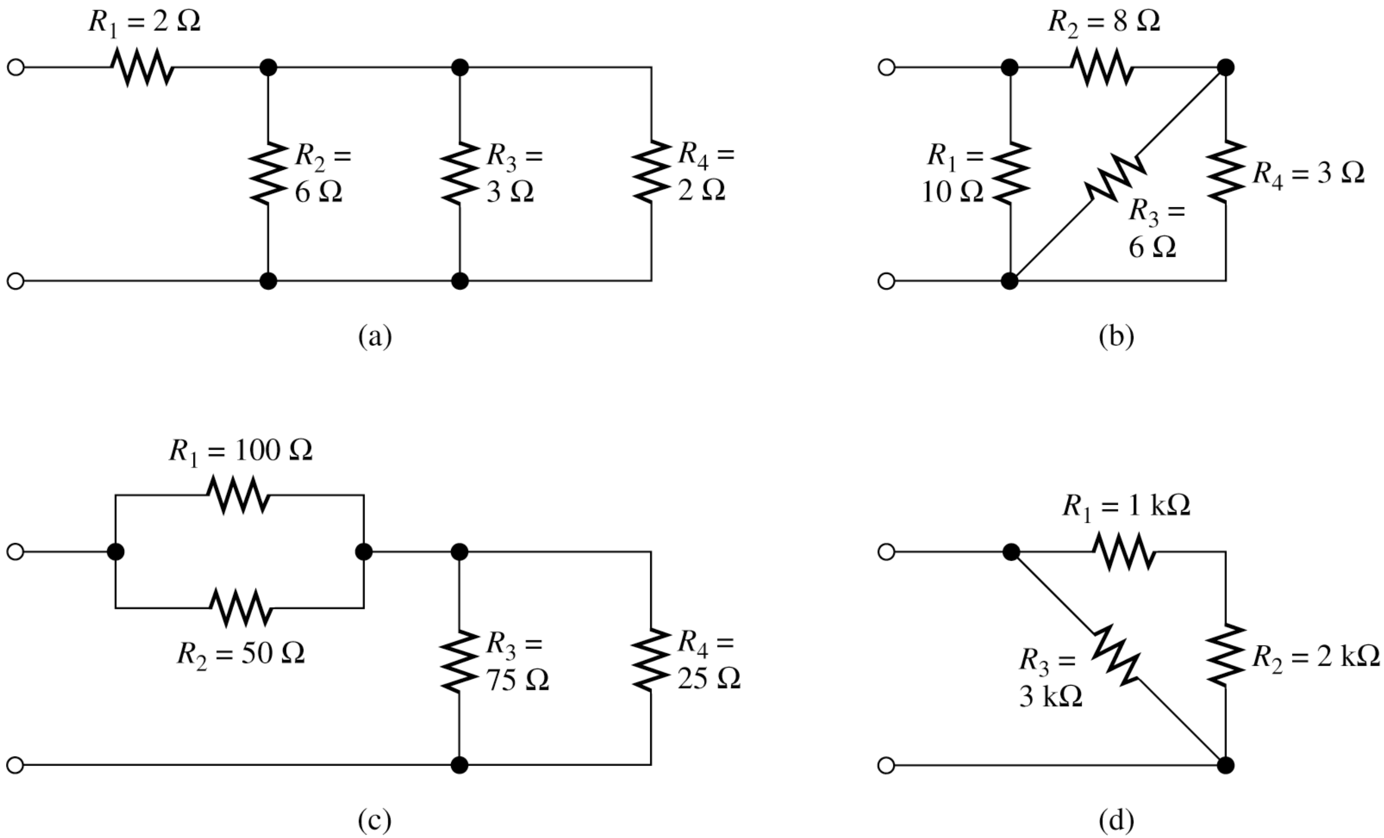
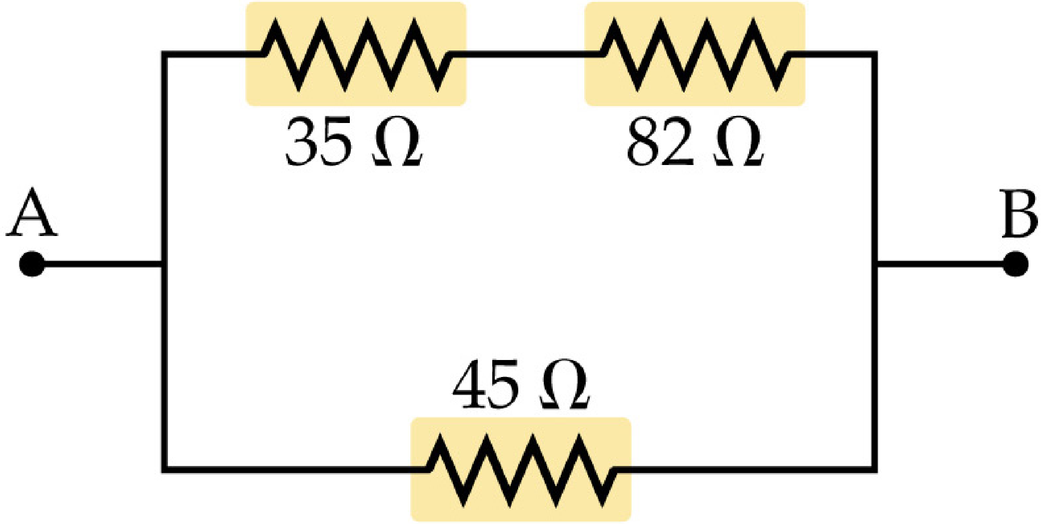


Figure 2.4 Resistive networks for Exercise 2.1.

What is the equivalent resistance?



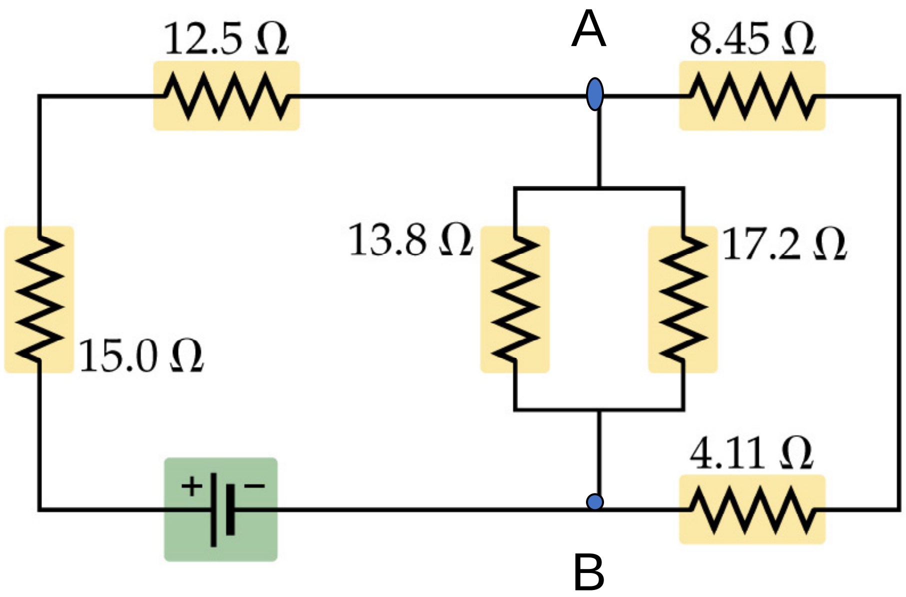
$$R_{12} = 35\Omega + 82\Omega = 117\Omega$$

$$\frac{1}{R_{eq}} = \frac{1}{R_{12}} + \frac{1}{R_3} = \frac{1}{117\Omega} + \frac{1}{45\Omega} = 0.0307\Omega^{-1}$$

$$R_{eq} = 32.5\Omega$$

Calculation Example

The current in the 13.8 Ω resistor is 0.750 A.
Find the current in other resistors in the circuit.



$V_{AB} = 13.8W\ 0.75A = 10.35\ V$

$I_{17.2} = V_{AB}\ /17.2W = 0.602A$

$I_{8.45+4.11} = V_{AB}\ /(8.45W+4.11W)$

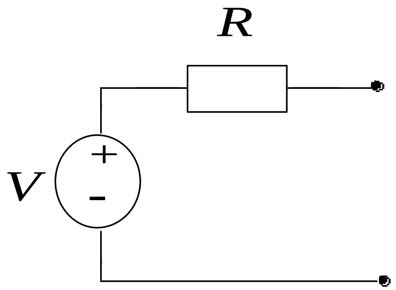
$= (10.35V)/(12.56W)$

$= 0.824A$

$I_{total} = 0.75\ A + 0.602\ A + 0.824A = 2.18\ A$

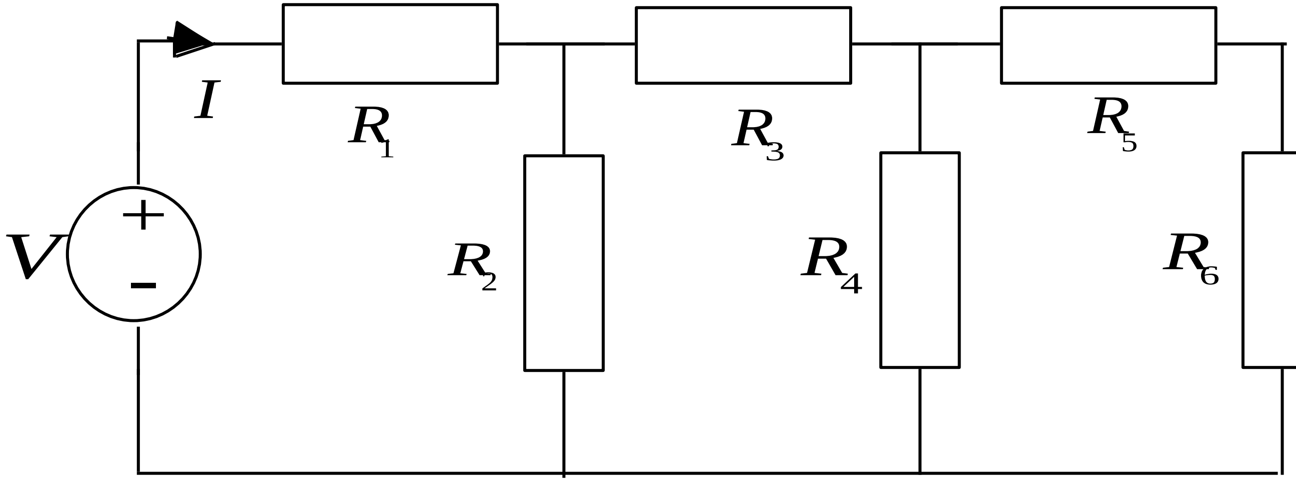
Sec.1.9 Terminal or Gate

- A point which provides a connection to external circuits is called a terminal (or pole).
- Two terminals form a gate.
- An example is a car battery with internal resistance.



Sec.1.10 Node

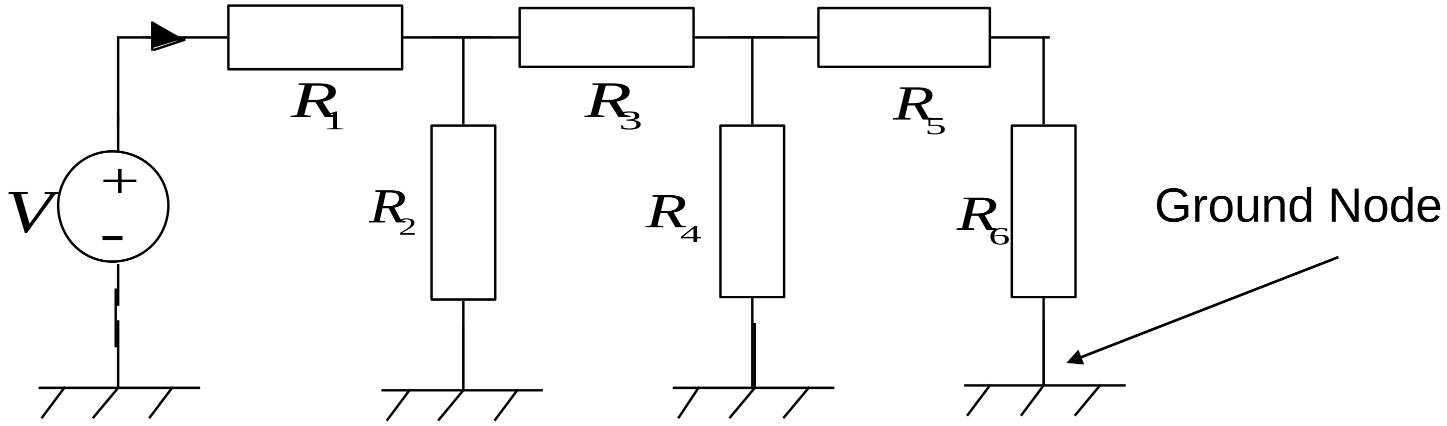
- A node means an area in the circuit where there are no potential differences, or alternatively a place where two or more circuit elements meet.



How many Nodes ?

Ground

- One of the nodes in the circuit can be appointed the ground node.
- By selecting one of the nodes to be the ground node, the circuit diagram usually appears cleaner and less cluttered.

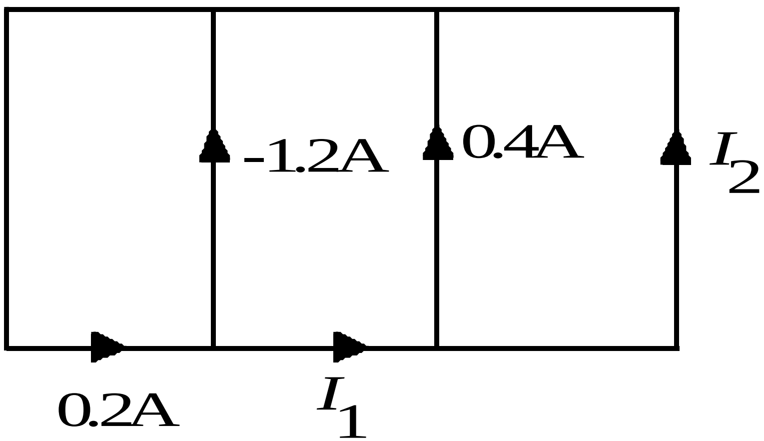


An example of ground node

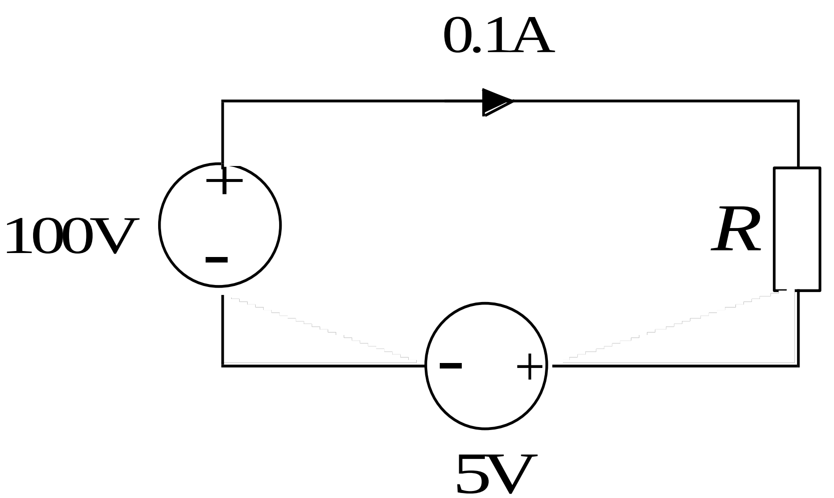
- A car battery is connected to the chassis of the car. Therefore it is convenient to handle the chassis as the ground node.
- When we say ”the voltage of this node is 12 volts” it means that the voltage between that node and the ground node is 12 volts.

Problems

1.1 Find I_1 and I_2



1.2 Find *R*.



1.3 Show an equation of two parallel resistors, R_1 and R_2 is

$$R = R_1 \,//\, R_2 = \frac{1}{\frac{\cdot 1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

1.4 Calculate total value of resistance consist of three resistors, $1\Omega, 10\Omega \text{ and } 100\Omega$, which are connected in series.

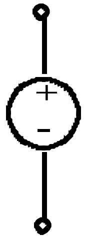
1.5 Calculate total value of resistance consist of three resistors, $1\Omega, 10\Omega \text{ and } 100\Omega$, which are connected in parallel.

Chapter 2

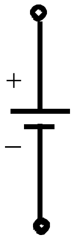
Power Sources and Measurement

Independent Voltage Source

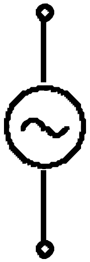
- A power source provides voltage whose value is independent from any current or voltage component in the circuit.



Voltage source used for both time–invariant (DC) and time-varying (AC) voltage



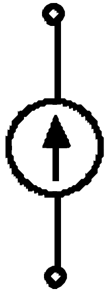
Voltage source used for time-invariant (DC) voltage



Voltage source used for time-varying (AC) voltage

Independent Current Source

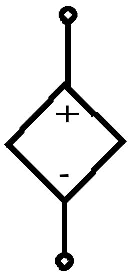
- A power source provides current whose value is independent from any current or voltage component in the circuit.



Current source used for both time –invariant (DC) and time-varying (AC) current

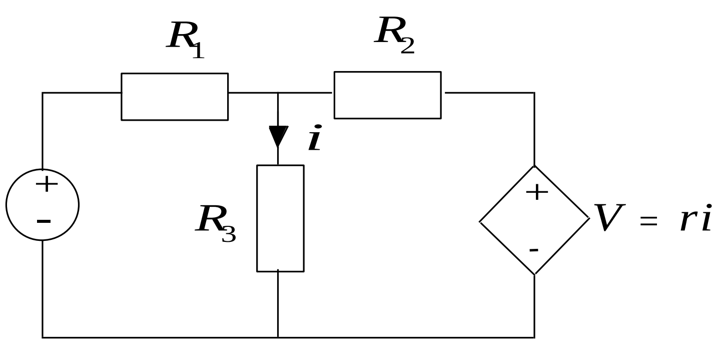
Controlled (or : Dependent) Voltage Source

- A power source provides voltage whose value is controlled by another voltage or current in the circuit



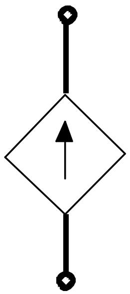
Used for both time –invariant (DC) and time-varying (AC) voltage

An example circuit with controlled voltage source



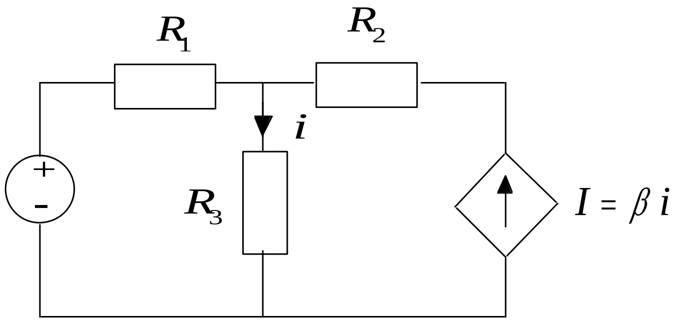
Controlled (or : Dependent) Current Source

- A power source provides current whose value is controlled by another voltage or current in the circuit



Used for both time –invariant (DC) and time-varying (AC) voltage

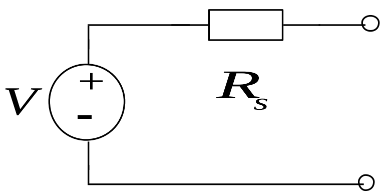
An example circuit with controlled current source



Ideal Power Sources and Internal Resistance

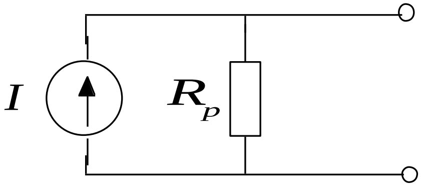
Voltage Source

- Real voltage source has an internal resistance R_s in series.
- A voltage source eliminated R_s is called an ideal voltage source.
- The internal resistance of an ideal voltage source is zero.



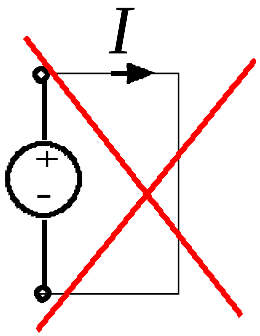
Current Source

- Real current source has an internal resistance R_p in parallel.
- A current source eliminated R_p is called an ideal current source.
- The internal resistance of an ideal current source is infinite.

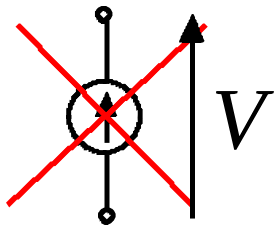


Important Remarks

- The ideal voltage source should not be shorten, because flowing current *I* becomes infinite.

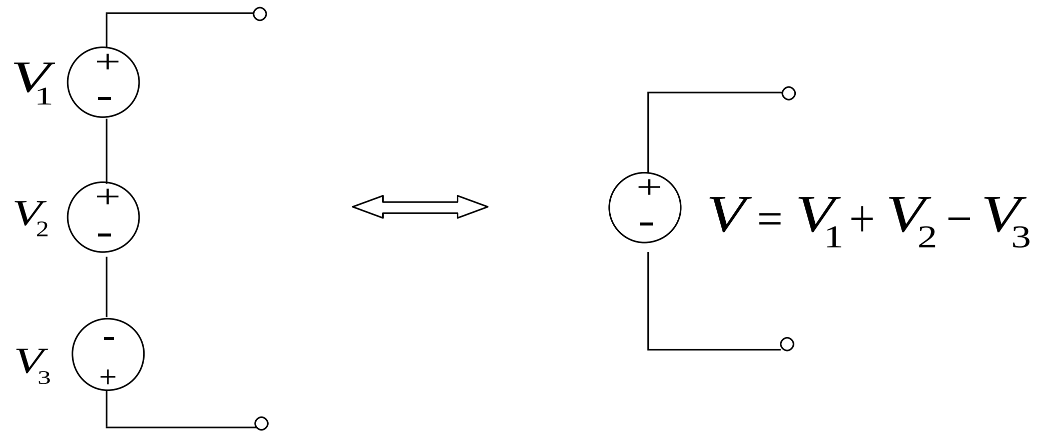


- The ideal current source should not be used in as open circuit, because voltage *V* between two terminals becomes infinite.

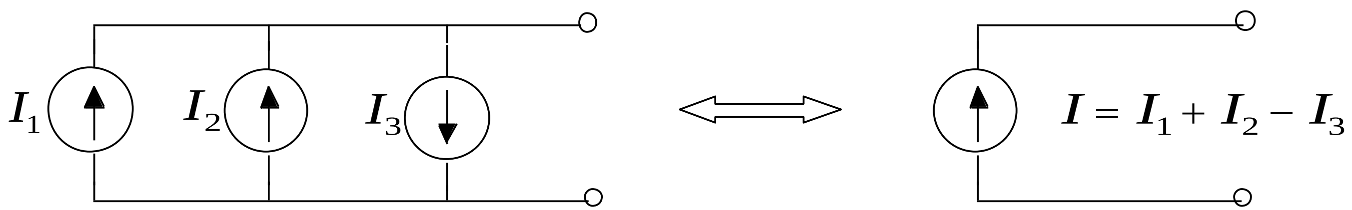


Connection of Power Sources

Voltage Sources in Series

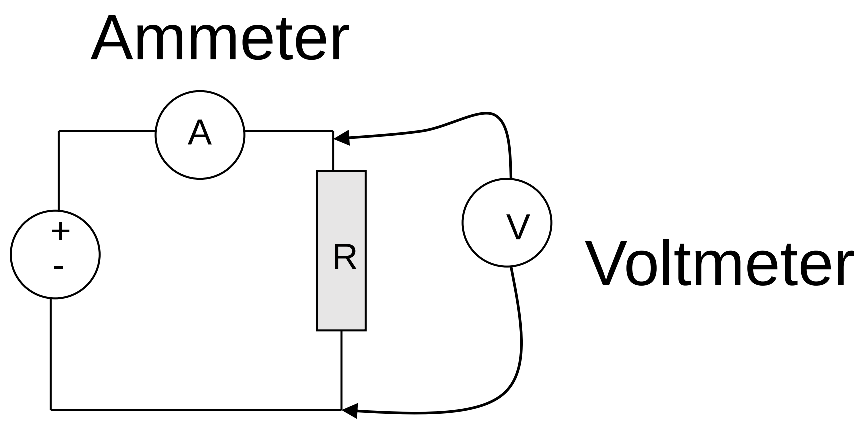


Current Sources in Parallel



Note : Parallel connection of ideal voltage sources and series connection of ideal current sources are inhibited.

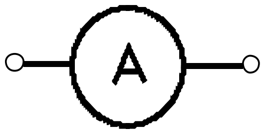
Voltage and Current Measurement



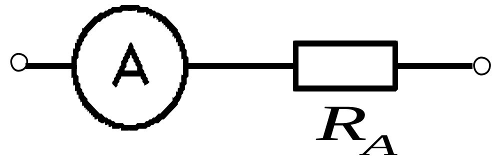
- Ammeter measures current and must be in series to the component being tested.

- Voltmeter measures voltage and must be in parallel.

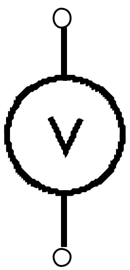
Internal (or : Equivalent) Resistance of the Meter



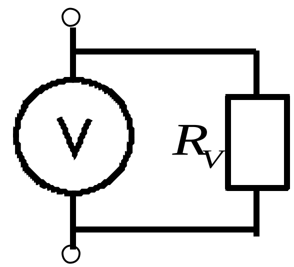
A real Ammeter



An ideal Ammeter + an internal resistance in series



A real Voltmeter



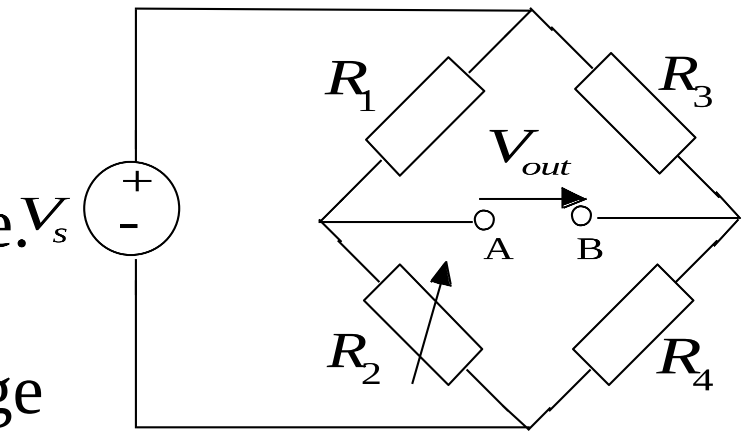
An ideal Voltmeter + an internal resistance in parallel

Notes for Word of “Ideal”

- As mentioned above, power sources and meters have internal resistances. Main parts of these circuit elements eliminated the internal resistances are called with “ideal” .
- However, in almost cases of electric circuits, circuit elements are represented as ideal elements. Therefore, word of “ideal” is not attached usually except in the cases to enhance that is “ideal”.

Wheatstone bridge

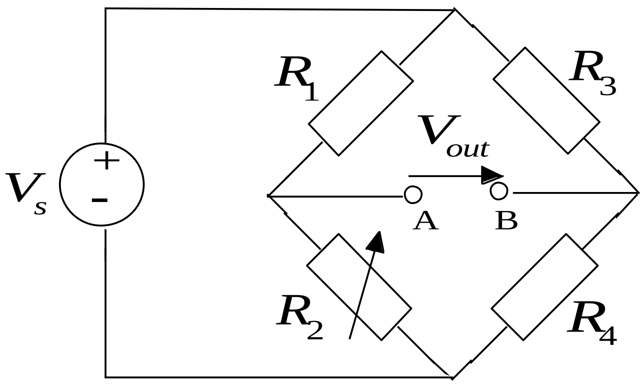
The Wheatstone bridge consists of four resistive arms forming two voltage dividers and a voltage source. The output is taken between the dividers. Frequently, one of the bridge resistors is adjustable.



When the bridge is balanced, the output voltage is zero and the products of resistances in the opposite diagonal arms are equal.

Balanced State

$$V_{out} = \left(\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right) V_s$$



When $V_{out} = 0$,

$$R_4 (R_1 + R_2) = R_2 (R_3 + R_4),$$

$$R_4 R_1 + R_4 R_2 = R_2 R_3 + R_2 R_4 .$$

Then

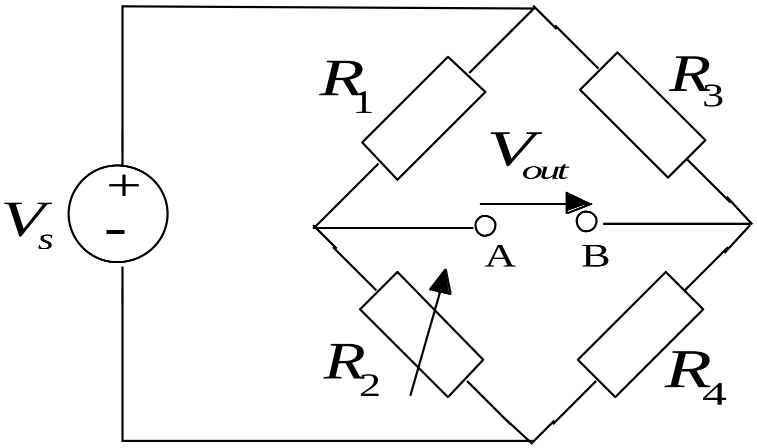
$$R_1 R_4 = R_2 R_3$$

Unbalanced State

$$V_{out} = \left(\frac{R_4}{R_3 + R_4} - \frac{R_2 + \Delta R_2}{R_1 + R_2 + \Delta R_2} \right) V_s$$

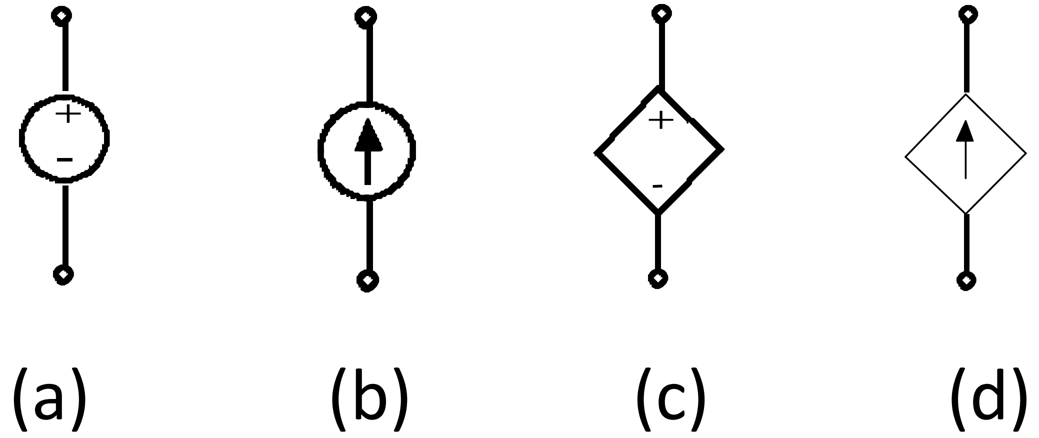
$$= -\frac{\Delta R_2 R_3}{(R_3 + R_4)(R_1 + R_2 + \Delta R_2)}$$

$$\approx -\frac{\Delta R_2 R_3}{(R_1 + R_2)(R_3 + R_4)}$$



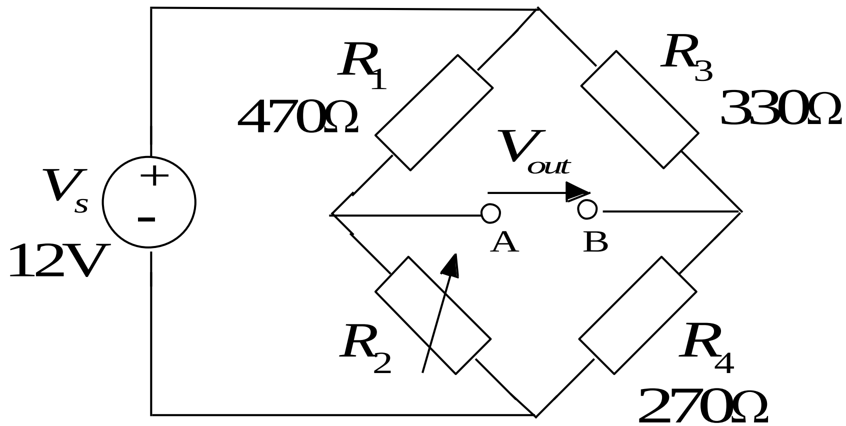
Problems

2.1 Give names of following circuit elements.



2.2 Explain the reason why parallel connection of ideal voltage sources is inhibited.

2.3 What is the value of R_2 if the bridge is balanced?



CHAPTER 3

Circuit Analysis

Nodal Analysis

1. Define current components I_i with flowing direction for each branch circuit.
2. Formulate relations among current components based on the Kirchhoff's Current Law.
3. Formulate relations to coincide the voltage at each node based on the Kirchhoff's Voltage Law and Ohm's Law.
4. Check number of equations to coincide with number of unknown variables.
5. Reform equations to reduce unknown variable.

   Application of the relations of series and parallel connection is helpful.

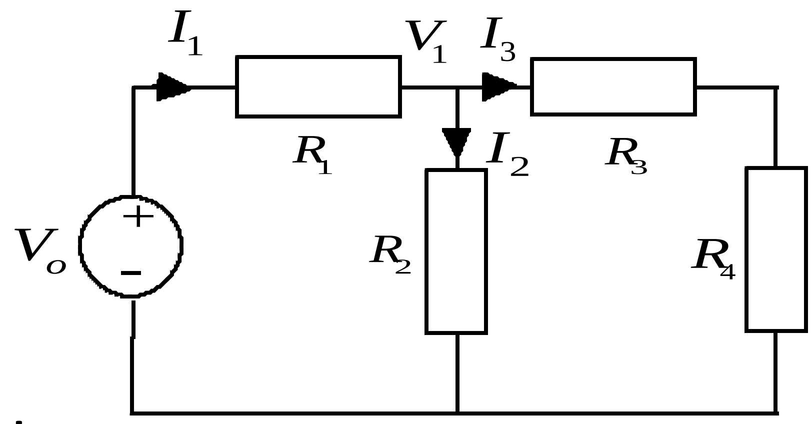
Find current components I_1, I_2 and I_3 and voltage V_1 .

$$I_1 = I_2 + I_3$$

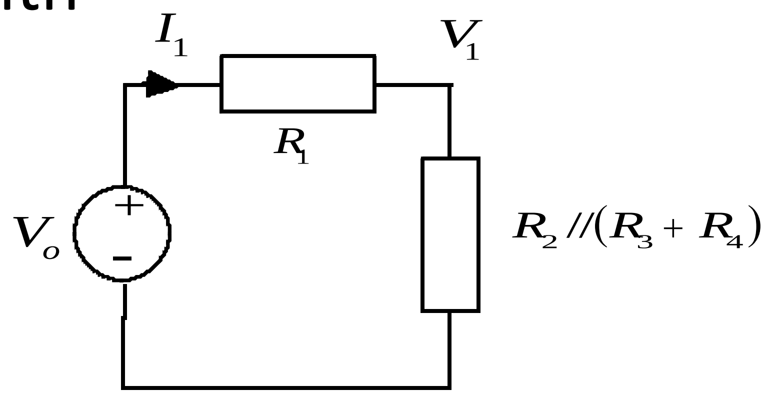
$$V_o = R_1 I_1 + V_1$$

$$V_1 = R_2 I_2$$

$$V_1 = (R_3 + R_4) I_3$$



- Unknown variables are 4, and equations are 4.
- Look at right part of the circuit with R_2 , R_3 and R_4 , whose value is $R_2 // (R_3 + R_4)$.
- Then, the circuit is rewrite with series connection of R_1 and $R_2 // (R_3 + R_4)$.



$$I_1 = \frac{V_o}{R_1 + \left\{R_2 //(R_3 + R_4)\right\}} = \frac{V_o}{R_1 + \dfrac{1}{\dfrac{1}{R_2} + \dfrac{1}{R_3 + R_4}}}$$

$$= \frac{V_0}{R_1 + \dfrac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}} = \frac{(R_2 + R_3 + R_4)V_o}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}$$

$$V_1 = I_1\left\{R_2 //(R_3 + R_4\right\} = \frac{\left\{R_2 //(R_3 + R_4\right\}V_o}{R_1 + \left\{R_2 //(R_3 + R_4)\right\}}$$

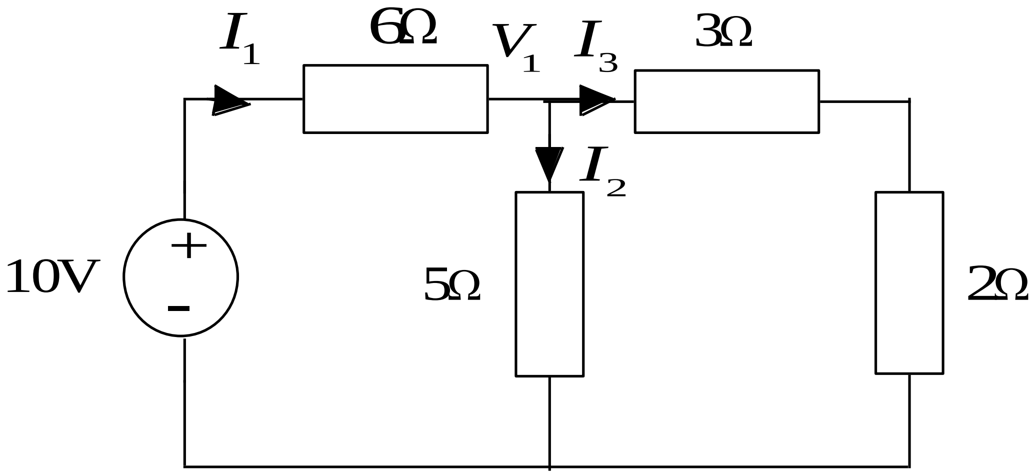
$$I_2 = \frac{V_1}{R_2} = \frac{\{R_2 // (R_3 + R_4\} V_o}{R_2\left[R_1 + \{R_2 // (R_3 + R_4)\}\right]}$$

$$I_3 = I_1 - I_2 = \frac{V_o}{R_1 + \{R_2 // (R_3 + R_4)\}} - \frac{\{R_2 // (R_3 + R_4\} V_o}{R_2\left[R_1 + \{R_2 // (R_3 + R_4)\}\right]}$$

$$= \frac{\left[R_2 - \{R_2 // (R_3 + R_4\}\right] V_o}{R_2\left[R_1 + \{R_2 // (R_3 + R_4)\}\right]}$$

$$= \frac{R_2 V_o}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}$$

Ex.3.2 Find numerical values of current components I_1, I_2 and I_3 and voltage V_1 .



Answer: $I_1 = 1.176\,\text{A}, I_2 = 0.589\text{A}, I_3 = 0.589\text{A}$ and $V_1 = 2.944\text{V}$

Mesh Analysis

A circuit consists with several closed loops, which are called meshes.

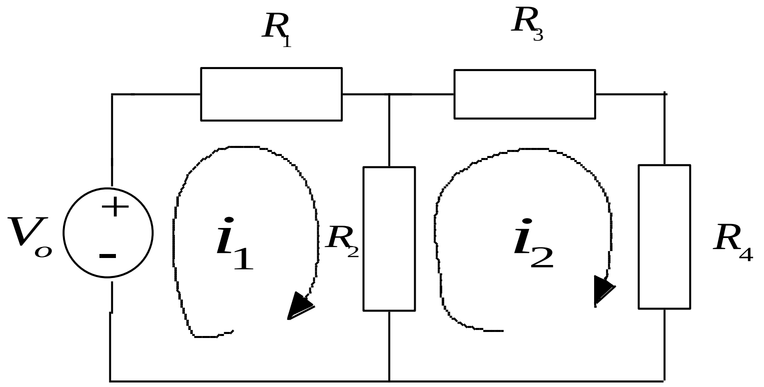
1. Define loop current for each mech. The Kirchhoff's Current Law is kept by definition of loop current.
2. Formulate voltage variation for each mesh based on the Kirchhoff's Voltage Law.
3. The number of equation is same as the number of mesh.
4. Reform equations to matrix form between the current and voltage.
5. Analyze current components by using inverse matrix transformation.

Ex.3.3 An example of mesh analysis

$$V_o = R_1 i_1 + R_2 (i_1 - i_2) = (R_1 + R_2) i_1 - R_2 i_2$$

$$0 = R_2 (i_2 - i_1) + (R_3 + R_4) i_2 = -R_2 i_1 + (R_2 + R_3 + R_4) i_2$$

$$\begin{bmatrix} V_o \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \equiv [Z] \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$



$$Z_{11} = R_1 + R_2, \quad Z_{12} = -R_2$$

$$Z_{21} = -R_2 \quad , \quad Z_{22} = R_2 + R_3 + R_4$$

Meanwhile

$$[Z]^{-1} \begin{bmatrix} V_o \\ 0 \end{bmatrix} = [Z]^{-1} [Z] \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$[Z]^{-1} \equiv [Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}}, \quad Y_{12} = -\frac{Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{21} = -\frac{Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}}, \quad Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_o \\ o \end{bmatrix}$$

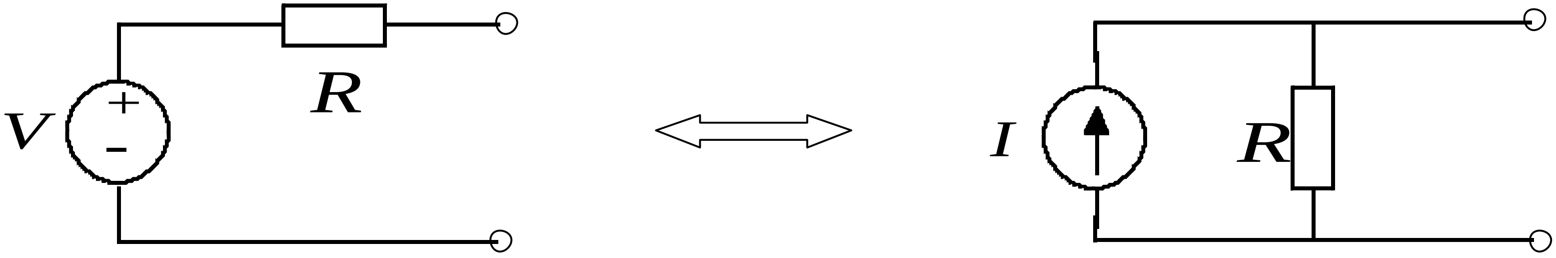
$$i_1 = Y_{11}V_o = \frac{(R_2 + R_3 + R_4)V_o}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}$$ (= I_1 in EX.3.1)

$$i_2 = Y_{21}V_o = \frac{R_2 V_o}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}$$ (= I_3 in EX.3.1)

Comparison between the node analysis and mesh analysis

- The node analysis is more fundamental as the circuit analysis.
- If you are familiar with calculation of the inverse matrix by help of the computer calculation, the mesh analysis would be easier forming equation and practice analysis .

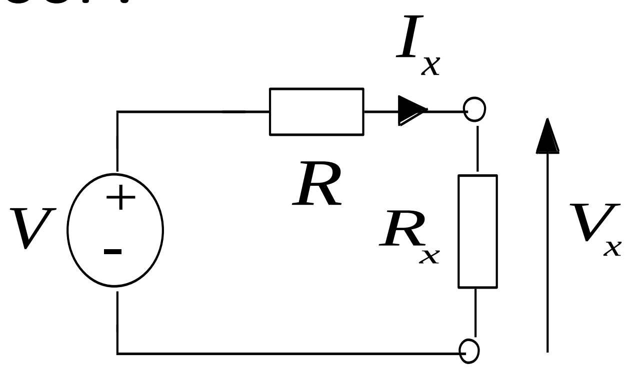
Source Transformation



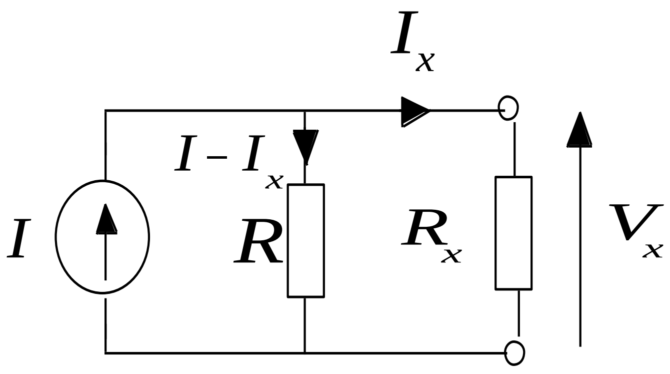
A voltage source with series resistance acts just like a current source with parallel resistance, where

$$V = R\,I, \quad I = \frac{V}{R}$$

Proof :



(a)



(b)

In Fig.(a)

$$I_x = \frac{V}{R + R_x}, \quad V_x = I_x R_x = \frac{V R_x}{R + R_x}$$

In Fig.(b)

$$V_x = I_x R_x = (I - I_x) R,$$

$$I_x (R_x + R) = I R, \quad I_x = \frac{I R}{R + R_x}$$

$$V_x = \frac{I R R_x}{R + R_x}$$

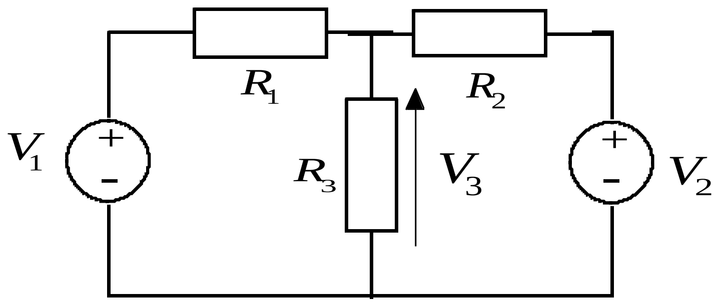
$$\therefore \quad V = IR$$

Important Remarks

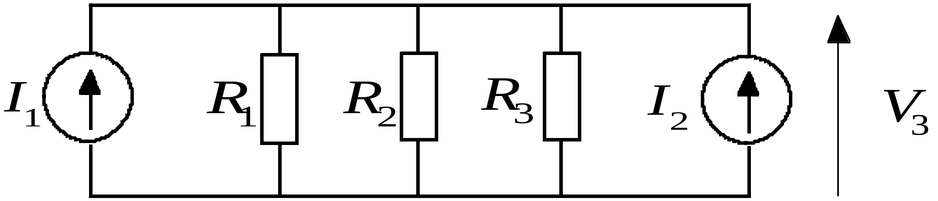
- An ideal voltage or current source can not be transformed.
- The voltage source to be transformed must have series resistance and the current source must have parallel resistance.

Ex.3.4 An example for application of the source transformation

Find V_3 in Fig.(a) .



(a)



(b)

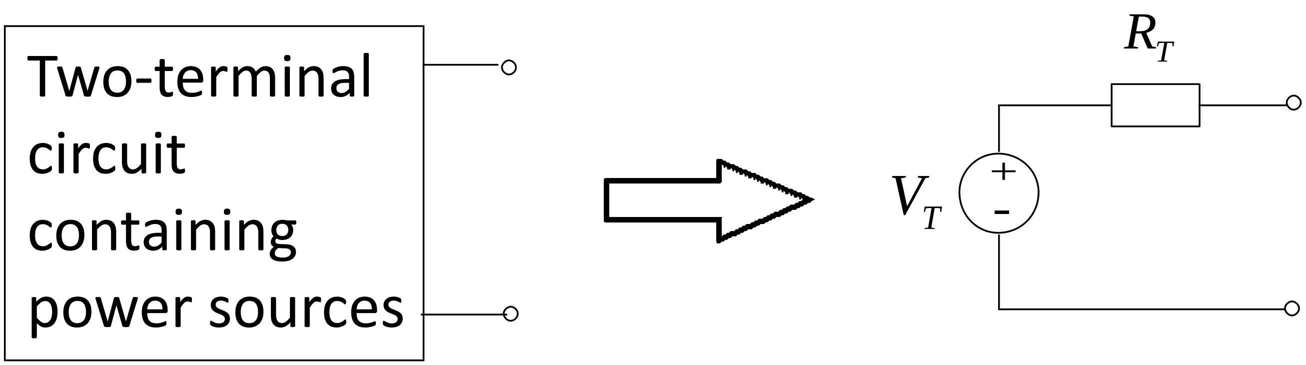
Answer: Fig.(a) is transformed to Fig.(b), where

$$I_1 = \frac{V_1}{R_1}, \quad I_2 = \frac{V_2}{R_2}$$

Then

$$V_3 = (I_1 + I_2)(R_1 // R_2 // R_3) = \frac{\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

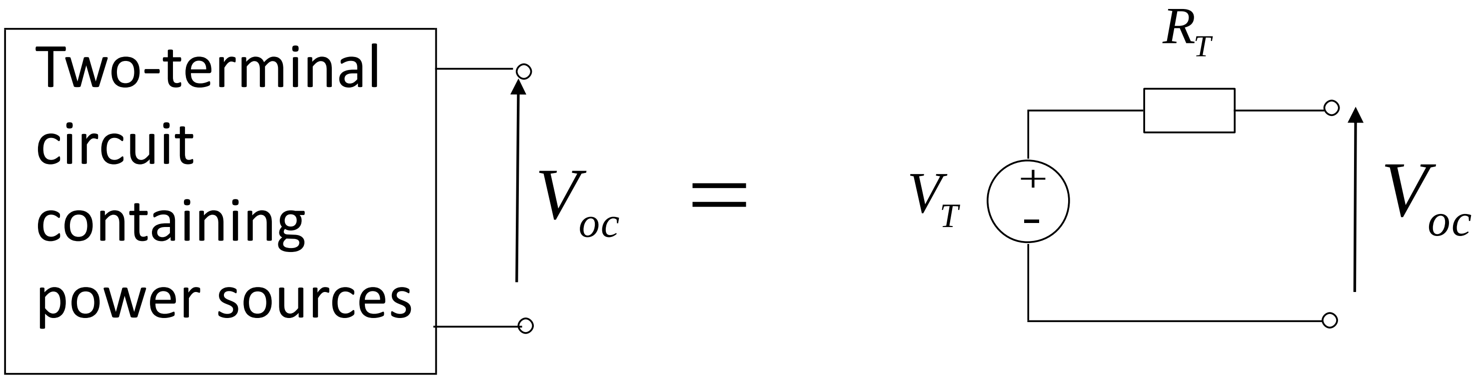
Thevenin's Theorem



Thevenin equivalent circuit

A two-terminal circuit containing power sources can be replaced by an equivalent circuit consisting of a voltage source V_T in series with a resister R_T .

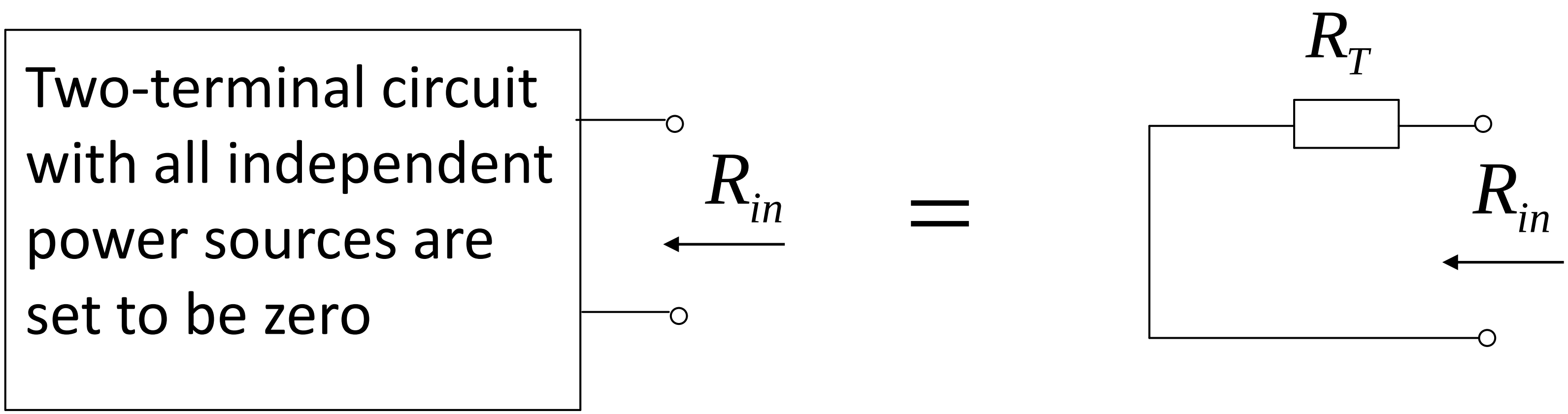
How to find V_T ?



V_{oc} is voltage for open terminal. Then

$$V_T = V_{oc}$$

How to find R_T ? : Method 1

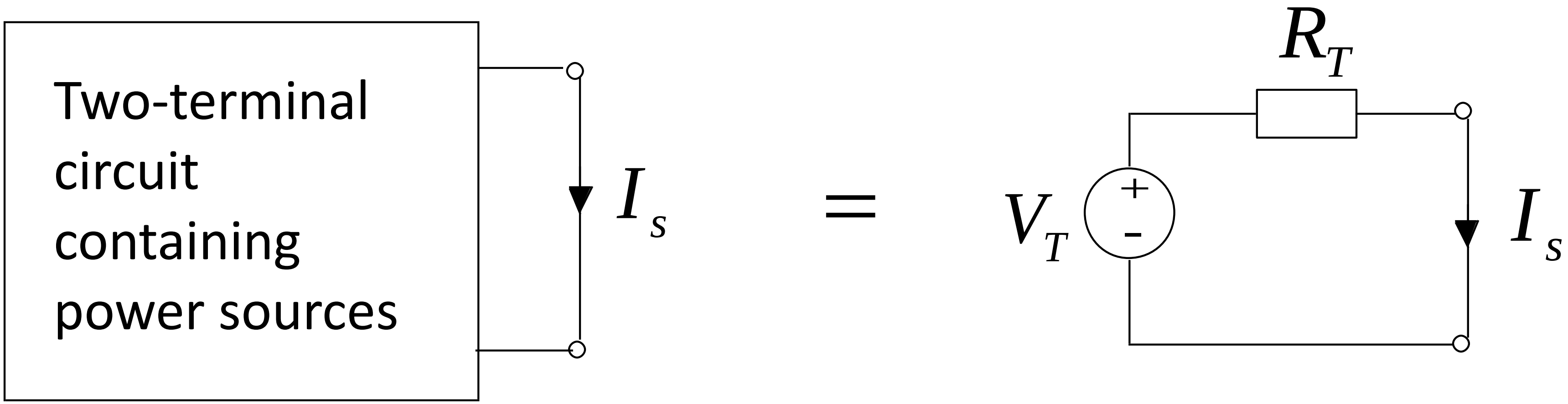


All independent power sources in the two-terminal circuit are set to be zero. Then, measure the internal impedance R_{in} from the terminals.

Therefore

$$R_T = R_{in}$$

How to find R_T ? : Method 2

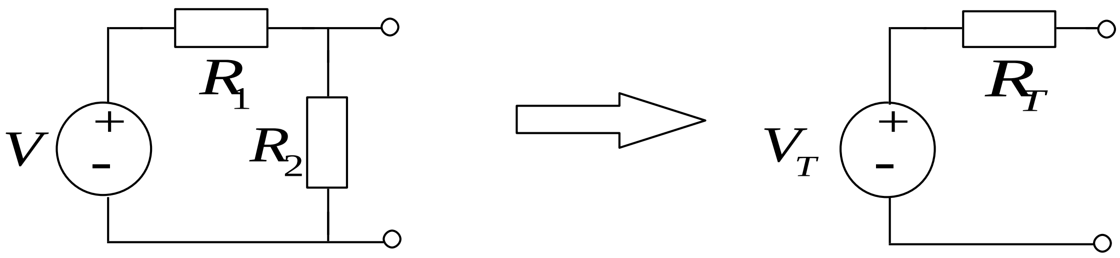


Short the terminals and measure the current I_s .
Then,

$$R_T = \frac{V_T}{I_S}$$

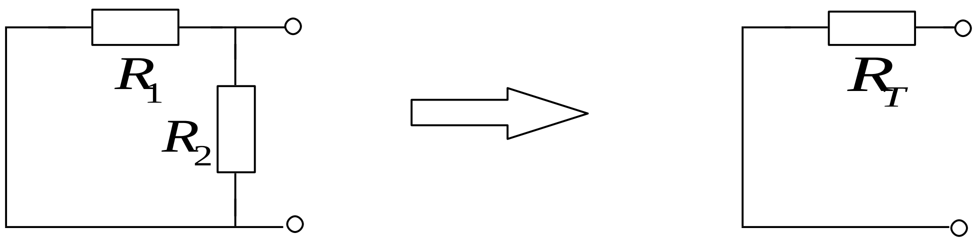
An example of Thevenin's theorem

Find V_T and R_T .



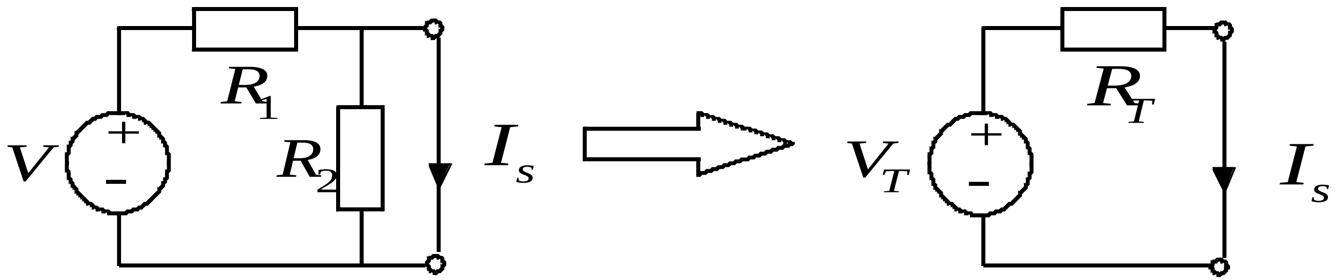
$$V_T = \frac{R_2}{R_1 + R_2} V$$

R_T : Method 1



$$R_T = R_1 // R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

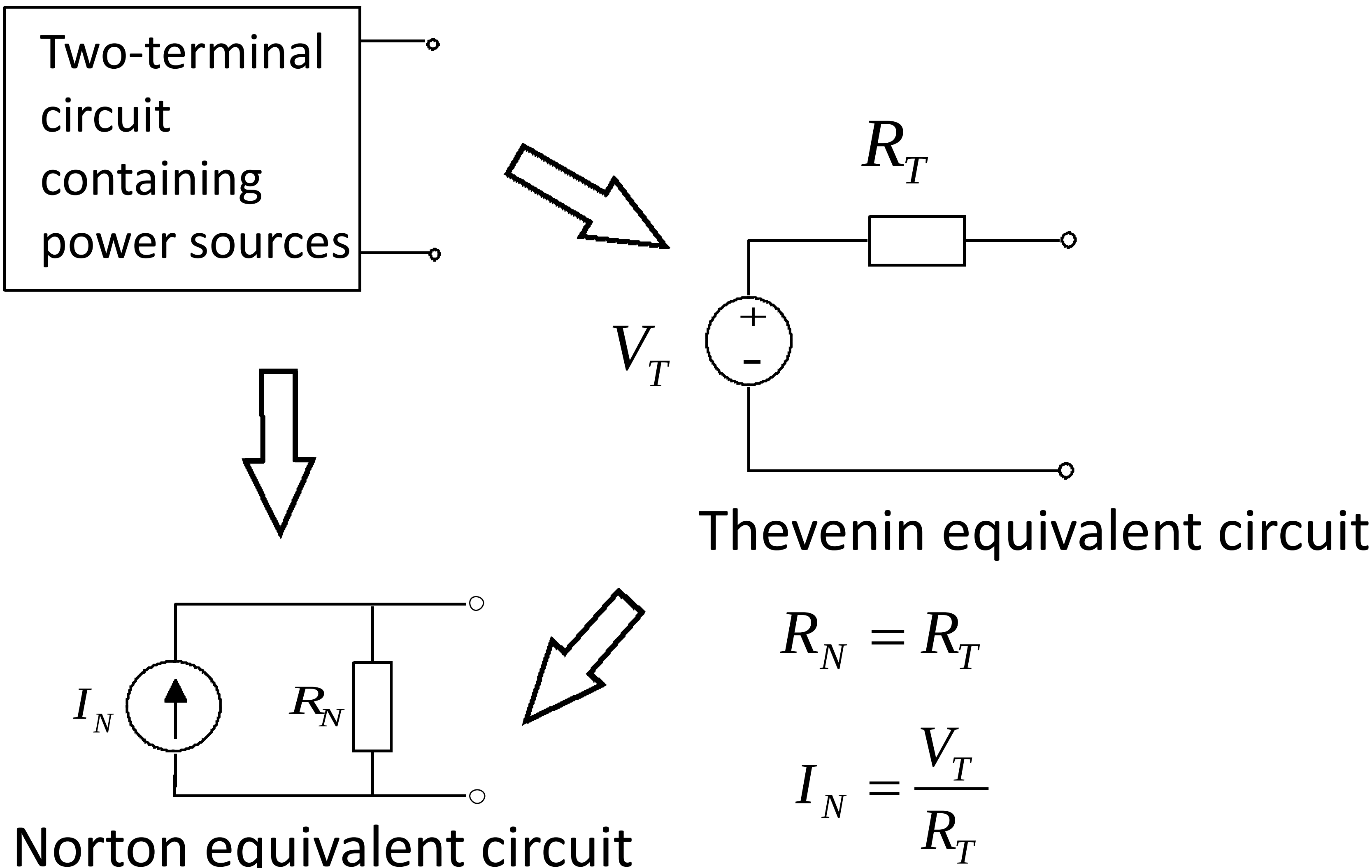
R_T : Method 2



$$I_s = \frac{V}{R_1} = \frac{V_T}{R_T}$$

$$R_T = \frac{V_T}{I_s} = \frac{\frac{R_2}{R_1 + R_2}V}{\frac{V}{R_1}} = \frac{R_1 R_2}{R_1 + R_2}$$

Norton's Theorem



$$R_N = R_T$$

$$I_N = \frac{V_T}{R_T}$$

How to find I_N and R_N in the Norton equivalent circuit?

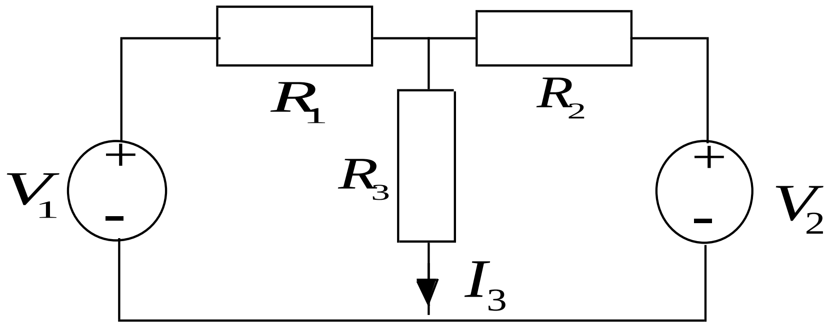
1. One method is to use source transformation from the Thevenin equivalent circuit.
2. Another method is directly determine I_N and R_N by using the shorten circuit and the analysis to find the internal resistance.

Superposition Principle

- When a circuit consists of multi-numbers of power sources, flowing currents in the circuit are obtained as summed values of current component for each source is turned-on and other sources are turned-off.
- Turned-off the voltage source : the source is shorten = voltage is zero and resistance is zero.
- Turned-off the current source : remove the source = current is zero and resistance is infinite.

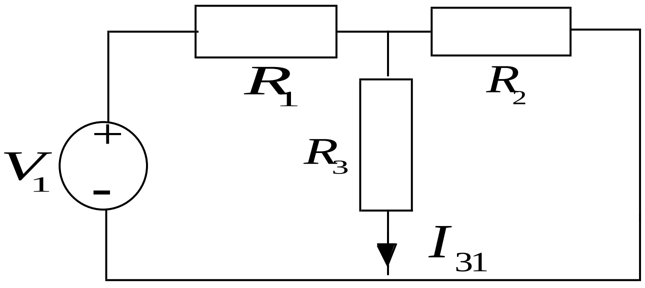
Ex.3.6 An application example of superposition principle

Find current I_3 by using the superposition principle.



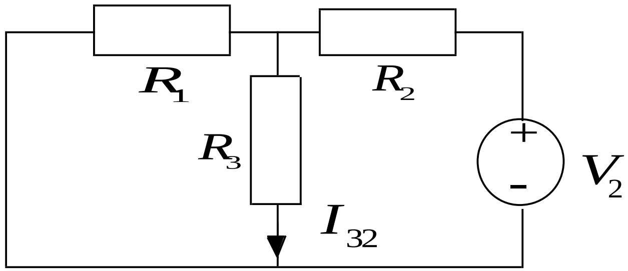
Answer :

First, we turn off the voltage source V_2 .



$$I_{31} = \frac{V_1}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}} \cdot \frac{R_2}{R_2 + R_3} = \frac{R_2 V_1}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

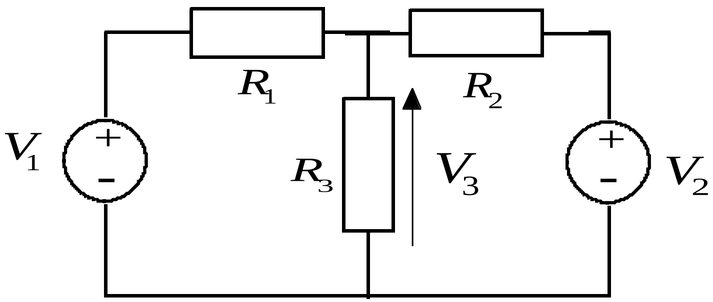
Second, we turn off the voltage source V_1 .



$$I_{32} = \frac{V_2}{R_2 + \frac{R_1 R_3}{R_1 + R_3}} \cdot \frac{R_1}{R_1 + R_3} = \frac{R_1 V_2}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

As the result, the current I_3 is obtained by summing the partial currents I_{31} and I_{32}.

$$I_3 = I_{31} + I_{32} = \frac{R_2 V_1 + R_1 V_2}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$



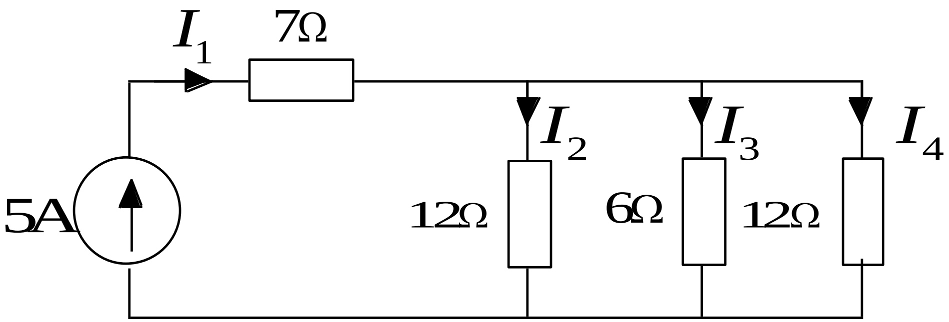
$$V_3 = I_3 R_3 = \frac{(R_2 V_1 + R_1 V_2) R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1} = \frac{\dfrac{V_1}{R_1} + \dfrac{V_2}{R_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

This equation is same with answer of Ex.3.4.

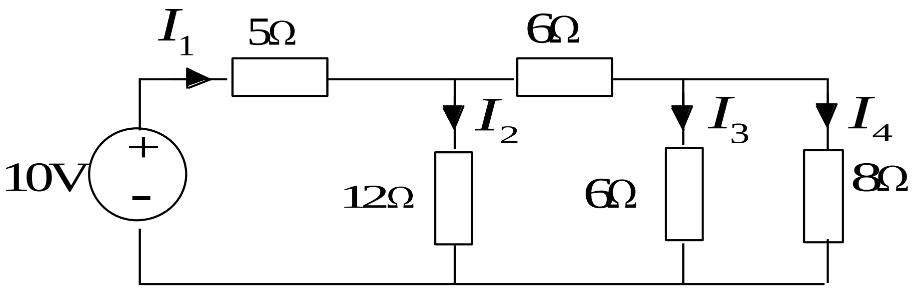
Problems

Calculate the current in each branch and the voltage across each resistor in the following circuits.

3.1



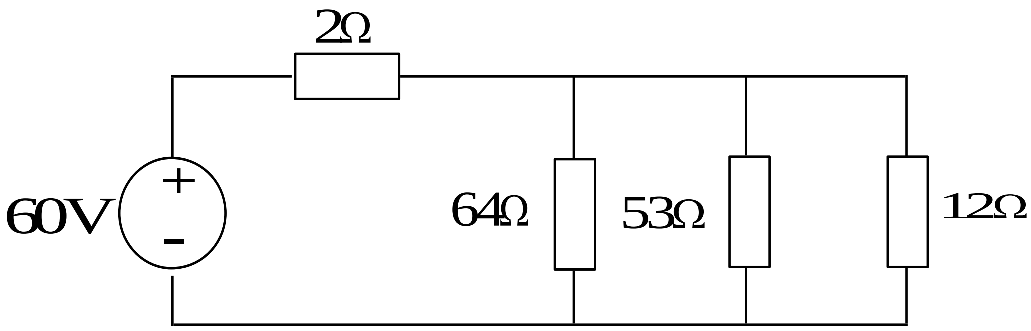
3.2



3.3 A resistive network consists of 2 resistors in parallel followed by a 1000 Ω series resistor. The two parallel resistors have values of 330 Ω and 500 Ω respectively. If 0.2 A flows in the 330 Ω resistor find the total power dissipated in the resistive network.

3.4 Obtain the total power supplied by the 60 V source and the power absorbed in each resistor in Fig.3.4

Fig.3.4



3.5. Find *I* in Fig.3.5.

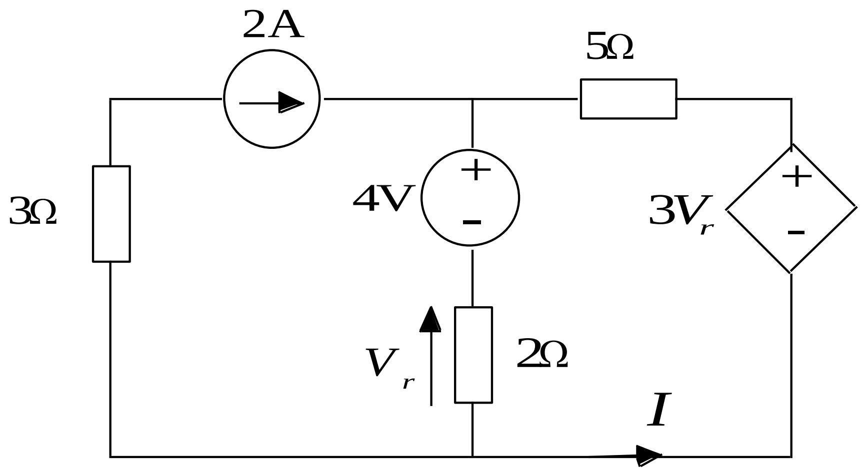


Fig.3.5

3.6 Find the Thevenin and Norton Equivalent circuits for Fig.3.6. What are values in the circuits, if R_1=100 Ω, R_2 = 50 Ω, R_3 = 15 Ω, R_4 =16 Ω, *R5* = 200 Ω and *I* = 0.1A.

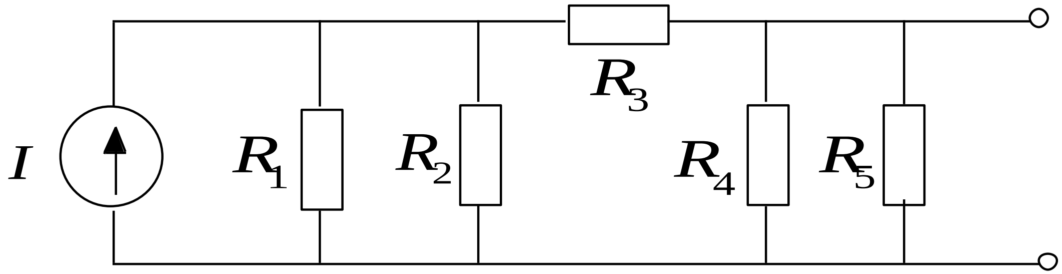
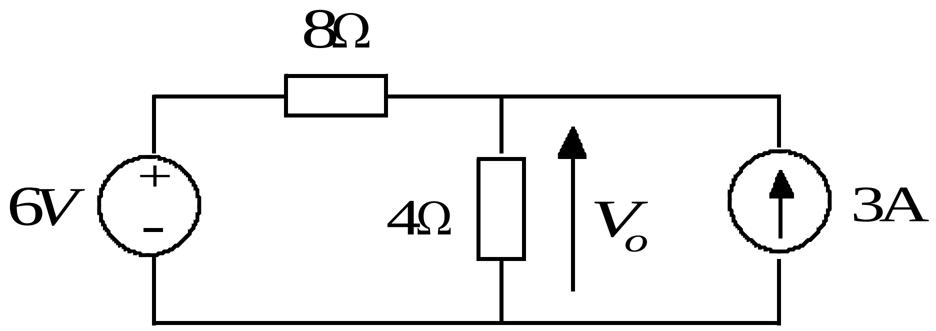


Fig.3.6

3.7 Use the superposition principle to find V_o in the circuit shown in Fig. 3.7.

Fig.3.7



3.8 In Fig.3.8, find the voltage V_1 when all resistors have the value 10 Ω, V = 10V and I = 1A using

a) nodal analysis,

b) superposition.

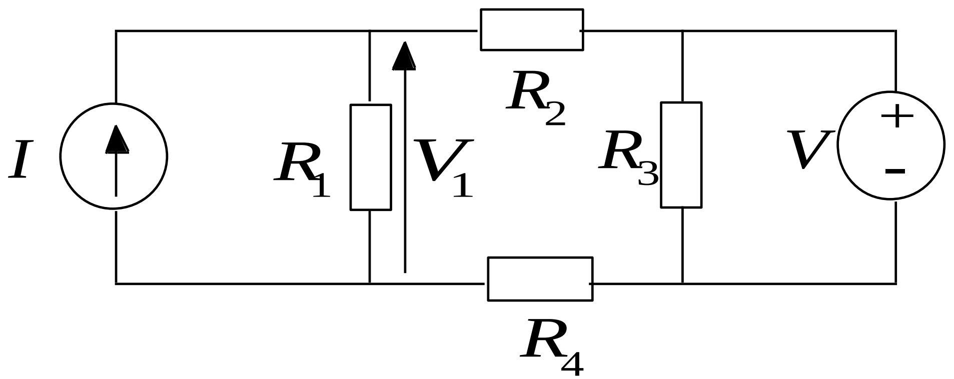
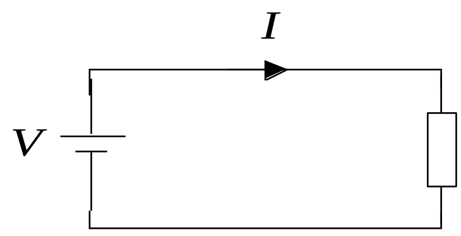


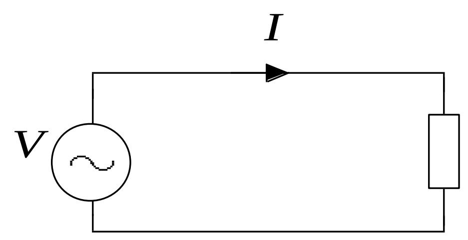
Fig.3.8

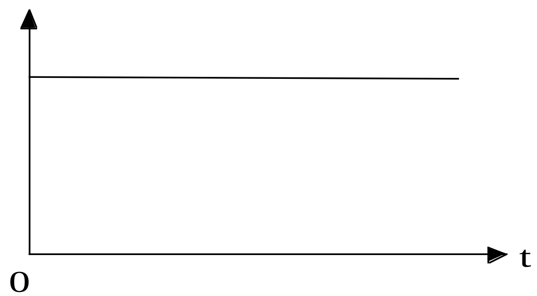
Chapter 4

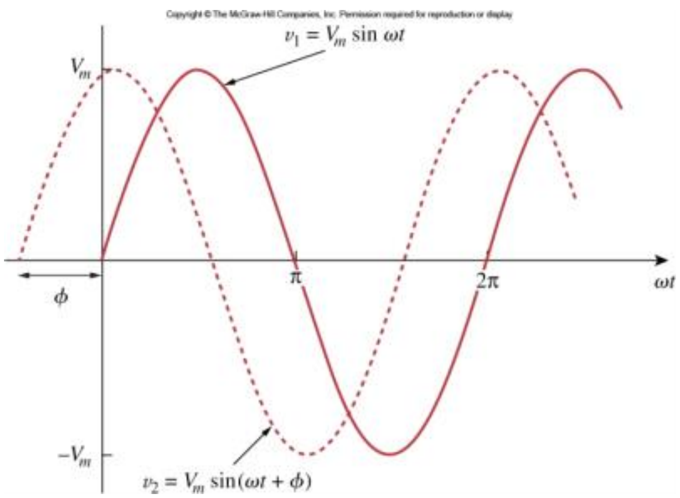
Alternating Current
and Passive Circuit Elements

Sec.4.1 Alternating Current (AC)









DC: Direct Current

AC : Alternating Current

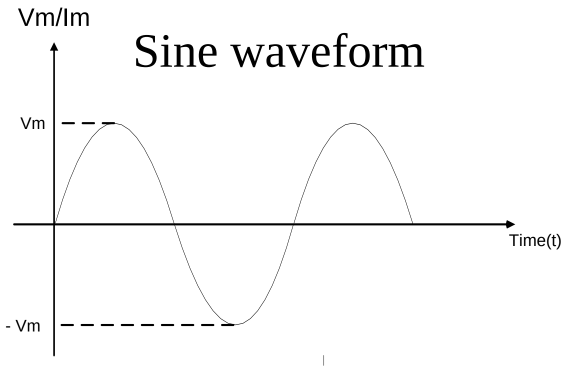
f : frequency [Hz]

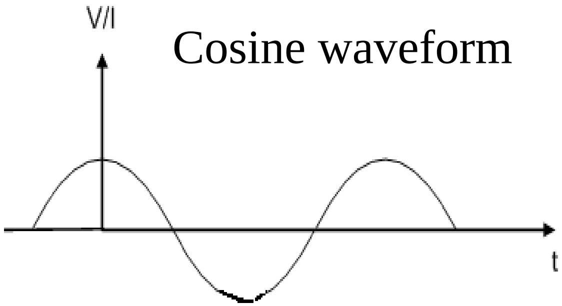
$\omega = 2\pi f$: angular frequency [rad/s]

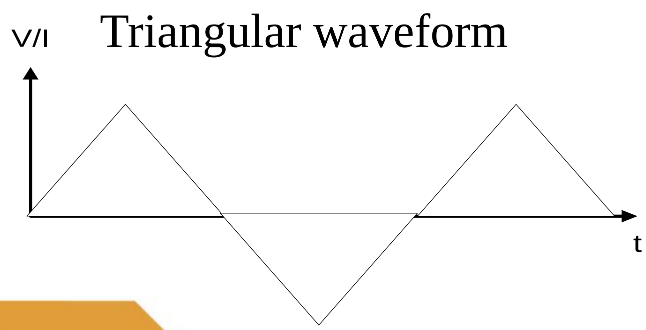
ϕ : initial phase [rad] or [deg]

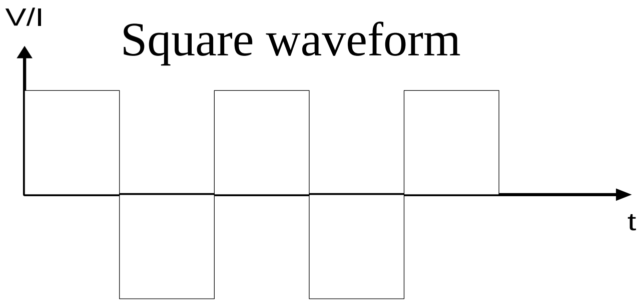
Sinusoidal Waveform

- **Alternating signal** is a signal that **varies with respect to time**.
- This voltage and current have **positive** and **negative** value.
- Sinus and cosines waveform is an **important** because it is a basic waveform for electric supply in **transmission system**.

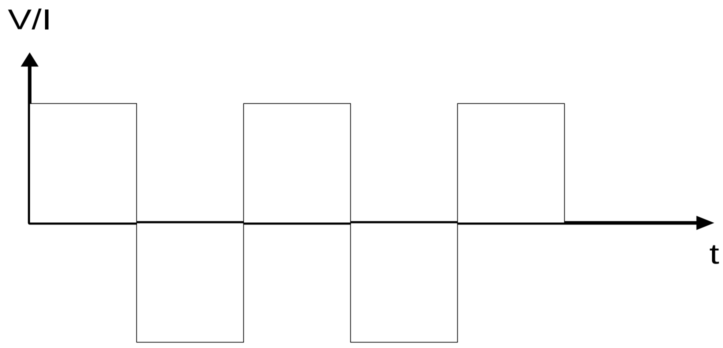




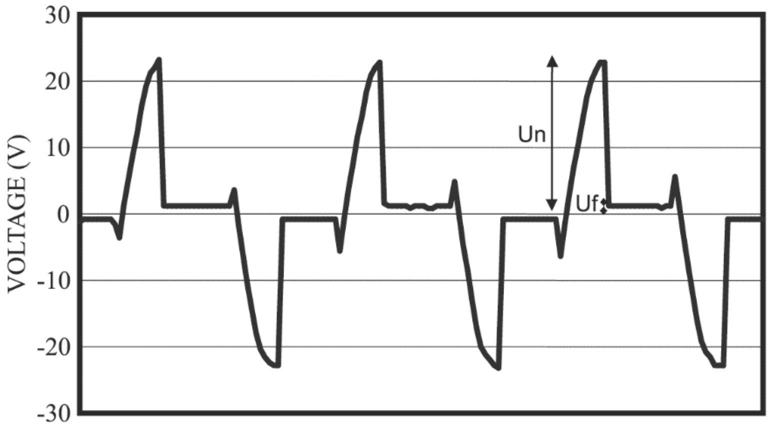




- There are **four types** of alternating waveform:
  - Symmetry
  - Non Symmetry
  - Periodic
  - Non Periodic



Periodic and Symmetry

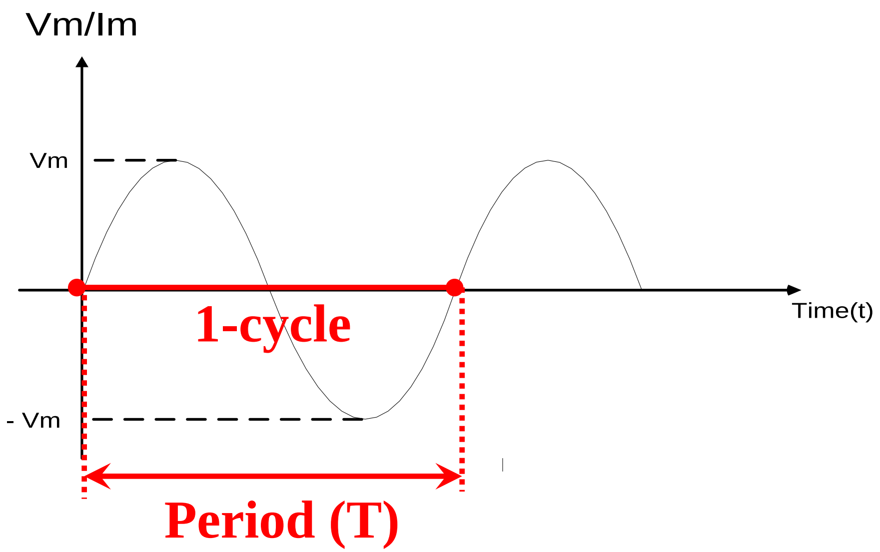


Periodic and Non-symmetry

Specification in sinusoidal waveform

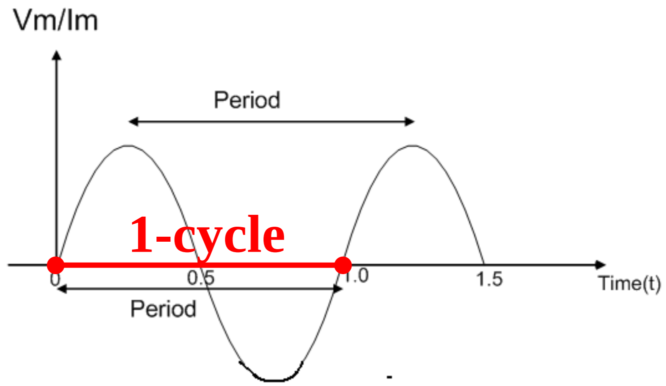
- Voltage and current value is represented by vertical axis and time represent by horizontal axis.

- **Period (T)**: Time taken to complete 1-cycle (Unit: second, ..).

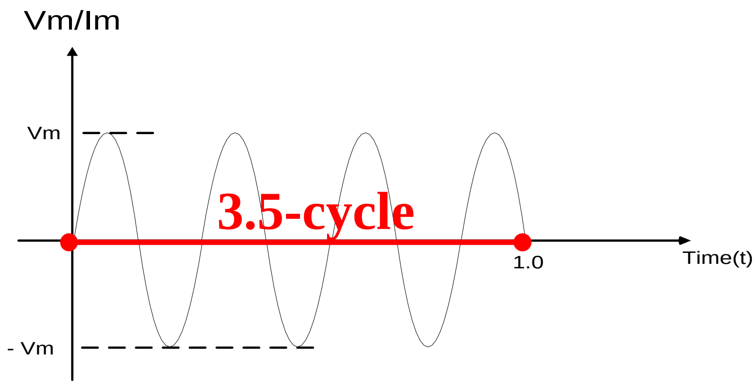


- **Frequency:** Number of cycles in 1 second (Unit: Hz).

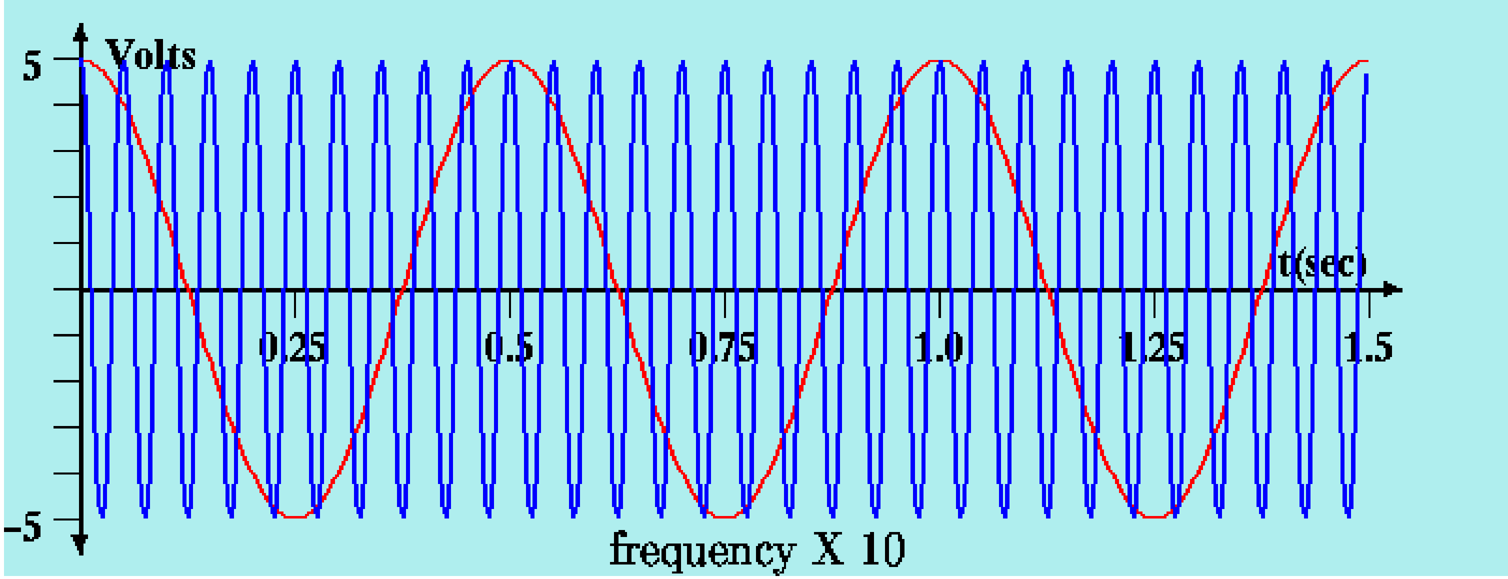
$$f = \frac{1}{T}$$



Signal with lower frequency



Signal with higher frequency

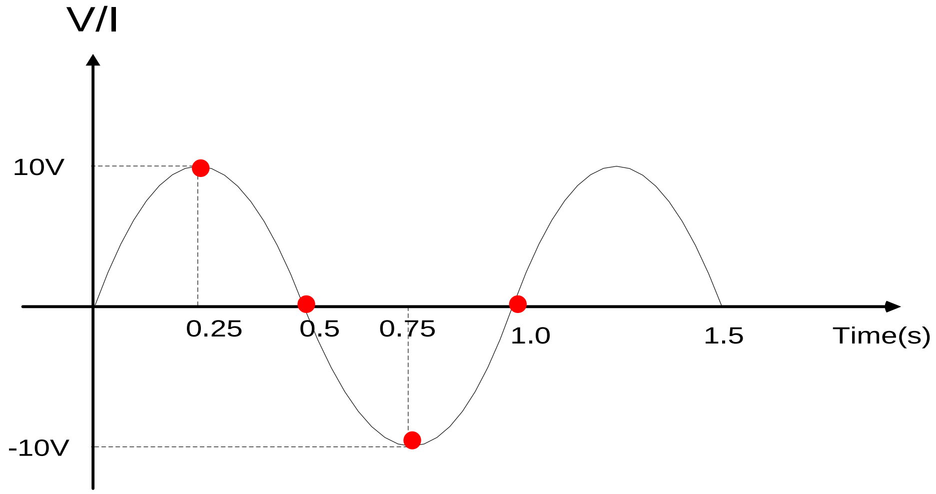


- The red wave shown above is given by $V = 5 \cos(4\pi t)$.
- Determine the frequency for both waveforms.

RED → 2 Hz

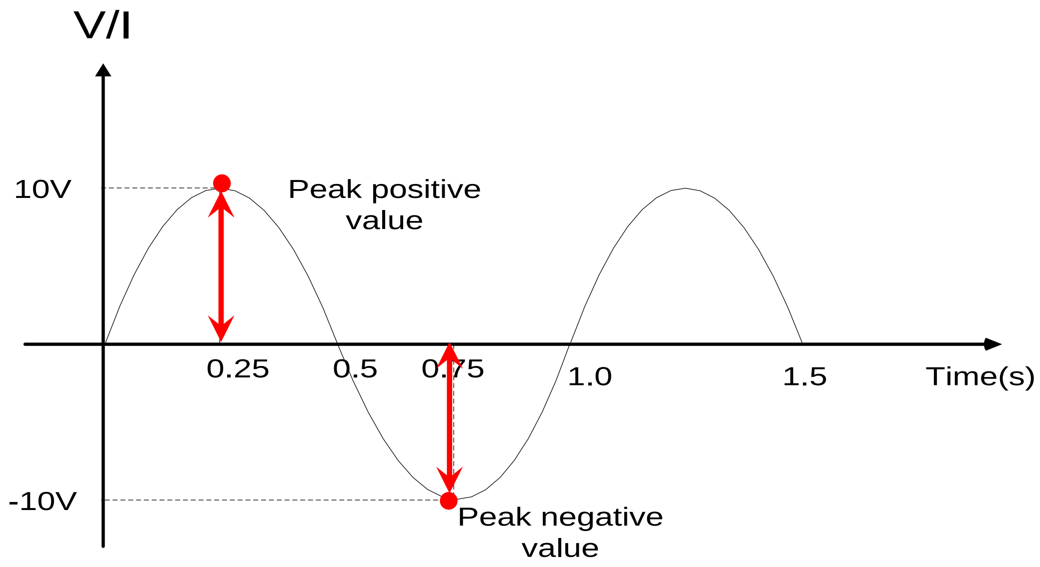
BLUE → 20 Hz

- **Instantaneous value**: Magnitude value of waveform at one specific time.
- Symbol for instantaneous value of voltage is v(t) and current is i(t).

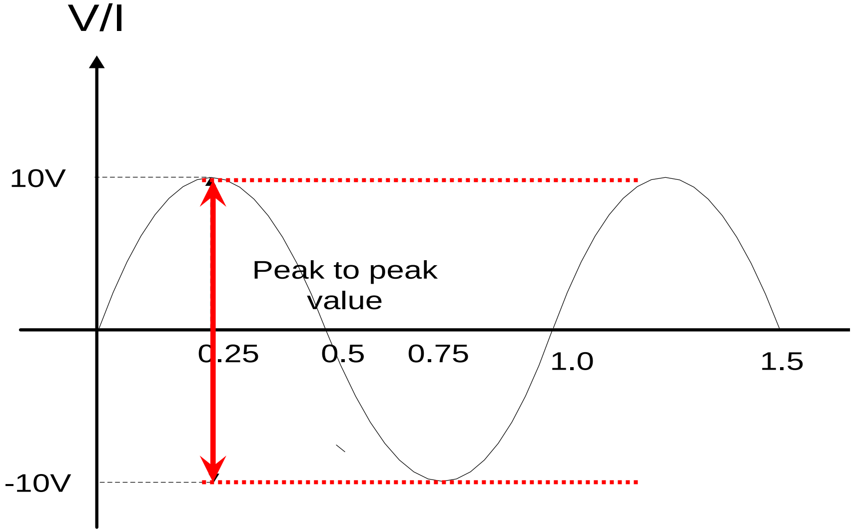


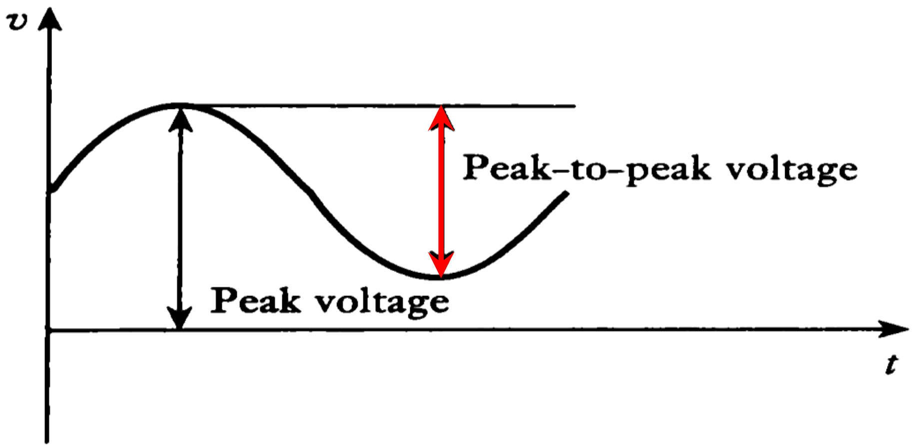
$$v(0.25) = 10V$$
$$v(0.5) = 0V$$
$$v(0.75) = -10V$$
$$v(1.0) = 0V$$

- **Peak value (V_p)**: Maximum value measured from reference axis of a waveform.
- For one complete cycle, there are two peak value that is positive peak value and negative peak value.



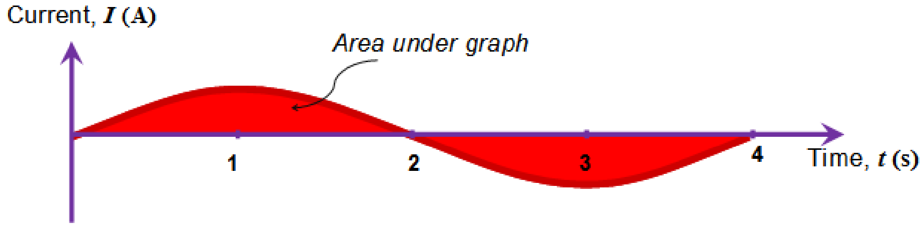
- **Peak to peak (V_{p-p}):** value is a maximum amplitude value of waveform that is calculated from negative peak value to positive peak value.





- **Average value:** Average value for all instantaneous value in half or one complete waveform cycle.
- It can be calculated in two ways:

1. Calculate the area under the graph



2. Use integral method

$$Average = \frac{1}{T}\int_0^T v(t)dt$$

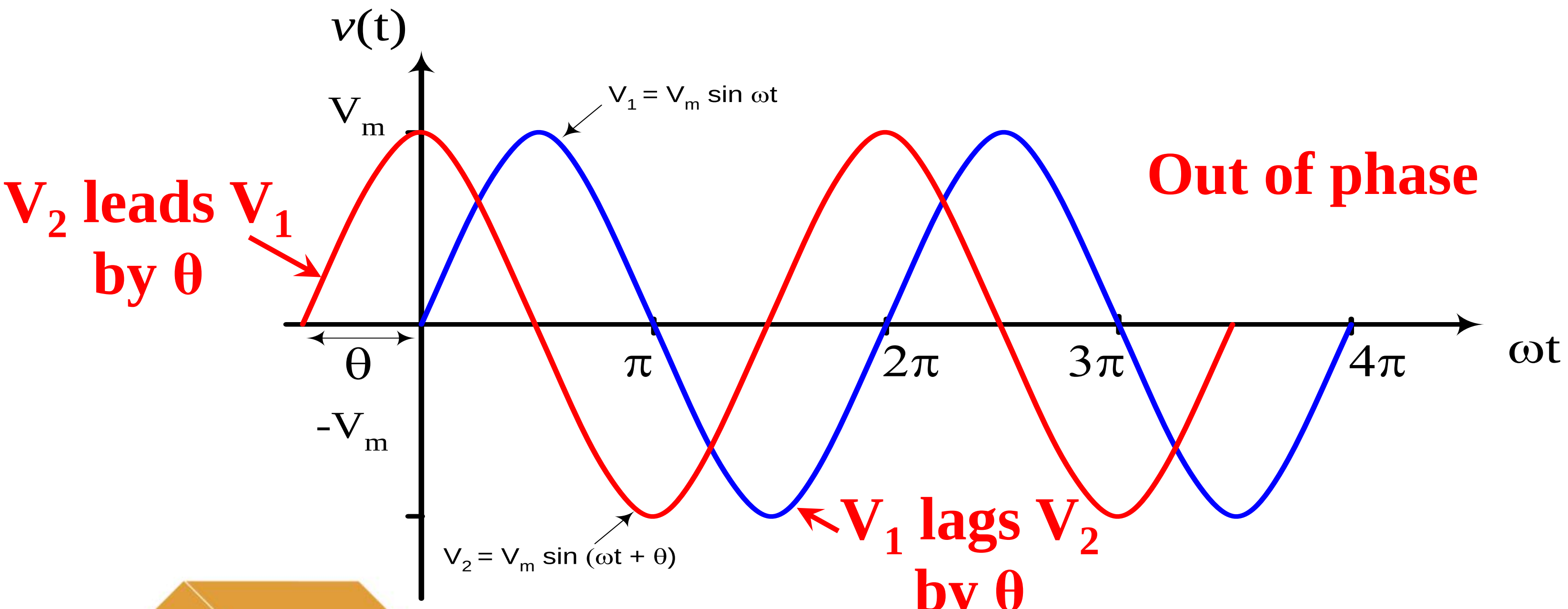
- **Effective value (RMS value)**: Specifying the amount of sine wave of voltage or current by relating it into dc voltage and current that will produce the same heat effect.
- Can be computed as follows:

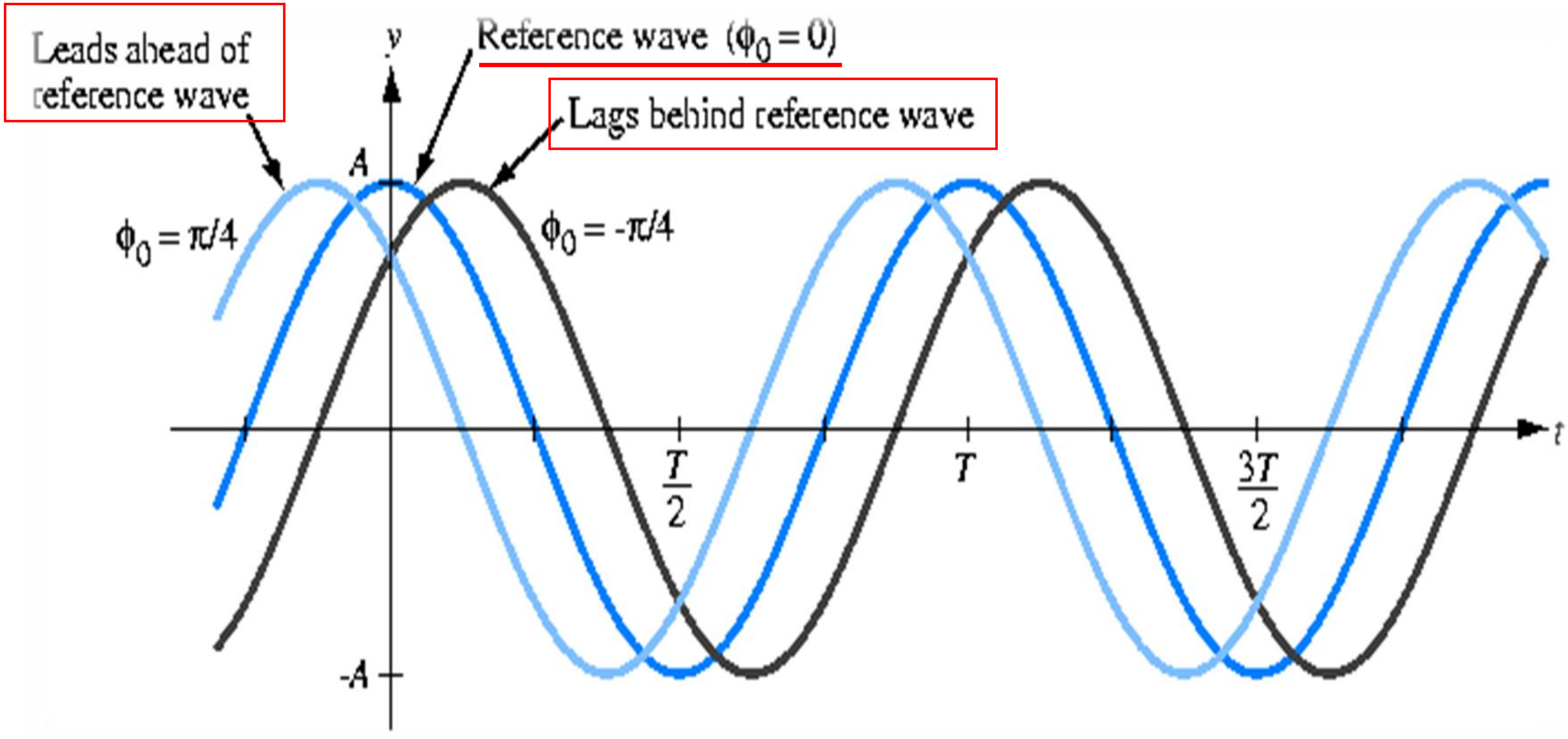
$$V_{rms} = \frac{v_m}{\sqrt{2}} = 0.7071 v_m$$

$$I_{rms} = \frac{i_m}{\sqrt{2}} = 0.7071 i_m$$

where I_m & V_m are peak values

- **Phase angle (θ or Φ)** is a shifted angle waveform from the reference origin (Unit: ° or radian).
- Two waveforms are called in phase if both have a same phase degree or different phase is zero
- Two waveforms are called out of phase if both have a different phase or different phase is not zero.




Leads ahead of reference wave
y
Reference wave ($\phi_0 = 0$)
Lags behind reference wave
A
$\phi_0 = \pi/4$
$\phi_0 = -\pi/4$
-A
t
$\frac{T}{2}$
T
$\frac{3T}{2}$

Expression in sinusoidal waveform

- A sinusoid can be expressed in either ***sine*** or ***cosine*** form.
- When comparing two sinusoids, it is recommended to express both as either sine or cosine with positive amplitudes.

- We can transform a sinusoid from sine to cosine form or vice versa using this relationship:

$$- \sin \omega t = \sin (\omega t \pm 180^o)$$

$$- \cos \omega t = \cos (\omega t \pm 180^o)$$

$$\cos \omega t = \sin (\omega t + 90^o)$$

$$\sin \omega t = \cos (\omega t - 90^o)$$

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

Trigonometric

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$2 \sin x \cos y = \sin(x + y) + \sin(x - y)$$

$$2 \sin x \sin y = -\cos(x + y) + \cos(x - y)$$

$$2 \cos x \cos y = \cos(x + y) + \cos(x - y)$$

Logarithm

$$a^x = e^{x \ln a}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Hiperbolic

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$= 2\cosh^2 x - 1$$

$$= 1 + 2\sinh^2 x$$

$$\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$$

Inverse Hiperbolic

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right), \quad x \geq 1$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right), \quad -1 < x < 1$$

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k] = 0, \quad k$ constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
$\frac{d}{dx}[\ln \lvert x\rvert] = \frac{1}{x}$	$\int \frac{dx}{x} = \ln \lvert x\rvert + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\operatorname{cosec} x] = -\operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}[e^x] = e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x] = \sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x] = \cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$	$\int \operatorname{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x] = -\operatorname{cosech}^2 x$	$\int \operatorname{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$	$\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$
$\frac{d}{dx}[\operatorname{cosech} x] = -\operatorname{cosech} x \coth x$	$\int \operatorname{cosech} x \coth x\,dx = -\operatorname{cosech} x + C$

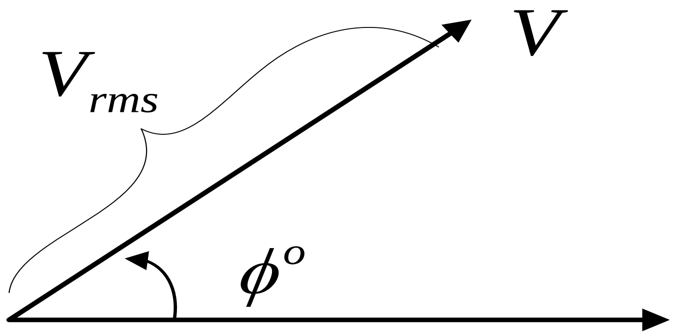
Phasor Diagram

- Sinusoids are easily expressed in terms of phasors.
- Phasor is a complex number that represent magnitude and phase angle for a sine wave.

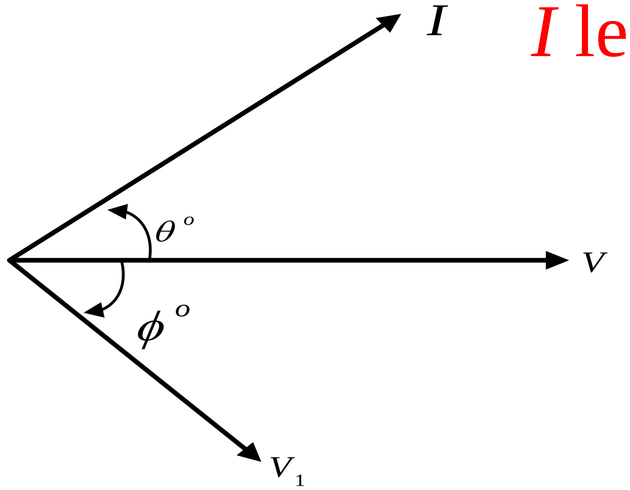
RMS value

$$v(t) = V_m \sin(\omega t + \theta) \quad \Rightarrow \quad \bar{V} = \frac{V_m}{\sqrt{2}} \angle \theta$$

- Phasor diagram is a vector line that represent magnitude and phase angle of a sine wave.



Phasor Diagram Expression



I leads V by θ^{o}

I leads V_1 by $(\phi^{o} + \theta^{o})$

V leads V_1 by ϕ^{o}

V lags I by θ^{o}

V_1 lags V by ϕ^{o}

V_1 lags I by $(\phi^{o} + \theta^{o})$

Operation in Phasor

- Adding phasors is equivalent to adding the corresponding time function for each phasor.
- One way is to dissolve the phasor to complex numbers and then adding then up according to the real & imaginary values.

$$\overline{V_1} = V_1\angle\theta \qquad \Rightarrow \qquad \overline{V_1} = V_1 cos\theta + jV_1 sin\theta$$

$$\overline{V_2} = V_2\angle\phi \qquad \qquad \overline{V_2} = V_2 cos\phi + jV_2 sin\phi$$

$$\overline{V_1} + \overline{V_2} = V_1 cos\theta + jV_1 sin\theta + V_2 cos\phi + jV_2 sin\phi$$

$$\overline{V_1} + \overline{V_2} = \underbrace{(V_1 cos\theta + V_2 cos\phi)}_{\text{Real part}} + j\underbrace{(V_1 sin\theta + V_2 sin\phi)}_{\text{Imaginary part}}$$

$$z = x + jy = r\angle\phi, \qquad z_1 = x_1 + jy_1 = r_1\angle\phi_1$$
$$z_2 = x_2 + jy_2 = r_2\angle\phi_2$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle \phi_1 - \phi_2$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r}\angle -\phi$$

Square Root:

$$\sqrt{z} = \sqrt{r}\angle \phi/2$$

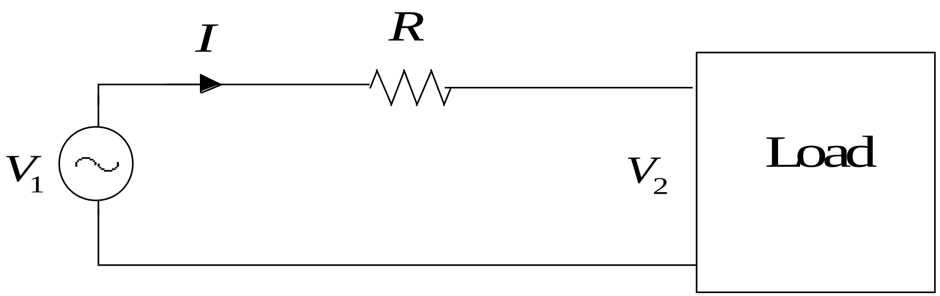
Complex Conjugate:

$$z^* = x - jy = r\angle -\phi = re^{-j\phi}$$

Sec.4.1.1. Why we study AC? (1st reason)

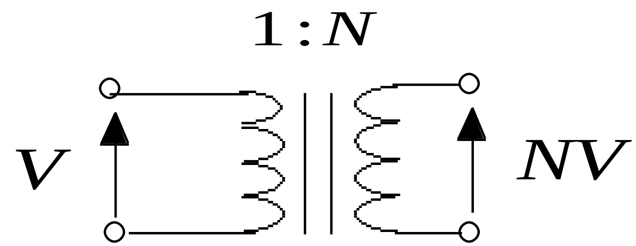
We need to DC power source in the electronics equipment, isn't it ? Why there still exist AC lines?

- The reason is to reduce power loss during power transmission from the power station to the consuming areas with long distance.



- When the power station send the electricity with voltage V_1 with current I , the received voltage is $V_2 = V_1 - RI$ and available power is $P_2 = V_2 I = V_1 I - I^2 R$, because transmitting wire has a resistance R .
- To get same value of P_2 operation with the higher value of V_2 and the smaller value of I is the more efficient.

Voltage conversion

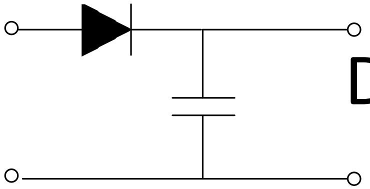


- Voltage conversion of AC is very easy by using the voltage transformer for both up and down conversion without energy loss.
- However, voltage conversion of DC is difficult. Voltage down conversion by a voltage divider losses energy. Voltage up conversion of DC is impossible in the principle.
- The DC-DC voltage converter is exist but is using the method of DC - AC - DC conversion.

Rectification

- AC to DC conversion is very easy by using a rectifying circuit.
- DC to AC conversion is rather difficult, because we require the switching or the oscillation circuit.

AC input



DC output

Rectifying circuit made of a diode and a capacitor

Why we study AC ? (2nd reason)

- The information or the signal is represented as change of certain physical value in the electronics.
- Any temporally (timely) variable function can be represented with linear combination of temporally varying periodic functions.

$$A(t) = \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega$$

where, $A(t)$ is a temporally variable function and $F(\omega)$ is a complex function for periodically varying with angular frequency ω .

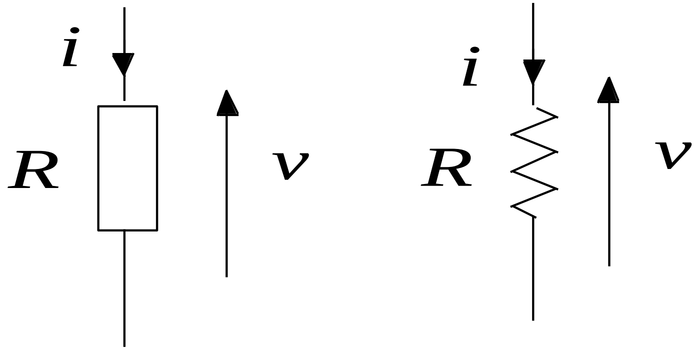
- Therefore, study of alternative current (AC) circuit is an important basis to understand the electronics.

Passive Circuit Elements

- In electric or electronic circuit, there are two types of circuit elements.
- One is called “passive circuit elements “, which never increase the electronic energy by themselves.

  Resistor, Inductor and Capacitor are typical passive elements.
- Others are called “active circuit elements” or “functional devices”, in which energy in the circuit or the “signal” for communication is generated and amplified.
- In this course of “Fundamental Electric Engineering”, we treat “resistor “, “inductor” and “capacitor” as the passive element.
- We also treat the operational amplifier as the active element in this course.

Resistor

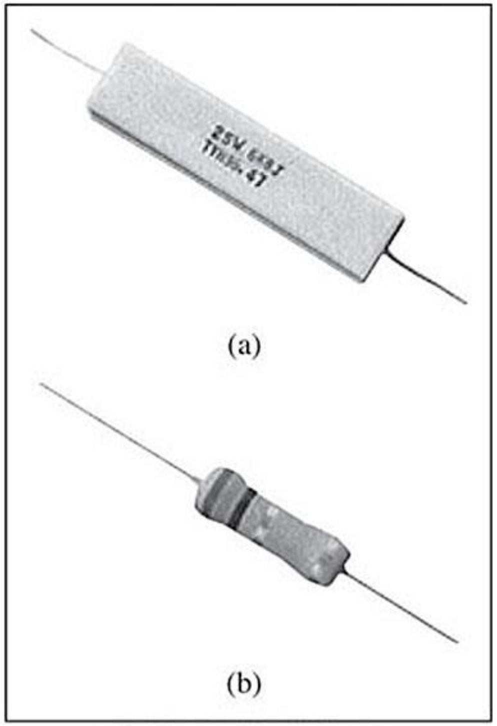


$v = Ri$ Ohm's Law

R : Resistance [Ω]

The consuming power is

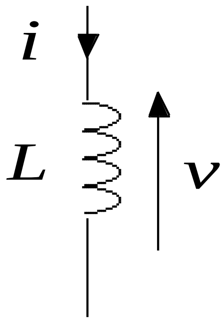
$$P = vi = \frac{v^2}{R} = Ri^2$$



Courtesy of Tech America

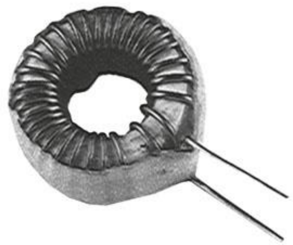
Note: "consuming" means loss of potential energy, which are changed other energy such as thermal energy know as the Joule heat.

Inductor



$$v = L\frac{di}{dt} \qquad (1)$$

(a)



(b)

(c)

Courtesy of Tech America

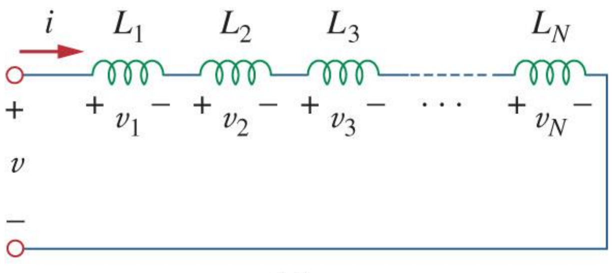
L: inductance [H]

The stored magnetic energy is

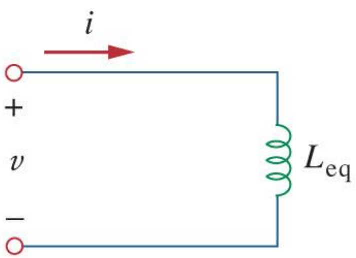
$$w = \frac{1}{2}Li^2$$

Note : “stored energy” means a potential energy, which is held inside or around the circuit element and is not consumed.

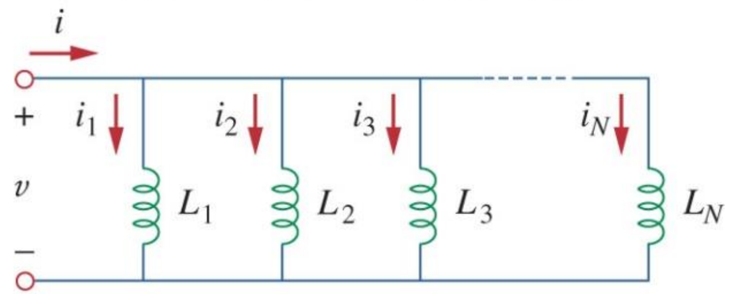
Series and parallel connections of *L*



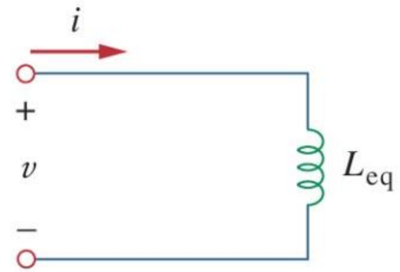
(a)



(b)



(a)



(b)

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

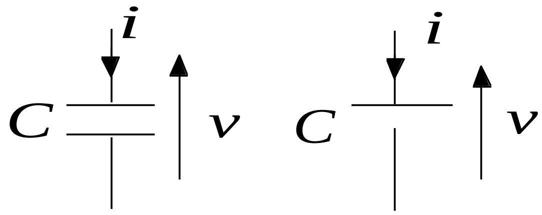
$$= \left(\sum_{k=1}^{N} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

$$i = \left(\sum_{k=1}^{N}\frac{1}{L_k}\right)\int_{t_0}^{t} v\,dt + \sum_{k=1}^{N} i_k(t_0) = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt + i(t_0)$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

Capacitor



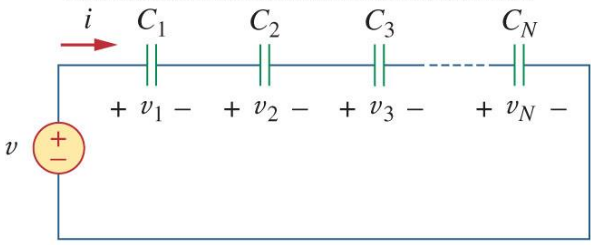
$$C \equiv \frac{Q}{v} = \frac{1}{v}\left[\int_{o}^{t} i(\tau)\,d\tau + Q(0)\right]$$: Capacitance [F]

Q : amount of the charge at the electrode [C]

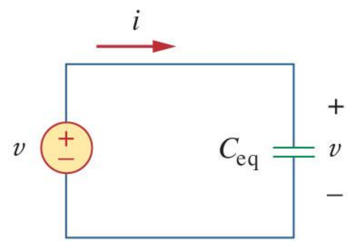
$$i = C\frac{dv}{dt} \qquad (2)$$

The stored electric energy is $w = \frac{1}{2}Cv^2$

Series and parallel connections of *C*



(a)

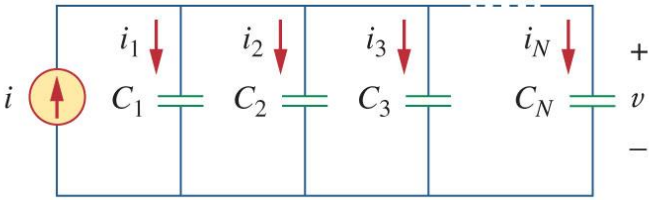


(b)

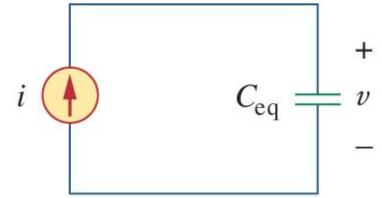
$$v = \sum_{k=1}^{N}\left[\frac{1}{C_N}\int_{t_o}^{t} i(\tau)\,d\tau + v_N(t_o)\right]$$

$$\equiv \frac{1}{C_{eq}}\int_{t_o}^{t} i(\tau)\,d\tau + v(t_o)$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$



(a)



(b)

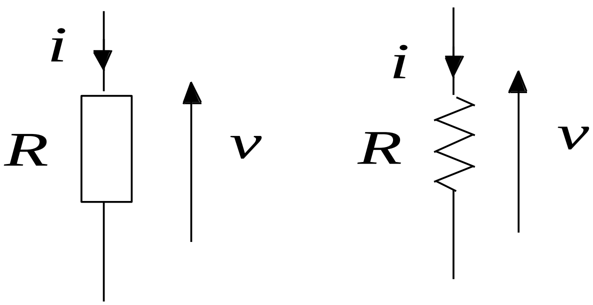
$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$

$$= \left(\sum_{k=1}^{N} C_k\right)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N}$$

Current flow and Phase

Current flow in a resistor



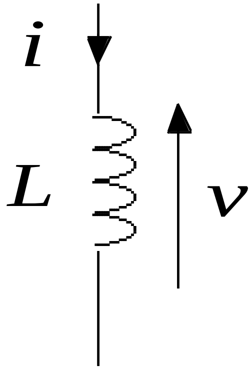
$$v(t) = v_m \cos \omega t$$

$$i(t) = \frac{v}{R} = \frac{v_m}{R} \cos \omega t$$

- Voltage and current have same angular frequency.
- Voltage and current are in the same phase.

There is no phase difference between the applied voltage and flowing current.

Current flow in an inductor



$$v = v_m \cos \omega t$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v dt + i(t_0) = \frac{v_m}{L}\int_{t_0}^{t} \cos \omega t \, dt + i(t_0)$$

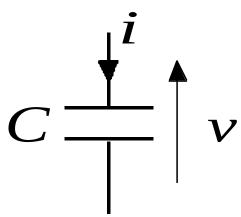
$$= \frac{v_m}{\omega L}\left[\sin \omega t\right]_{t_0}^{t} + i(t_0) = \frac{v_m}{\omega L}\left(\sin \omega t - \sin \omega t_o\right) + i(t_0)$$

AC component of this current is

$$i(t) = \frac{v_m}{\omega L}\sin \omega t = \frac{v_m}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

- Voltage and current have same angular frequency.
- The higher frequency current the less flows an inductor.
- The phase of the flowing current in an inductor is $\pi / 2 = 90^\circ$ delayed from that of the voltage.

Current flow in a capacitor



$$v = v_m \cos \omega t$$

$$i = C\frac{dv}{dt} = -\omega C v_m \sin \omega t = \omega C v_m \cos\left(\omega t + \frac{\pi}{2}\right)$$

- Voltage and current have same angular frequency.
- The higher frequency current the more flows an inductor.
- The phase of the flowing current in a capacitor $\pi / 2 = 90°$ proceeds from that of the voltage.

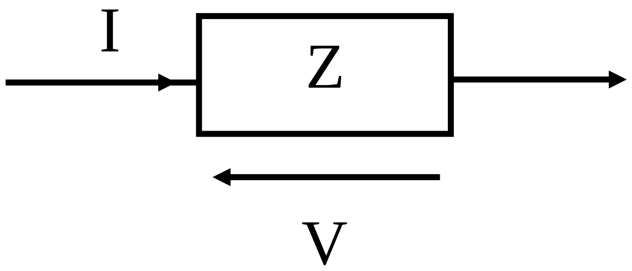
AC Impedances

Impedance Z

- It is a ratio between voltage and current.

$$Z = \frac{V}{I}$$

- Also known as complex resistance
- Units: Ohm (Ω)

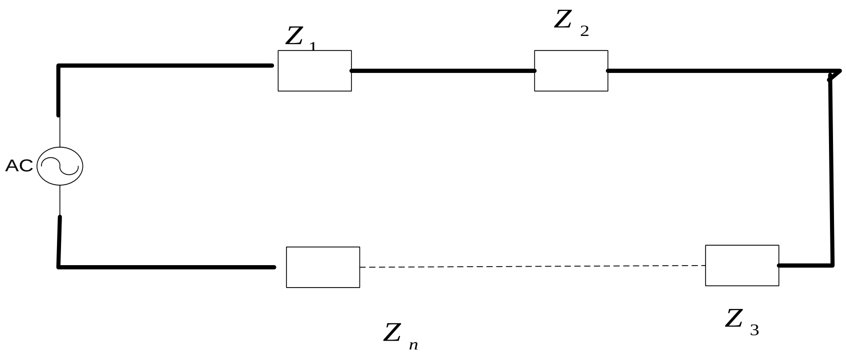


Impedance symbol

Series and parallel connection of impedance Z

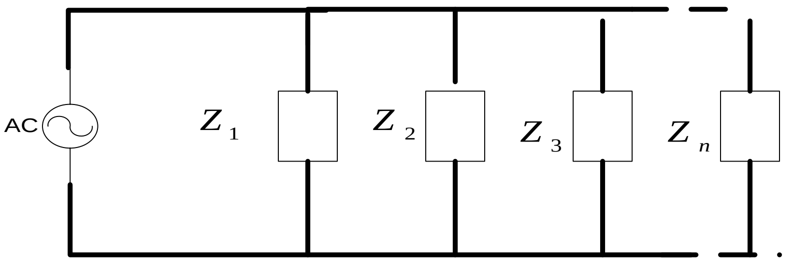
- Impedance Z in series

$$Z_{series} = Z_1 + Z_2 + Z_3 \ldots Z_n (\Omega)$$



- Impedance Z in parallel

$$Z_{parallel} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \ldots \frac{1}{Z_n}} (\Omega)$$



$$Z = \frac{V}{I} \Rightarrow Z = \frac{V\angle\theta}{I\angle\phi} = |Z|\angle\varphi \Rightarrow Z = R \pm jX$$

$$Z = \underline{R} \pm j\underline{X}$$

Real component
Resistance R

Imaginary component
Reactance X

- Reactance can be inductance or capacitance.

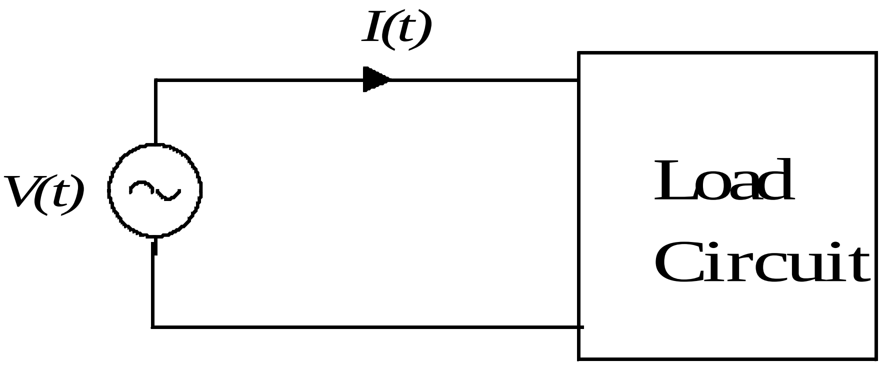
Effective Power & Effective Voltage and Current

If we suppose the voltage

$$V(t) = V_m \cos(\omega t + \theta_1)$$

is applied a load circuit and

current $I(t) = I_m \cos(\omega t + \theta_2)$



flows, effective power *P* consumed in the load circuit is

$$P = \lim_{T\to\infty} \frac{1}{T}\int_0^T V(t)\, I(t)\, dt$$

$$= V_m I_m \lim_{T\to\infty} \frac{1}{T}\int_0^T [\cos(\omega t + \theta_1)][\cos(\omega t + \theta_2)] dt$$

$$= V_m I_m \lim_{T\to\infty} \frac{1}{T}\int_0^T \frac{\cos(2\omega t + \theta_1 + \theta_2) + \cos(\theta_1 - \theta_2)}{2} dt$$

$$= \frac{V_m I_m}{2} \cos(\theta_1 - \theta_2)$$

Effective voltage and current

To make good correspondence with case of DC, we define effective voltage and current to be

$$V_e = \frac{V_m}{\sqrt{2}} \approx 0.707 V_m, \quad I_e = \frac{I_m}{\sqrt{2}} \approx 0.707\, I_m$$

as the RMS (root mean square) values, and the phase difference

$$\phi \equiv \theta_1 - \theta_2 \; .$$

Then, effective power *P* is given as

$$P = V_e I_e \cos\phi \quad [W]$$

where

V_m : maximum voltage

V_e : effective voltage

I_m : maximum current

I_e : effective current

$\cos\phi$: power factor

Commercially supplied electricity is expressed with effective voltage and frequency.

- Malaysia : $V_e = 220V$ and $f = 50\,Hz$
- Japan : $V_e = 100V$ and $f = 50\,Hz$ or $60Hz$
- U.S.A : $V_e = 110V$ and $f = 60Hz$
- European countries :

$$V_e = 220 \text{ to } 250V \text{ and } f = 50\,Hz$$

Problems

4.1 Why electric power is transmitted in the form of AC with very high voltage ?

4.2 Does a capacitor consume energy ? Also explain the reason.

4.3 When voltage $V(t) = V_m \sin \omega t$ is applied on a load circuit, current $I(t) = I_m \sin(\omega t - \phi)$ is flowed. Determine, the effective power, the effective voltage, the effective current and the power factor.

Chapter 5

Representation of Alternating Current Circuit with Complex Numbers and Circuit analysis

Sec.5.1 Mathematical Review of the Complex numbers

$$c \equiv a + jb$$

c : complex number

a, b : real number

$j = \sqrt{-1}$: imaginary unit

$$a = \mathrm{Re}(c)$$

$$b = \mathrm{Im}(c)$$

$$g \equiv d + jh$$

g : complex number

d, h : real number

$$c+g=(a+jb)+(d+jh)=a+d+j(b+h): \text{addition}$$

$$c-g=(a+jb)-(d+jh)=a-d+j(b-h): \text{subtraction}$$

$$c\,g=(a+jb)(d+jh)=ad-bh+j(ah+bd): \text{multiplication}$$

$$\frac{g}{c}=\frac{d+jh}{a+jb}=\frac{(d+jh)(a-jb)}{(a+jb)(a-jb)}=\frac{ad+bh+j(ah-bd)}{a^2+b^2}: \text{division}$$

$$c^*=a-jb: \text{complex conjugate}$$

$$|c|^2=c\,c^*=(a+jb)(a-jb)=a^2+b^2+j(-ab+ba)=a^2+b^2$$

$$c = a + jb = |c| e^{j\theta}$$

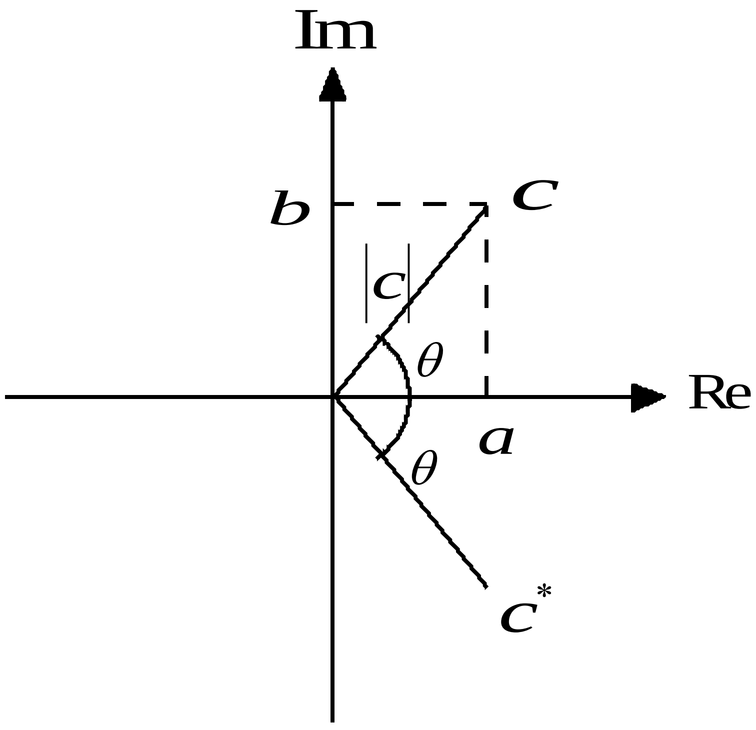
$$g = d + jd = |g| e^{j\phi}$$

$$|c|, |g| : \text{absolute value}$$

$$\theta, \phi : \text{argument}$$

$$\theta = \tan^{-1} \frac{b}{a} = \tan^{-1} \frac{\operatorname{Im} c}{\operatorname{Re} c}$$

$$\phi = \tan^{-1} \frac{h}{d} = \tan^{-1} \frac{\operatorname{Im} g}{\operatorname{Re} g}$$



$$cg = |c||g| e^{j(\theta+\phi)} = |cg| e^{j(\theta+\phi)}$$

$$\frac{g}{c} = \frac{|g|}{|c|} e^{j(\phi-\theta)} = \left|\frac{g}{c}\right| e^{j(\phi-\theta)}$$

Euler's formula

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

This equation is rewritten as

$$\cos\omega t = \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right),\ \sin\omega t = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)$$

From the Euler's formula, we obtain simple equations.

$$\frac{d}{dt}e^{j\omega t} = \frac{d}{dt}\left(\cos\omega t + j\sin\omega t\right) = -\omega\sin\omega t + j\omega\cos\omega t$$

$$= j\omega e^{j\omega t}$$

$$\int e^{j\omega t}dt = \int\left(\cos\omega t + j\sin\omega t\right)dt = \frac{\sin\omega t}{\omega} - \frac{j}{\omega}\cos\omega t$$

$$= \frac{1}{j\omega}e^{j\omega t}$$

Complex number representation of AC circuit

We represent the voltage and the current in the complex number form.

$$V = V_m e^{j\omega t} \qquad [V]$$

$$I = I_m e^{j\omega t} \qquad [A]$$

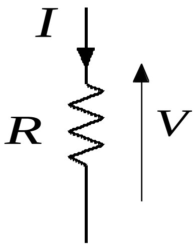
Here, V, V_m, I and I_m complex numbers. Then,

$$V = L\frac{dI}{dt} = j\omega L I\ ,\ \ I = \frac{1}{L}\int V\,dt = \frac{V}{j\omega L} \qquad \text{for inductor}$$

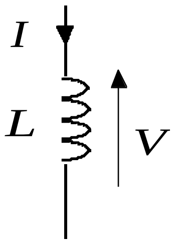
$$V = \frac{1}{C}\int I dt = \frac{I}{j\omega C}\ ,\ I = C\frac{dV}{dt} = j\omega C V \qquad \text{for capacitor}$$

Impedance

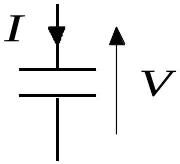
Ratio of voltage *V* to current *I* is called an impedance.



Resistor : $$\frac{V}{I} = Z = R$$



Inductor : $$\frac{V}{I} = Z = j\omega L$$



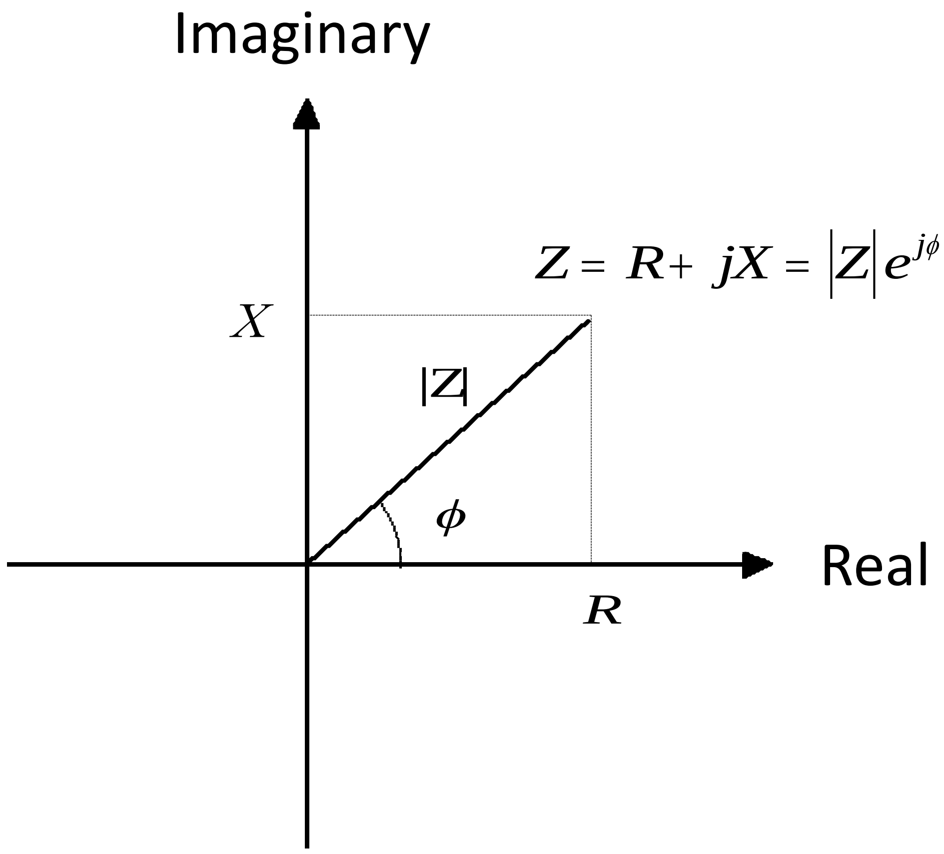
Capacitor : $$\frac{V}{I} = Z = \frac{1}{j\omega C}$$

General expression of the impedance

$$Z = R + jX \quad [\Omega]$$

R : Resistance

X : Reactance



Admittance

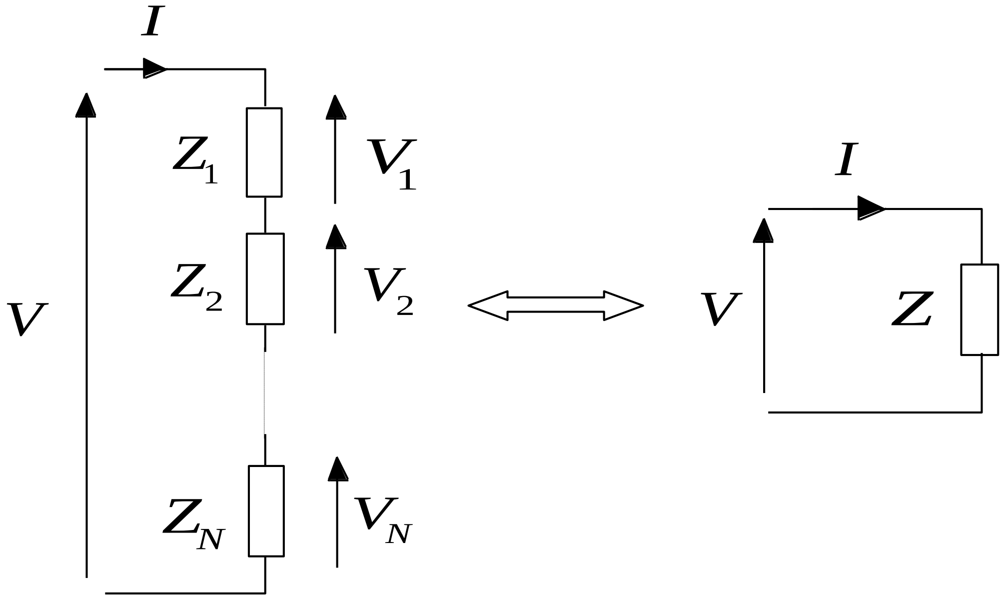
$$Y = \frac{1}{Z} = G + jB \quad [S]$$

G : Conductance

B : Susceptance

Series and Parallel connections of the impedance

Series connection



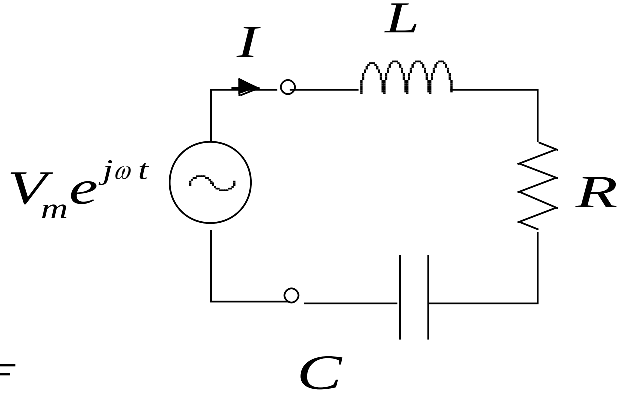
$$Z = \frac{V}{I} = Z_1 + Z_2 + \cdots + Z_N = \sum_{i=1}^{N} Z_i$$

LCR series resonance circuit

$$V = V_m e^{j\omega t}$$

$$Z = j\omega L + R + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

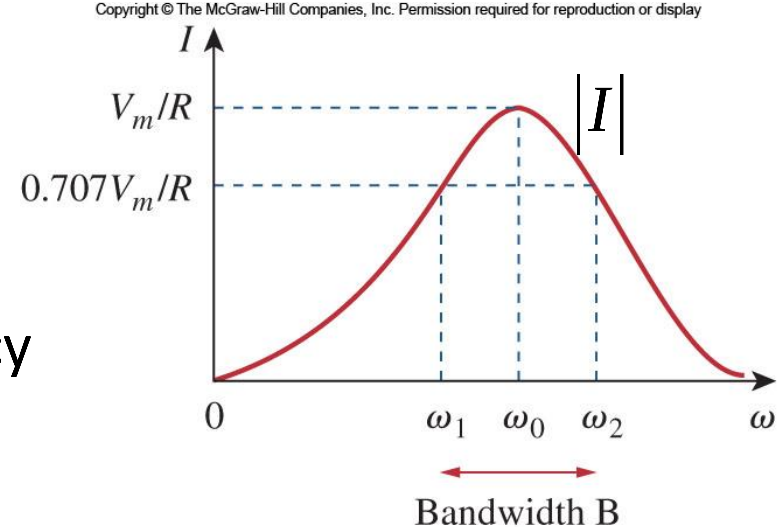
$$I = \frac{V}{Z} = \frac{V_m e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}, \quad |I| = \frac{|V_m|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



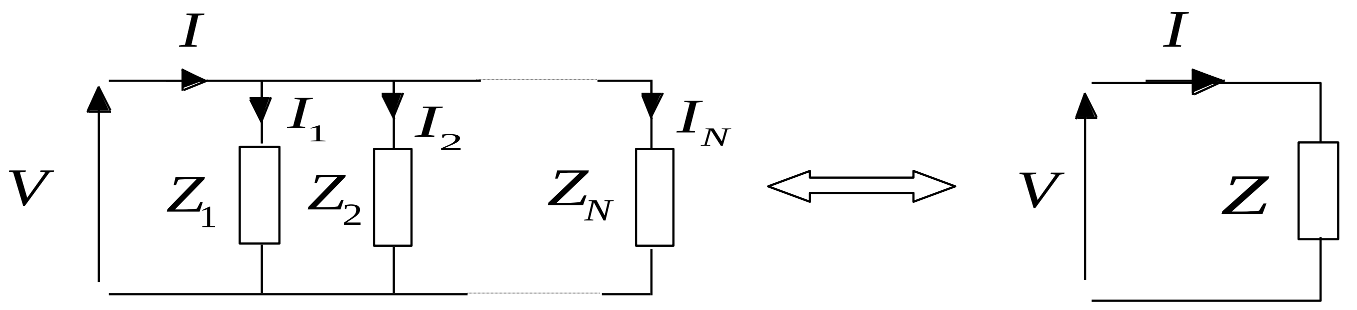
$$\omega_o^2 = \frac{1}{LC}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{[rad/s]}$$: resonance angular frequency

$$f_o = \frac{1}{2\pi\sqrt{LC}} \text{[Hz]}$$: resonance frequency



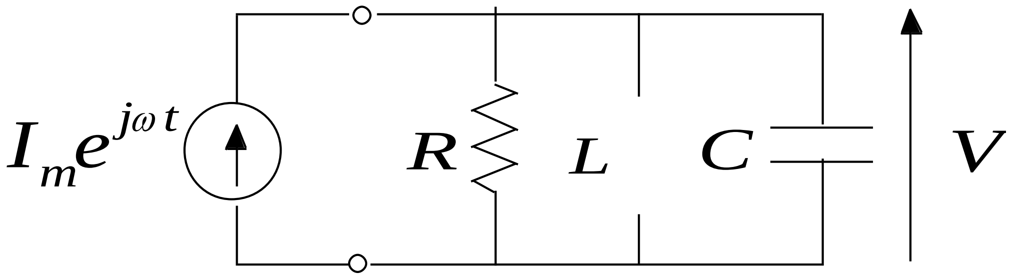
Parallel connection



$$\frac{1}{Z} = \frac{I}{V} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N} = \sum_{i=1}^{N} \frac{1}{Z_i}$$

$$Y = \frac{1}{Z} = Y_1 + Y_2 + \cdots + Y_N = \sum_{i=1}^{N} Y_i$$

LCR parallel resonance circuit



Exercise

Determine voltage v.s. angular frequency characteristics when R,L,C, parallel circuit is driven with a current source with angular frequency ω.

Power calculation with complex number

Let' suppose complex voltage and current as

$$V = V_m e^{j\omega t} = |V_m| e^{j(\omega t + \theta_1)}$$

$$I = I_m e^{j\omega t} = |I_m| e^{j(\omega t + \theta_2)}$$

Temporally averaged value of *V I* becomes 0 and does not have any meaning.

The following equation can represent the effective power.

$$P_{eff} = \frac{1}{2}\mathrm{Re}(V I^*) = \frac{1}{2}|V_m||I_m| \mathrm{Re}\, e^{j(\theta_1 - \theta_2)}$$

$$= \frac{1}{2}|V_m||I_m| \cos(\theta_1 - \theta_2)$$

$$= V_e I_e \cos\phi \qquad [\mathrm{W}]$$

where, $|V_m|$: maximum voltage

$V_e = |V_m|/\sqrt{2}$: effective voltage

$|I_m|$: maximum current

$I_e = |I_m|/\sqrt{2}$: effective current

$\cos\phi$: power factor

We call

$P_{eff} = \frac{1}{2}\mathrm{Re}(V I^*) = V_e I_e \cos\phi$: effective power, active power [W]

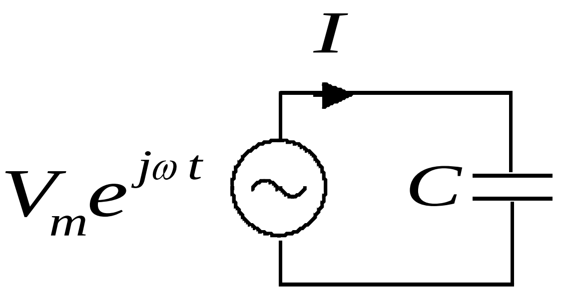
$P_{app} = \frac{1}{2}|VI^*| = V_e I_e$: apparent power [VA]

$P_{rea} == \frac{1}{2}\mathrm{Im}(V I^*) = V_e I_e \sin\phi$: reactive power [Var]

- Efficiency of electric power consumption is indicated with the power factor.
- The electric power capacity is usually given with the apparent power with [VA].

Power consumption in a capacitor

$$V = V_m e^{j\omega t}\ ,\ Z = \frac{1}{j\omega C}$$



$$I = \frac{V}{Z} = j\omega C V = j\omega C V_m e^{j\omega t}$$

$$I^* = -j\omega C V_m^* e^{-j\omega t},\ V I^* = -j\omega C |V_m|^2$$

$$|V| = |V_m|,\quad |I| = \omega C |V_m|$$

$$|V||I| = \omega C |V_m|^2$$

The effective power is $P_{eff} = \frac{1}{2}\mathrm{Re}\left(V I^*\right) = 0$

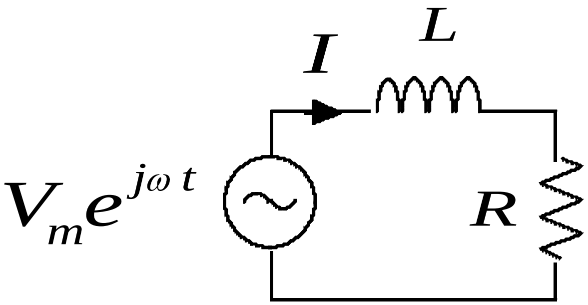
The apparent power is $P_{app} = V_e I_e = \frac{1}{2}\omega C |V_m|^2 = \omega C V_e^{\,2}$

The power factor is $\cos\phi = 0$.

There is no power consumption in the capacitor.

LR series circuit

Find current, effective power, apparent power and power factor.



$$V = V_m e^{j\omega t}$$

$$Z = R + j\omega L$$

$$I = \frac{V}{Z} = \frac{V}{R + j\omega L} = \frac{V_m e^{j\omega t}}{R + j\omega L}$$

$$I^* = \frac{V^*}{R - j\omega L}, \quad V I^* = \frac{|V|^2}{R - j\omega L} = \frac{|V_m|^2 (R + j\omega L)}{R^2 + (\omega L)^2}$$

$$P_{eff} = \frac{1}{2}\mathrm{Re}(V I^*) = \frac{|V_m|^2 R}{2\{R^2 + (\omega L)^2\}} = \frac{R V_e^{\ 2}}{R^2 + (\omega L)^2}$$

: effective power

$$|V| = |V_m|, \quad |I| = \frac{|V_m|}{\sqrt{R^2 + (\omega L)^2}}$$

$$P_{app} = V_e I_e = \frac{1}{2}|V||I| = \frac{|V_m|^2}{2\sqrt{R^2 + (\omega L)^2}}$$: apparent power

$$\cos\phi = \frac{P_{eff}}{P_{app}} = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$: power factor

Note : Amount of flowing current is determined by absolute value of the circuit impedance. However, the effective power is determined by consumed power by the resistor.

AC Circuit Analysis

Similar as the DC circuit, the following methods are applicable for AC circuit by replacing *R* to *Z* and DC power source to AC power source.

- Nodal analysis
- Mesh analysis
- Source transformation
- Thevenin's theorem
- Norton's theorem
- Superposition principle

Notice for AC circuit analysis

- When the circuit include multiple power sources, angular frequency of each sources should be identical.

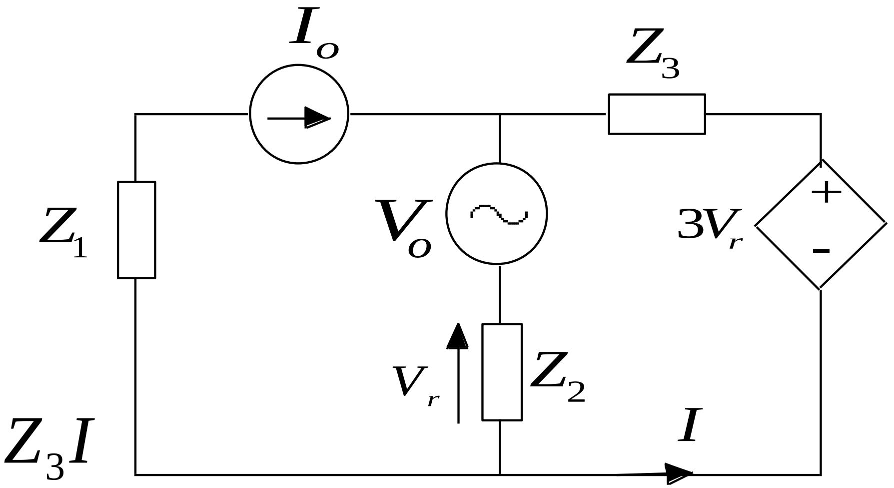
  Otherwise, we can not define the impedances of the inductor and the capacitor uniquely.
- Power calculation is not same as the DC circuit.

  The effective power is given by

$$P_{eff} = \frac{1}{2}\mathrm{Re}(V I^*) = V_e I_e \cos\phi$$

An example of circuit analysis

Find *I* and consumed power in Z_3 .



$$V_o + V_r = 3V_r - Z_3 I$$

$$V_r = Z_2(I_o + I)$$

$$\therefore V_o = 2V_r - Z_3 I = 2Z_2\left(I_o + I\right) - Z_3 I$$

$$= \left(2Z_2 - Z_3\right)I + 2I_o$$

$$\therefore I = \frac{V_o - 2Z_2 I_o}{2Z_2 - Z_3}$$

Voltage drop at Z_3 is $Z_3 I$. Then consumed power in Z_3 is

$$P = \frac{1}{2}\mathrm{Re}\left(Z_3 I I^*\right) = \frac{1}{2}|I|^2 \mathrm{Re}\left(Z_3\right) = \frac{1}{2}\left|\frac{V_o - 2Z_2 I_o}{2Z_2 - Z_3}\right|^2 \mathrm{Re}\left(Z_3\right)$$

Problems

5.1 Show the conductance G and the susceptance B in the admittance Y with the resistance R and the reactance X in the impedance.

5.2 Find the input impedance of the circuit in Fig.5.2. Assume that the circuit operates at $\omega =$ 50 rad/s.

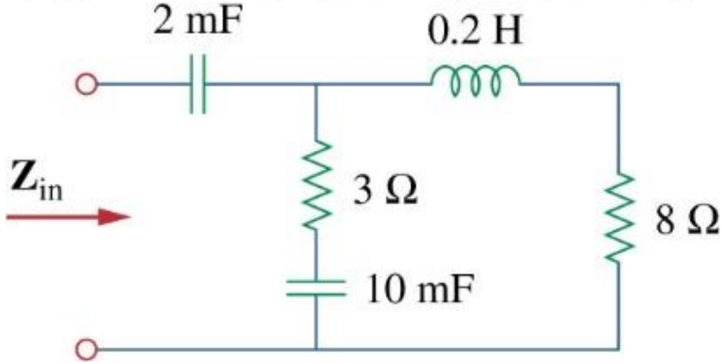


Fig.5.2

5.3 Derive the effective power consumed in the LRC series circuit shown in the Ex.5.1.

5.4 Derive the effective power consumed in the LRC parallel circuit shown in the Ex.5.2

5.5 Find V_3 in Fig.5.4 by using the source transformation.

Fig.5.5