



# ACRG INTERNATIONAL LECTURE SERIES

(Algebra and Combinatorics Research Group)

"Topological Indices of Algebraic Graphs  
Associated to Groups and Rings"



15 MONDAY 20  
JULY 24



08.00 WITA



Aula FMIPA



With:  
**Prof. Nor Haniza Sarmin**  
Department of Mathematical Sciences  
Faculty of Science  
Universiti Teknologi Malaysia

Presented by:  
Program Studi Matematika  
Fakultas MIPA Universitas Mataram



Jl. Majapahit No. 62 Telp. dan Faks 0370 646506 Mataram 83125 NTB



# Topological Indices of Algebraic Graphs Associated to Groups and Rings

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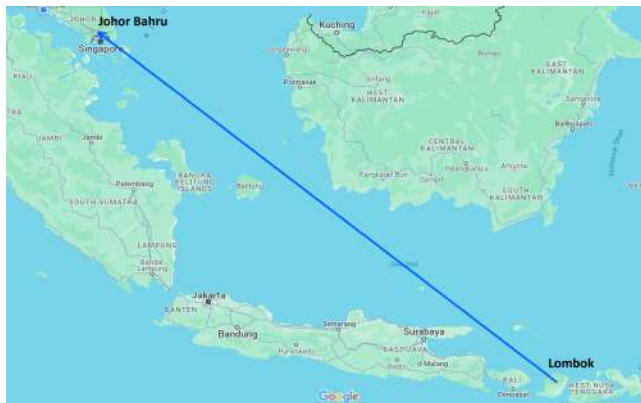
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*Algebra and Combinatorics Research Group (ACRG) International Lecture Series*

15 July 2024



# From Lombok to Johor Bahru



- The distance between Lombok and Johor Bahru is estimated at 2600 km or 1616 miles.
- Flight: LOP – KUL – JHB (one stop) ~ 3 hours  
LOP – SIN (direct) – Johor Bahru (by land transportation)
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# Location of Universiti Teknologi Malaysia (UTM)



# Faculties in UTM

12

Faculties

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- Energy Engineering



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- Electronic and Computing Engineering
- Electrical Power Engineering

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- Urban and Regional Planning
- Landscape Architecture
- Geoinformation
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- Accounting
- Marketing
- Technology Management
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- Information Systems

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- School of Education
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## Faculty of Artificial Intelligence

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- Science, Management & Design
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- Perdana Centre

## Azman Hashim International Business School

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- Mechanical Precision Engineering
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UTM  
QS WORLD UNIVERSITY RANKINGS by Subject

Ranked in QS World University Rankings by Subject 2024

**TOP 200**  
Environmental Sciences  
Materials Sciences  
Mathematics

**TOP 250**  
Chemistry

**TOP 300**  
Physics and Astronomy

**TOP 450**  
Biological Sciences

**#2** In Malaysia: Mathematics, Physics and Astronomy

**#Joint 2** In Malaysia: Environmental Sciences, Materials Sciences

**#Joint 3** In Malaysia: Chemistry

**#5** In Malaysia: Biological Sciences

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TOP 40

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**56**

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**195**

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**8**

**RESEARCH CENTRE &  
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Bachelor of Science  
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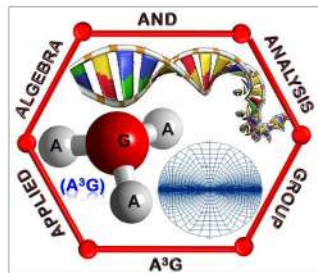
Bachelor of Science (Industrial  
Mathematics) with Honours

<https://science.utm.my/mathematics/>

The department is divided into 5 broad research areas/**academic panels**:

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2. Applied Mathematics
3. Numerical & Computational
4. Operations Research
5. Statistics

**Members** belong to specific **research groups** focused on diverse studies.



Joined Department of Mathematics,  
Faculty of Science, UTM on  
2 May 1991



# My Background



State University of New York at Binghamton  
(now known as Binghamton University), New York, USA



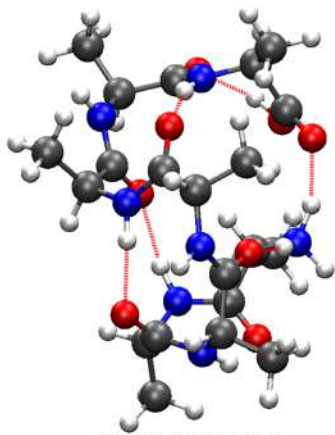
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# Topological Indices of Algebraic Graphs Associated to Groups and Rings

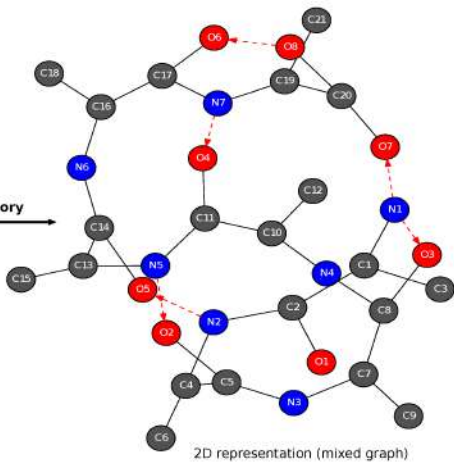
## Abstract

Topological index (TI) is a function that assigns a numeric value to a (molecular) graph that predicts its various physical and structural properties such as volume, density, pressure, weight, boiling point, freezing point, vaporisation point, heat of formation, and heat of evaporation. In this presentation, the most significant types of topological indices of graphs are degree-based and distance-based topological indices. The degree-based topological indices studied in this research include the first Zagreb index, the second Zagreb index, the general zeroth-order Randić index, and the Sombor index. The distance-based topological indices include the Wiener index, the Szeged index, and the Harary index. The relationship between topological indices of graphs associated to groups and rings is of interest. The graphs considered are the non-commuting graph and the coprime graph associated to the dihedral groups, the generalized quaternion groups, and the quasidihedral groups. Another graph discussed is the zero divisor graph associated to some commutative rings.

# Motivation of the Research



Graph theory



# Presentation Outline

- 1 Introduction
- 2 Topological Indices
- 3 Graphs Associated to Groups
- 4 Graphs Associated to Rings
- 5 Topological Indices of Graphs Associated to Groups
- 6 Topological Indices of Graphs Associated to Rings
- 7 Conclusion
- 8 Future Research Recommendations
- 9 References
- 10 Publication
- 11 Collaborators
- 12 Acknowledgement

# INTRODUCTION

# INTRODUCTION

- Topological indices provide **numerical descriptors** that capture important **structural features of molecules**.
- They serve as **powerful tools** for the analysis and prediction of various **physicochemical properties and biological activities**.
- The significance of topological indices lies in their **ability to transform** complex molecular structures into numerical representations, **enabling the development** of computational models and the **efficient exploration** of chemical space for various applications in drug discovery, materials science, and reaction chemistry [1].
- Various types of topological indices have been developed based on either **chemistry or mathematical perspectives**.

[1] I. Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry, Springer Science and Business Media*, 2012.

# TOPOLOGICAL INDICES



Figure 1: Different Types of Topological Indices

# DEFINITIONS OF TOPOLOGICAL INDICES

## Wiener Index

- The **first type** of topological index has been discovered by Wiener [2] in 1947, in which the concept of Wiener number considering the **path in a graph** is introduced.
- The Wiener number of some **paraffins** are determined and their **boiling points** are also predicted.
- Then, Hosoya [3] reformulated the formula of Wiener number, known as Wiener index of a graph,  $W(\Gamma)$ , and its formula is given in the following.

$$W(\Gamma) = \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m d(i, j),$$

where  $d(i, j)$  is the **distance** between vertices  $i$  and  $j$ , and  $m$  is the total number of vertices in a graph  $\Gamma$ .

[2] K. Wiener, *Structural determination of paraffin boiling points*, *Journal of the American Chemical Society*, **69(1)** (1947), 17-20.

[3] H. Hosoya, *Topological index. A newly proposed quantity characterizing the topological nature of structural isomers of saturated hydrocarbons*, *Bulletin of the Chemical Society of Japan*, **44(9)** (1971), 2332-2339.



# Example of Wiener Index

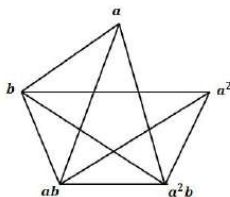


Figure 2: The non-commuting graph  $D_6$

$$\begin{aligned} W(\Gamma) &= \frac{1}{2} \sum_{i=1}^5 \sum_{j=1}^5 d(i, j) \\ &= \frac{1}{2} [d(a, a^2) + d(a, b) + d(a, ab) + d(a, a^2b) + d(a^2, b) + \\ &\quad d(a^2, ab) + d(a^2, a^2b) + d(b, ab) + d(ab, a^2b) + d(b, a^2b)] \\ &= 2 + 9(1) \\ &= 11. \end{aligned}$$

## Proposition 1.1 [4]

Let  $G$  be a finite group and  $\Gamma_G^{\text{NC}}$  be the non-commuting graph. Then, the Wiener index of the non-commuting graph of  $G$  is given as

$$W(\Gamma_G^{\text{NC}}) = \frac{1}{2} [(|G| - |Z(G)|) (|G| - 2|Z(G)| - 2) + |G| (k(G) - |Z(G)|)].$$

[4]A. Azad and M. Eliahi, *Distance in the non-commuting graph of groups. Ars Comb.* **99** (2011), 279-287.

## Zagreb Index

In 1972, Gutman and Trinajstić [5] introduced the **degree-based topological index**, Zagreb index, which is divided into two types; first Zagreb index,  $M_1$ , and second Zagreb index,  $M_2$ , defined as follows.

$$M_1(\Gamma) = \sum_{v \in v(\Gamma)} (\deg(v))^2$$

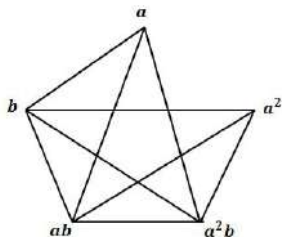
and

$$M_2(\Gamma) = \sum_{\{u,v\} \in E(\Gamma)} \deg(u)\deg(v).$$

[5] I. Gutman and N. Trinajstić, *Graph theory and molecular orbitals. Total  $\pi$ -electron energy of alternant hydrocarbons*, *Chemical Physics Letters*, **17**(4) (1972), 535-538.

# Example of First Zagreb Index

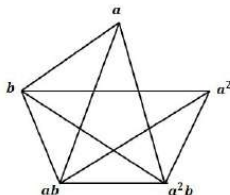
Based on Figure 2,



$$\begin{aligned}M_1(\Gamma) &= \sum_{v \in v(\Gamma)} (\deg(v))^2 \\ &= \deg(a)^2 + \deg(a^2)^2 + \deg(b)^2 + \deg(ab)^2 + \deg(a^2b)^2 \\ &= 3 + 3 + 4 + 4 + 4 \\ &= 18.\end{aligned}$$

# Example of Second Zagreb Index

Based on Figure 2,



$$\begin{aligned}M_2(\Gamma) &= \sum_{\{u,v\} \in E(\Gamma)} \deg(u)\deg(v) \\&= \deg(a)\deg(b) + \deg(a)\deg(ab) + \deg(a)\deg(a^2b) + \deg(a^2)\deg(b) + \\&\quad \deg(a^2)\deg(ab) + \deg(a^2)\deg(a^2b) + \deg(b)\deg(ab) + \deg(ab)\deg(a^2b) \\&\quad + \deg(b)\deg(a^2b) \\&= 3(4) + 3(4) + 3(4) + 3(4) + 3(4) + 3(4) + 4(4) + 4(4) + 4(4) \\&= 120.\end{aligned}$$

# Zagreb Index of the Non-commuting Graph

## Proposition 1.2 [6]

Let  $G$  be a finite group and  $\Gamma_G^{\text{NC}}$  be the non-commuting graph of  $G$ . Then, the first Zagreb index of the non-commuting graph of  $G$ ,

$$M_1(\Gamma_G^{\text{NC}}) = |G|^2(|G| + |Z(G)| - 2k(G)) - \sum_{x \in G - Z(G)} |C_G(x)|^2.$$

## Proposition 1.3 [6]

Let  $G$  be a finite group and  $\Gamma_G^{\text{NC}}$  be the non-commuting graph. Then, the second Zagreb index of the non-commuting graph of  $G$ ,

$$M_2(\Gamma_G^{\text{NC}}) = -|G|^2|E(\Gamma_G^{\text{NC}})| + |G|M_1(\Gamma_G^{\text{NC}}) + \sum_{x, y \in E(\Gamma_G^{\text{NC}})} |C_G(x)||C_G(y)|.$$

[6] M. Mizargar and A. Ashrafi, *Some distance-based topological indices of a non-commuting graph*. Hacettepe Journal of Mathematics and Statistics. **41(4)** (2012), 515-526.

# DEFINITIONS OF TOPOLOGICAL INDICES

## Szeged Index

Let  $\Gamma$  be a simple connected graph with vertex set  $V(\Gamma) = \{1, 2, \dots, n\}$ . The Szeged index,  $Sz(\Gamma)$  is given as in the following :

$$Sz(\Gamma) = \sum_{e \in E(\Gamma)} n_1(e|\Gamma)n_2(e|\Gamma),$$

where the summation embraces all edges of  $\Gamma$ ,

$$n_1(e|\Gamma) = |\{v|v \in V(\Gamma), d(v, x|\Gamma) < d(v, y|\Gamma)\}|$$

and

$$n_2(e|\Gamma) = |\{v|v \in V(\Gamma), d(v, y|\Gamma) < d(v, x|\Gamma)\}|$$

which means that  $n_1(e|\Gamma)$  counts the  $\Gamma$ 's vertices are closer to one edge's terminal  $x$  than the other while  $n_2(e|\Gamma)$  is vice versa [7].

[7] P.V. Khadikar, N.V. Deshpande, V. Narayan, P. Kale, P. Prabhakar, A. Dobrynin, I. Gutman, and G. Domotor, *The Szeged index and an analogy with the Wiener index*, *Journal of Chemical Information and Computer Sciences*, **35**(3) (1995), 547-550.

# Example of Szeged Index

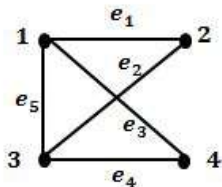


Figure 3: A simple connected graph

Note that  $N_1(e_i|\Gamma)$  is the vertices of  $\Gamma$  lying closer to one endpoint  $x$  of the edge  $e_i$  than to its other endpoint  $y$  while  $N_2(e_i|\Gamma)$  is vice versa. First,  $N_1(e_i|\Gamma)$  and  $N_2(e_i|\Gamma)$  are calculated for all  $i$ .

For  $e_1 = \{1, 2\}$ ,

$$N_1(e_1|\Gamma) = \{x \in V(\Gamma) : d(x, 1) < d(x, 2)\}, \quad n_1(e_1|\Gamma) = 2, \\ = \{1, 4\},$$

$$N_2(e_1|\Gamma) = \{y \in V(\Gamma) : d(y, 1) > d(y, 2)\}, \quad n_2(e_1|\Gamma) = 1. \\ = \{2\},$$



## Example of Szeged Index (CONT.)

For  $e_2 = \{2, 3\}$ ,

$$N_1(e_2|\Gamma) = \{x \in V(\Gamma) : d(x, 2) < d(x, 3)\}, \quad n_1(e_2|\Gamma) = 1, \\ = \{2\},$$

$$N_2(e_2|\Gamma) = \{y \in V(\Gamma) : d(y, 2) > d(y, 3)\}, \quad n_2(e_2|\Gamma) = 2. \\ = \{3, 4\},$$

For  $e_3 = \{1, 4\}$ ,

$$N_1(e_3|\Gamma) = \{x \in V(\Gamma) : d(x, 1) < d(x, 4)\}, \quad n_1(e_3|\Gamma) = 2, \\ = \{1, 2\},$$

$$N_2(e_3|\Gamma) = \{y \in V(\Gamma) : d(y, 1) > d(y, 4)\}, \quad n_2(e_3|\Gamma) = 1. \\ = \{4\},$$

## Example of Szeged Index (CONT.)

For  $e_4 = \{3, 4\}$ ,

$$N_1(e_4|\Gamma) = \{x \in V(\Gamma) : d(x, 3) < d(x, 4)\}, \quad n_1(e_4|\Gamma) = 2, \\ = \{2, 3\},$$

$$N_2(e_4|\Gamma) = \{y \in V(\Gamma) : d(y, 3) > d(y, 4)\}, \quad n_2(e_4|\Gamma) = 1. \\ = \{4\},$$

For  $e_5 = \{1, 3\}$ ,

$$N_1(e_5|\Gamma) = \{x \in V(\Gamma) : d(x, 1) < d(x, 3)\}, \quad n_1(e_5|\Gamma) = 1, \\ = \{1\},$$

$$N_2(e_5|\Gamma) = \{y \in V(\Gamma) : d(y, 1) > d(y, 3)\}, \quad n_2(e_5|\Gamma) = 1. \\ = \{3\},$$

# Example of Szeged Index (CONT.)

Hence,

$$\begin{aligned}Sz(\Gamma) &= \sum_{i=1}^5 n_1(e_i|\Gamma)n_2(e_i|\Gamma) \\ &= n_1(e_1|\Gamma)n_2(e_1|\Gamma) + n_1(e_2|\Gamma)n_2(e_2|\Gamma) + n_1(e_3|\Gamma)n_2(e_3|\Gamma) + \\ &\quad n_1(e_4|\Gamma)n_2(e_4|\Gamma) + n_1(e_5|\Gamma)n_2(e_5|\Gamma) \\ &= (2)(1) + (1)(2) + (2)(1) + (2)(1) + (1)(1) \\ &= 9.\end{aligned}$$

# DEFINITIONS OF TOPOLOGICAL INDICES

## Harary Index

Let  $\Gamma$  be a connected graph with vertex set  $V = \{1, 2, \dots, n\}$ . Half the elements' sum in the reciprocal distance matrix,  $D^r = D^r(\Gamma)$ , is what is known as the Harary index, written as

$$H = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n D^r(i, j),$$

where

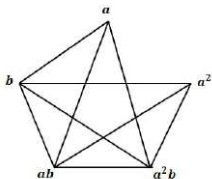
$$D^r(i, j) = \begin{cases} \frac{1}{d(i, j)} & \text{if } i \neq j, \\ 0 & \text{if } i = j, \end{cases}$$

and  $d(i, j)$  is the shortest distance between vertex  $i$  and  $j$  [8].

[8] D. Plavšić, S. Nikolić, N. Trinajstić, and Z. Mihalić, *On the Harary index for the characterization of chemical graphs*, *Journal of Mathematical Chemistry*, **12** (1993), 235-250.

# Example of Harary Index

Based on Figure 2,



$$\begin{aligned} H &= \frac{1}{2} \sum_{i=a}^{a^2b} \sum_{j=a}^{a^2b} D^r(i, j) \\ &= \frac{1}{2} [D^r(a, a) + D^r(a, a^2) + D^r(a, b) + D^r(a, ab) + D^r(a, a^2b) + \\ &D^r(a^2, a) + D^r(a^2, a^2) + D^r(a^2, b) + D^r(a^2, ab) + D^r(a^2, a^2b) + \\ &D^r(b, a) + D^r(b, a^2) + D^r(b, b) + D^r(b, ab) + D^r(b, a^2b) + \\ &D^r(ab, a) + D^r(ab, a^2) + D^r(ab, b) + D^r(ab, ab) + D^r(ab, a^2b) + \\ &D^r(a^2b, a) + D^r(a^2b, a^2) + D^r(a^2b, b) + D^r(a^2b, ab) + D^r(a^2b, a^2b)] \end{aligned}$$

# Example of Harary Index (CONT.)

$$\begin{aligned} H &= \frac{1}{2} \sum_{i=a}^{a^2b} \sum_{j=a}^{a^2b} D^r(i, j) \\ &= \frac{1}{2} \left[ 0 + \frac{1}{2} + 1 + 1 + 1 + \frac{1}{2} + 0 + 1 + 1 + 1 + \right. \\ &\quad \left. 1 + 1 + 0 + 1 + 1 + 1 + 1 + 1 + 0 + 1 + 1 + 1 + 1 + 1 + 0 \right] \\ &= 9.5 \end{aligned}$$

## Randić Index

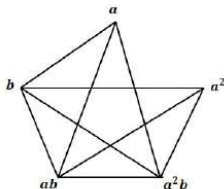
- The Randić index is a **graph-theoretical descriptor** that quantifies the complexity or branching structure of a molecular graph.
- It was introduced by Milan Randić [9] in 1975 and has found applications in various fields of chemistry.
- It is defined as the sum of the reciprocal square roots of the product of **the degrees of connected pairs of vertices**, written as

$$R(\Gamma) = \sum_{u,v \in E(\Gamma)} \frac{1}{\sqrt{\deg(u)\deg(v)}}.$$

[9] M. Randić, *Characterization of molecular branching. Journal of the American Chemical Society*, **97(23)** (1975), 6609-6615.

# Example of Randić Index

Based on Figure 2,



$$\begin{aligned} R(\Gamma) &= \sum_{u,v \in E(\Gamma)} \frac{1}{\sqrt{\deg(u)\deg(v)}} \\ &= \frac{1}{\sqrt{(3)(4)}} + \frac{1}{\sqrt{(3)(4)}} + \frac{1}{\sqrt{(3)(4)}} + \frac{1}{\sqrt{(3)(4)}} + \frac{1}{\sqrt{(3)(4)}} \\ &\quad \frac{1}{\sqrt{(3)(4)}} + \frac{1}{\sqrt{(4)(4)}} + \frac{1}{\sqrt{(4)(4)}} + \frac{1}{\sqrt{(4)(4)}} \\ &= 2.48 \end{aligned}$$



# DEFINITIONS OF TOPOLOGICAL INDICES

## General Zeroth Order Randić Index

The Randić index is modified and introduced a concept of general zeroth order Randić index, which is defined as

$${}^0R_\alpha = \sum_{u \in V(\Gamma)} (\deg(u))^\alpha,$$

where  $\alpha$  can be any non-zero real number [10].

## Sombor Index

Recently, in 2021, a new topological index, Sombor index has been established by Gutman [11]. The Sombor index of a graph,  $SO(\Gamma)$ , is defined as follows.

$$SO(\Gamma) = \sum_{u,v \in E(\Gamma)} \sqrt{\deg(u)^2 + \deg(v)^2}.$$

[10] H. Ahmed, A.A. Bhatti, and A. Ali, *Zeroth-order general Randić index of cactus graphs*. *AKCE International Journal of Graphs and Combinatorics*, **16(2)** (2019), 182-189.

[11] I. Gutman, *Geometric approach to degree-Based topological indices: Sombor indices*. *MATCH Commun. Math. Comput. Chem.*, **86** (2021), 11-16.

# Example of Sombor Index

Based on Figure 2,

$$\begin{aligned}SO(\Gamma) &= \sum_{u,v \in E(\Gamma)} \sqrt{\deg(u)^2 + \deg(v)^2} \\&= \sqrt{\deg(a)^2 + \deg(b)^2} + \sqrt{\deg(a)^2 + \deg(ab)^2} + \sqrt{\deg(a)^2 + \deg(a^2b)^2} + \\&\quad \sqrt{\deg(a^2)^2 + \deg(b)^2} + \sqrt{\deg(a^2)^2 + \deg(ab)^2} + \\&\quad \sqrt{\deg(a^2)^2 + \deg(a^2b)^2} + \sqrt{\deg(b)^2 + \deg(ab)^2} + \\&\quad \sqrt{\deg(b)^2 + \deg(a^2b)^2} + \sqrt{\deg(ab)^2 + \deg(a^2b)^2} \\&= \sqrt{3^2 + 4^2} + \sqrt{3^2 + 4^2} + \sqrt{3^2 + 4^2} + \sqrt{3^2 + 4^2} + \sqrt{3^2 + 4^2} \\&\quad \sqrt{3^2 + 4^2} + \sqrt{4^2 + 4^2} + \sqrt{4^2 + 4^2} + \sqrt{4^2 + 4^2} \\&= 46.97\end{aligned}$$

# GRAPHS ASSOCIATED TO GROUPS

## Definition 2.1 [12] The Non-commuting Graph

Let  $G$  be a finite group. The non-commuting graph of  $G$ , denoted by  $\Gamma_G^{\text{NC}}$ , is the graph of vertex set  $G - Z(G)$  and two distinct vertices  $x$  and  $y$  are joined by an edge whenever  $xy \neq yx$ .

Example :

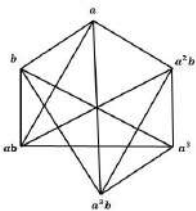


Figure 4: The non-commuting graph of  $D_8$

[12] A. Abdollahi, S. Akbari, H. Maimani, *Non-commuting graph of a group*. *Journal of Algebra*. **298** (2006), 468-492.

# THE NON-COMMUTING GRAPH ASSOCIATED TO SOME FINITE GROUPS

## Proposition 2.1 [13]

Let  $G$  be the dihedral group,  $D_{2n} = \langle a, b \mid a^n = b^2 = 1, bab = a^{-1} \rangle$  of order  $2n$  where  $n \geq 3, n \in \mathbb{N}$  and let  $\Gamma_G^{\text{NC}}$  be the non-commuting graph of  $G$ . Then,

$$\Gamma_G^{\text{NC}} = \begin{cases} \underbrace{K_{1, 1, \dots, 1, n-1}}_{n \text{ times}}, & \text{if } n \text{ is odd,} \\ \underbrace{K_{2, 2, \dots, 2, n-2}}_{\frac{n}{2} \text{ times}}, & \text{if } n \text{ is even.} \end{cases}$$

[13] R. Mahmoud, *Energy and Laplacian Energy of Graphs Related to a Family of Finite Groups*, Ph.D Thesis, Universiti Teknologi Malaysia. 2018.

# THE NON-COMMUTING GRAPH ASSOCIATED TO SOME FINITE GROUPS

## Proposition 2.2 [13]

Let  $G$  be the generalised quaternion group,  $Q_{4n} = \langle a, b \mid a^n = b^2, a^{2n} = b^4 = 1, bab = a^{-1} \rangle$  of order  $4n$  where  $n \geq 2, n \in \mathbb{N}$  and let  $\Gamma_G^{\text{NC}}$  be the non-commuting graph of  $G$ . Then,

$$\Gamma_G^{\text{NC}} = K_{\underbrace{2, 2, \dots, 2}_{n \text{ times}}, 2, 2n-2}.$$

## Proposition 2.3 [13]

Let  $G$  be the quasidihedral group,  $QD_{2n} = \langle a, b \mid a^{2^{n-1}} = b^2 = 1, bab = a^{2^{n-2}-1} \rangle$  of order  $2^n$  where  $n \geq 4, n \in \mathbb{N}$  and let  $\Gamma_G^{\text{NC}}$  be the non-commuting graph of  $G$ . Then,

$$\Gamma_G^{\text{NC}} = K_{\underbrace{2, 2, \dots, 2}_{2^{n-2} \text{ times}}, 2, 2^{n-1}-2}.$$

# GRAPHS ASSOCIATED TO GROUPS

## Definition 2.2 [14] The Coprime Graph

Coprime graph of a group  $G$  is a graph that consists the elements in  $G$  as the set of vertices where two distinct vertices are adjacent if and only if the order of both vertices are coprime.

Example:

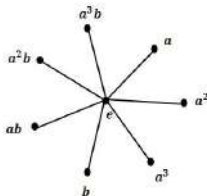


Figure 5: The coprime graph of  $D_8$

[14] X.L. Ma, H.Q. Wei, and L.Y. Yang, *The coprime graph of a group*. *International Journal of Group Theory*, **3(3)** (2014), 13-23.

# THE COPRIME GRAPH ASSOCIATED TO DIHEDRAL GROUPS

In 2014, Ma *et al.* [14] generalized the coprime graph for certain order of dihedral groups, as stated in the following propositions.

## Proposition 2.4 [14]

Let  $G$  be the dihedral groups of order  $2n$  and the coprime graph of  $G$  is denoted as  $\Gamma_G^{\text{CO}}$ . Then,  $\Gamma_G^{\text{CO}}$  is isomorphic to a multipartite graph  $K_{1,n-1,n}$  if  $n$  is an odd prime.

## Proposition 2.5 [14]

Let  $G$  be the dihedral groups of order  $2n$  and the coprime graph of  $G$  is denoted as  $\Gamma_G^{\text{CO}}$ . Then,  $\Gamma_G^{\text{CO}}$  is isomorphic to a star graph,  $K_{1,2^{k+1}-1}$  if  $n = 2^k$  for some positive integer  $k$ .

[14] X.L. Ma, H.Q. Wei, and L.Y. Yang, *The coprime graph of a group. International Journal of Group Theory*, **3(3)** (2014), 13-23.

# GRAPHS ASSOCIATED TO RINGS

## Zero Divisor Graph [15]

Let  $R$  be a commutative ring with identity,  $Z(R)$  its set of zero divisors. The zero divisor graph of  $R$  is  $\Gamma(R) = Z(R) - 0$ , with distinct vertices  $a$  and  $b$  adjacent if and only if  $ab = 0$  or  $ba = 0$ .

Example :

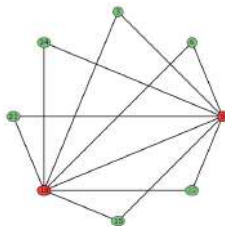


Figure 6: The zero divisor graph for  $\mathbb{Z}_{27}$ ,  $\Gamma(\mathbb{Z}_{27})$

[15] D.F. Anderson and P.S. Livingston, *The zero-divisor graph of a commutative ring*. *Journal of Algebra*, **217**(2) (1999), 434–447.



# TOPOLOGICAL INDICES OF GRAPHS ASSOCIATED TO GROUPS

# The Generators and Relations for Some Finite Groups

## Dihedral Group [16]

The dihedral group, denoted by  $D_{2n}$ , is a group of symmetries of a regular polygon, which include rotations and reflections. The order of  $D_{2n}$  is  $2n$ , where  $n \geq 3$  is an integer. The dihedral group can be presented in the form of generators and relations given as follows:

$$D_{2n} = \langle a, b : a^n = b^2 = 1, bab = a^{-1} \rangle,$$

where  $a$  and  $b$  are the generators of  $D_{2n}$ .

## Generalized Quaternion Group [16]

The generalized quaternion group,  $Q_{4n}$  is a group of order  $4n$ ,  $n \in \mathbb{N}$  and the group presentation of  $Q_{4n}$  is presented as follows:

$$Q_{4n} = \langle a, b \mid a^n = b^2, a^{2n} = b^4 = 1, bab = a^{-1} \rangle,$$

where  $n \geq 2$ .

## Quasidihedral Group [16]

The Quasidihedral Group,  $QD_{2^n}$ , is a non-abelian group of order  $2^n$  with group presentation given as follows::

$$QD_{2^n} = \langle a, b \mid a^{2^{n-1}} = b^2 = 1, bab = a^{2^{n-2}-1} \rangle,$$

where  $n \in \mathbb{N}$  and  $n \geq 4$ .

[16] J. Humphreys, *A course in group theory, vol. 6, Oxford University Press on Demand, 1996.*

# The Wiener Index of the Non-commuting Graph for Some Finite Groups

## Theorem 1 [17]

Let  $G$  be the dihedral groups,  $D_{2n}$  of order  $2n$  where  $n \geq 3$ ,  $\Gamma_G$  is the non-commuting graph of  $G$  and  $W(\Gamma_G^{NC})$  is the Wiener index of  $\Gamma_G$ . Then,

$$W(\Gamma_G^{NC}) = \frac{1}{2}(5n^2 - 9n + 4).$$

[17] N.I. Alimon, *Topological Indices of a Class of Graphs of Some Finite Groups and Applications to Molecular Structures*, Ph.D Thesis. Universiti Teknologi Malaysia (2021).

## Proof

For  $n$  is odd and  $n \geq 3$ ,

$$\begin{aligned}W(\Gamma_G^{\text{NC}}) &= \frac{1}{2} [(|G| - |Z(G)|) (|G| - 2|Z(G)| - 2) + |G| (k(G) - |Z(G)|)] \\&= \frac{1}{2} \left[ (2n - 1) (2n - 2(1) - 2) + (2n) \left( \frac{n+3}{2} - 1 \right) \right] \\&= \frac{1}{2} [(2n - 1)(2n - 4) + n(n + 1)] \\&= \frac{1}{2} [4n^2 - 8n - 2n + 4 + n^2 + n] \\&= \frac{1}{2} (5n^2 - 9n + 4).\end{aligned}$$

## Proof (Cont.)

For  $n$  is even and  $n \geq 4$ ,

$$\begin{aligned}W(\Gamma_G^{\text{NC}}) &= \frac{1}{2} [(|G| - |Z(G)|) (|G| - 2|Z(G)| - 2) + |G| (k(G) - |Z(G)|)] \\&= \frac{1}{2} \left[ (2n - 2) (2n - 2(2) - 2) + (2n) \left( \frac{n+6}{2} - 2 \right) \right] \\&= \frac{1}{2} [(2n - 2)(2n - 6) + n(n + 2)] \\&= \frac{1}{2} [4n^2 - 12n - 4n + 12 + n^2 + 2n] \\&= \frac{1}{2} [5n^2 - 14n + 12].\end{aligned}$$



# The Wiener Index of the Non-commuting Graph for Some Finite Groups

## Theorem 2 [18]

Let  $G$  be the generalised quaternion group,  $Q_{4n}$  of order  $4n$  where  $n \geq 2$ ,  $\Gamma_G$  is the non-commuting graph of  $G$  and  $W(\Gamma_G^{NC})$  is the Wiener index of  $\Gamma_G$ . Then,

$$W(\Gamma_G^{NC}) = 2n(5n - 7) + 6.$$

## Theorem 3 [17]

Let  $G$  be the quasidihedral group,  $QD_{2^n}$  of order  $2^n$  where  $n \geq 4$ ,  $\Gamma_G$  is the non-commuting graph of  $G$  and  $W(\Gamma_G^{NC})$  is the Wiener index of  $\Gamma_G$ . Then,

$$W(\Gamma_G^{NC}) = 2^{2n-1} + 2^{2n-3} - 7(2^{n-1}) + 6.$$

[17] N.I. Alimon, *Topological Indices of a Class of Graphs of Some Finite Groups and Applications to Molecular Structures*, Ph.D Thesis. Universiti Teknologi Malaysia (2021).

[18] N.H. Sarmin, N.I. Alimon, and A. Erfanian, *Topological indices of the non-commuting graph for generalised quaternion group*. *Bulletin of the Malaysian Mathematical Sciences Society*, **43(5)** (2020), 3361-3367.

# The Wiener Index of the Coprime Graph for Some Finite Groups

The cases are only limited to  $n$  is an odd prime and  $n = 2^k, k \in \mathbb{Z}$  since the coprime graph associated to the dihedral groups of the other cases of  $n$  cannot be generalized.

## Theorem 4 [19]

Let  $G$  be the dihedral group,  $D_{2n}$  of order  $2n$  where  $n$  is an odd prime. Then, the Wiener index of the coprime graph of  $G$ ,  $\Gamma_G^{\text{CO}}$  is stated as follows :

$$W(\Gamma_G^{\text{CO}}) = (n - 1)(3n - 1) + n.$$

## Proof

- The coprime graph of  $D_{2n}$ , when  $n$  is an odd prime, is  $K_{1,n-1,n}$ .
- Then, the total number of vertices in  $K_{1,n-1,n}$  is  $1 + n - 1 + n = 2n$  vertices. Its coprime graph has three independent sets which are  $1, n - 1$  and  $n$  elements, respectively.

[19] N.I. Alimon, N.H. Sarmin, and A. Erfanian, *The Szeged and Wiener indices for coprime graph of dihedral groups*. In *AIP Conference Proceedings*, **2266(1)** (2020), 060006.



## Proof (Cont.)

- The coprime graph of dihedral groups has  $n^2 + n - 1$  edges where the number of edges for a complete graph,  $K_{2n}$  minus the number of edges for  $K_{n-1}$  and  $K_n$ , as shown in the following.

$$\begin{aligned} |E(K_{1,n-1,n})| &= \frac{2n(2n-1)}{2} - \frac{n(n-1)}{2} - \frac{(n-1)(n-2)}{2} \\ &= n^2 + n - 1. \end{aligned}$$

Thus, there are  $n^2 + n - 1$  edges which have a distance of 1, while  $\frac{(n-1)(n-2)}{2}$  and  $\frac{n(n-1)}{2}$  edges have a distance of 2.

By using the definition of the Wiener index,

$$\begin{aligned} W(\Gamma_G^{\text{CO}}) &= \frac{1}{2} \sum_{i=1}^{2n} \sum_{j=1}^{2n} d(i, j) \\ &= 1 \times [n^2 + n - 1] + 2 \times \left[ \frac{n(n-1)}{2} \right] + 2 \times \left[ \frac{(n-1)(n-2)}{2} \right] \\ &= (n-1)(3n-1) + n. \end{aligned}$$

# The Wiener Index of the Coprime Graph for Some Finite Groups

## Theorem 5 [19]

Let  $G$  be the dihedral group,  $D_{2n}$  of order  $2n$  where  $n = 2^k, k \in \mathbb{Z}^+$ . Then, the Wiener index of the coprime graph for  $G$ ,

$$W(\Gamma_G^{\text{CO}}) = (2n - 1)^2.$$

## Theorem 6 [17]

Let  $G$  be the generalized quaternion group,  $Q_{4n}$  of order  $4n$  where  $n = 2^{k-1}, k \geq 2$ . Then, the Wiener index of the coprime graph for  $G$ ,  $\Gamma_G^{\text{CO}}$  is stated as follows :

$$W(\Gamma_G^{\text{CO}}) = (4n - 1)^2.$$

[17] N.I. Alimon, *Topological Indices of a Class of Graphs of Some Finite Groups and Applications to Molecular Structures*, Ph.D Thesis. Universiti Teknologi Malaysia (2021).

[19] N.I. Alimon, N.H. Sarmin, and A. Erfanian, *The Szeged and Wiener indices for coprime graph of dihedral groups*. In *AIP Conference Proceedings*, **2266(1)** (2020), 060006.

# The Wiener Index of the Coprime Graph for Some Finite Groups

## Theorem 7 [17]

Let  $G$  be the quasidihedral group,  $Q_{2^n}$  of order  $2^n$  where  $n \geq 4$ . Then, the Wiener index of the coprime graph for  $G$ ,  $\Gamma_G^{\text{CO}}$  is stated as follows :

$$W(\Gamma_G^{\text{CO}}) = (2^n - 1)^2.$$

[17] N.I. Alimon, *Topological Indices of a Class of Graphs of Some Finite Groups and Applications to Molecular Structures*, Ph.D Thesis. Universiti Teknologi Malaysia (2021).

# The Zagreb Index of the Non-commuting Graph for Some Finite Groups

## Theorem 8[20]

Let  $G$  be a dihedral group,  $D_{2n}$  of order  $2n$  where  $n \geq 3$ . Then,

$$M_1(\Gamma_G^{NC}) = \begin{cases} n(5n - 4)(n - 1) & \text{if } n \text{ is odd,} \\ n(5n - 8)(n - 2) & \text{if } n \text{ is even.} \end{cases}$$

[20] N.I. Alimon, N.H., Sarmin, and A. Erfanian, *Topological indices of non-commuting graph of dihedral groups. Malaysian Journal of Fundamental and Applied Sciences*, (2018), 473-476.

## Proof

For  $n$  is odd,

$$\begin{aligned}M_1(\Gamma_G^{\text{NC}}) &= |G|^2 (|G| + |Z(G)| - 2k(G)) - \sum_{x \in G - Z(G)} |C_G(x)|^2 \\&= 4n^2 \left[ 2n + 1 - 2 \left( \frac{n+3}{2} \right) \right] - 2^2 n + n^2(n-1) \\&= n(5n-4)(n-1).\end{aligned}$$

For  $n$  is even,

$$\begin{aligned}M_1(\Gamma_G^{\text{NC}}) &= |G|^2 (|G| + |Z(G)| - 2k(G)) - \sum_{x \in G - Z(G)} |C_G(x)|^2 \\&= 4n^2 \left[ 2n + 1 - 2 \left( \frac{n+6}{2} \right) \right] - 4^2 n + n^2(n-2) \\&= n(5n-8)(n-2).\end{aligned}$$



# The Zagreb Index of the Non-commuting Graph for Some Finite Groups

## Theorem 9 [20]

Let  $G$  be a dihedral group,  $D_{2n}$  of order  $2n$  where  $n \geq 3$ . Then,

$$M_2(\Gamma_G^{NC}) = \begin{cases} 2n(n-1)^2(2n-1) & \text{if } n \text{ is odd,} \\ 4n(n-2)^2(n-1) & \text{if } n \text{ is even.} \end{cases}$$

## Theorem 10 [18]

Let  $G$  be the generalised quaternion group,  $Q_{4n}$  of order  $4n$  where  $n \geq 2$ . Then,

$$M_1(\Gamma_G^{NC}) = 8n(5n^2 - 9n + 4),$$

and

$$M_2(\Gamma_G^{NC}) = 32n(2n^3 - 5n^2 + 4n - 1).$$

[20] N.I. Alimon, N.H., Sarmin, and A. Erfanian, *Topological indices of non-commuting graph of dihedral groups. Malaysian Journal of Fundamental and Applied Sciences*, (2018), 473-476.

[18] N.H. Sarmin, N.I. Alimon, and A. Erfanian, *Topological indices of the non-commuting graph for generalised quaternion group. Bulletin of the Malaysian Mathematical Sciences Society*, **43(5)** (2020), 3361-3367.

# The Zagreb Index of the Non-commuting Graph for Some Finite Groups

## Theorem 11 [17]

Let  $G$  be the quasidihedral group,  $QD_{2^n}$  of order  $2^n$  where  $n \geq 4$ . Then,

$$M_1(\Gamma_G^{\text{NC}}) = [5(2^{3n-3}) - 9(2^{2n-1}) + 8(2^n)] ,$$

and

$$M_2(\Gamma_G^{\text{NC}}) = [2^{4n-2} - 5(2^{3n-1}) + 8(3^n) - 8(2^n)] .$$

[17] N.I. Alimon, *Topological Indices of a Class of Graphs of Some Finite Groups and Applications to Molecular Structures*, Ph.D Thesis. Universiti Teknologi Malaysia (2021).

# The Szeged Index of the Non-commuting Graph for $D_{2n}$

## Theorem 12 [21]

Let  $G$  be the dihedral groups,  $D_{2n}$ , where  $n \geq 3$ . Then, the Szeged index of the non-commuting graph of  $G$ ,

$$Sz(\Gamma_G^{\text{NC}}) = \begin{cases} n(n-1)(n-\frac{1}{2}), & \text{if } n \text{ is odd,} \\ 2n(n-2)(n-1), & \text{if } n \text{ is even.} \end{cases}$$

## Proof

By Proposition 2.1, the non-commuting graph of  $D_{2n}$  is

$$\Gamma_G^{\text{NC}} = \begin{cases} \underbrace{K_{1,1,\dots,1,n-1}}_{n \text{ times}}, & \text{if } n \text{ is odd,} \\ \underbrace{K_{2,2,\dots,2,n-2}}_{\frac{n}{2} \text{ times}}, & \text{if } n \text{ is even.} \end{cases}$$

[21] N.I. Alimon, N.H., Sarmin, and A. Erfanian, *On the Szeged index and its non-commuting graph*, *Jurnal Teknologi*, **85(3)** (2023), 105-110.



## Proof (Cont.)

For  $n$  is odd and  $n \geq 3$ :

- There are  $n(n-1)$  edges have  $n_1(e|\Gamma) = 1$  since there is only one element of vertices which is closer to a vertex  $a^i$  of the edge than the other vertex of the edge,  $a^j b$ , where  $i = \{1, 2, \dots, n-1\}$  and  $j = \{0, 1, \dots, n-1\}$ .
- Then,  $n_2(e|\Gamma) = n-1$  since  $n-1$  elements of vertices which are closer to  $a^k b$  than to  $a^l b$ , where  $k, l = \{0, 1, \dots, n-1\}$ . Meanwhile, the rest of the edges have  $n_1(e|\Gamma) = n_2(e|\Gamma) = 1$ .

Thus, by definition of the Szeged index,

$$\begin{aligned} Sz(\Gamma_G^{\text{NC}}) &= n(n-1)[1 \times (n-1)] + (|E(\Gamma_G)| - n(n-1))[1 \times 1] \\ &= n(n-1)(n-1) + \left[ \frac{|G|^2 - k(G)|G|}{2} - n(n-1) \right] \\ &= n(n-1)(n-1) + \left[ 2n^2 - n \frac{n+3}{2} - n(n-1) \right] \\ &= n(n-1)^2 + \frac{n^2}{2} - \frac{n}{2} \\ &= n(n-1) \left( n - \frac{1}{2} \right). \end{aligned}$$

## Proof (Cont.)

For  $n$  is even and  $n \geq 3$  :

- There are  $2n(n-2)$  edges have  $n_1(e|\Gamma) = 2$  since there are two elements of vertices which are closer to a vertex of edge,  $a^i$  than the other vertex of edge,  $a^j b$ , where  $a^i$  is non-central elements and  $j = \{0, 1, \dots, n-1\}$ .

Then,  $n_2(e|\Gamma) = n-2$  since there is  $n-2$  elements of vertices which are closer to  $a^k b$  than to  $a^l b$ , where  $k, l = \{0, 1, \dots, n-1\}$ . Meanwhile, the rest of the edges have  $n_1(e|\Gamma) = n_2(e|\Gamma) = 2$ . Thus, by the definition of Szeged index:

$$\begin{aligned}Sz(\Gamma_G) &= n(n-2)[2 \times (n-2)] + (|E(\Gamma_G)| - n(n-2))[2 \times 2] \\ &= 2n(n-2)(n-2) + \left[ \frac{|G|^2 - k(G)|G|}{2} - n(n-2) \right] [2 \times 2] \\ &= 2n(n-2)^2 + \left[ \frac{4n^2 - (n+6)(n)}{2} - n(n-2) \right] [2 \times 2] \\ &= 2n(n-2)^2 + 2[4n^2 - n(n+6) - 2n(n-2)] \\ &= 2n(n-2)^2 + 2n(n-2) \\ &= 2n(n-2)(n-1).\end{aligned}$$

# The Szeged Index of the Non-commuting Graph for $Q_{4n}$ and $QD_{2n}$

## Theorem 13 [21]

Let  $G$  be the generalized quaternion groups,  $Q_{4n}$  where  $n \geq 2$ . Then, the Szeged index of the non-commuting graph of  $G$ ,

$$Sz(\Gamma_G^{\text{NC}}) = 8n(2n - 1)(n - 1).$$

## Theorem 14 [21]

Let  $G$  be the quasidihedral groups of order  $2^n$  where  $n \geq 4$ . Then, the Szeged index of the non-commuting graph of  $G$ ,

$$Sz(\Gamma_G^{\text{NC}}) = [2^{3n-2} - 3(2^{2n-1}) + 2^{n+1}].$$

[21] N.I. Alimon, N.H., Sarmin, and A. Erfanian, *On the Szeged index and its non-commuting graph*, *Jurnal Teknologi*, **85(3)** (2023), 105-110.

# The Szeged Index of the Coprime Graph for $D_{2n}$

## Theorem 15 [19]

Let  $G$  be the dihedral groups,  $D_{2n}$  where  $n \geq 3$ . If  $n$  is an odd prime, then the Szeged index of the coprime graph of  $G$ ,

$$Sz(\Gamma_G^{CO}) = n^4 - 2n^3 + 3n^2 - 2n + 1.$$

## Theorem 16 [19]

Let  $G$  be a dihedral group,  $D_{2n}$  and  $\Gamma_G^{CO}$  is coprime graph of  $G$ . Then, if  $n = 2^k$ , where  $k \in \mathbb{Z}^+$ , the Szeged index of coprime graph for  $D_{2n}$  is as follows :

$$Sz(\Gamma_G^{CO}) = 4n^2 - 4n + 1.$$

[19] N.I. Alimon, N.H. Sarmin, and A. Erfanian, *The Szeged and Wiener indices for coprime graph of dihedral groups*. In *AIP Conference Proceedings*, **2266(1)** (2020), 060006.

# The Szeged Index of the Coprime Graph for $Q_{4n}$ and $QD_{2n}$

## Theorem 17 [17]

Let  $G$  be the generalized quaternion groups of order  $4n$  where  $n \geq 2$ . If  $n = 2^{k-1}$ ,  $k \geq 2$ , then the Szeged index of the coprime graph of  $G$ ,

$$Sz(\Gamma_G^{\text{CO}}) = (4n - 1)^2.$$

## Theorem 18 [17]

Let  $G$  be the quasidihedral groups of order  $2^n$  where  $n \geq 4$ . Then, the Szeged index of the coprime graph of  $G$ ,

$$Sz(\Gamma_G^{\text{CO}}) = (2^n - 1)^2.$$

[17] N.I. Alimon, *Topological Indices of a Class of Graphs of Some Finite Groups and Applications to Molecular Structures*, Ph.D Thesis. Universiti Teknologi Malaysia (2021).

# The Harary Index of the Non-commuting Graph for $D_{2n}$

## Theorem 19 [22]

Let  $G$  be a dihedral group,  $D_{2n}$  and  $\Gamma_G^{NC}$  is a non-commuting graph of  $G$ . Then,

$$H(\Gamma_G^{NC}) = \begin{cases} \frac{1}{4} [(n-2)(7n-3) + n] & \text{if } n \text{ is even,} \\ \frac{1}{4} [(n-1)(7n-2)] & \text{if } n \text{ is odd.} \end{cases}$$

[22] N.I. Alimon, N.H. Sarmin, and A. Erfanian, *The Harary index of the non-commuting graph for dihedral groups. Southeast Asian Bull. Math.*, **44(6)** (2020), 763-768.

## Proof

The Harary index of the non-commuting graph for dihedral group where  $n$  is odd is the same as half of the total of all entries in its distance matrix,  $D^r$ . The entries of  $D^r$  in this case is either 1 if two vertices are connected or  $\frac{1}{2}$  if two vertices are not connected to each other. Hence, for  $n$  is even and  $n \geq 4$ ,

$$\begin{aligned}H(\Gamma_G^{NC}) &= |E(\Gamma_G^{NC})| + \left(\frac{1}{2} \times \frac{n}{2}\right) + \left(\frac{(n-2)(n-3)}{2} \times \frac{1}{2}\right) \\&= \frac{3}{2}n(n-2) + \frac{n}{4} + \frac{(n-2)(n-3)}{4} \\&= \frac{1}{4} [6n(n-2) + n + (n-2)(n-3)] \\&= \frac{1}{4} [(n-2)(7n-3) + n],\end{aligned}$$

## Proof (Cont.)

for  $n$  is odd and  $n \geq 3$ ,

$$\begin{aligned}H(\Gamma_G^{NC}) &= |E(\Gamma_G^{NC})| + \frac{1}{2} \left[ \frac{(n-1)(n-2)}{2} \right] \\&= \frac{3n}{2}(n-1) + \frac{1}{4}(n-1)(n-2) \\&= \frac{1}{2}(n-1) \left( 3n + \frac{1}{2}(n-2) \right) \\&= \frac{1}{2}(n-1) \left( \frac{7}{2}n - 1 \right) \\&= \frac{1}{4}(n-1)(7n-2).\end{aligned}$$

Therefore,

$$H(\Gamma_G^{NC}) = \begin{cases} \frac{1}{4}[(n-2)(7n-3) + n], & \text{if } n \text{ is even,} \\ \frac{1}{4}(n-1)(7n-2), & \text{if } n \text{ is odd.} \end{cases} \blacksquare$$



# The Harary Index of the Non-commuting Graph for $Q_{4n}$ and $QD_{2n}$

## Theorem 20 [22]

Let  $G$  be the generalized quaternion groups of order  $4n$ , where  $n \geq 2$ . Then, the Harary index of the non-commuting graph of  $G$ ,

$$H(\Gamma_G^{\text{NC}}) = 7n^2 - 8n + \frac{3}{2}.$$

## Theorem 21 [22]

Let  $G$  be the quasidihedral groups of order  $2^n$ , where  $n \geq 4$ . Then, the Harary index of the non-commuting graph of  $G$ ,

$$H(\Gamma_G^{\text{NC}}) = 7(2^{2n-4}) - 2^{n+1} + \frac{3}{2}.$$

[22] N.I. Alimon, N.H. Sarmin, and A. Erfanian, *The Harary index of the non-commuting graph for dihedral groups*. *Southeast Asian Bull. Math*, **44**(6) (2020), 763-768.

# The Randić Index of the Non-commuting Graph for Some Finite Groups

## Theorem 22 [23]

Let  $G$  be a dihedral group,  $D_{2n}$  and  $\Gamma_G^{NC}$  is a non-commuting graph of  $G$ . Then,

$$R(\Gamma_G^{NC}) = \begin{cases} \frac{n\sqrt{2n(n-1)+4n(n-1)}}{4\sqrt{2n(n-1)}} & \text{if } n \text{ is odd,} \\ \frac{n\sqrt{2n(n-2)+4n(n-2)}}{4\sqrt{2n(n-2)}} & \text{if } n \text{ is even.} \end{cases}$$

[23] S.R.D. Rosly, N.F.A.Z. Ab Halem, N.S.S. Zailani, and N.I. Alimon, *Generalization of Randić index of the non-commuting graph for a family of finite groups*. *Malaysian Journal of Fundamental and Applied Sciences*, Malaysian Journal of Fundamental and Applied Sciences, **19(5)** (2023), 762-768.

# The Randić Index of the Non-commuting Graph for Some Finite Groups

## Theorem 23 [23]

Let  $G$  be the generalised quaternion group,  $Q_{4n}$  and  $\Gamma_G^{NC}$  is a non-commuting graph of  $G$ . Then,

$$R(\Gamma_G^{NC}) = \frac{4n(n-1)}{\sqrt{8n(n-1)}} + \frac{n}{2}.$$

## Theorem 24 [23]

Let  $G$  be the quasidihedral group,  $QD_{2^n}$  and  $\Gamma_G^{NC}$  is a non-commuting graph of  $G$ . Then,

$$R(\Gamma_G^{NC}) = \frac{2^{n-1}(2^{n-1} - 2)}{\sqrt{(2^{n-1})(2^n - 4)}} + \frac{2^{n-1}(2^{n-2} - 1)}{2^n - 4}.$$

[23] S.R.D. Rosly, N.F.A.Z. Ab Halem, N.S.S. Zailani, and N.I. Alimon, *Generalization of Randić index of the non-commuting graph for a family of finite groups*. *Malaysian Journal of Fundamental and Applied Sciences*, Malaysian Journal of Fundamental and Applied Sciences, **19(5)** (2023), 762-768.

# The Sombor Index of the Non-commuting Graph for Some Finite Groups

## Theorem 25 [24]

Let  $\Gamma_G$  be the non-commuting graph of  $G$  where  $G$  is the dihedral groups of order  $2n$ ,  $n \geq 3$ . Then, the Sombor index of  $\Gamma_G^{NC}$ ,

$$SO(\Gamma_G^{NC}) = \begin{cases} (n-1)[\sqrt{2}(n-1) + \sqrt{4(n-1)^2 + n^2}] & \text{if } n \text{ is odd,} \\ (n-2)[\sqrt{2}(n-2) + \sqrt{4(n-2)^2 + n^2}] & \text{if } n \text{ is even.} \end{cases}$$

## Theorem 26 [24]

Let  $\Gamma_G$  be the non-commuting graph of  $G$  where  $G$  is the generalized quaternion groups of order  $4n$ ,  $n \geq 2$ . Then, the Sombor index of  $\Gamma_G^{NC}$ ,

$$SO(\Gamma_G^{NC}) = n(n-1)[\sqrt{2}(n-1) + \sqrt{4(n-1)^2 + n^2}].$$

[24] S.M.S. Khasraw, N.H. Sarmin, N.I. Alimon, N. Najmuddin, and G. Semil @ Ismail, *The Sombor index and Sombor polynomial of the power graph associated to some finite groups*, *Journal of Advanced Research in Applied Sciences and Engineering Technology*, **42(2)**, (2024), 112-121.

# The Sombor Index of the Non-commuting Graph for Some Finite Groups

## Theorem 27 [24]

Let  $\Gamma_G$  be the non-commuting graph of  $G$  where  $G$  is the quasidihedral groups of order  $2^n$ ,  $n \geq 4$ . Then, the Sombor index of  $\Gamma_G^{NC}$ ,

$$SO(\Gamma_G^{NC}) = \sqrt{2}(2^n - 4) (2^{2n-3} - 2^{n+1}) + (2^{2n-2} - 2) \sqrt{(2^n - 4)^2 + 2^{2n-2}}.$$

[24] S.M.S. Khasraw, N.H. Sarmin, N.I. Alimon, N. Najmuddin, and G. Semil @ Ismail, *The Sombor index and Sombor polynomial of the power graph associated to some finite groups*, *Journal of Advanced Research in Applied Sciences and Engineering Technology*, **42(2)**, (2024), 112-121.

# TOPOLOGICAL INDICES OF GRAPHS ASSOCIATED TO RINGS

# Preliminaries in Ring Theory

## Ring [25]

A ring  $R$  is defined as a nonempty set with two operations  $+$ ,  $\cdot : R \times R \rightarrow R$  with the properties, such as for all  $a, b, c \in R$ :

- 1  $\langle R, + \rangle$  is an Abelian group;
- 2  $\langle R, \cdot \rangle$  is associative, i.e.  $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ ;
- 3 the distributive laws are valid:  $(a + b) \cdot c = (a \cdot c) + (b \cdot c)$  and  $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ .

## Commutative Ring [26]

A ring  $R$  is commutative if  $ab = ba$  for all  $a, b \in R$

[25] J.B. Fraleigh, *A first course in abstract algebra*, Pearson Education India, 2003.

[26] R. Wisbauer, *Foundations of module and ring theory*, Routledge, 2018.

# The First Zagreb Index of the Zero Divisor Graph for the Ring $\mathbb{Z}_{p^k}$

## Proposition 1 [27]

Let  $p$  be a prime number,  $k \in \mathbb{N}$  and  $a \in \mathbb{Z}_{p^k}$  with  $\gcd(a, p^k) = p^i$  for  $i = 1, 2, \dots, k$ . Then, the degree of vertex  $a$  of the zero divisor graph for the ring  $\mathbb{Z}_{p^k}$  is

$$\deg(a) = \begin{cases} p^i - 1, & \text{for } i \leq \left\lfloor \frac{k-1}{2} \right\rfloor, \\ p^i - 2, & \text{for } i > \left\lfloor \frac{k-1}{2} \right\rfloor, \end{cases}$$

where  $k \geq 3$  for  $p = 2$  and  $k \geq 2$  for odd primes  $p$ , and  $\lfloor k \rfloor$  denotes the floor function of  $k$ .

## Proposition 2 [27]

Let  $a \in V(\Gamma(\mathbb{Z}_{p^k}))$  thus  $a \in Z(\mathbb{Z}_{p^k})$  where  $\gcd(a, p^k) = p^i$ . Then  $|V(\mathbb{Z}_{p^k})| = p^{k-i} - p^{k-(i+1)}$  for  $1 \leq i \leq k-1$  where  $k \geq 3$  for  $p = 2$  and  $k \geq 2$  for odd primes  $p$ .



# The First Zagreb Index of the Zero Divisor Graph for the Ring $\mathbb{Z}_{p^k}$

## Theorem 28 [27]

The first Zagreb index of the zero divisor graph for the ring  $\mathbb{Z}_{p^k}$ ,

$$M_1(\Gamma(\mathbb{Z}_{p^k})) = 2(p^{k-1} - p^k) \left(k - 1 + \lceil \frac{k-1}{2} \rceil\right) + (p^k + 1)(p^{k-1} - 1) + 3 \left(p^{\lceil \frac{k-1}{2} \rceil} - 1\right)$$

where  $k \geq 3$  for  $p = 2$  and  $k \geq 2$  for odd primes  $p$ .

## Proof.

Using definition of the first Zagreb index, Proposition 1, and Proposition 2,

$$\begin{aligned} M_1(\Gamma(\mathbb{Z}_{p^k})) &= \sum_{u \in V(\Gamma(\mathbb{Z}_{p^k}))} (\deg(u))^2 \\ &= \sum_{i=1}^{\lfloor \frac{k-1}{2} \rfloor} \left(p^{k-i} - p^{k-(i+1)}\right) (p^i - 1)^2 + \sum_{i=1+\lfloor \frac{k-1}{2} \rfloor}^{k-1} \left(p^{k-i} - p^{k-(i+1)}\right) \\ &\quad (p^i - 2)^2 \end{aligned}$$

# The First Zagreb Index of the Zero Divisor Graph for the Ring $\mathbb{Z}_{p^k}$

Proof.

$$\begin{aligned} &= \sum_{i=1}^{k-1} \left( p^{k-i} - p^{k-(i+1)} \right) (p^{2i} - 2p^i + 1) \\ &\quad + \sum_{i=1+\lfloor \frac{k-1}{2} \rfloor}^{k-1} \left( p^{k-i} - p^{k-(i+1)} \right) (3 - 2p^i). \end{aligned}$$

Using the summation rules and the geometric sequences,

$$\begin{aligned} M_1(\Gamma(\mathbb{Z}_{p^k})) &= 2(p^{k-1} - p^k) \sum_{i=1}^{k-1} 1 + p^k \sum_{i=1}^{k-1} \left( p^i + \frac{1}{p^i} - p^{i-1} - \frac{1}{p^{i+1}} \right) \\ &\quad + 2(p^{k-1} - p^k) \sum_{i=1+\lfloor \frac{k-1}{2} \rfloor}^{k-1} 1 + \sum_{i=1+\lfloor \frac{k-1}{2} \rfloor}^{k-1} \left( \frac{1}{p^i} - \frac{1}{p^{i+1}} \right). \end{aligned}$$

# The First Zagreb Index of the Zero Divisor Graph for the Ring $\mathbb{Z}_{p^k}$

Proof.

Therefore, the first Zagreb index of the zero divisor graph for the ring  $\mathbb{Z}_{p^k}$ ,

$$M_1(\Gamma(\mathbb{Z}_{p^k})) = 2(p^{k-1} - p^k) \left( k - 1 + \left\lceil \frac{k-1}{2} \right\rceil \right) + (p^k + 1)(p^{k-1} - 1) + 3(p^{\lceil \frac{k-1}{2} \rceil} - 1),$$

where  $k \geq 3$  for  $p = 2$  and  $k \geq 2$  for odd primes  $p$ . □

[27] G. Semil @ Ismail, N.H. Sarmin, N.I. Alimon, and F. Maulana, *The first Zagreb index of zero divisor graph for the ring of integers modulo power of primes*, *Malaysian Journal of Fundamental and Applied Sciences*, **19(5)** (2023), 892-900.

# The First Zagreb Index of the Zero Divisor Graph for the Ring $\mathbb{Z}_{2^k q}$

## Theorem 29 [28]

The first Zagreb index of the zero divisor graph for the ring  $\mathbb{Z}_{2^k q}$ ,  $M_1(\Gamma(\mathbb{Z}_{p^k})) = (q-1) [(2^k - 2^{k-1})(q - 2k + 1) + (2^k - 1)(2^k + 2^{k-1} - 1) - 1] + q^2 2^k (2^{k-2} - 1) + (2^{k-1} q - 2)^2 - 2q(2^k - 2^{k-1})(k - 2 + \lceil \frac{k-3}{2} \rceil) - 2 + 2^{k-1} + 3 \left( \frac{2^k - 2^{\lfloor \frac{k+3}{2} \rfloor}}{2^{\lfloor \frac{k+1}{2} \rfloor}} \right)$  where  $q$  is an odd prime number and  $k$  is a positive integer.

[28] G. Semil @ Ismail, N.H. Sarmin, N.I. Alimon, and F. Maulana, *The First Zagreb Index of the Zero Divisor Graph for the Ring of Integers Modulo  $2^k q$* , AIP Conf. Proc., In press.

# The General Zeroth-Order Randić Index of the Zero Divisor Graph for $\mathbb{Z}_{p^k}$

## Theorem 30 [29]

The general zeroth-order Randić index of the zero divisor graph for  $\mathbb{Z}_{p^k}$  when  $\alpha = 1$ ,  $R_1^0(\Gamma(\mathbb{Z}_{p^k})) = (p^k - p^{k-1})(k-1) - p^{\lceil \frac{k-1}{2} \rceil} - p^{k-1} + 2$  where  $k \geq 3$  for  $p = 2$  and  $k \geq 2$  for odd prime  $p$ .

Notice that the general zeroth-order Randić index of the zero divisor graph for the ring  $\mathbb{Z}_{p^k}$  when  $\alpha = 2$  is equal to the first Zagreb index of the zero divisor graph for the ring  $\mathbb{Z}_{p^k}$ ,  $R_2^0(\Gamma(\mathbb{Z}_{p^k})) = M_1(\Gamma(\mathbb{Z}_{p^k}))$ , as shown in the following theorem.

## Theorem 31 [29]

The general zeroth-order Randić index of the zero divisor graph for  $\mathbb{Z}_{p^k}$  when  $\alpha = 2$ ,  $R_2^0(\Gamma(\mathbb{Z}_{p^k})) = M_1(\Gamma(\mathbb{Z}_{p^k})) = 2(p^{k-1} - p^k)(k-1 - \lceil \frac{k-1}{2} \rceil) + p^{2k-1} + p^{k-1} - p^k + 3(p^{\lceil \frac{k-1}{2} \rceil} - 1) - 1$  where  $k \geq 3$  for  $p = 2$  and  $k \geq 2$  for odd prime  $p$ .

# The General Zeroth-Order Randić Index of the Zero Divisor Graph for $\mathbb{Z}_{p^k}$

## Theorem 32 [29]

The general zeroth-order Randić index of the zero divisor graph for  $\mathbb{Z}_{p^k}$  when  $\alpha = 3$ ,  $R_3^0(\Gamma(\mathbb{Z}_{p^k})) = \frac{p^{3k-1} - p^{k+1}}{p+1} + 3(p^k - p^{k-1})(k - 1 + 3\lceil \frac{k-1}{2} \rceil) - 6p^{2k-1} + 3p^k - p^{k-1} + 3p^{\lfloor \frac{3k-1}{2} \rfloor} - 7p^{\lfloor \frac{k}{2} \rfloor} + 8$  where  $k \geq 3$  for  $p = 2$  and  $k \geq 2$  for odd prime  $p$ .

[29] G. Semil @ Ismail, N.H. Sarmin, N.I. Alimon, and F. Maulana, *General zeroth-order Randić index of zero divisor graph for the ring of integers modulo  $p^k$* , AIP Conf. Proc. **2975**, 020002 (2023).

# The General Zeroth-Order Randić Index of $\Gamma(\mathbb{Z}_p \times \mathbb{Z}_{q^2})$

## Theorem 33 [30]

The general zeroth-order Randić index of  $\Gamma(\mathbb{Z}_p \times \mathbb{Z}_{q^2})$ ,  $R_\alpha^0(\Gamma(\mathbb{Z}_p \times \mathbb{Z}_{q^2})) = (q-1)(pq-2)^\alpha + (p-1)(q^2-1)^\alpha + (q^2-q)(p-1)^\alpha + (pq-p-q+1)(q-1)^\alpha$  where  $p, q$  are primes and  $\alpha \in \mathbb{R}$ .

[30] Nurhabibah, A.G. Syarifudin, I.M. Alamsyah, E. Suwastika, N.H. Sarmin, N.I. Alimon, and G. Semil @ Ismail, *Topological Indices of the Zero Divisor Graph of Direct Product of Integers Modulo Ring*, AIP Conf. Proc., Submitted.

# CONCLUSION

- Some topological indices, which are the Wiener index, the Zagreb index, the Szeged index, the Harary index and the Randić index of some graphs associated to some finite groups are found.
- Based on the results, the higher the order of the groups, the higher the value of topological indices. This is due to the increasing of the number of vertices and edges.
- The degree-based topological indices, which are the first Zagreb index and the general zeroth-order Randić index of the zero divisor graph for the rings  $\mathbb{Z}_{p^k}$  and  $\mathbb{Z}_{2^k q}$  are determined.



# RECOMMENDATIONS

- Other types of topological indices of some graphs associated to groups can be computed.
- Similar to other types of topological indices of some graphs associated to rings can also be computed.
- The topological indices for graphs representing chemical structures in drugs are a valuable approach to determine both the physicochemical properties and biological activities of these molecules can be constructed.

# REFERENCES I



I. Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer Science and Business Media, 2012.



K. Wiener, *Structural determination of paraffin boiling points*, *Journal of the American Chemical Society*, **69(1)** (1947), 17-20.



H. Hosoya, *Topological index. A newly proposed quantity characterizing the topological nature of structural isomers of saturated hydrocarbons*, *Bulletin of the Chemical Society of Japan*, **44(9)** (1971), 2332–2339.



A. Azad and M. Eliasi, *Distance in the non-commuting graph of groups*. *Ars Comb.* **99** (2011), 279-287.



I. Gutman and N. Trinajstić, *Graph theory and molecular orbitals. Total  $\pi$ -electron energy of alternant hydrocarbons*, *Chemical Physics Letters*, **17(4)** (1972), 535-538.



M. Mizargar and A. Ashrafi, *Some distance-based topological indices of a non-commuting graph*. *Hacetatepe Journal of Mathematics and Statistics*. **41(4)** (2012), 515-526.



P.V. Khadikar, N.V. Deshpande, V. Narayan, P. Kale, P. Prabhakar, A. Dobrynin, I. Gutman, and G. Domotor, *The Szeged index and an analogy with the Wiener index*, *Journal of Chemical Information and Computer Sciences*, **35(3)** (1995), 547-550.



D. Plavšić, S. Nikolić, N. Trinajstić, and Z. Mihalić, *On the Harary index for the characterization of chemical graphs*, *Journal of Mathematical Chemistry*, **12** (1993), 235-250.



M. Randić, *Characterization of molecular branching*. *Journal of the American Chemical Society*, **97(23)** (1975), 6609-6615.

# REFERENCES II



H. Ahmed, A.A. Bhatti, and A. Ali, *Zeroth-order general Randić index of cactus graphs*. *AKCE International Journal of Graphs and Combinatorics*, **16(2)** (2019), 182-189.



I. Gutman, *Geometric approach to degree-Based topological indices: Sombor indices*. *MATCH Commun. Math. Comput. Chem.*, **86** (2021), 11–16.



A. Abdollahi, S. Akbari, H. Maimani, *Non-commuting graph of a group*. *Journal of Algebra*. **298** (2006), 468-492.



R. Mahmoud, *Energy and Laplacian Energy of Graphs Related to a Family of Finite Groups*, Ph.D Thesis, Universiti Teknologi Malaysia. 2018.



X.L. Ma, H.Q. Wei, and L.Y. Yang, *The coprime graph of a group*. *International Journal of Group Theory*, **3(3)** (2014), 13-23.



D.F. Anderson and P.S. Livingston, *The zero-divisor graph of a commutative ring*. *Journal of Algebra*, **217(2)** (1999), 434–447.



J. Humphreys, *A course in group theory, vol. 6*, Oxford University Press on Demand, 1996.



N.I. Alimon, *Topological Indices of a Class of Graphs of Some Finite Groups and Applications to Molecular Structures*, Ph.D Thesis. Universiti Teknologi Malaysia (2021).



N.H. Sarmin, N.I. Alimon, and A. Erfanian, *Topological indices of the non-commuting graph for generalised quaternion group*. *Bulletin of the Malaysian Mathematical Sciences Society*, **43(5)** (2020), 3361-3367.

# REFERENCES III



N.I. Alimon, N.H. Sarmin, and A. Erfanian, *The Szeged and Wiener indices for coprime graph of dihedral groups*. In *AIP Conference Proceedings*, **2266(1)** (2020), 060006.



N.I. Alimon, N.H., Sarmin, and A. Erfanian, *Topological indices of non-commuting graph of dihedral groups*. *Malaysian Journal of Fundamental and Applied Sciences*, (2018), 473-476.



N.I. Alimon, N.H., Sarmin, and A. Erfanian, *On the Szeged index and its non-commuting graph*, *Jurnal Teknologi*, **85(3)** (2023), 105-110.



N.I. Alimon, N.H. Sarmin, and A. Erfanian, *The Harary index of the non-commuting graph for dihedral groups*. *Southeast Asian Bull. Math*, **44(6)** (2020), 763-768.



S.R.D. Roslly, N.F.A.Z. Ab Halem, N.S.S. Zailani, and N.I. Alimon, *Generalization of Randić index of the non-commuting graph for a family of finite groups*. *Malaysian Journal of Fundamental and Applied Sciences*, **19(5)** (2023), 762-768.



S.M.S. Khasraw, N.H. Sarmin, N.I. Alimon, N. Najmuddin, and G. Semil @ Ismail *The Sombor index and Sombor polynomial of the noncommuting graph associated to some finite groups*, *Journal of Advanced Research in Applied Sciences and Engineering Technology*, **42(2)** (2024), 112-121.



J.B. Fraleigh, *A first course in abstract algebra*, Pearson Education India, 2003.



R. Wisbauer, *Foundations of module and ring theory*, Routledge, 2018.



G. Semil @ Ismail, N.H. Sarmin, N.I. Alimon, and F. Maulana, *The first Zagreb index of zero divisor graph for the ring of integers modulo power of primes*, *Malaysian Journal of Fundamental and Applied Sciences*, **19(5)** (2023), 892-900.

# REFERENCES IV



G. Semil @ Ismail, N.H. Sarmin, N.I. Alimon, and F. Maulana, *The First Zagreb Index of the Zero Divisor Graph for the Ring of Integers Modulo  $2^k q$* , AIP Conf. Proc., In press.



G. Semil @ Ismail, N.H. Sarmin, N.I. Alimon, and F. Maulana, *General zeroth-order Randić index of the zero divisor graph for some commutative rings*, AIP Conf. Proc. **2905**, 070005 (2024).



Nurhabibah, A.G. Syarifudin, I.M. Alamsyah, E. Suwastika, N.H. Sarmin, N.I. Alimon, and G. Semil @ Ismail, *Topological Indices of the Zero Divisor Graph of Direct Product of Integers Modulo Ring*, AIP Conf. Proc., Submitted.

# PUBLICATION

<https://people.utm.my/nizarmin/journal-papers/>

<https://people.utm.my/nizarmin/conference-proceedings/>



- 1 Sanhan Muhammad Salih Khasraw, [Nor Haniza Sarmin](#), Nur Idayu Alimon, Nabilah Najmuddin and Ghazali Semil @ Ismail. "[Sombor Index and Sombor Polynomial of the Noncommuting Graph Associated to Some Finite Groups](#)", Journal of Advanced Research in Applied Sciences and Engineering Technology, 42(2): 112-121, 2024.
- 2 Fariz Maulana, Muhammad Zulfikar Aditya, Erma Suwastika, Intan Muchtadi-Alamisyah, Nur Idayu Alimon, [Nor Haniza Sarmin](#). "[On The Topological Indices of Zero Divisor Graphs of Some Commutative Rings](#)", Journal of Applied Mathematics and Informatics, 42(3): 663-680, 2024.
- 3 Nur Idayu Alimon, [Nor Haniza Sarmin](#) and Ahmad Erfanian. "[On the Szeged Index and Its Noncommuting Graph](#)", Jurnal Teknologi, 85(3), 105-110, 2023.

- 4 Nur Idayu Alimon, [Nor Haniza Sarmin](#) and Ahmad Erfanian. “[On Topological Indices of a Graph Associated to the Direct Product of Two Groups](#)”, Kongzhi yu Juece/Control and Decision Journal, 38(03), 1173-1181, 2023.
- 5 [Nor Haniza Sarmin](#), Nur Idayu Alimon and Ahmad Erfanian. “[Topological Indices of the Non-commuting Graph for Generalised Quaternion Group](#)”, Bulletin of the Malaysian Mathematical Sciences Society, 43, 3361–3367, 2020.
- 6 Nur Idayu Alimon, [Nor Haniza Sarmin](#) and Ahmad Erfanian, “[The Harary Index of the Non-commuting Graph for Dihedral Groups](#)”, Southeast Asian Bulletin of Mathematics, 44(6), 763-768, 2020.
- 7 Nur Idayu Alimon, [Nor Haniza Sarmin](#), Ahmad Erfanian. “[The Topological Indices of the Non-commuting Graph for Symmetric Groups](#)”, ASM Sc. J., Vol. 13, 2020, 1-5.
- 8 Nur Idayu Alimon, [Nor Haniza Sarmin](#) and Ahmad Erfanian, “[The Szeged and Wiener Indices for Coprime Graph of Dihedral Groups](#)”, AIP Conference Proceedings 2266, 060006, 2020.
- 9 Nur Idayu Alimon, [Nor Haniza Sarmin](#), Ahmad Erfanian. “[The Wiener and Zagreb Indices of Conjugacy Class Graph of the Dihedral Groups](#)”, MATEMATIKA, 35(1), 51-57, 2019.





- 1 Ghazali Semil @ Ismail, [Nor Haniza Sarmin](#), Nur Idayu Alimon and Fariz Maulana. "General Zeroth-Order Randić Index of the Zero Divisor Graph for Some Commutative Rings", AIP Conf. Proc. 2905, 070005, 2024.
- 2 Ghazali Semil @ Ismail, [Nor Haniza Sarmin](#), Nur Idayu Alimon and Fariz Maulana. "The First Zagreb Index of the Zero Divisor Graph for the Ring of Integers Modulo Power of Primes", Malaysian Journal of Fundamental and Applied Sciences, 19(5), 892-900, 2023.
- 3 Ghazali Semil @ Ismail, [Nor Haniza Sarmin](#), Nur Idayu Alimon and Fariz Maulana. "General Zeroth-Order Randić Index of Zero Divisor Graph for the Ring of Integers Modulo  $p^n$ ", AIP Conference Proceedings, 2975, 020002(1-6), 2023.

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# Active Collaborator on Topological Indices of Graphs



**Group Photo with Team Members of the Institut Teknologi Bandung (ITB) International Research Grant**

**Collaborate on the research papers**

# My Collaborators Around the World



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