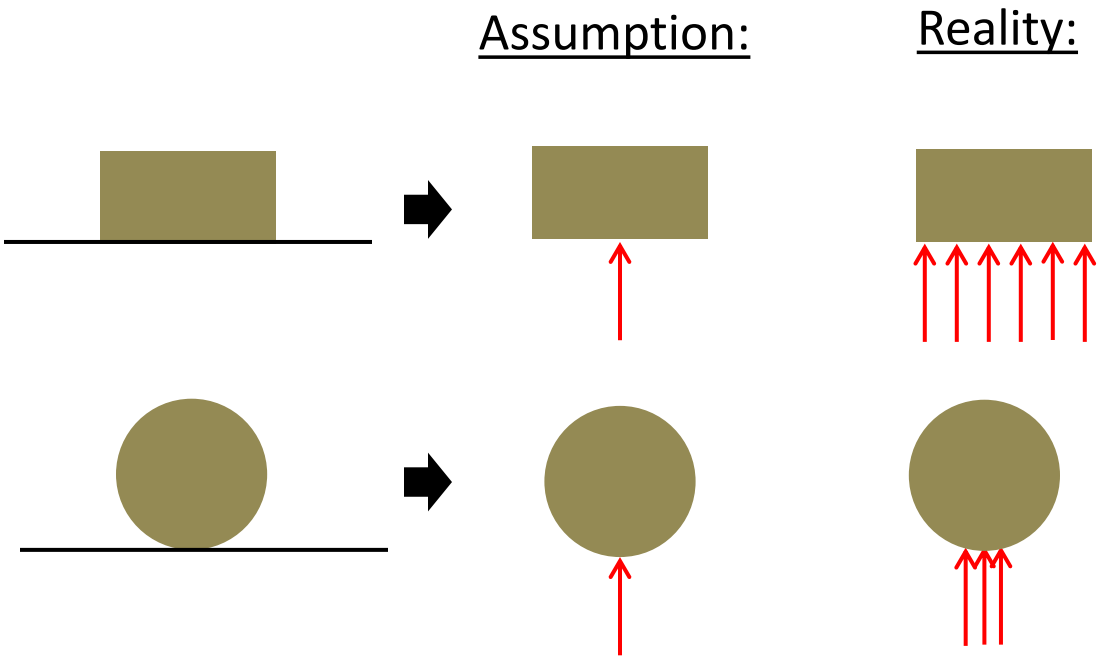
CENTROIDS & CENTRES OF GRAVITY

CHAPTER

5

CENTROIDS & CENTRES OF GRAVITY

Distributed Forces:



Assumption – Attraction by the earth on a body is represented by a single force applied at centre of gravity of the body.

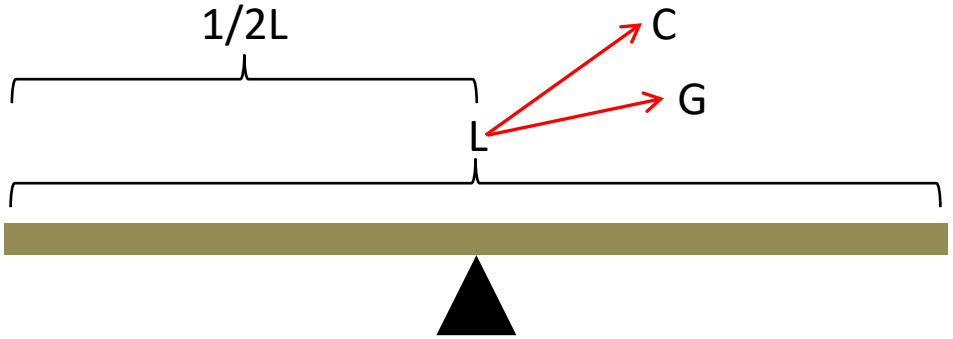
Reality - This single force is actually the resultant of large number of forces distributed over the entire body.

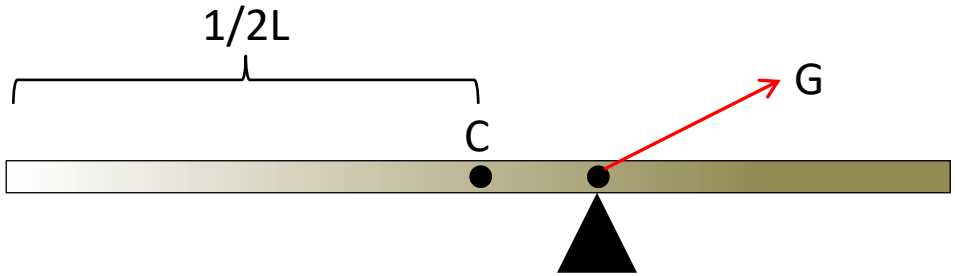
2D – Centroid of Area
- Centroid of Lines

} Normal Method

- Centroid an Area & Lines by Integration
- Theorem of Pappus-Galdinus
- Solving Problems

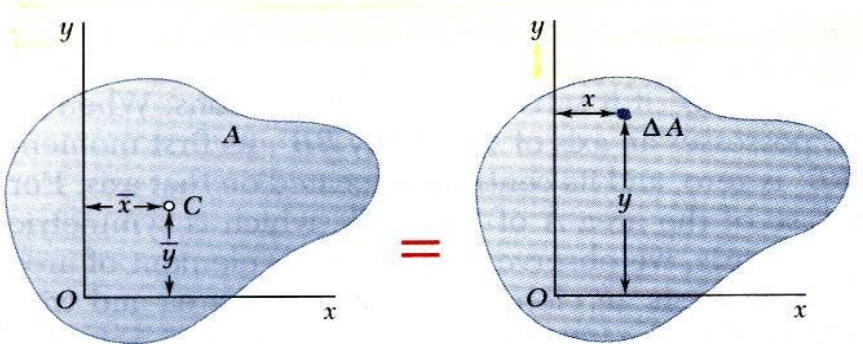
3D – Centre of a 3D Shape
- Centre of Volume





- Not necessary the centre of gravity, G is at the centroid, C/mid point of the body

- Centroid of an Area



$$\bar{x}W = \int x\,dW$$

$$\bar{x}(\gamma A t) = \int x(\gamma t)\,dA$$

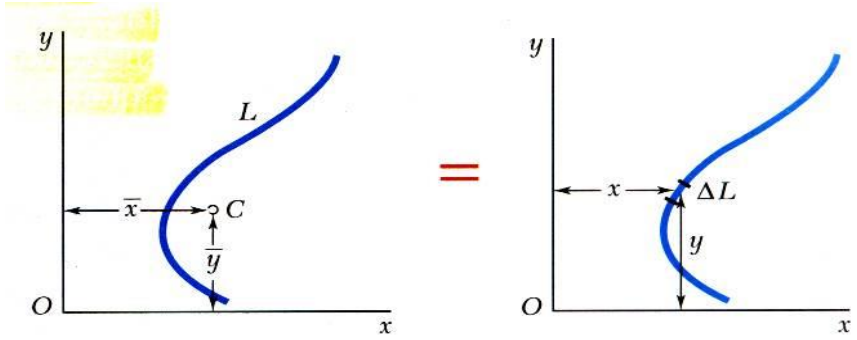
$$\bar{x}A = \int x\,dA = Q_y$$

$$= \text{first moment with respect to y}$$

$$\bar{y}A = \int y\,dA = Q_x$$

$$= \text{first moment with respect to x}$$

- Centroid of a Line



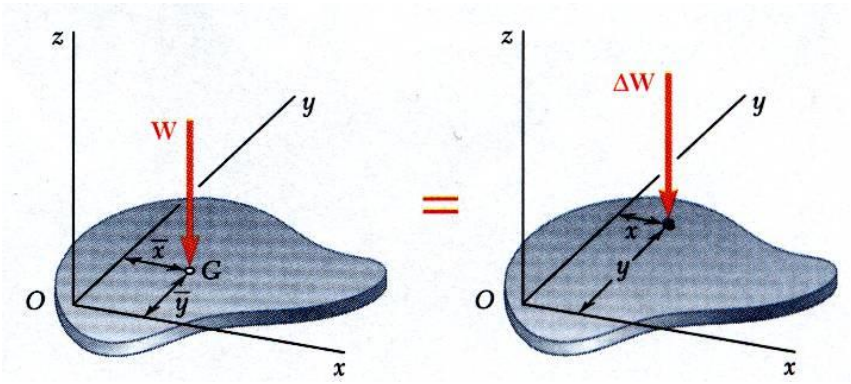
$$\bar{x}W = \int x\,dW$$

$$\bar{x}(\gamma L a) = \int x(\gamma a)\,dL$$

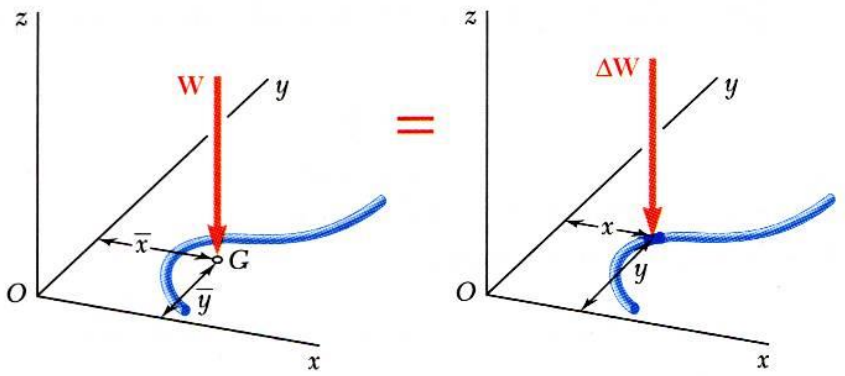
$$\bar{x}L = \int x\,dL$$

$$\bar{y}L = \int y\,dL$$

- Center of gravity of a plate



- Center of gravity of a wire



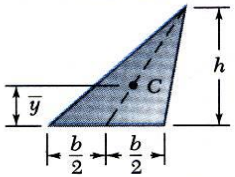
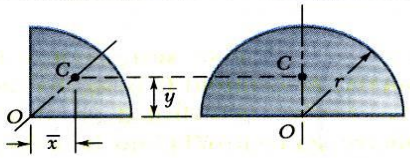
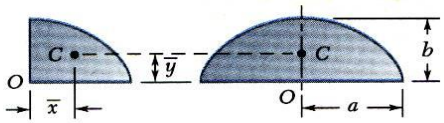
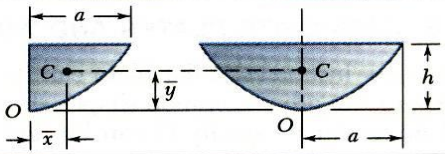
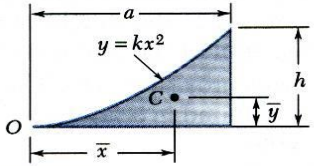
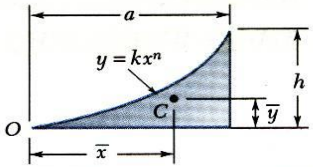
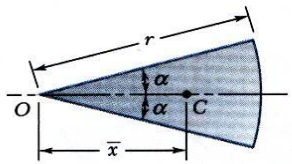
$$\sum M_y \quad \bar{x}W = \sum x\Delta W$$

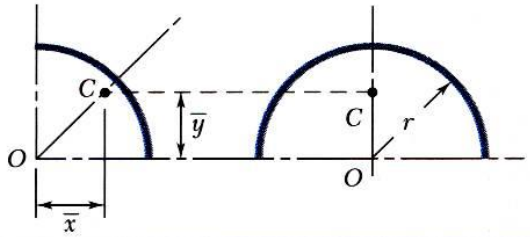
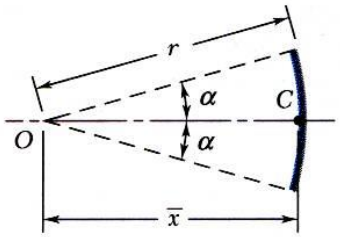
$$= \int x\,dW$$

$$\sum M_y \quad \bar{y}W = \sum y\Delta W$$

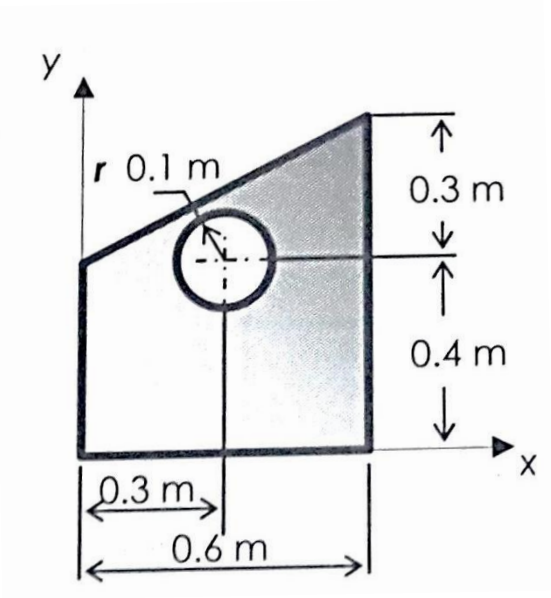
$$= \int y\,dW$$

Common shapes of areas & lines

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

1. Centroid (C)/ Centre of Gravity (G) of An Area



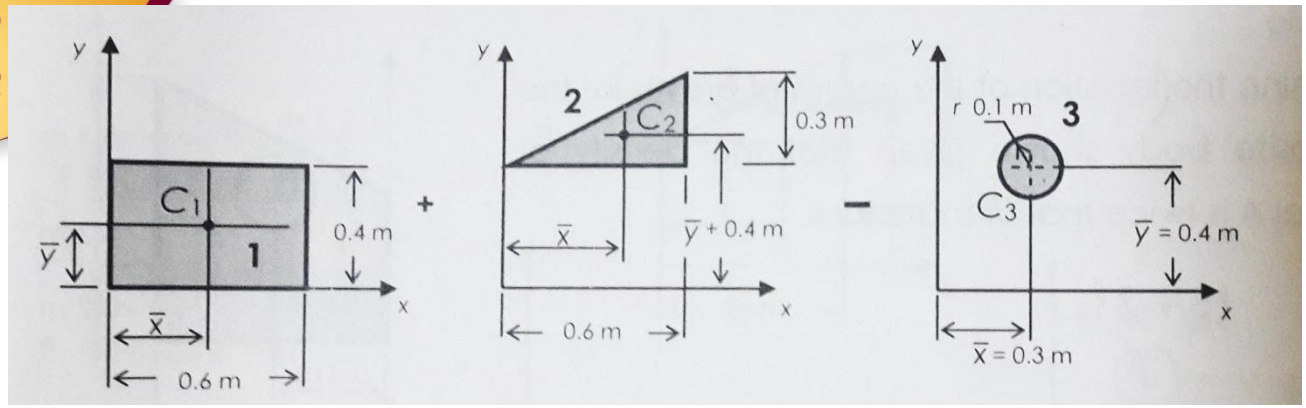
Ex. 1: Determine the location of centroid of the flat plate shown in the diagram.

Solution:

1. Divide into few shapes
2. Construct a table – Find each of the info

   - A (m^2), $\bar{x}$ (m), $\bar{y}$ (m), $\bar{x}$A (m^3), $\bar{y}$A (m^3)

3. Solve using; $\bar{X} = \frac{\sum \bar{x}A}{\sum A}$ $\bar{Y} = \frac{\sum \bar{y}A}{\sum A}$



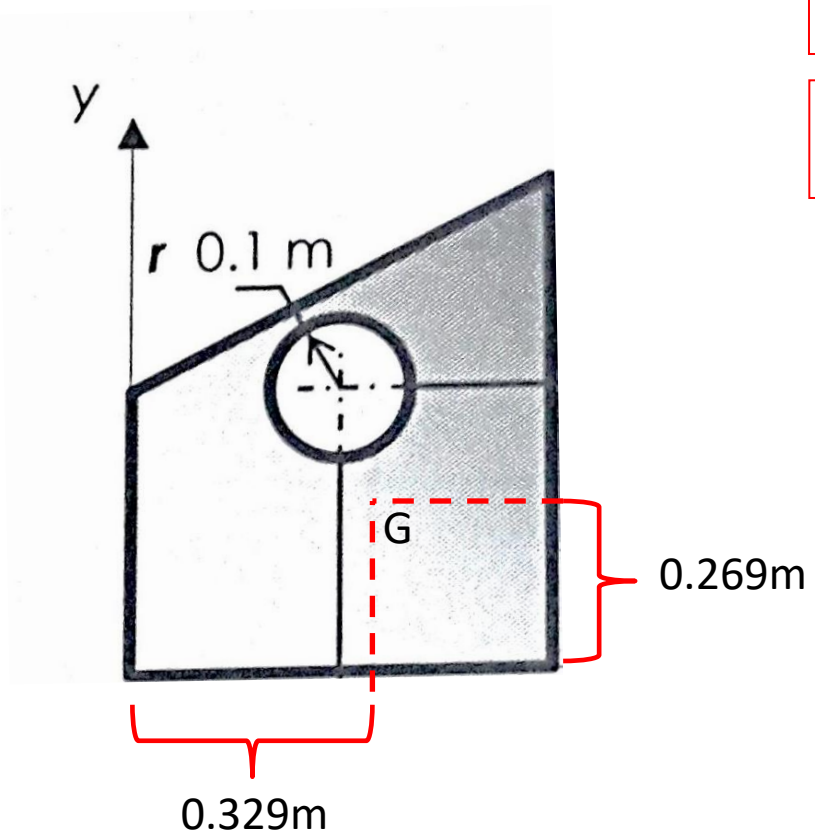
Component	A (m^2)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$A (m^3)	$\bar{y}$A (m^3)
1 (+)	(0.6) (0.4) = 0.24	0.3	0.2	(0.24) (0.3) = 0.072	(0.24) (0.2) = 0.048
2 (+)	½ (0.6) (0.3) = 0.09	2/3 (0.6) = 0.4	0.4 + 1/3(0.3) = 0.5	(0.09) (0.4) = 0.036	(0.09) (0.5) = 0.045
3 (-)	- π $(0.1)^2$ = - 0.0314	0.3	0.4	- (0.0314) (0.3) = 0.00984	- (0.0314) (0.4) = 0.0126
Total	0.2986			0.0982	0.0804

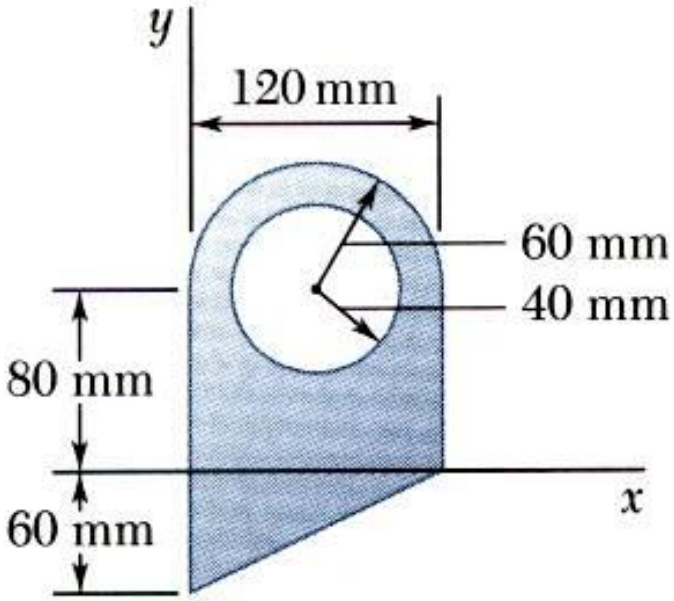
$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{0.0982 \quad m^3}{0.2986 \quad m^2} = 0.329m$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.0804m^{\ 3}}{0.2986 \quad m^2} = 0.269m$$

$$\bar{X} = 0.329\ \text{m}$$

$$\bar{Y} = 0.269\ \text{m}$$





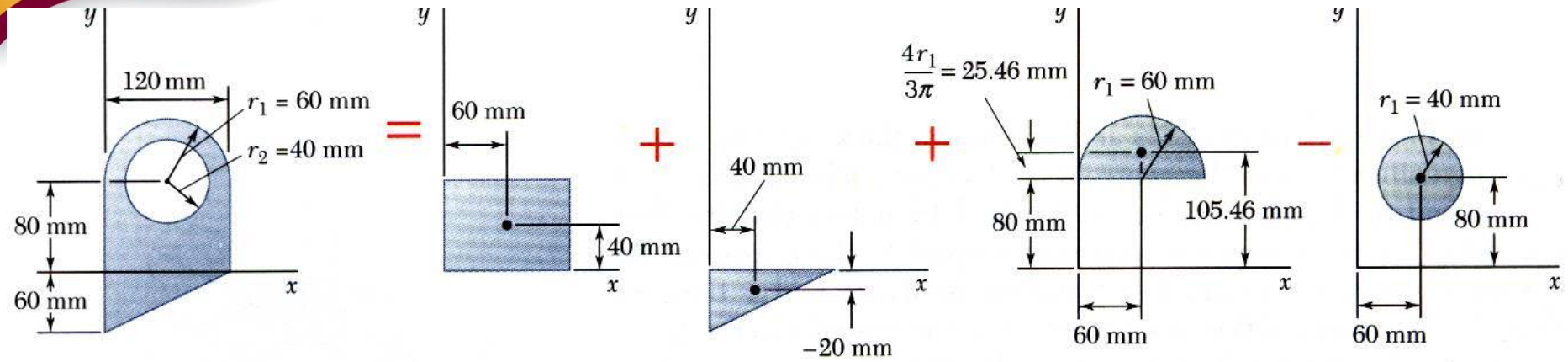
Ex. 2: For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid.

Solution:

1. Divide into few shapes
2. Construct a table – Find each of the info

   - A (m^2), $\bar{x}$ (m), $\bar{y}$ (m), $\bar{x}$A (m^3), $\bar{y}$A (m^3)

3. Solve using; $\bar{X} = \frac{\sum \bar{x}A}{\sum A}$ $\bar{Y} = \frac{\sum \bar{y}A}{\sum A}$



Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma \bar{x}A = +757.7 \times 10^3$	$\Sigma \bar{y}A = +506.2 \times 10^3$

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{+757.7 \times 10^3 \text{ mm}^3}{13.828 \times 10^3 \text{ mm}^2}$$

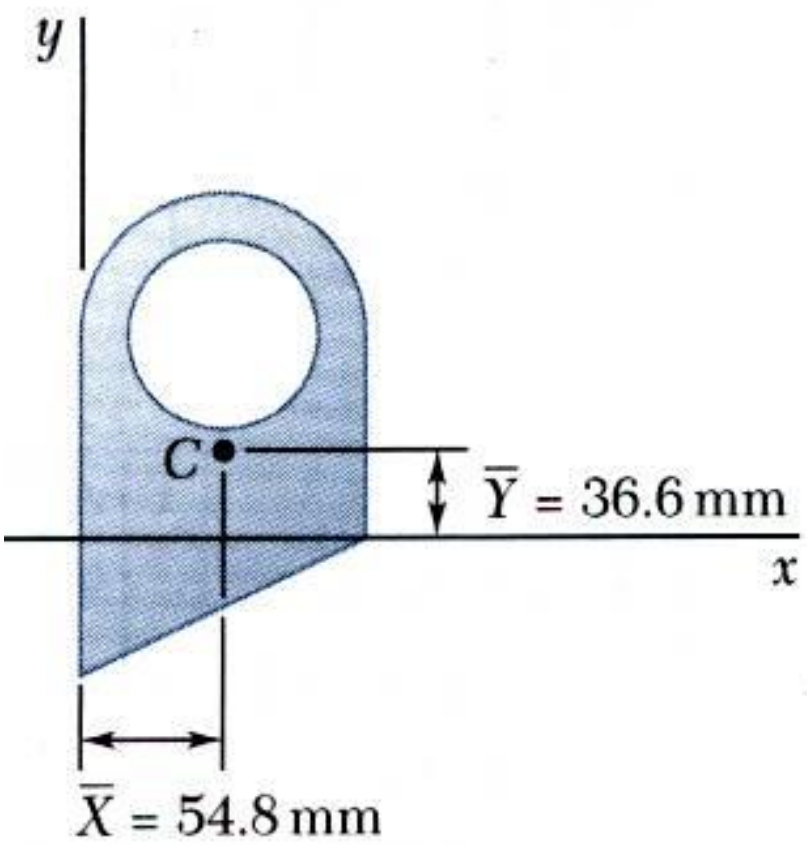
$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{+506.2 \times 10^3 \text{ mm}^3}{13.828 \times 10^3 \text{ mm}^2}$$

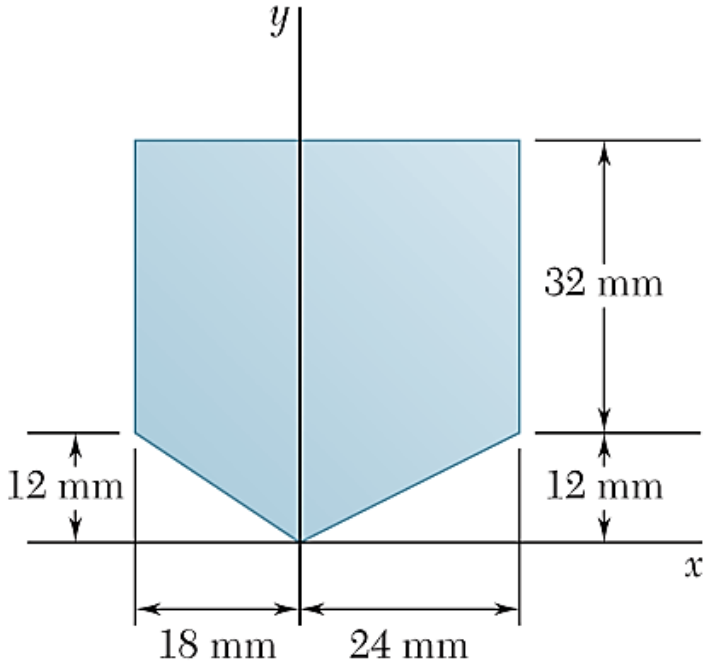
$\bar{X} = 54.8 \text{ mm}$

$\bar{Y} = 36.6 \text{ mm}$

$$\bar{X} = 54.8\,\text{mm}$$

$$\bar{Y} = 36.6\,\text{mm}$$





$\bar{x}$ = 2.84mm

$\bar{y}$ = 24.8mm

Ex. 3: Locate the centroid of the plane area shown

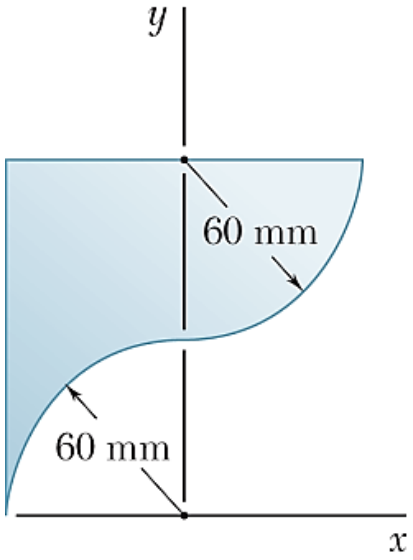
Solution:

1. Divide into few shapes
2. Construct a table – Find each of the info

   - A (m^2), $\bar{x}$ (m), $\bar{y}$ (m), $\bar{x}$A (m^3), $\bar{y}$A (m^3)

3. Solve using;

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} \qquad \bar{Y} = \frac{\sum \bar{y}A}{\sum A}$$



$\bar{x}$ = -10mm

$\bar{y}$ = 87.5mm

Ex. 4: Locate the centroid of the plane area shown

Solution:

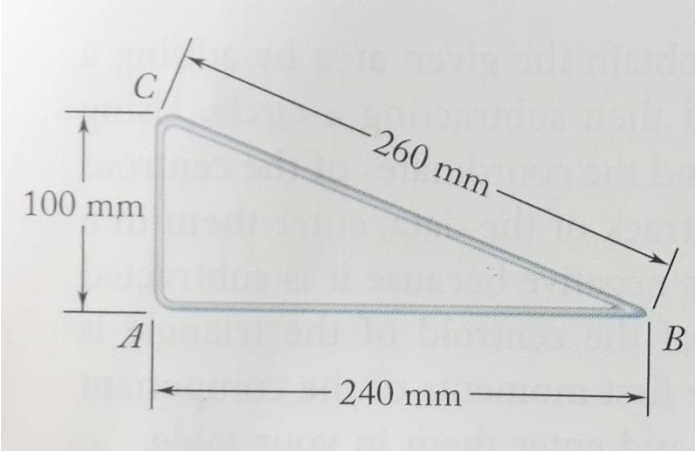
1. Divide into few shapes
2. Construct a table – Find each of the info

   - A (m^2), $\bar{x}$ (m), $\bar{y}$ (m), $\bar{x}$A (m^3), $\bar{y}$A (m^3)

3. Solve using;

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} \qquad \bar{Y} = \frac{\sum \bar{y}A}{\sum A}$$

2. Centroid (C)/ Centre of Gravity (G) of Lines



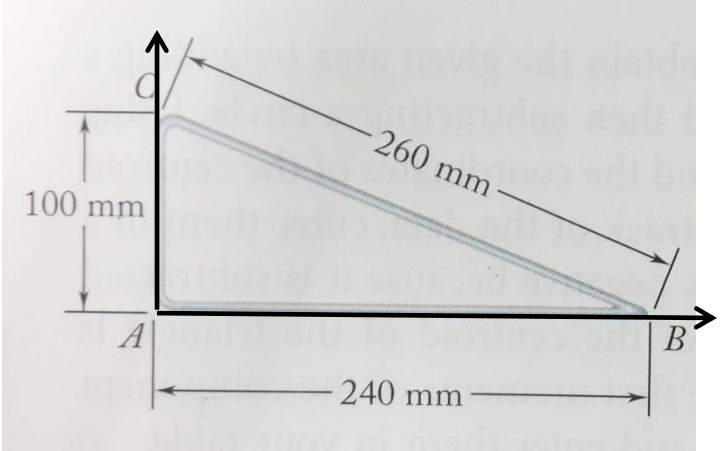
Ex. 5: The figure shown is made from a piece of thin, homogeneous wire. Determine the location of its center of gravity .

Solution:

1. Divide into few lines
2. Construct a table – Find each of the info

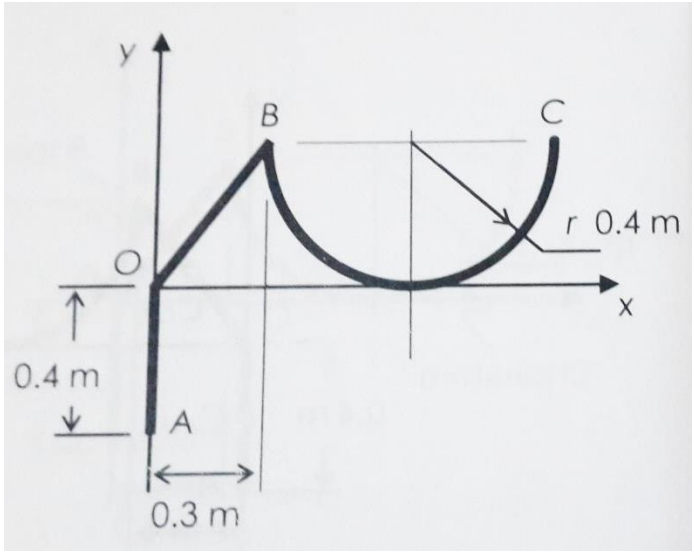
   - L (m), $\bar{x}$(m), $\bar{y}$ (m), $\bar{x}$ L (m^2), $\bar{y}$ L(m^2)

3. Solve using; $\bar{X} = \dfrac{\sum \bar{x}L}{\sum L}$ $\bar{Y} = \dfrac{\sum \bar{y}L}{\sum L}$



Lines	L (m)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$L (m^2)	$\bar{y}$L (m^2)
AC (+)	0.1	0	0.05	0	(0.1) (0.05) = 0.005
AB (+)	0.24	0.12	0	(0.24) (0.12) = 0.0288	0
BC (+)	0.26	0.12	0.05	(0.26) (0.12) = 0.0312	(0.26) (0.05) = 0.013
Total	0.6			0.06	0.018

$$\bar{X} = \frac{\sum \bar{x}\Delta L}{\sum \Delta L} = \frac{0.06 m^2}{0.6 m} = 0.1 m \qquad \bar{Y} = \frac{\sum \bar{y} A}{\sum A} = \frac{0.018 m^2}{0.6\ m} = 0.03 m$$



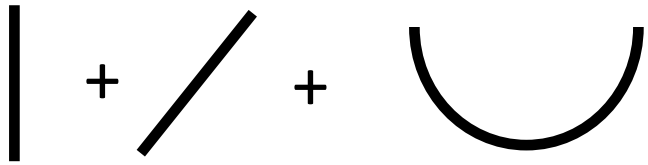
Ex. 6: Determine the location of centroid of the line body shown in the diagram.

Solution:

1. Divide into few lines
2. Construct a table – Find each of the info

   - L (m), $\bar{x}$(m), $\bar{y}$(m), $\bar{x}$ L (m^2), $\bar{y}$ L(m^2)

3. Solve using; $\bar{X} = \dfrac{\sum \bar{x}L}{\sum L}$ $\bar{Y} = \dfrac{\sum \bar{y}L}{\sum L}$

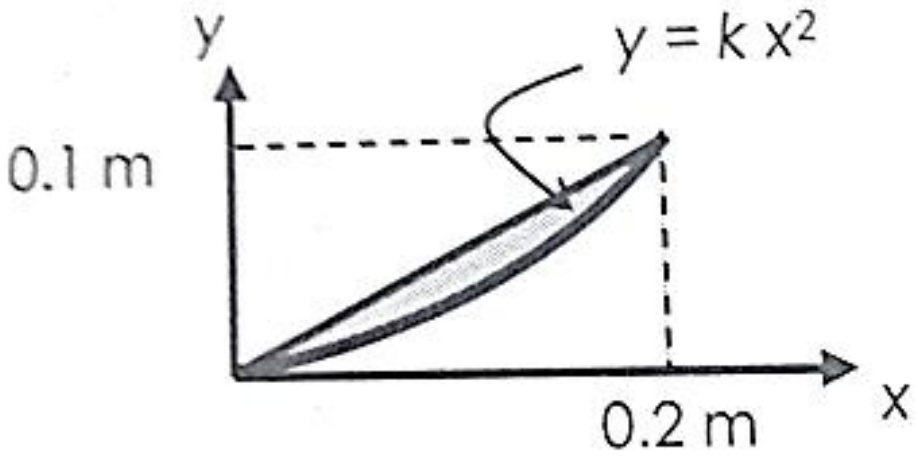


Lines	L (m)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$L (m^2)	$\bar{y}L$ (m^2)
OA (+)	0.4	0	-0.2	(0.4) (0) = 0	(0.4) (-0.2) = -0.08
OB (+)	$\sqrt{(0.3)^2 (0.4)^2}$ = 0.5	0.15	0.2	(0.5) (0.15) = 0.075	(0.5) (0.2) = 0.1
BC (+)	π (0.4) = 1.257	0.3 + 0.4 = 0.7	0.4 - 2(0.4)/π = 0.1453	(1.257) (0.7) = 0.88	(1.257) (0.1453) = 0.1826
Total	2.157			0.955	0.2026

$$\bar{X} = \frac{\sum \bar{x}L}{\sum L} = \frac{0.955 \quad m^2}{2.157 \quad m} = 0.443m$$

$$\bar{Y} = \frac{\sum \bar{y}L}{\sum L} = \frac{0.2026m^{\ 2}}{2.157 \quad m} = 0.096m$$

3.1 Centroid (C)/ Centre of Gravity (G) of an Area by Integration



Ex. 7: Determine the centroid of the shaded area by integration

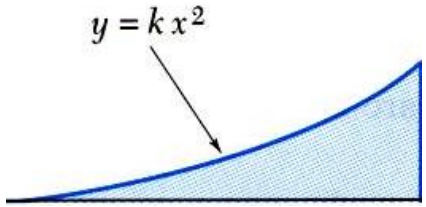
Solution:

1. Divide into few shapes
2. Find A, $\bar{x}$, and $\bar{y}$ of the shape by integration
3. Construct a table – Find each of the info

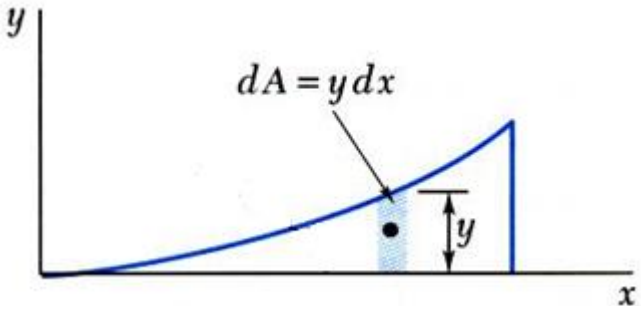
   - A(m^2), $\bar{x}$(m), $\bar{y}$ (m), $\bar{x}$A(m^3), $\bar{y}$A(m^3)
4. Solve using; $\bar{X} = \frac{\sum \bar{x} L}{\sum L}$ $\bar{Y} = \frac{\sum \bar{y} L}{\sum L}$

Vertical Element Method

Horizontal Element Method

1.  - 

2. Vertical Element Method:



1. Choose any point on the graph – Draw a vertical line from chosen point to x-axis.

2. A = x times y: Any change of A must be due to change of x or y - dx

   So, (dA = dx y) OR (dA = y dx)

3. $A = \int dA$

$= \int_0^{0.2} y\ dx$

$= \int_0^{0.2} 2.5x^2 dx$

$= \left[\frac{5}{6}x^3\right]_0^{0.2}$

$= 0.0067$

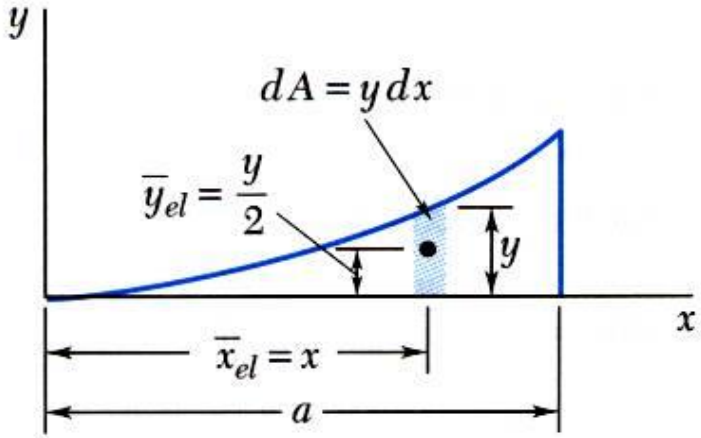
$y = kx^2$

When x = 0.2m and y = 0.1m;

$0.1 = k\,(0.2)^2$

$k = 2.5$

So, $y = 2.5\,x^2$



$$\bar{x}_{el} = x$$

$$\bar{y}_{el} = \frac{y}{2}$$

$$\bar{x}A = \int \bar{x}_{el}\,dA$$

$$= \int x(y\,dx)$$

$$= \int 2.5x^3\,dx$$

$$= \left[\frac{5}{8}x^4\right]_0^{0.2}$$

$$= 0.001\text{m}$$

$$\text{So}, \bar{x} = \frac{0.001}{0.0067}$$

$$= 0.15\text{m}$$

$$\bar{y}A = \int \bar{y}_{el}\,dA$$

$$= \int \frac{y}{2}(y dx)$$

$$= \int \frac{2.5x^2}{2}(2.5x^2)dx$$

$$= \int 3.125x^4\,dx$$

$$= [0.625x5]_0^{0.2}$$

$$= 0.0002\text{m}$$

$$\text{So}, \bar{y} = \frac{0.0002}{0.0067}$$

$$= 0.03\text{m}$$



3.

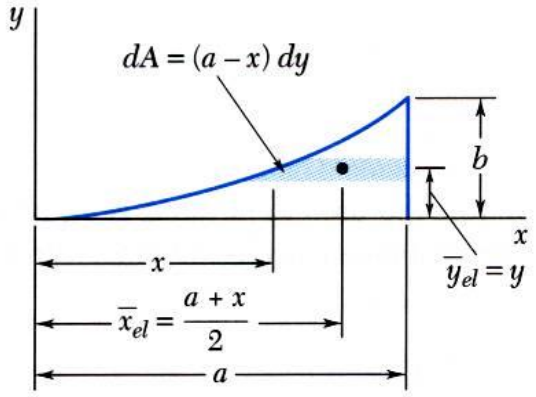
Component	A (m^2)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$A (m^3)	$\bar{y}$A (m^3)
1 (+)	½ (0.2)(0.1) = 0.01	0.133	0.033	(0.01) (0.133) = 0.0013	(0.01) (0.033) = 0.0003
2 (-)	- 0.0067	0.15	0.03	(- 0.0067) (0.15) = - 0.001	(- 0.0067) (0.03) = - 0.0002
Total	0.0033			0.0003	0.0001

4.

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{0.0003}{0.0033} = 0.091 \qquad \bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.0001}{0.0033} = 0.03$$

Centroid = (0.09, 0.03)

2. Horizontal Element Method:



1. Choose any point on the graph – Draw a horizontal line from chosen point to vertical line.

2. A = x times y: Any change of A must be due to change of x or y - dy

   So, (dA = x dy) but must remember, the measurement must be taken from reference y-axis. So, dA = (a-x) dy.

3. $A = \int dA$

$$= \int_0^{0.1} (a - x)\,dy$$

$$= \int_0^{0.1} 0.2 - \sqrt{\frac{y}{2.5}}\,dy$$

$$= \left[0.2y - \frac{y^{3/2}}{2.372} \right]_0^{0.1}$$

$$= 0.02 - 0.0133$$

$$= 0.0067$$

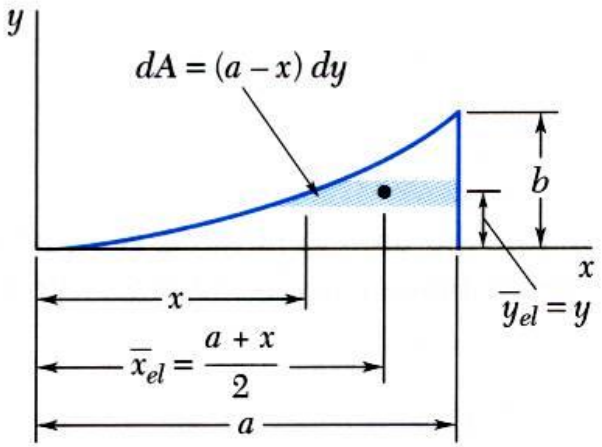
$y = kx^2$

When x = 0.2m and y = 0.1m;

$0.1 = k\,(0.2)^2$

$k = 2.5$

So, $y = 2.5\,x^2$ and $x = \sqrt{(y/2.5)}$



$$\bar{x}_{el} = \frac{a+x}{2}$$

$$\bar{y}_{el} = y$$

$$\bar{x}A = \int \bar{x}_{el}\,dA$$

$$= \int \frac{a+x}{2}(a-x)dy$$

$$= \int \frac{a^2 - x^2}{2}\,dy$$

$$= \int \frac{(0.2)^2 - [(y/2.5)^{1/2}]^2}{2}\,dy$$

$$= \int 0.02 - \frac{y}{5}\,dy$$

$$= \left[0.02y - \frac{y^2}{10}\right]_0^{0.1}$$

$$= 0.002 - 0.001$$

$$= 0.001$$

$$\text{So, } \bar{x} = \frac{0.001}{0.0067}$$

$$= 0.15\text{m}$$

$$\bar{y}A = \int \bar{y}_{el}\,dA$$

$$= \int y(a-x)dy$$

$$= \int y(0.2 - \sqrt{\frac{y}{2.5}})dy$$

$$= \int 0.2y - \frac{y^{3/2}}{\sqrt{2.5}}\,dy$$

$$= \left[0.1y^2 - \frac{y^{5/2}}{3.953}\right]_0^{0.1}$$

$$= 0.0002\text{m}$$

$$\text{So, } \bar{y} = \frac{0.0002}{0.0067}$$

$$= 0.03\text{m}$$



3.

Component	A (m^2)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$A (m^3)	$\bar{y}$A (m^3)
1 (+)	½ (0.2)(0.1) = 0.01	0.133	0.033	(0.01) (0.133) = 0.0013	(0.01) (0.033) = 0.0003
2 (-)	- 0.0067	0.15	0.03	(-0.0067) (0.15) = - 0.001	(-0.0067) (0.03) = - 0.0002
Total	0.0033			0.0003	0.0001

4.

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{0.0003}{0.0033} = 0.091 \qquad \bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.0001}{0.0033} = 0.03$$

Centroid = (0.09, 0.03)

4. Theorem of Pappus-Galdinus

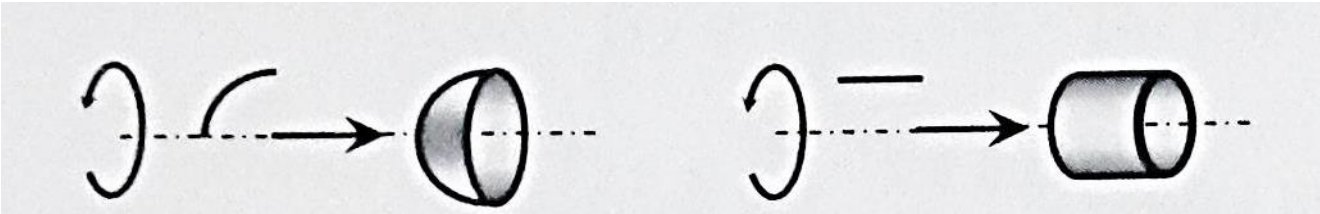
- A theorem about Surface Area & Volume of revolution

Surface Area of revolution – The area produced by generating a line about an axis

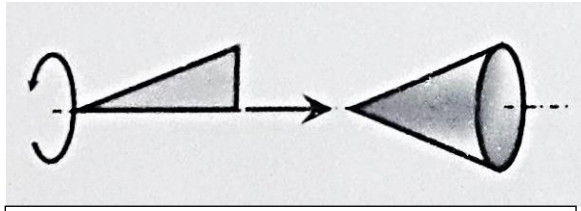


It will generate a hollow cone (an ice-cream)

It will generate a cone with a base



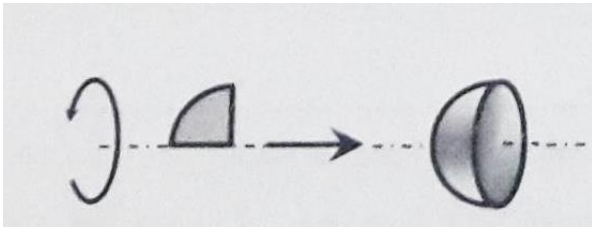
Volume of revolution – The volume produced by generating a shape or an area about an axis

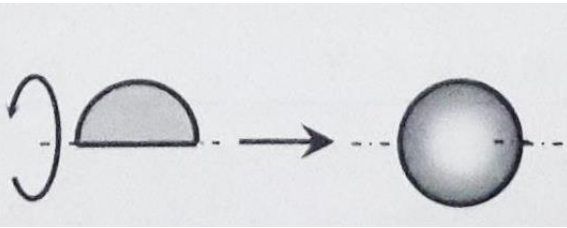


It will generate a volume of a cone



It will generate a volume of a cylinder





Theorem 1:

The generated surface area = The total length of the line x Distance covered by the centroid of that line

If a line rotate about x-axis; $A = 2\pi\bar{y}\sum L = 2\pi \sum \bar{y}L$

If a line rotate about y-axis; $A = 2\pi\bar{x}\sum L = 2\pi \sum \bar{x}L$

Theorem 2:

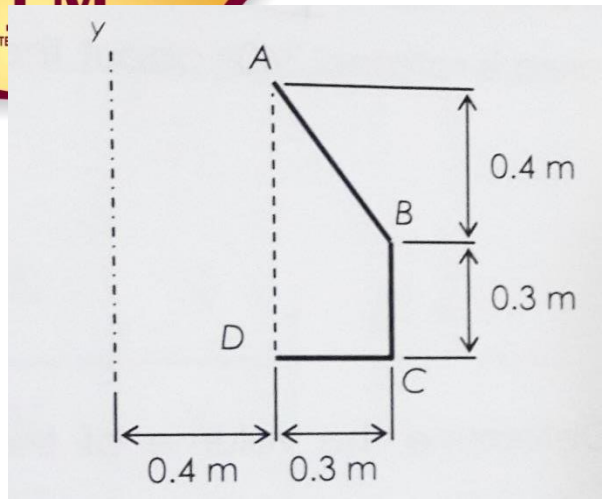
The generated volume = The total area x Distance covered by the centroid of that area

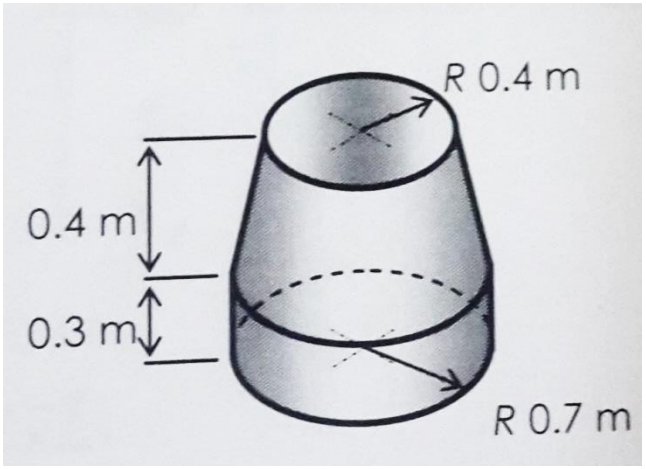
If an area rotate about x-axis; $V = 2\pi\bar{y}\sum A = 2\pi \sum \bar{y}A$

If an area rotate about y-axis; $V = 2\pi\bar{x}\sum A = 2\pi \sum \bar{x}A$

*Revolution of a slanting line about an axis = Surface area

Revolution of an area = Volume





Ex. 8: Determine the centroid of the composite line and surface area generated if the line is rotated 360^o about the y-axis

Solution:

1. Divide into few lines
2. Construct a table – Find each of the info

   - L (m), $\bar{x}$ (m), $\bar{y}$(m), $\bar{x}$L (m^2), $\bar{y}$L(m^2)

3. Solve using; $\bar{X} = \dfrac{\sum \bar{x}L}{\sum L}$ $\bar{Y} = \dfrac{\sum \bar{y}L}{\sum L}$

Lines	L (m)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$L (m^2)	$\bar{y}$L (m^2)
AB (+)	√(0.3)^2 (0.4)^2 = 0.5	0.15+0.4 = 0.55	0.2+0.3 = 0.5	(0.5) (0.55) = 0.275	(0.5) (0.5) = 0.25
BC (+)	0.3	0.7	0.15	(0.3) (0.7) = 0.21	(0.3) (0.15) = 0.045
CD (+)	0.3	0.15 + 0.4 = 0.55	0	(0.3) (0.55) = 0.165	0
Total	1.1			0.65	0.295

$$\bar{X} = \frac{\sum \bar{x}L}{\sum L} = \frac{0.65\ \text{m}^2}{1.1\ \text{m}} = 0.59m \qquad \bar{Y} = \frac{\sum \bar{y}L}{\sum L} = \frac{0.295\text{m}^2}{1.1\text{m}} = 0.27m$$

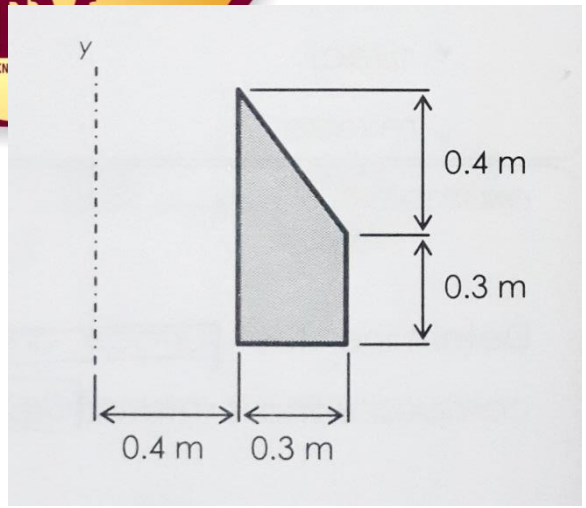
So, Centroid = (0.6, 0.3)

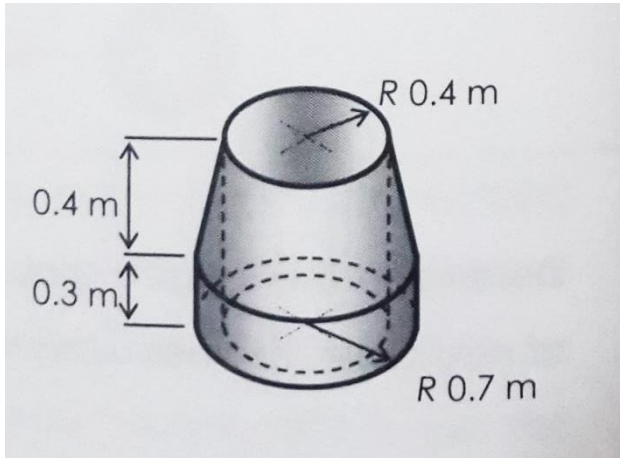
Surface Area: Since the line is rotating about y-axis;

$$A = 2\pi \sum \bar{x}L$$

$$= 2\,\pi\,(0.65)$$

$$= 4.08\text{m}^2$$





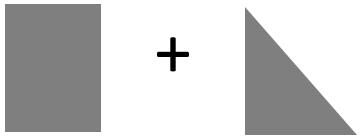
Ex. 9: Determine the centroid and volume generated if the shaded area is rotated about the y-axis

Solution:

1. Divide into few shapes
2. Construct a table – Find each of the info

   - A (m^2), $\bar{x}$(m), $\bar{y}$ (m), $\bar{x}$A (m^3), $\bar{y}$A (m^3)

3. Solve using; $\bar{X} = \dfrac{\sum \bar{x}A}{\sum A}$ $\bar{Y} = \dfrac{\sum \bar{y}A}{\sum A}$



Component	A (m^2)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$A (m^3)	$\bar{y}$A (m^3)
1 (+)	(0.3) (0.3) = 0.09	0.55	0.15	(0.09) (0.55) = 0.0495	(0.09) (0.15) = 0.0135
2 (+)	½ (0.4) (0.3) = 0.06	1/3 (0.3) + 0.4 = 0.5	1/3 (0.4) + 0.3 = 0.433	(0.06) (0.5) = 0.03	(0.06) (0.433) = 0.026
Total	0.15			0.0795	0.0395

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{0.0795\ \text{m}^3}{0.15\ \text{m}^2} = 0.53m$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{0.0395\text{m}^3}{0.15\ \text{m}^2} = 0.263m$$

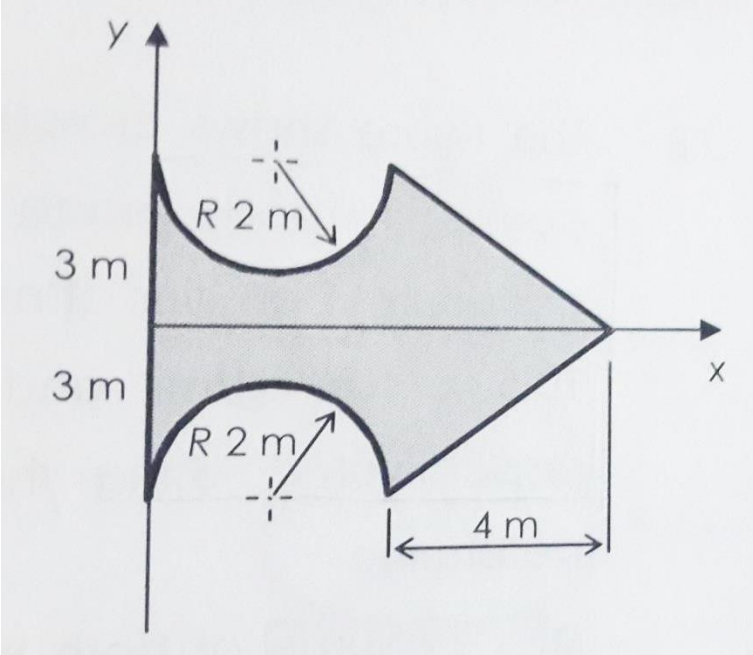
So, Centroid = (0.5, 0.3)

Volume: Since the area is rotating about y-axis;

A = $2\pi \sum \bar{x}A$

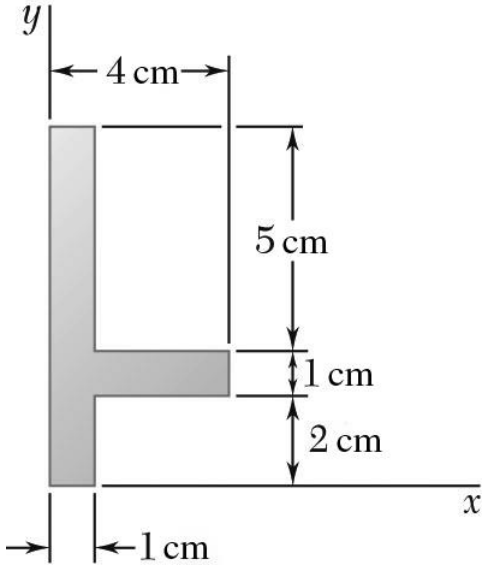
= $2\ \pi\ (0.0795\text{m}^2)$

= 0.5m^3



Ex. 10: Determine the centroid of the shaded area and the volume generated if rotated 90^o about y-axis.

$V = 136.5\ m^3$

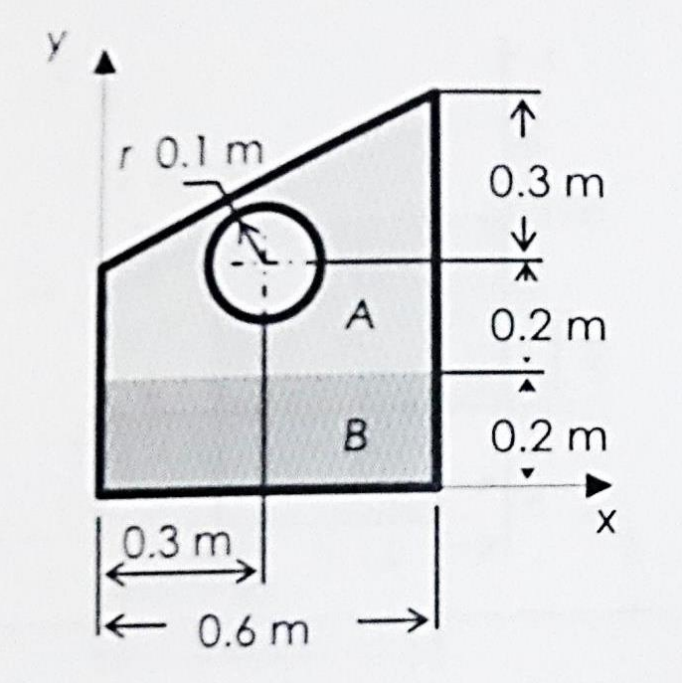


Ex. 11: Determine the volume of the shape shown by rotating the area about (*a*) the *x*-axis, (*b*) the *y*-axis.

(a) V = 248.2 cm^3

(b) V = 72.3 m^3

5. Solving Problems



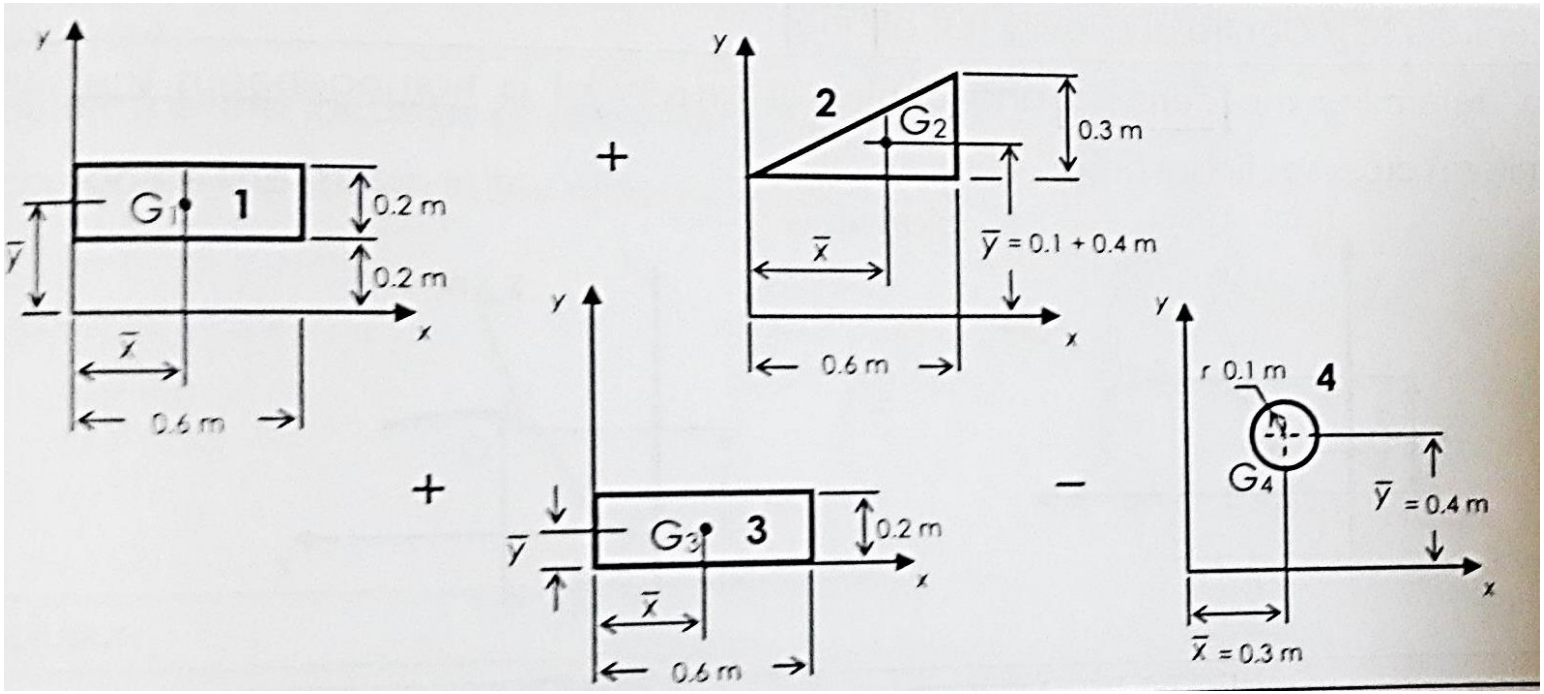
*If given 2 types of materials:

Ex. 12: Determine the location of center of gravity of the composite body shown that the density of material A is twice that of material B.

W = ρ A t g, where t (thickness) and g (9.81 m/s^2) are constant.

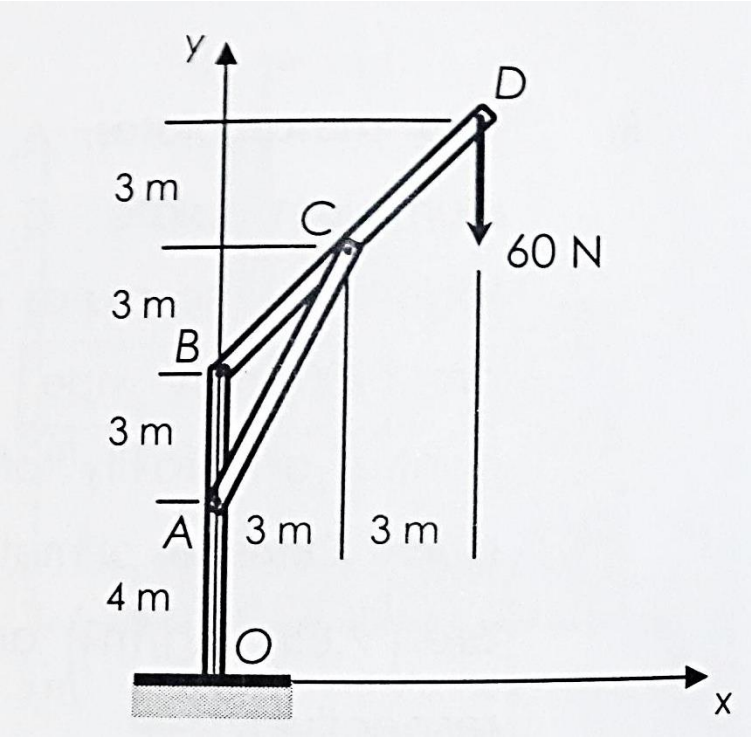
Analysis is focused on ρ and ΔA.

So, $\Delta W = \rho\, \Delta A\, t\, g$ and $\rho_A = 2\, \rho_B$

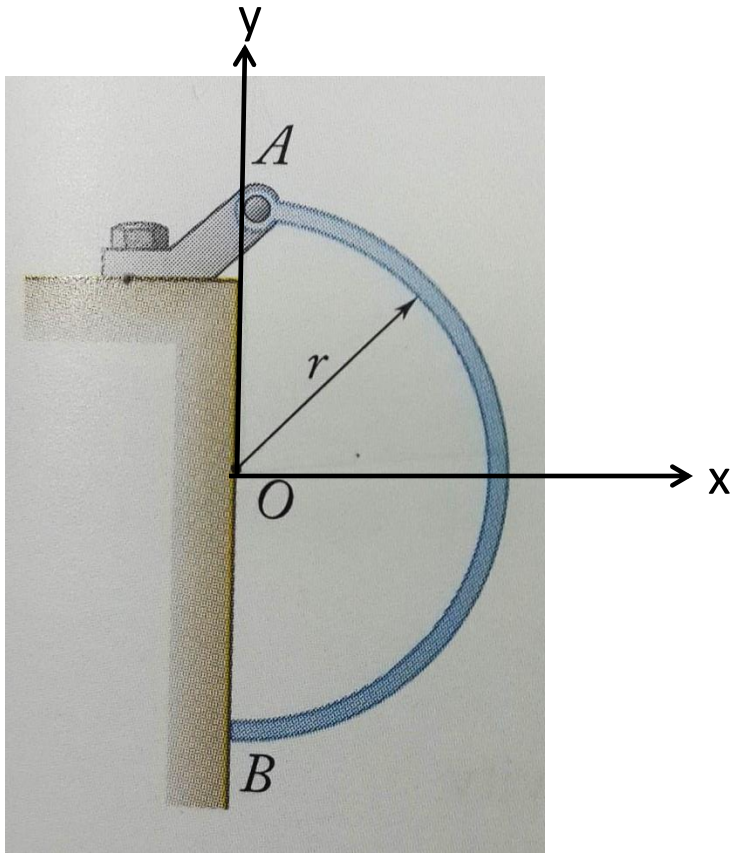


Comp.	W (kg/m)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$W (kg)	$\bar{y}$W (kg)
1 (B) (+)	(0.2) (0.6) ρ_B = 0.12 ρ_B	0.3	0.1	(0.12ρ_B) (0.1) = 0.036ρ_B	(0.12ρ_B) (0.1) = 0.012ρ_B
2 (A) (+)	(0.6) (0.2) ρ_A = 0.12ρ_A = 0.24ρ_B	0.3	0.2 + 0.1 = 0.3	(0.12ρ_A) (0.3) = 0.036ρ_A = 0.072ρ_B	(0.12ρ_A) (0.3) = 0.036ρ_A = 0.072ρ_B
3 (A) (+)	½ (0.6) (0.3) ρ_A = 0.09ρ_A = 0.18ρ_B	2/3 (0.6) = 0.4	1/3 (0.3) + 0.4 = 0.5	(0.09ρ_A) (0.4) = 0.036ρ_A = 0.072ρ_B	(0.09ρ_A) (0.5) = 0.045ρ_A = 0.09ρ_B
4 (A) (-)	- π(0.1)^2 ρ_A = - 0.0314ρ_A = - 0.0628ρ_B	0.3	0.4	-(0.0314ρ_A) (0.3) = - 0.0094ρ_A = - 0.0188ρ_B	(0.0314ρ_A)(0.4)= - 0.0126ρ_A = - 0.0251ρ_B
Total	0.4772ρ_B			0.1612ρ_B	0.1489 ρ_B

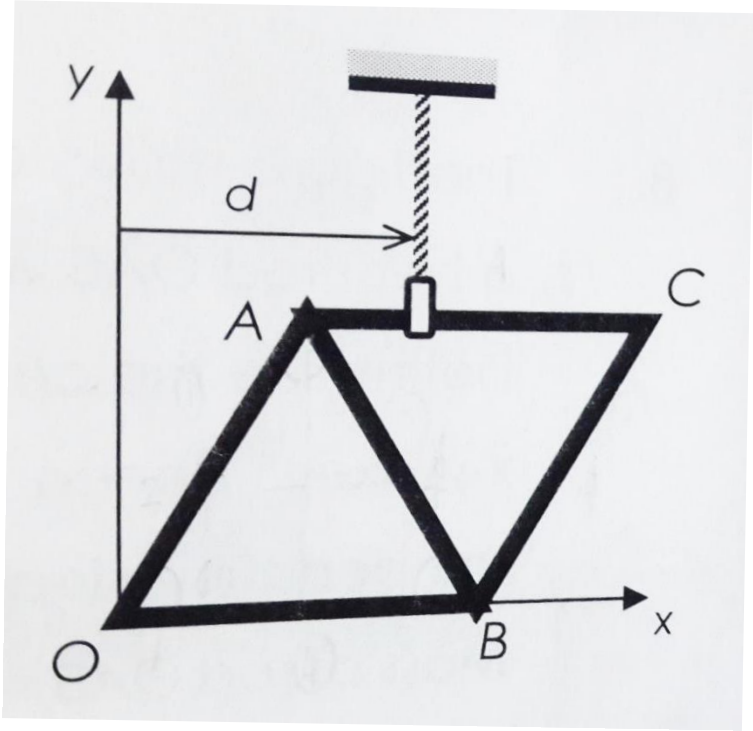
$$\bar{X} = \frac{\sum \bar{x}W}{\sum W} = \frac{0.1612\ \rho_B\ \text{kg}}{0.4772\ \rho_B\ \text{kg/m}} = 0.338\text{m} \qquad \bar{Y} = \frac{\sum \bar{y}W}{\sum W} = \frac{0.1489\ \rho_B\ \text{kg}}{0.4772\ \rho_B\ \text{kg/m}} = 0.312\text{m}$$



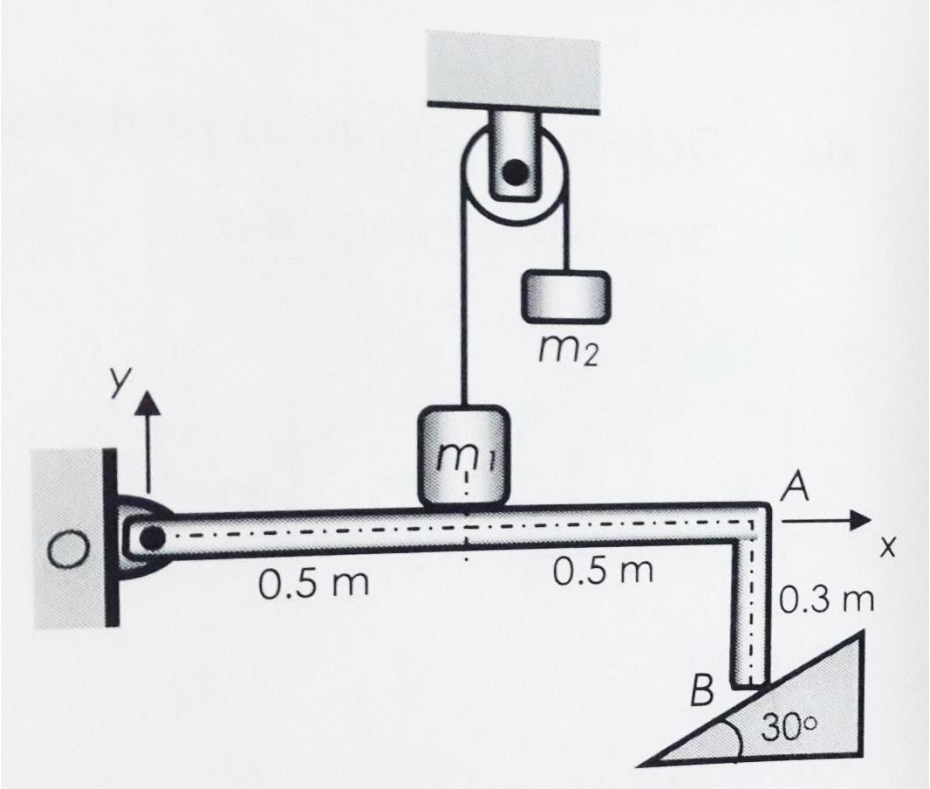
Ex. 13: The weight of the three members is 4N. Determine the centre of gravity, G. Hence, determine the reaction at O.



Ex. 14: A uniform semicircular rod of weight W and radius, r is attached to a pin at A and rests against a frictionless surface at B. Determine the reactions at A and B.



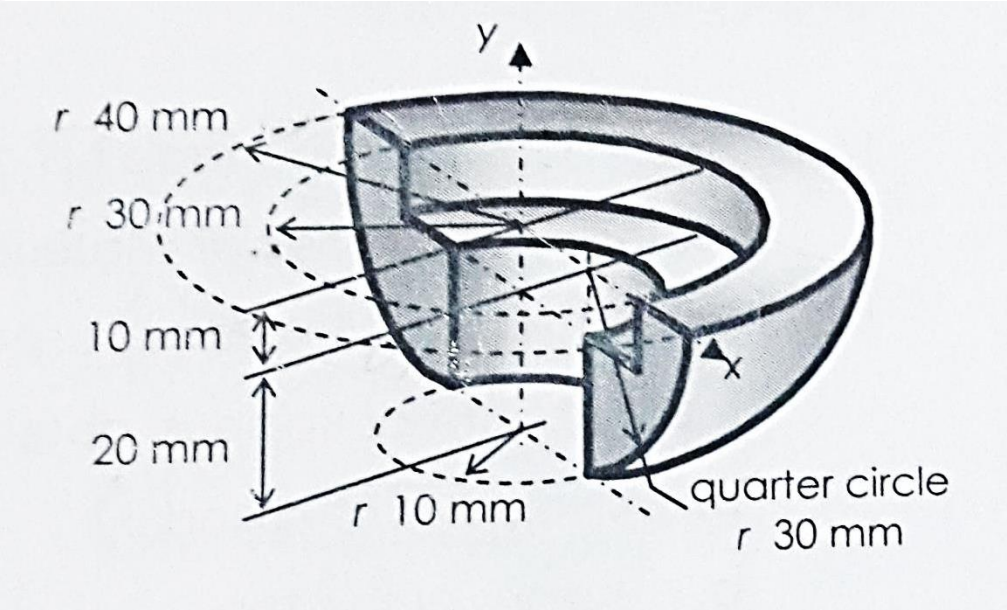
Ex. 15: The truss OABC is made up of five 4m length rods with a mass of 7kg/m is lifted by a cable. Determine the distance, d so that the truss maintain at the position shown.



Ex. 16: The 130kg bent rod OAB is supported as shown. Mass m_1 = 50kg and m_2 = 35kg. The surface at B is smooth. Determine;

(a) The centroid of the bent rod

(b) The reactions at O and B



Ex. 17: Determine the centroid and the volume generated of the component using Theorem of Pappus.

Solution:

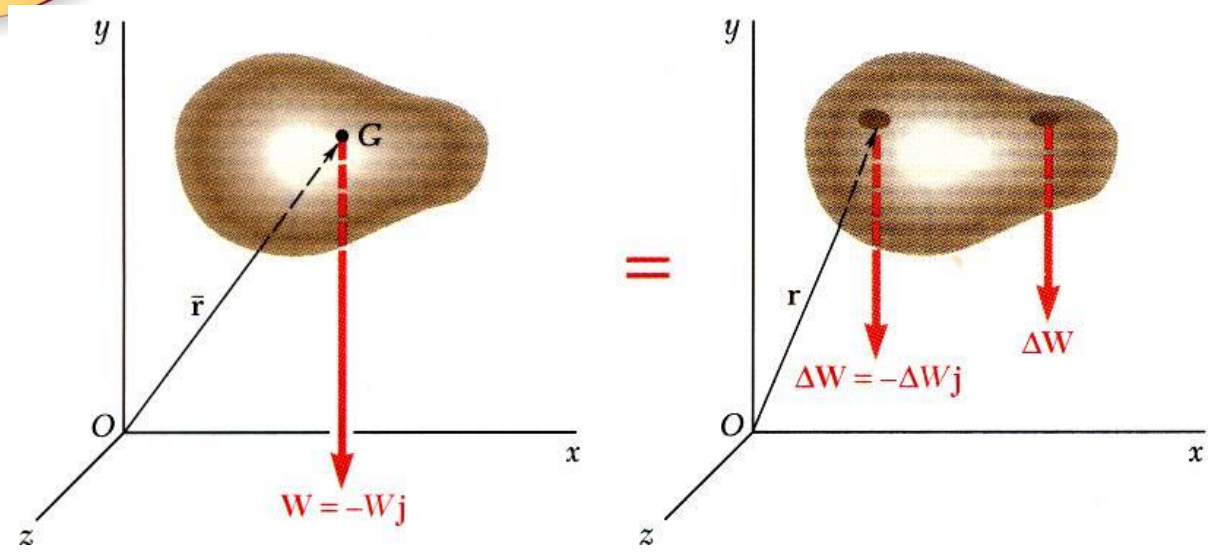
1. Divide into few shapes
2. Construct a table – Find each of the info

   - A (m^2), $\bar{x}$(m), $\bar{y}$ (m), $\bar{x}$A (m^3), $\bar{y}$A (m^3)

3. Solve using;

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} \qquad \bar{Y} = \frac{\sum \bar{y}A}{\sum A}$$

5. Centroid (C)/ Centre of Gravity (G) of 3D Shape/Volume



$$\bar{X} = \frac{\sum \bar{x}W}{\sum W} \qquad \bar{Y} = \frac{\sum \bar{y}W}{\sum W} \qquad \bar{Z} = \frac{\sum \bar{z}W}{\sum W}$$

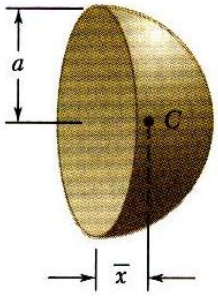
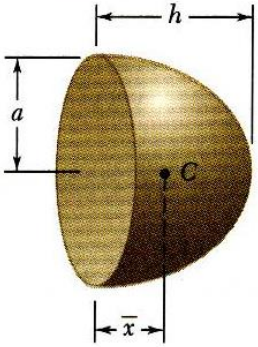
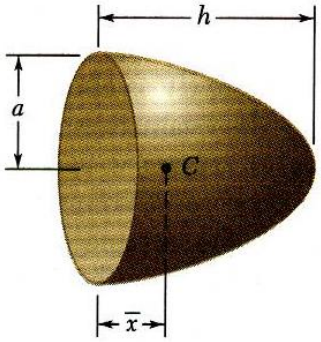
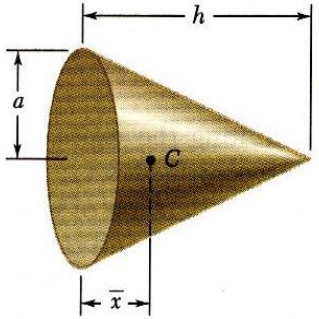
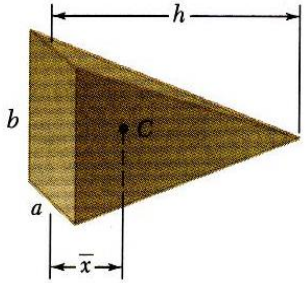
But, $W = \rho g V$

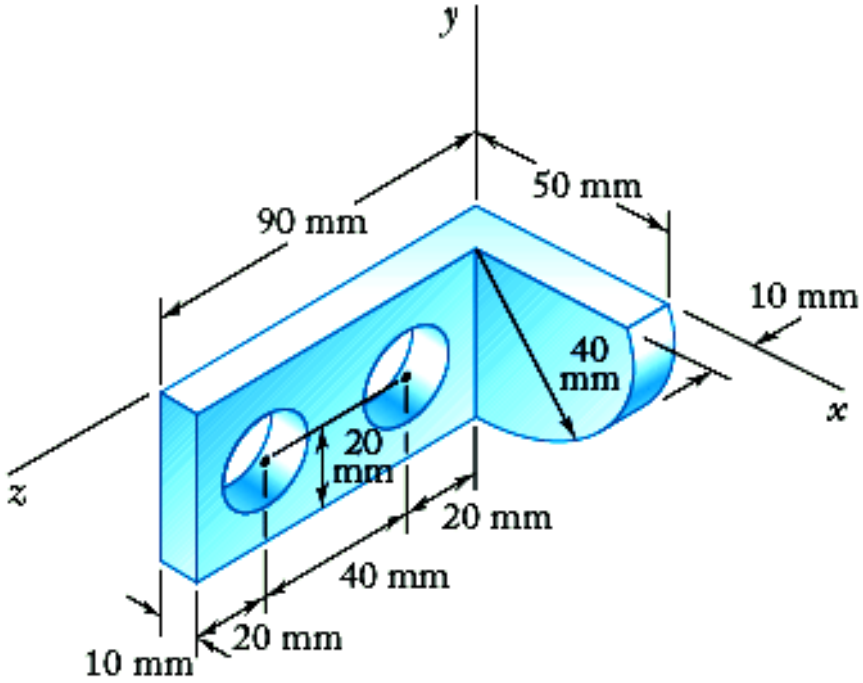
$\Delta W = \rho g \Delta V$

For homogeneous plate, ρ and g are constant and the equation can be simplified between ΔW and ΔV;

$$\bar{X} = \frac{\sum \bar{x}V}{\sum V} \qquad \bar{Y} = \frac{\sum \bar{y}V}{\sum V} \qquad \bar{Z} = \frac{\sum \bar{z}V}{\sum V}$$

Centroid of Common 3D Shapes

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$
Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$



Ex. 18: Locate the center of gravity of the steel machine element. The diameter of each hole is 20mm.

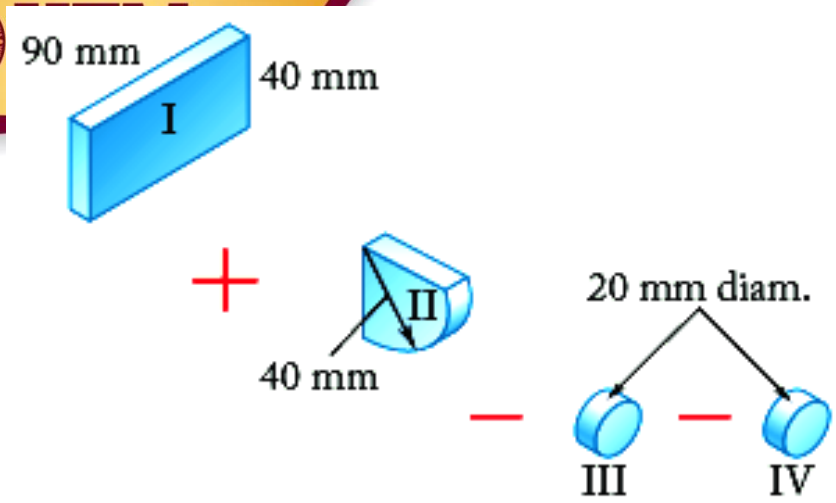
Solution:

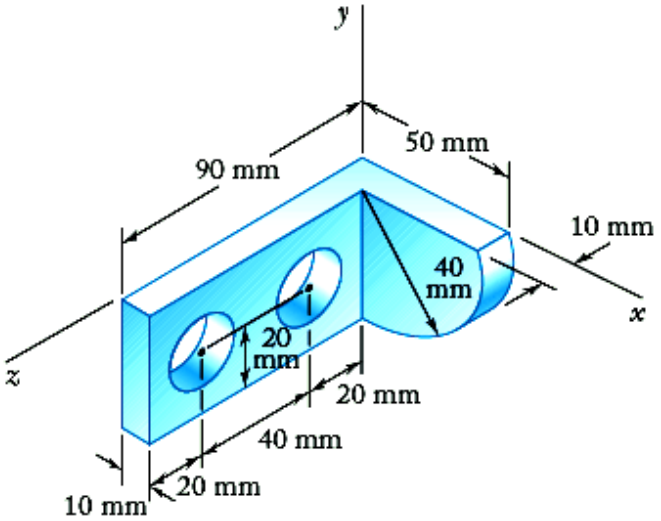
1. Divide into few shapes
2. Construct a table – Find each of the info

   - V (m^3), $\bar{x}$ (m), $\bar{y}$(m), $\bar{z}$ (m) ,$\bar{x}$V (m^4), $\bar{y}$V(m^4), $\bar{z}$V (m^4)

3. Solve using;

$$\bar{X} = \frac{\sum \bar{x}V}{\sum V} \qquad \bar{Y} = \frac{\sum \bar{y}V}{\sum V} \qquad \bar{Z} = \frac{\sum \bar{z}V}{\sum V}$$





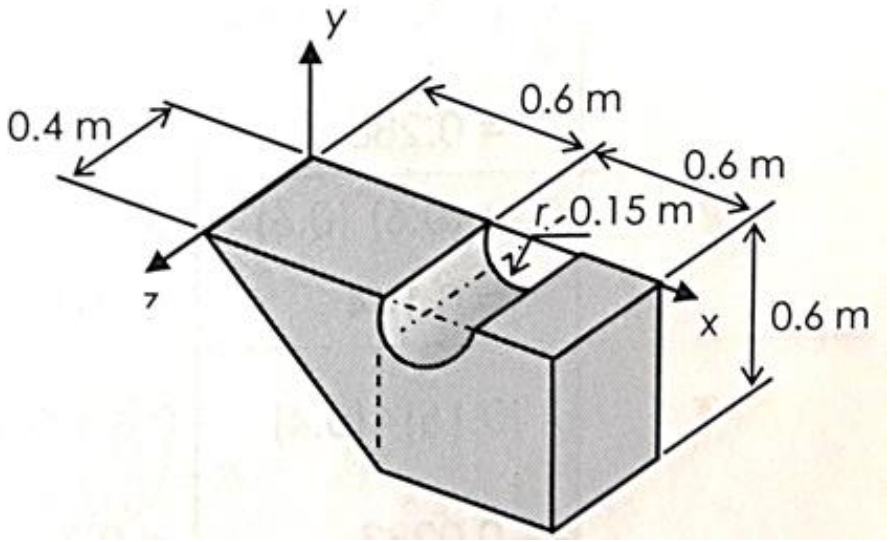
S	V (m^3)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{z}$ (m)	$\bar{x}V$ (m^4)	$\bar{y}V$ (m^4)	$\bar{z}V$ (m^4)
1 (+)	$3.6x^{-5}$	0.005	- 0.02	0.045	$1.8x^{-7}$	$- 7.2x^{-7}$	$1.62x^{-6}$
2 (+)	¼ π $(0.04)^2$ (0.01) = $1.257x^{-5}$	4(0.04)/3π + 0.01 = 0.027	-4(0.04) /3π = - 0.017	0.005	$3.394x^{-7}$	$- 2.14x^{-7}$	$6.285x^{-8}$
3 (-)	- π $(0.01)^2$ (0.01) = - $3.142x^{-6}$	0.005	- 0.02	0.07	$- 1.571x^{-8}$	$6.284x^{-8}$	$- 2.2x^{-7}$
4 (-)	- π $(0.01)^2$ (0.01) = - $3.142x^{-6}$	0.005	- 0.02	0.03	$- 1.571x^{-8}$	$6.284x^{-8}$	$- 9.426x^{-8}$
Total	$4.23x^{-5}$				$4.88x^{-7}$	$- 8.08x^{-7}$	$1.37x^{-6}$

$$\bar{X} = \frac{\sum \bar{x}V}{\sum V} = \frac{4.88x^{-7}\ m^4}{4.23x^{-5}\ m^3} = 0.0115m$$

$$\bar{Y} = \frac{\sum \bar{y}V}{\sum V} = \frac{-8.08x^{-7}m^4}{4.23x^{-5}\ m^3} = -0.019m$$

$$\bar{Z} = \frac{\sum \bar{z}V}{\sum V} = \frac{1.37x^{-6}m^4}{4.23x^{-5}m^3} = 0.0324m$$

$$(\bar{X}, \bar{Y}, \bar{Z}) = (0.0115,\ \ -0.019,\ \ 0.0324)\ \ m$$



Ex. 19: Determine the centroid of the component shown.

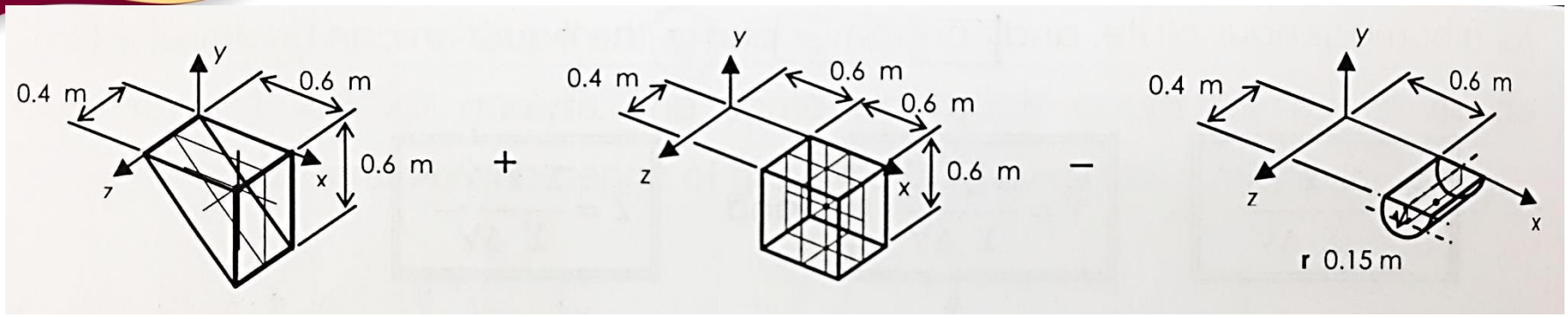
Solution:

1. Divide into few shapes
2. Construct a table – Find each of the info

   - V (m^3), $\bar{x}$ (m), $\bar{y}$(m), $\bar{z}$ (m) ,$\bar{x}$V (m^4), $\bar{y}$V(m^4), $\bar{z}$V (m^4)

3. Solve using;

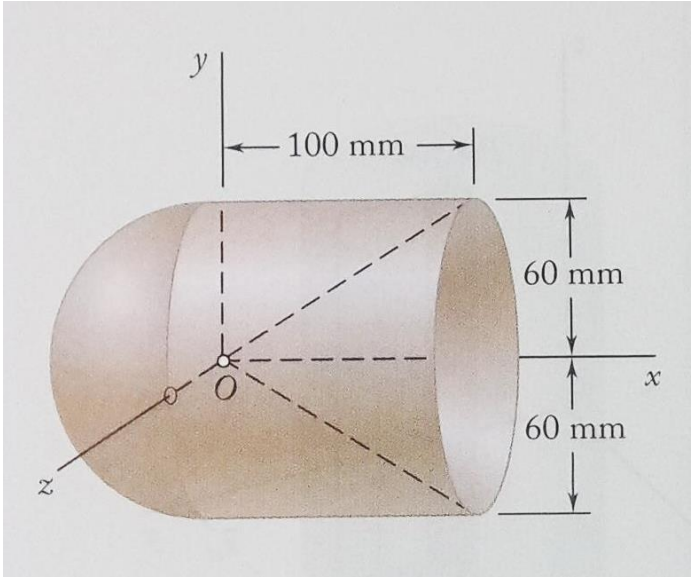
$$\bar{X} = \frac{\sum \bar{x}V}{\sum V} \qquad \bar{Y} = \frac{\sum \bar{y}V}{\sum V} \qquad \bar{Z} = \frac{\sum \bar{z}V}{\sum V}$$



S	V (m^3)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{z}$ (m)	$\bar{x}$V (m^4)	$\bar{y}$V (m^4)	$\bar{z}$V (m^4)
1 (+)							
2 (+)							
3 (-)							
Total							

$$\bar{X} = \frac{\sum \bar{x}V}{\sum V} \qquad \bar{Y} = \frac{\sum \bar{y}V}{\sum V} \qquad \bar{Z} = \frac{\sum \bar{z}V}{\sum V}$$

Centroid = (0.554,-0. 245,0.2) m



Ex. 20: Determine the location of the center of gravity of the homogeneous body of revolution shown that was obtained by joining a hemisphere and cylinder and carving out a cone.

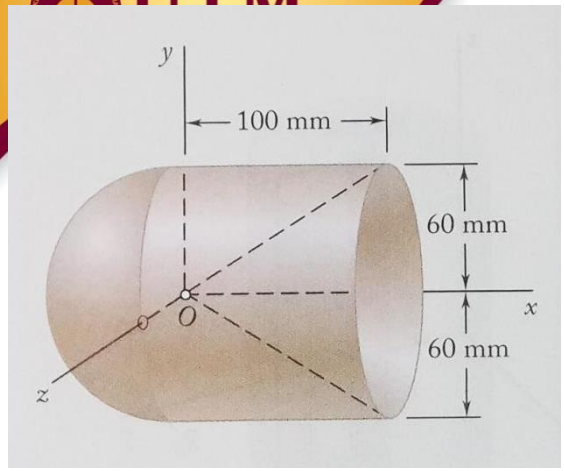
Solution:

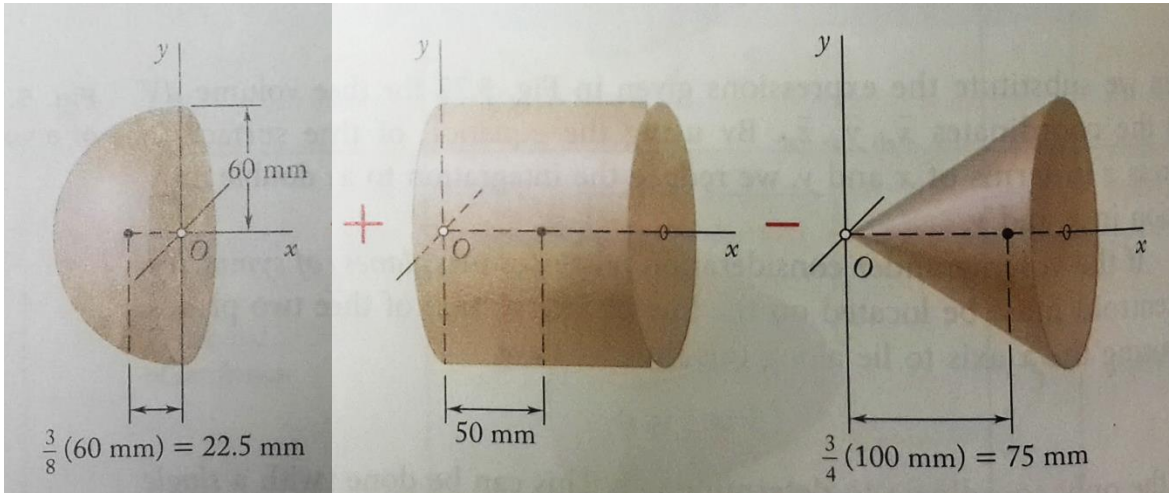
1. Divide into few shapes
2. Construct a table – Find each of the info

   - V (m^3), $\bar{x}$ (m), $\bar{y}$(m), $\bar{z}$ (m) ,$\bar{x}$V (m^4), $\bar{y}$V(m^4), $\bar{z}$V (m^4)

3. Solve using;

$$\bar{X} = \frac{\sum \bar{x}V}{\sum V} \qquad \bar{Y} = \frac{\sum \bar{y}V}{\sum V} \qquad \bar{Z} = \frac{\sum \bar{z}V}{\sum V}$$





S	V (m^3)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{z}$ (m)	$\bar{x}$V (m^4)	$\bar{y}$V (m^4)	$\bar{z}$V (m^4)
1 (+)							
2 (+)							
3 (-)							
Total							

$$\bar{X} = \frac{\sum \bar{x}V}{\sum V} \qquad \bar{Y} = \frac{\sum \bar{y}V}{\sum V} \qquad \bar{Z} = \frac{\sum \bar{z}V}{\sum V}$$

$$\text{Centroid} = (\bar{X}, \bar{Y}, \bar{Z})$$