CHAPTER

6

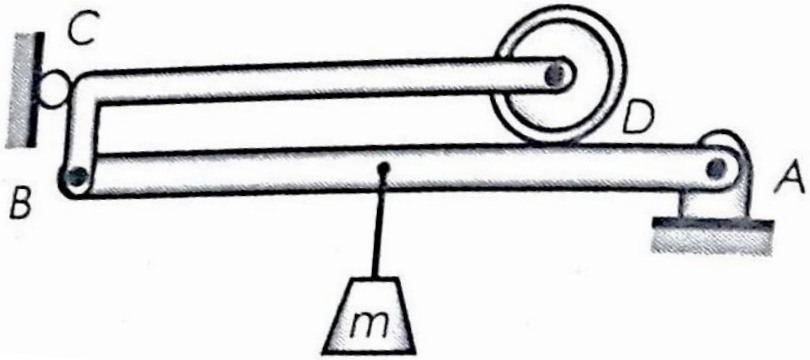
ANALYSIS OF STUCTURES

6.1 Analysis of Trusses

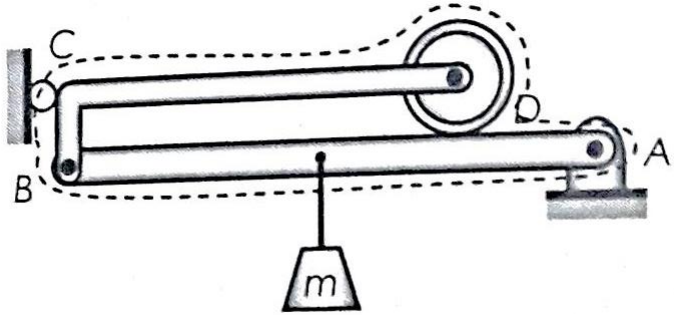
- Method of joints
- Method of sections

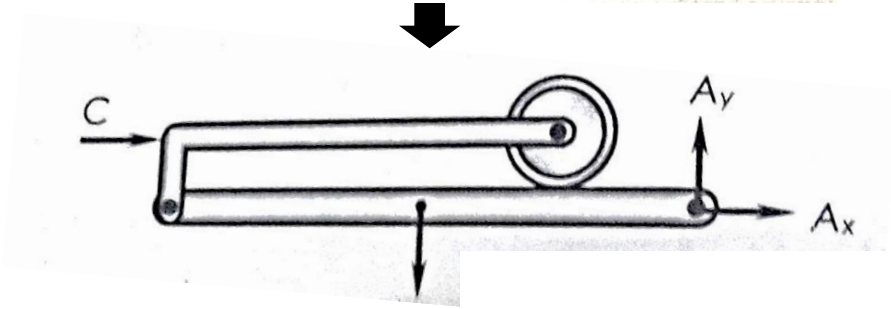
6.2 Analysis of Frames

Example 1: Draw FBD

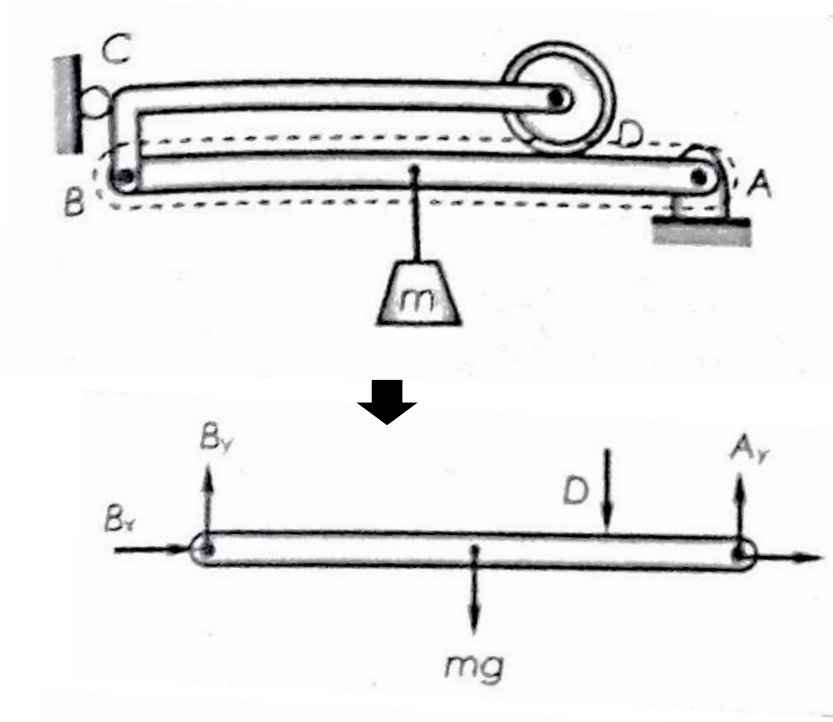


1. Draw overall FBD

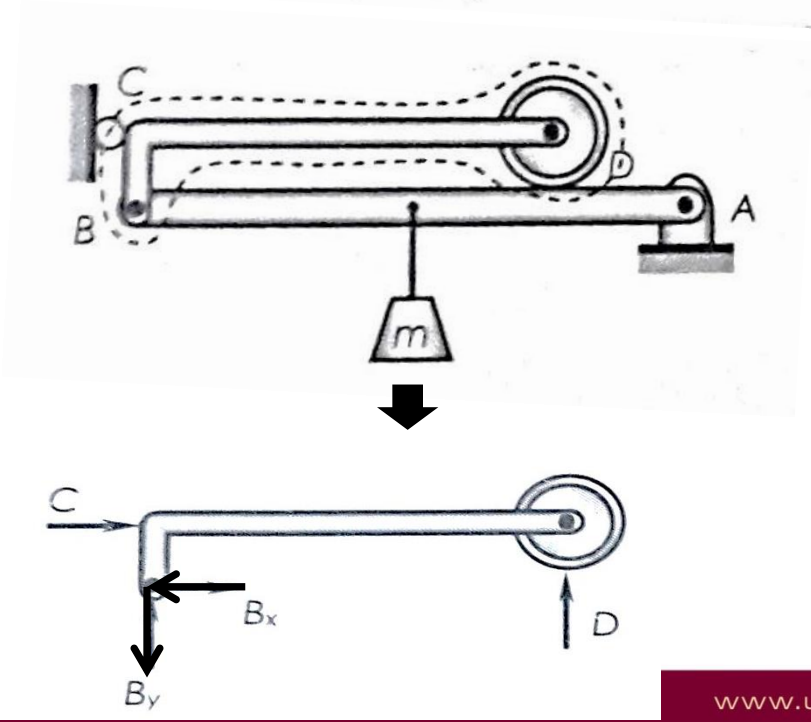




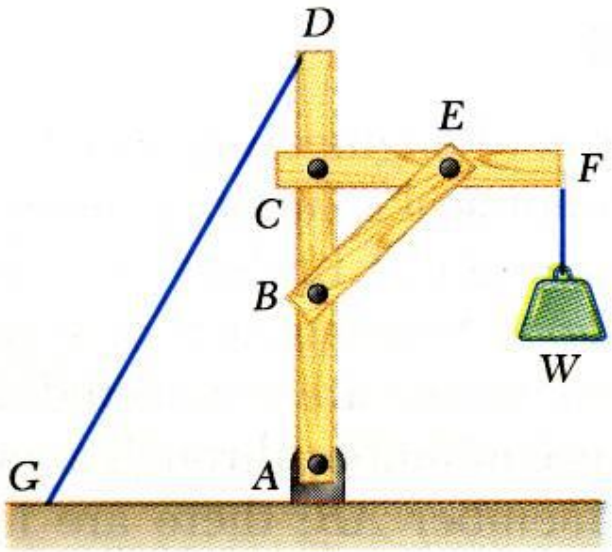
2. Draw FBD of member AB



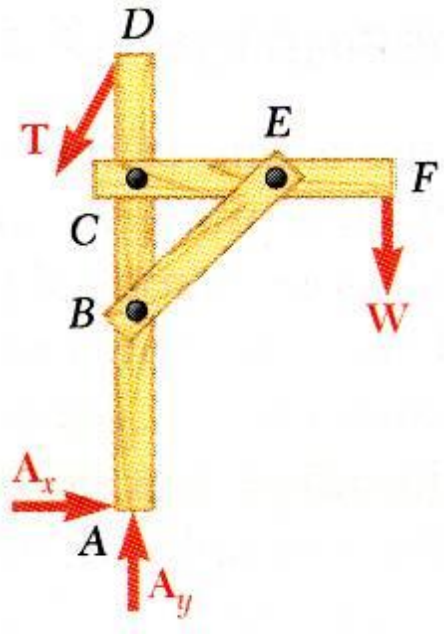
3. Draw FBD of member BCD



Example 2: Draw FBD

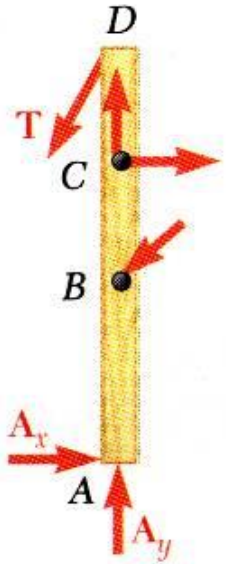


3rd Newton's law – Forces of action & reactions between 2 bodies in contact have the same magnitude, same line of action and opposite directions.

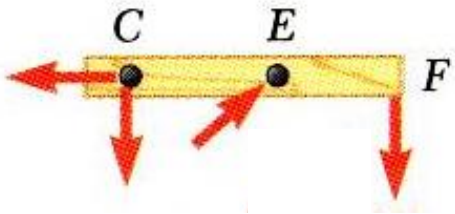


Draw FBD of the crane

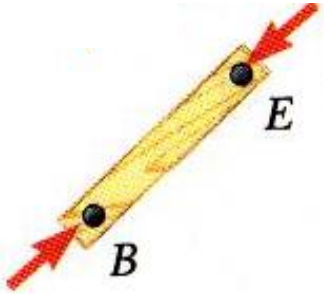
Draw FBD of member ABCD



Draw FBD of member CEF

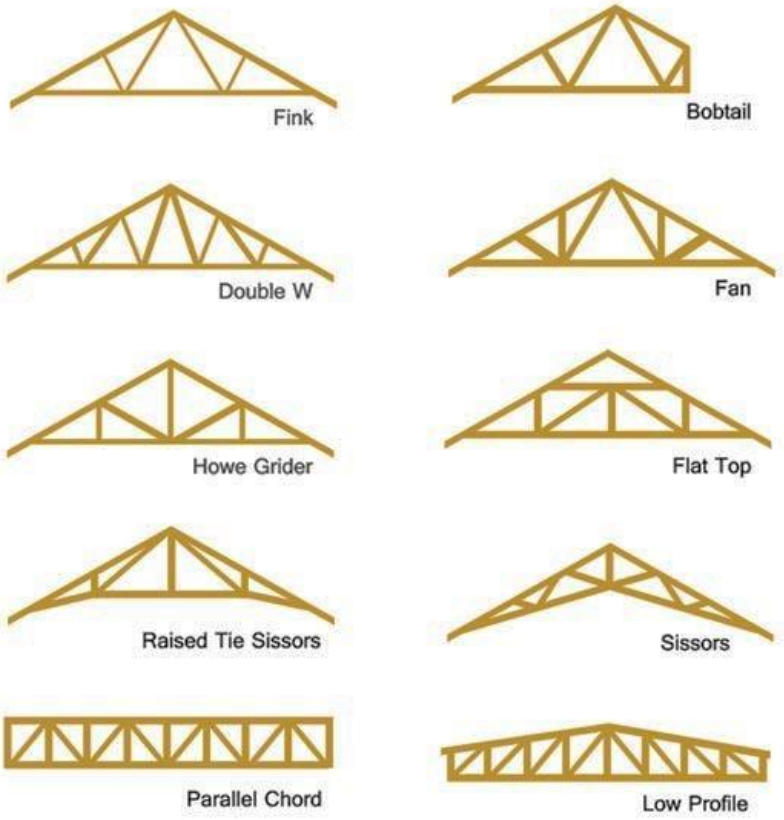


Draw FBD of member BE

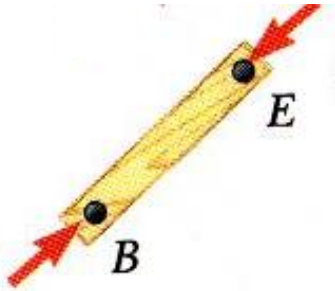


Internal Force – Hold various parts/members of the structure

- Three categories of engineering structures are considered:
  - *a)* *Trusses*: straight members with end point connections and forces act only at these end points (formed from *two-force members*)
    - To support loads & always stationary.

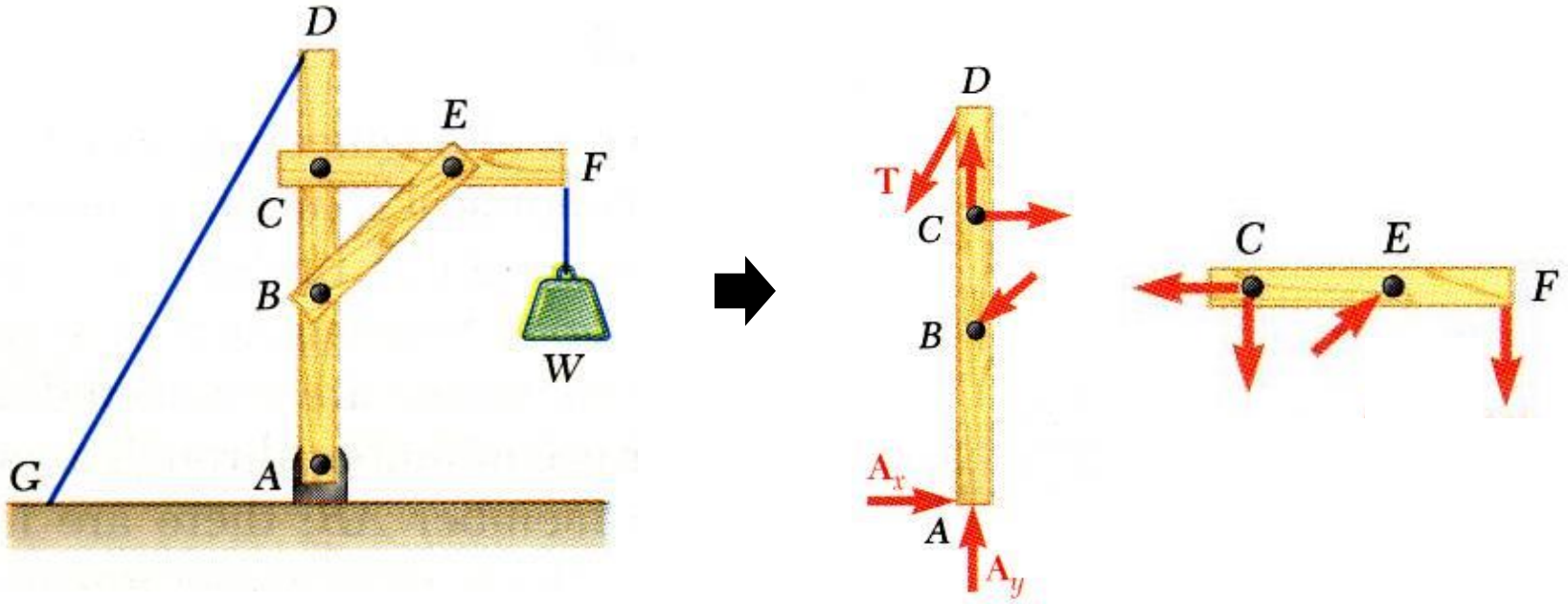


Two-force member – Members acted upon by two equal & opposite forces directed along the member.

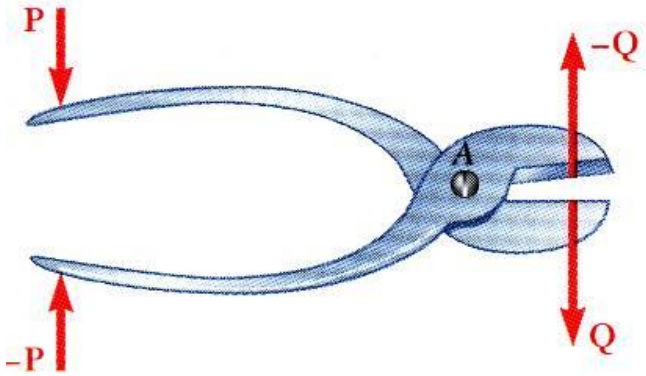


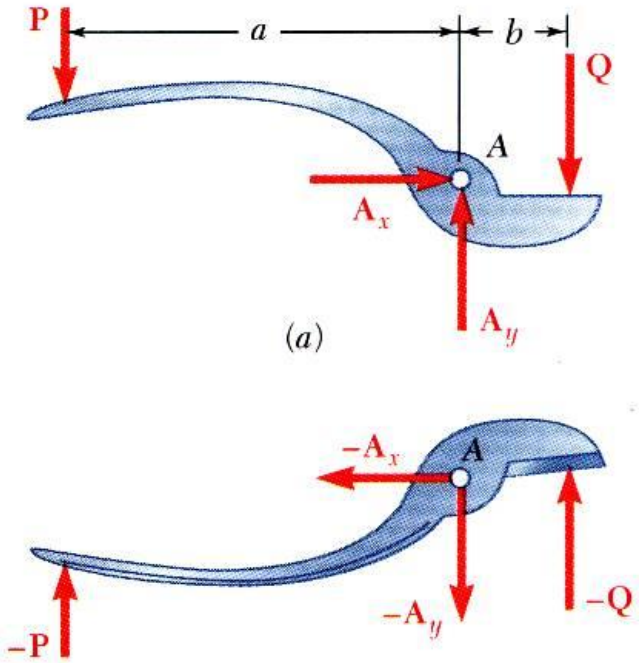
b) Frames: contain at least one multi-force member, i.e., member acted upon by 3 or more forces.

- To support loads & always stationary.

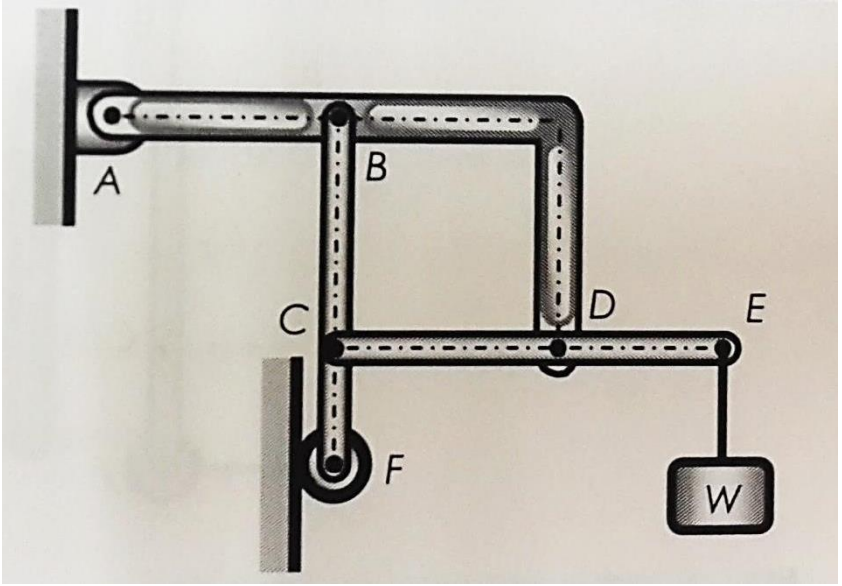


c) Machines: structures containing moving parts, designed to transmit and modify forces.





Exercise:

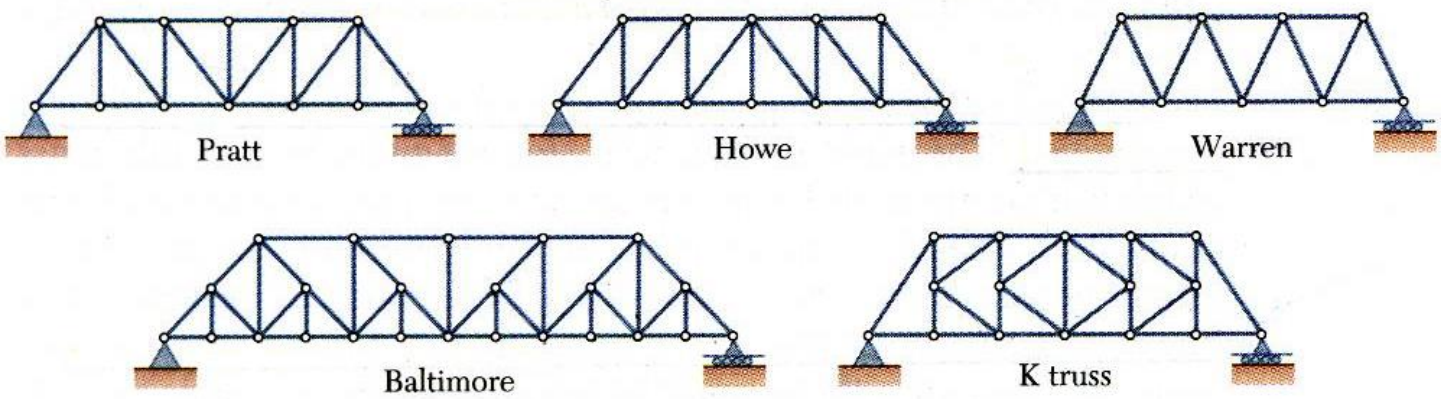


Draw FBD for each of the member:

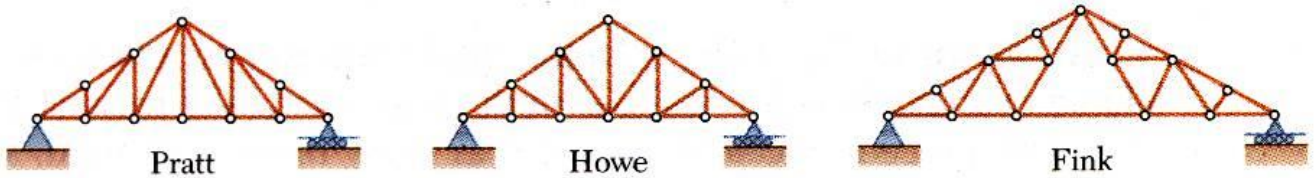
Draw FBD:

6.1 Trusses

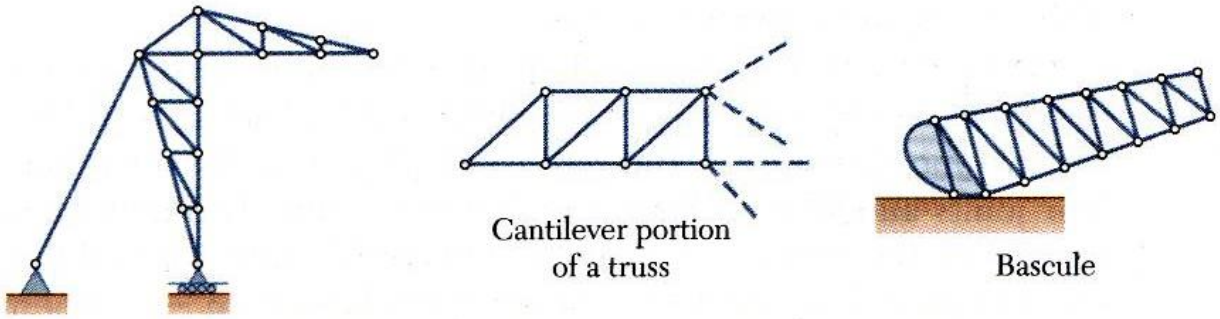
- Usually employed in designing bridges & buildings .



Typical Bridge Trusses



Typical Roof Trusses



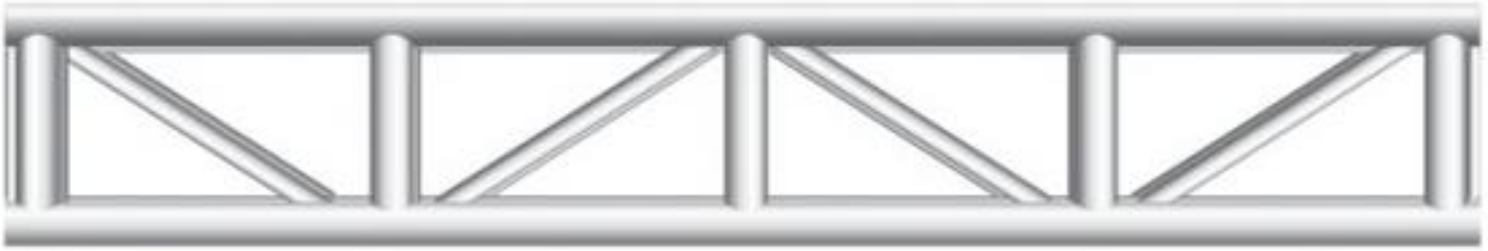
Other Types of Trusses

- Trusses are made up of members with end point connections only.

- Design to support loads in the same plane (2D – one plane).

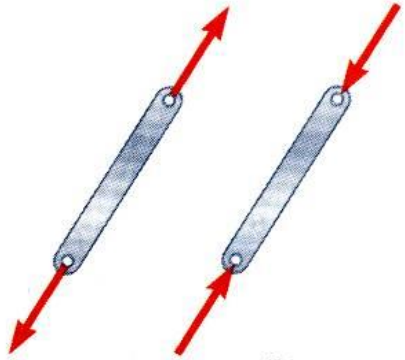
- All members are considered to act at the joints – pin jointed.

- **Formed from *two-force members*, i.e., straight members with end point connections and forces act only at these end points.**



Truss – As an integration of a group of joints, with a collection of two-force member

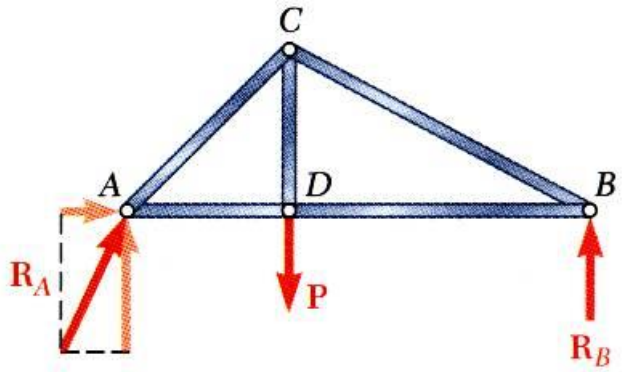
Tension

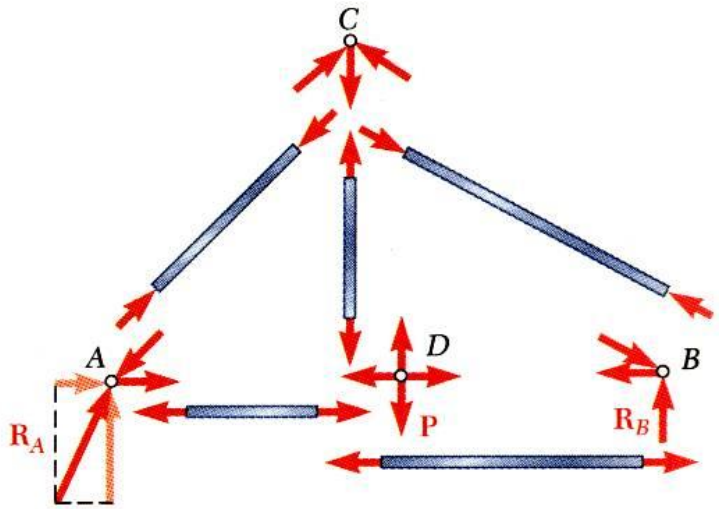


Compression

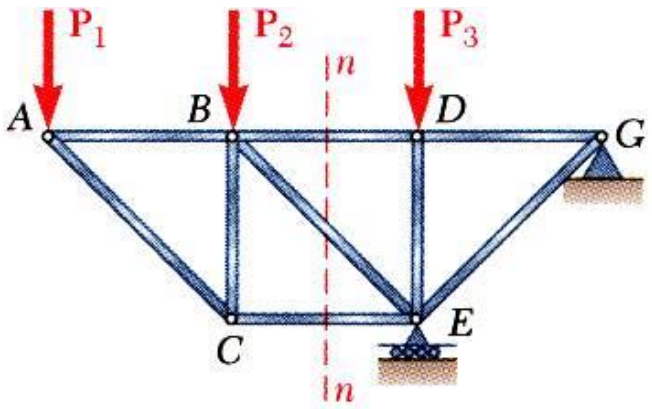
6.1 Analysis of Trusses

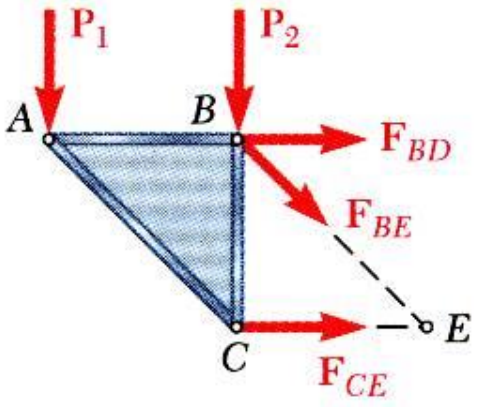
6.1.1 Method of Joint





6.1.2 Method of Section





6.1.1 Method of Joint

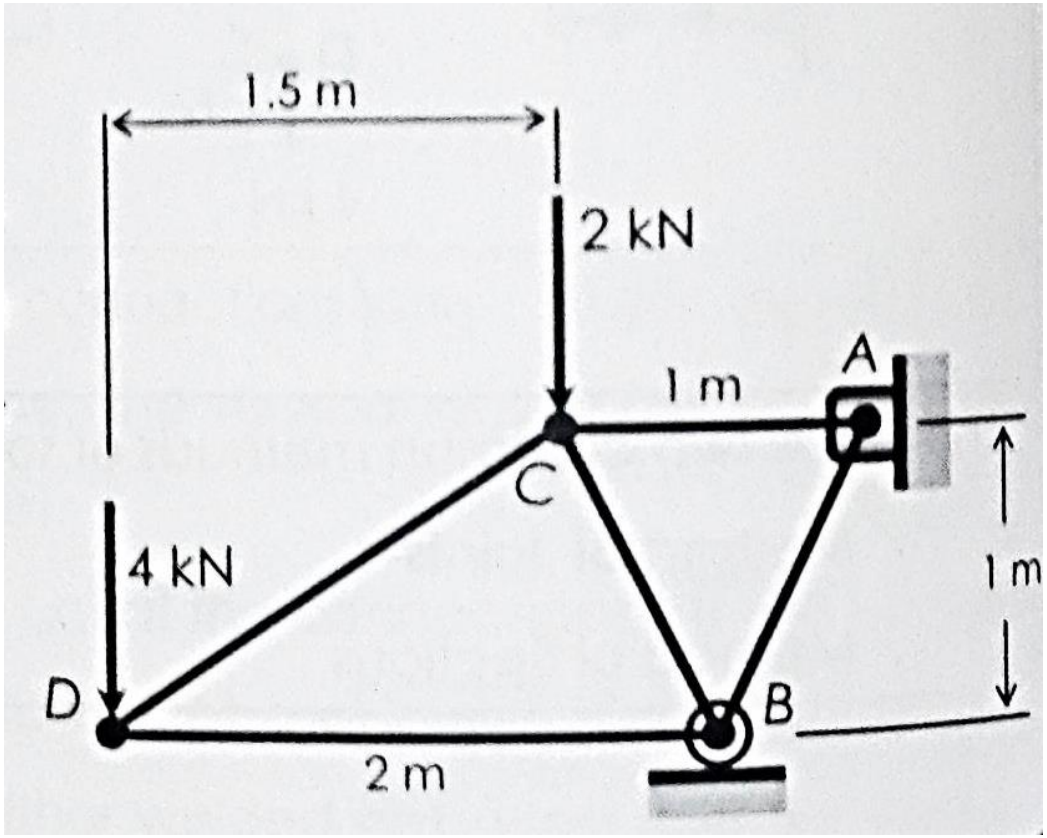
- Dismember the truss and create a FBD for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense -3rd Newton's Law.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite (2 force member).

6.1.1 Method of Joint

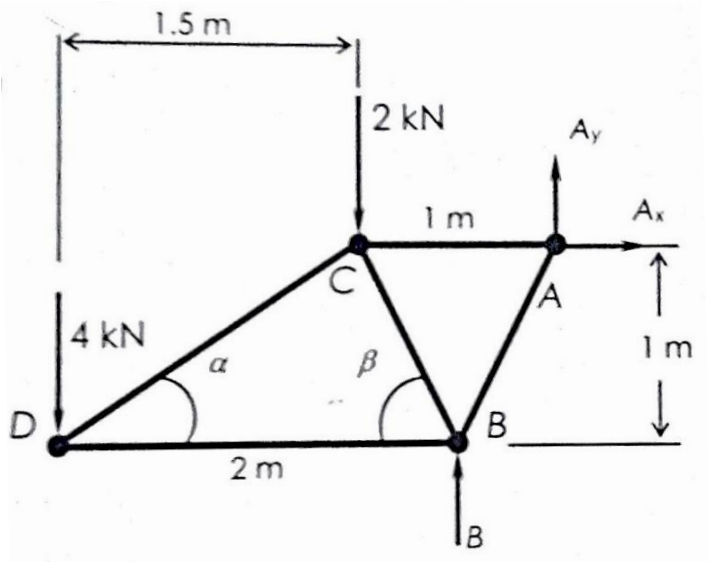
Strategy to solve:

1. Draw FBD of overall truss and solve for reactions.
2. Draw FBD for all joints. Solve for all joints. Start with least unknown forces.
3. Assume all the unknown forces (internal forces) are in tension (directing outward of the joint). A positive answer means that the assumption is correct (tension) but if negative means that the member is in compression (inward to the joint).
4. Solve using two equations ($\sum F_x = 0$, $\sum F_y = 0$)

Ex 1: Using the method of joints, determine the force in each member of the truss shown either compression or tension.



1. Overall FBD – To determine the external reactions & support



+ve $\sum M_A = 0;$

$4kN\ (2.5m) + 2kN\ (1m) - B_y\ (0.5m) = 0$

$B_y = 24kN\ (\ \ )$

$(\rightarrow)\ \sum F_x = 0;$

$A_x = 0$

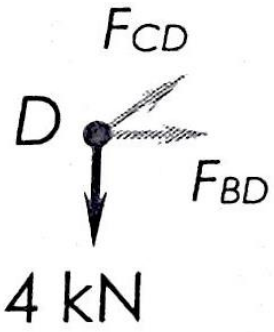
$(\uparrow)\ \sum F_y = 0;$

$A_y - 4kN - 2kN + 24kN = 0$

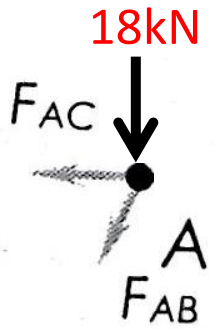
$A_y = -18kN$

$= 18kN\ (\ \ )$

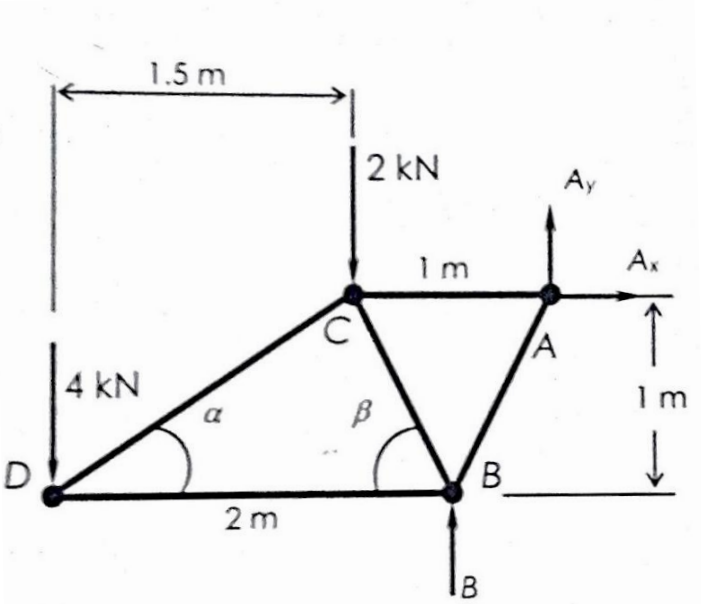
2. FBD for each of the joint:
*Declare tension as positive (Outward from the joints), if got negative answer, then the member is in compression (Inward to the joints)

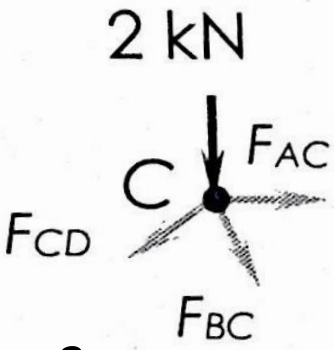


2 Unknowns

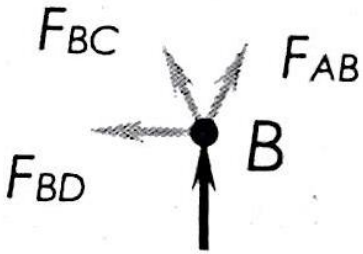


2 Unknowns





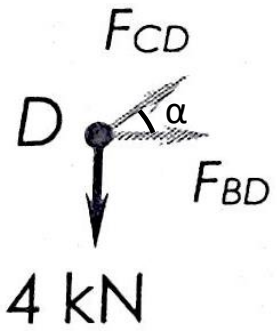
3 Unknowns



3 Unknowns

3. Solve for each joint – Represents the member is in tension or compression.

3.1 At joint D:



$(\Sigma F_x = 0)$;

$F_{DB} + F_{DC} \cos \alpha = 0$

$F_{DB} + 7.21\text{kN} \cos 33.69^o = 0$

$F_{DB} = -6\text{kN}$

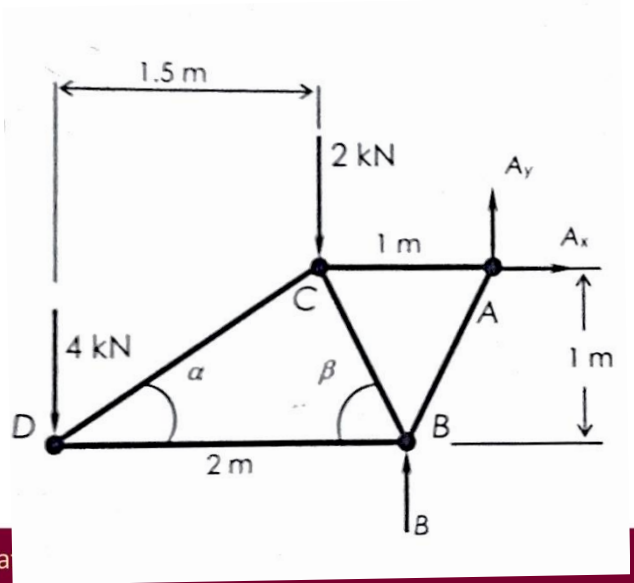
$F_{DB} = 6\text{kN}$ (Compression)

$(\Sigma F_y = 0)$;

$F_{DC} \sin \alpha - 4\text{kN} = 0$

$F_{DC} \sin 33.69^o = 0$

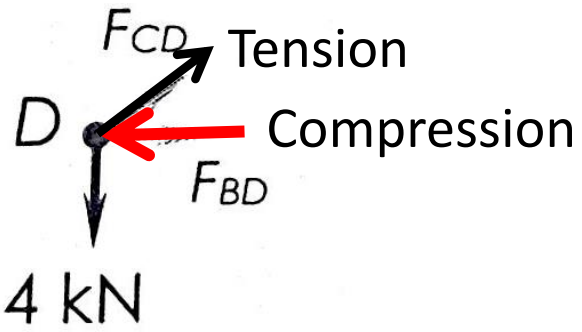
$F_{DC} = 7.21\text{kN}$ (Tension)



Angle α:

Tan α = 1m/1.5m

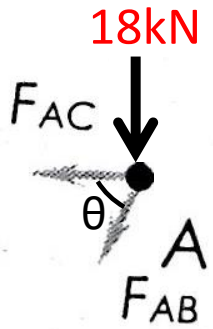
α = 33.69°



$F_{DC} = F_{CD} = 7.21kN$ (Tension)

$F_{DB} = F_{DB} = 6kN$ (Compression)

3.2 At joint A:



$(\rightarrow)\ \Sigma F_x = 0;$

$- F_{AC} - F_{AB} \cos\theta = 0$

$- F_{AC} - (-20.13kN) \cos 63.43^o = 0$

$F_{AC} = 9kN$

(Tension)

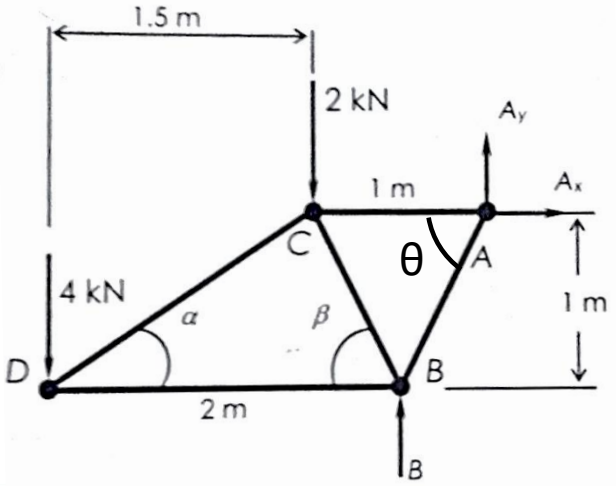
$(\uparrow)\ \Sigma F_y = 0;$

$- F_{AB} \sin\theta - 18kN = 0$

$- F_{AB} \sin 63.43^o - 18kN = 0$

$F_{AB} = - 20.13kN$

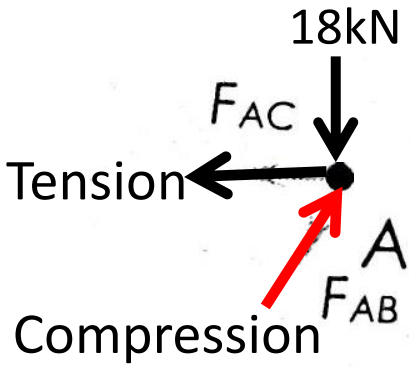
(Compression)



Angle θ:

Tan θ = 1m/0.5m

θ = 63.43°



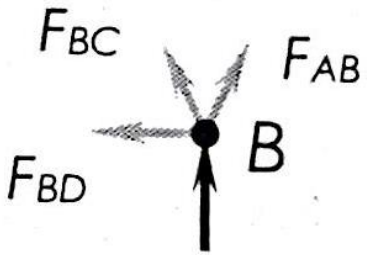
$F_{DC} = F_{CD} = 7.21kN$ (Tension)

$F_{DB} = F_{DB} = 6kN$ (Compression)

$F_{AB} = F_{BA} = 20.13kN$ (Compression)

$F_{AC} = F_{CA} = 9kN$ (Tension)

3.3 At joint B:



24kN

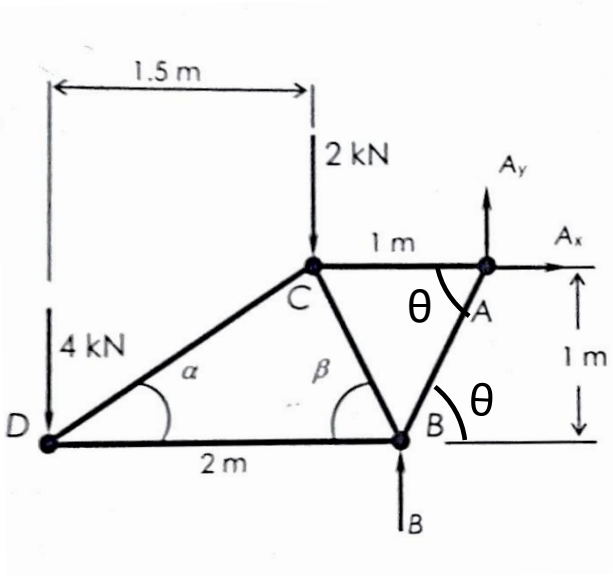
$(\longrightarrow) \sum F_x = 0;$

$- F_{BD} - F_{BC} \cos\beta + F_{AB} \cos\theta = 0$

$- (-6kN) - F_{BC} \cos 63.43^o + (-20.13kN) \cos 63.43^o = 0$

$F_{BC} = -6.72kN$

(Compression)

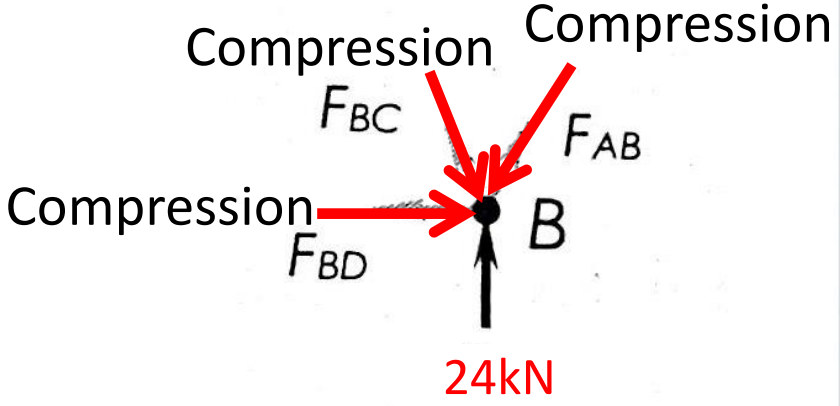


$\text{Tan } \beta = 1m/0.5m$

$\beta = 63.43^o$

$\text{Tan } \theta = 1m/0.5m$

$\theta = 63.43^o$



F_{BC} = 6.72kN (Compression)

F_{CD} = 7.21kN (Tension)

F_{AC} = 9kN (Tension)

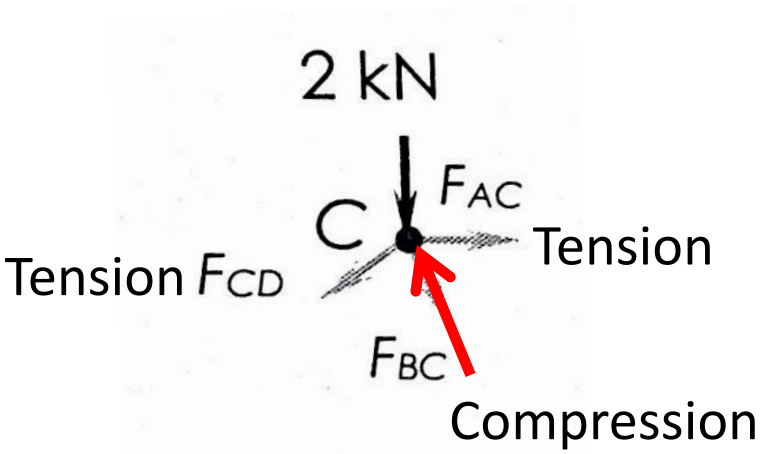
F_{AB} = 20.13kN (Compression)

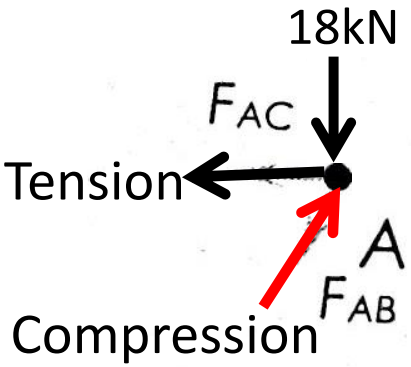
F_{BD} = 6kN (Compression)

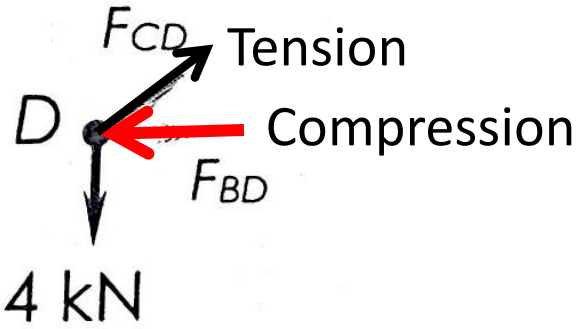
Ay = 18kN (↓)

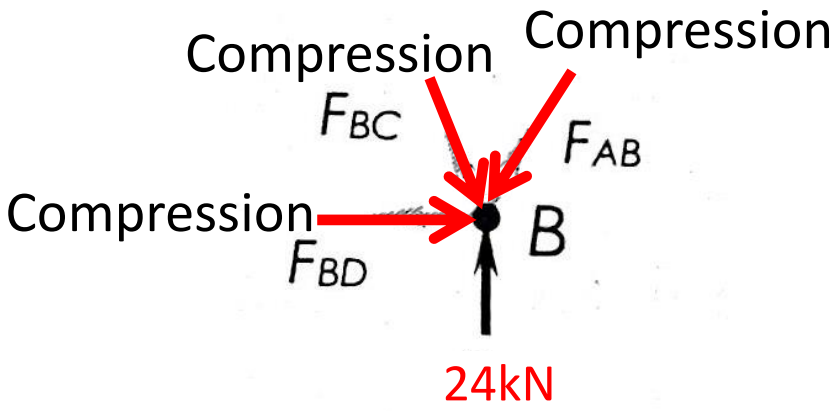
Ax = 0kN

B = 24kN (↓)

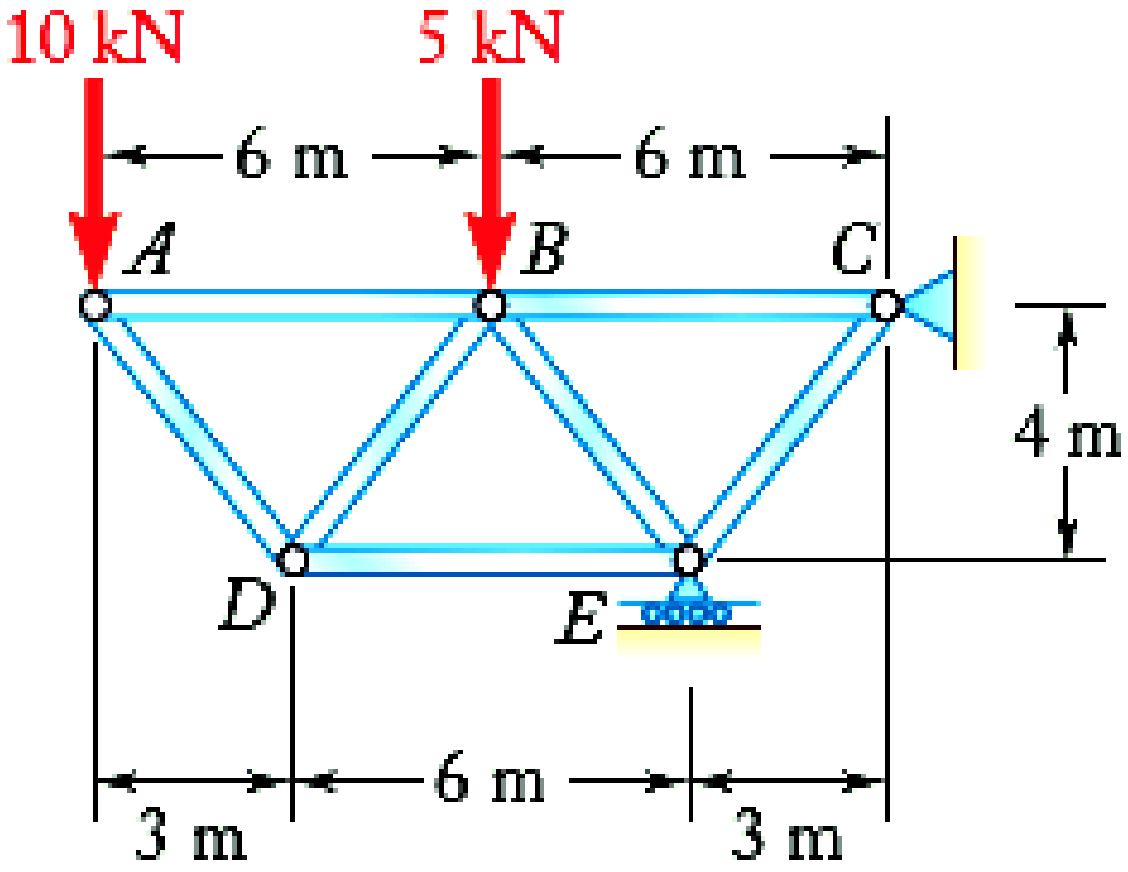






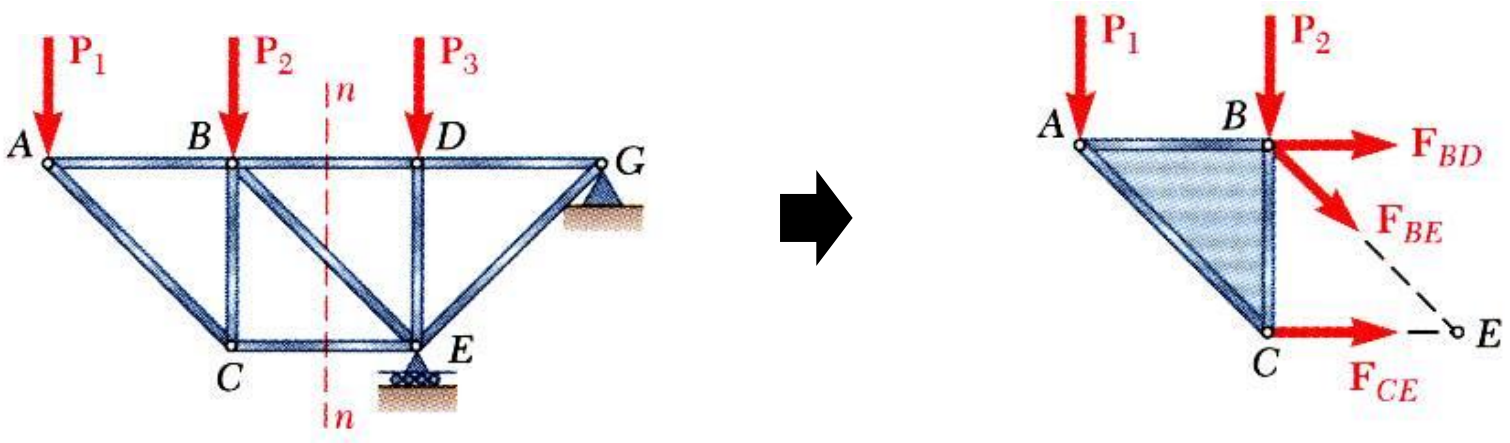


Ex 2: Using the method of joints, determine the force in each member of the truss shown.



6.1 Analysis of Trusses

6.1.2 Method of Section



- Usually used to find only few forces relating to some members of the truss.

- Method of joint is available, but it involves long and tedious calculations. More convenient to use method of section.

6.1.2 Method of Section

Strategy to solve:

1. Draw FBD of overall truss and solve for reactions.
2. Make a decisions of sectioning the truss:

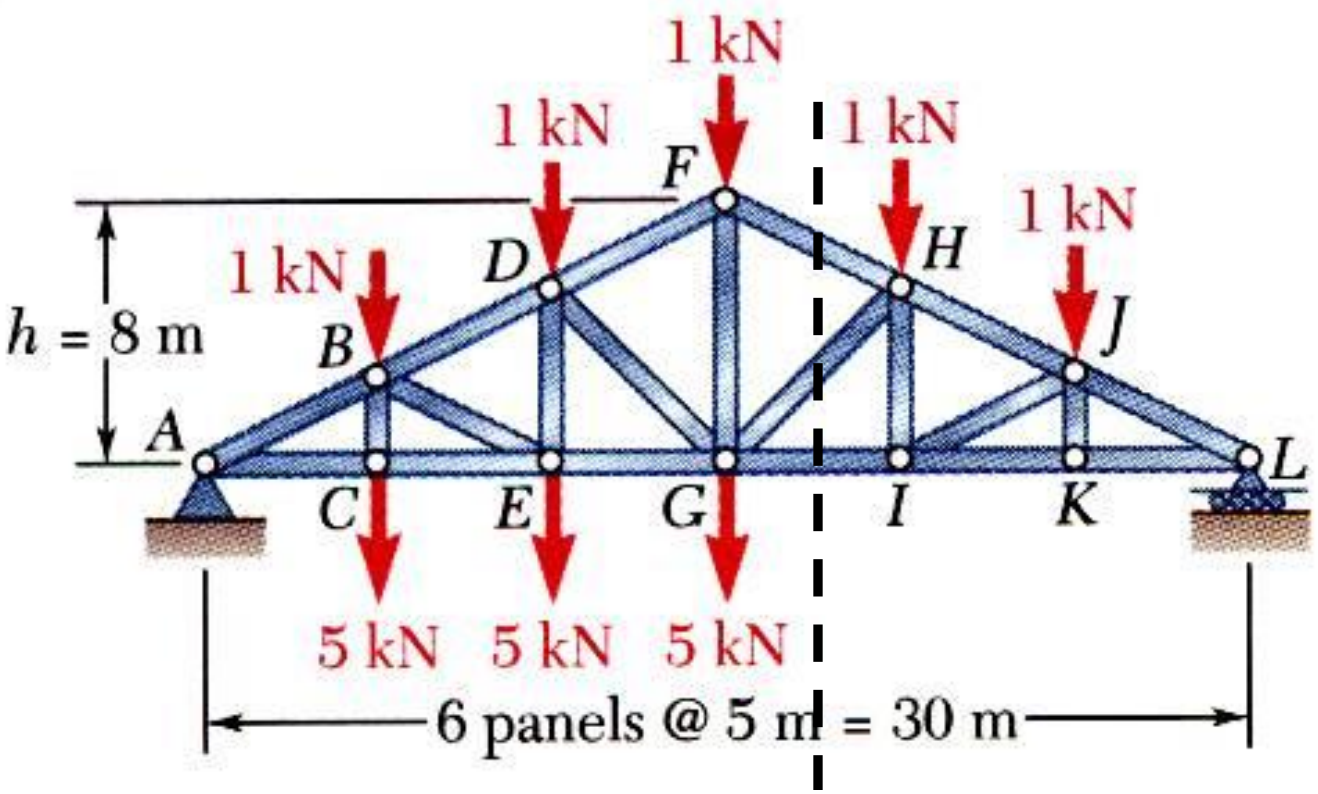
   - Cutting through the required members (Based on question/problem).

3. When the truss is divided into 2 sections, choose the simpler one to solve. 'Simpler' means;

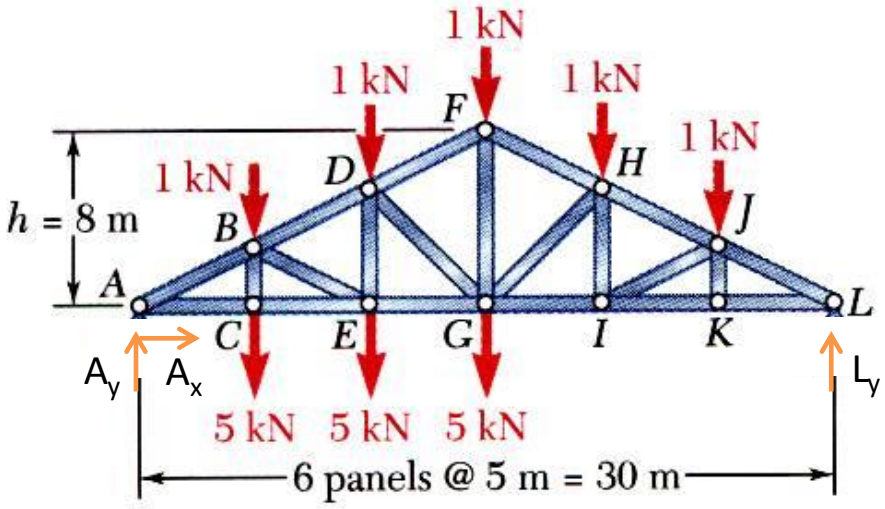
   - The sections with less forces.

   - Most of the forces involved are horizontal or vertical component.

4. Assume all the unknown forces (internal forces) are in tension (directing outward of the joint). A positive answer means that the assumption is correct (tension) but if negative means that the member is in compression.
5. Solve using three equations ($\sum F_x = 0$, $\sum F_y = 0$ and $\sum M_{Point} = 0$)



Ex. 1Determine the forces in member FH, GH and G*I* of the roof truss shown.



1. Overall FBD – To determine the external reactions & support

+ve

$\sum M_A = 0;$

$-1kN(5m) - 5kN(5m) - 1kN(10m) - 5kN(10m) - 1kN(15m) - 5kN(15m) - 1kN(20m) - 1kN(25m) + L_y(30m) = 0$

$L_y = 7.5kN\ (\uparrow)$

$(\rightarrow)$ $\sum F_x = 0;$

$A_x = 0$

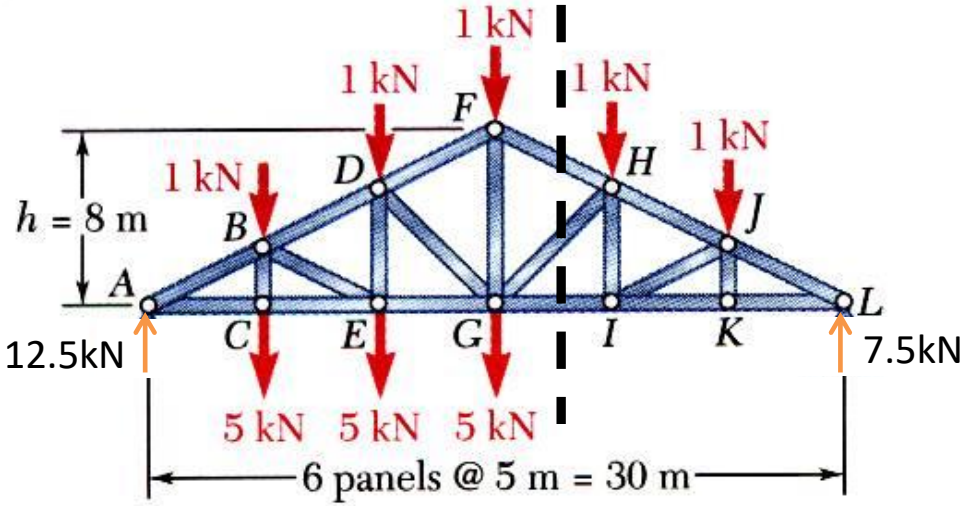
$(\uparrow)$ $\sum F_y = 0;$

$A_y - 6kN - 6kN - 6kN - 1kN - 1kN + L_y = 0$

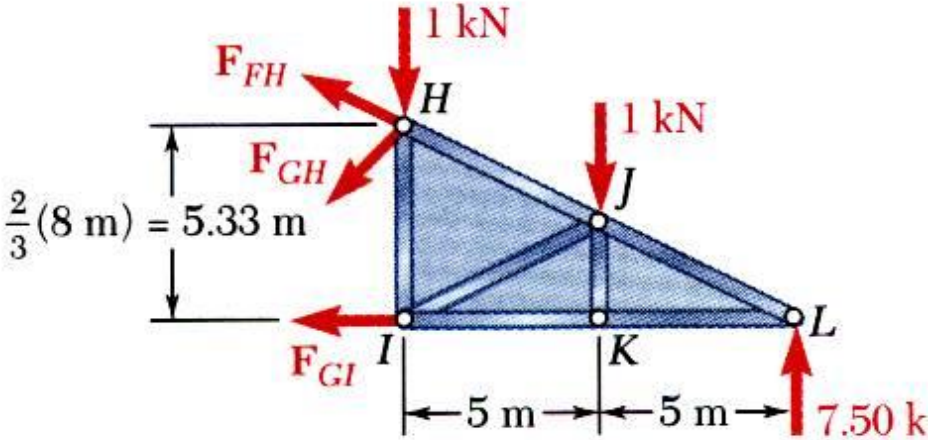
$A_y - 6kN - 6kN - 6kN - 1kN - 1kN + 7.5kN = 0$

$A_y = 12.5kN\ (\uparrow)$

2. Cut the required section

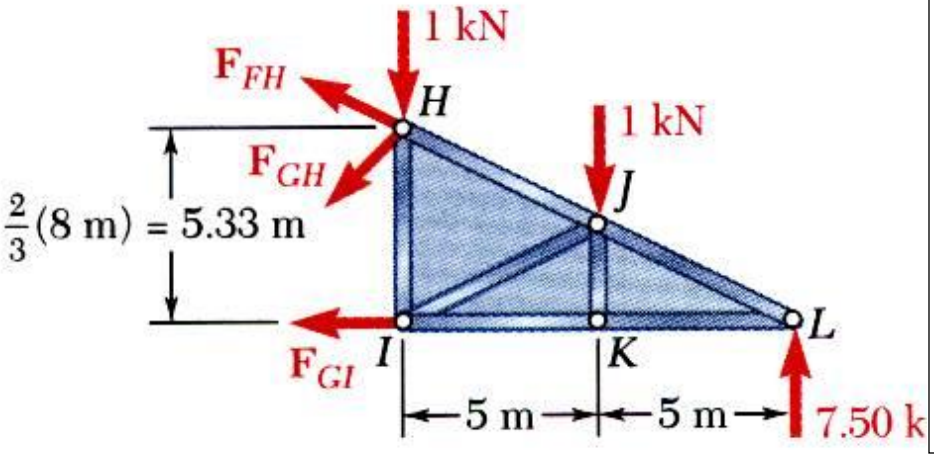


3. Choose the simpler section – Less unknown forces



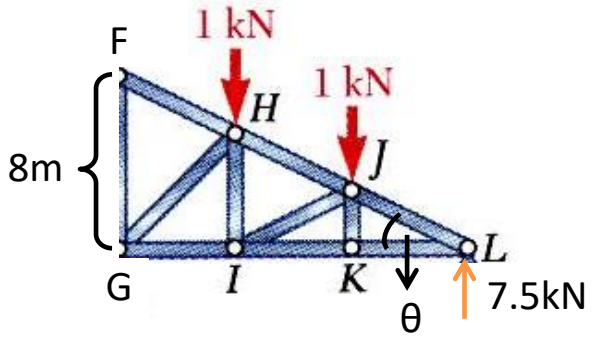
4. Assume the members are in tension (directing outward of the joint.
If +ve, the assumption is correct (tension) but if get –ve value, the member is in compression (inward to the point)

*Determine the length of member *HI*:



1st way:
There is 3 verticals members (either form left or right side) = 8m
3/3 = 8m

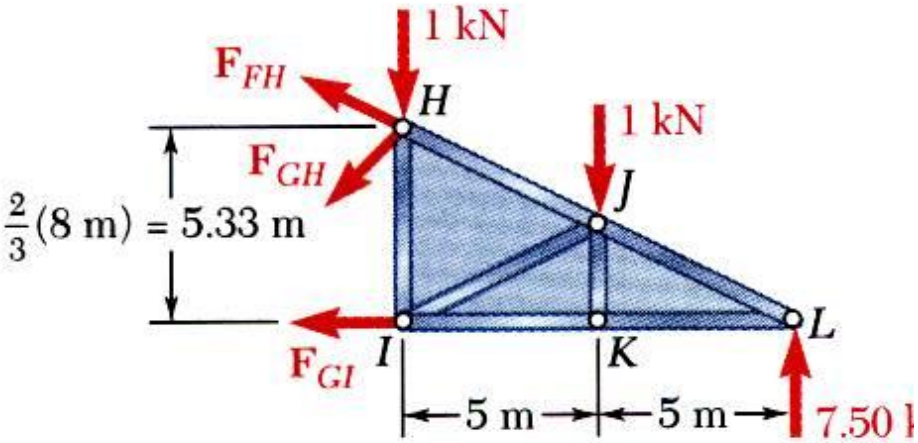
Member *HI* is 2nd out of the three members, for right side. So, length *HI*
2/3 (8m) = 5.33m

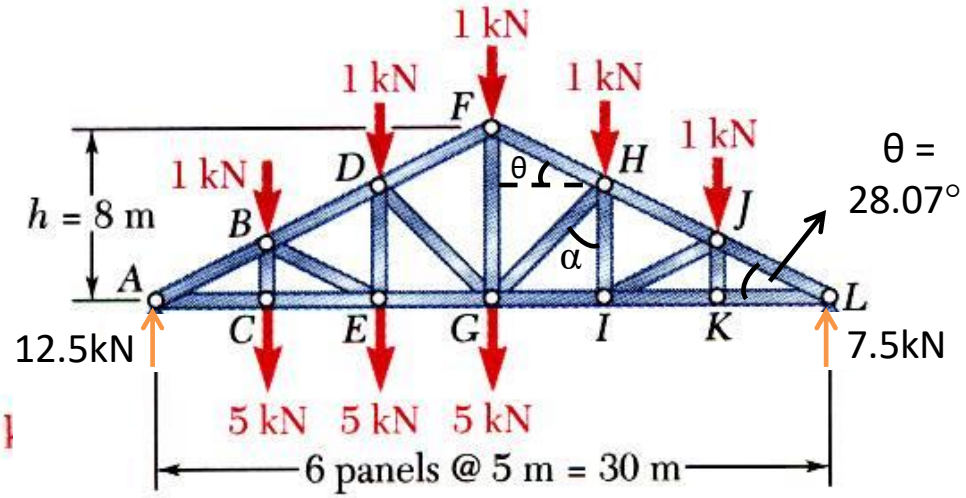


2nd way:
From the triangle;
Tan θ = Length *FG*/Length *GL*
= 8m/15m
θ = 28.07°

So,
Tan 28.07° = Length *HI*/10m
Length *HI* = 5.33m

5. Solve using ($\sum M_{Point} = 0$, $\sum F_x = 0$ and $\sum F_y = 0$)
– Determine the member is in tension or compression.





+ve

$\sum M_H = 0$;

$- F_{GI}\ (5.33\text{m}) - 1\text{kN}\ (5\text{m}) + 7.5\text{kN}\ (10\text{m}) = 0$

$F_{GI} = 13.13\text{kN}$ (Tension)

$(\rightarrow)$ $\sum F_x = 0$;

$- F_{GI} - F_{GH} \sin\alpha - F_{FH} \cos 28.07° = 0$

$- 13.13\text{kN} - F_{GH} \sin 43.17° - F_{FH} \cos 28.07° = 0$ ——— 1

$(\uparrow)$ $\sum F_y = 0$;

$- F_{GH} \cos\alpha - F_{FH} \sin\theta - 1\text{kN} - 1\text{kN} + 7.5\text{kN} = 0$

$- F_{GH} \cos 43.17° - F_{FH} \sin 28.07° - 1\text{kN} - 1\text{kN} + 7.5\text{kN} = 0$ ——— 2

Solve equations 1 and 2 simultaneously;

F_{FH} = -13.81kN (Compression)

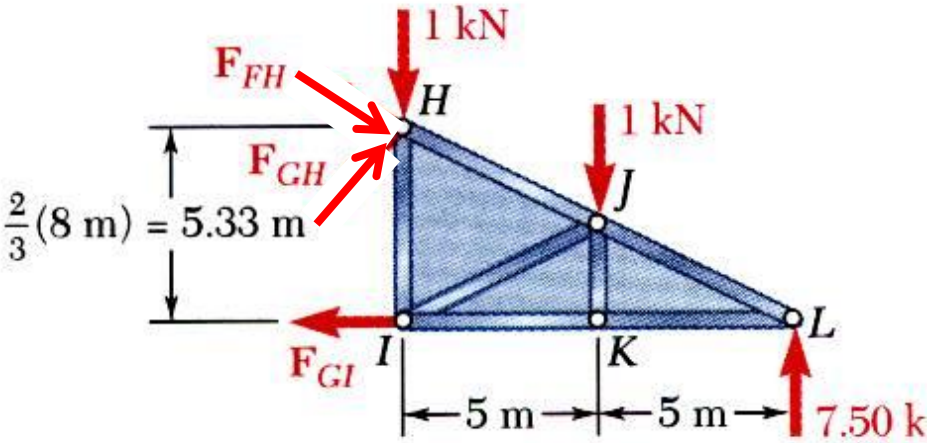
F_{GH} = -1.37kN (Compression)

So;

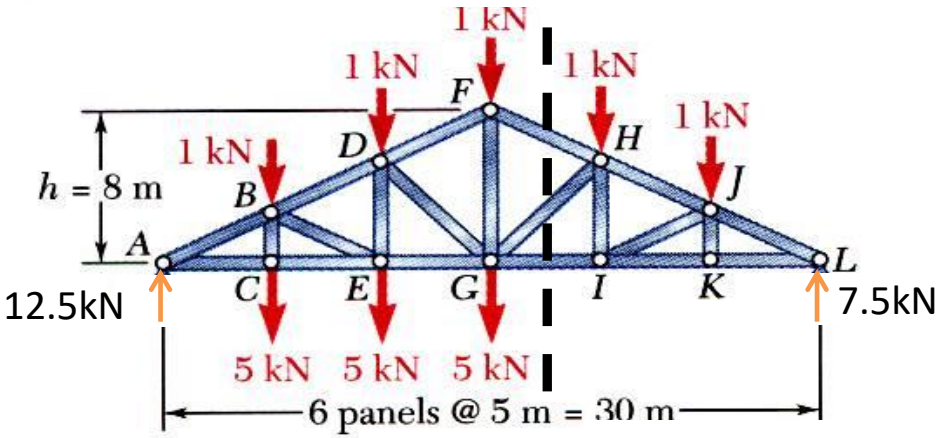
F_{GH} = 13.13kN (Tension)

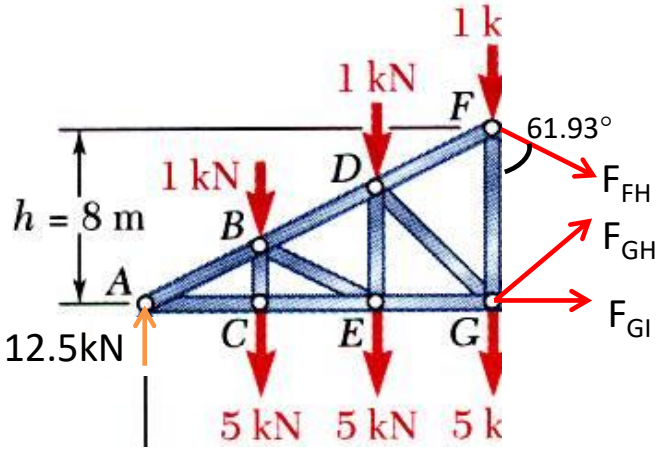
F_{FH} = 13.81kN (Compression)

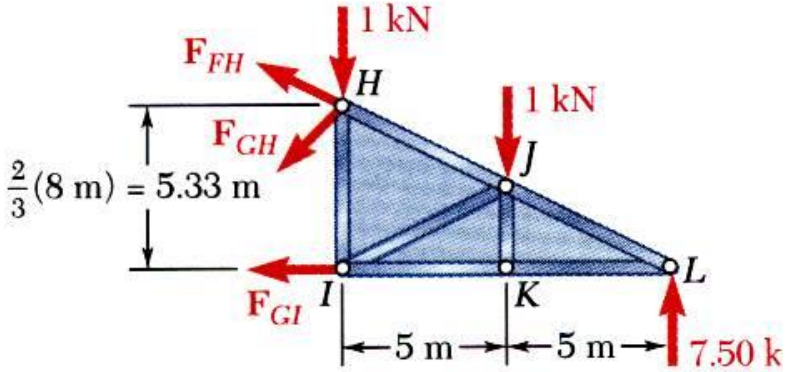
F_{GH} = 1.37kN (Compression)



Alternative way: Use the other section







+ve

$\sum M_H = 0$;

$-F_{GI}$ (5.33m) - 1kN (5m) + 7.5kN (10m) = 0

F_{GI} = 13.13kN (Tension)

+ve

$\sum M_G = 0$;

$-F_{FH}$ sin 61.93° (8m) - 6kN (5m) – 6kN (10m) + 12.5kN (15m) = 0

F_{FH} = -13.81kN (Compression)

(→) $\sum F_x = 0$;

$F_{GI} + F_{GH} \sin \alpha + F_{FH} \sin 61.93° = 0$

13.13kN + F_{GH} sin 43.17° + (-13.81kN) sin 61.93° = 0

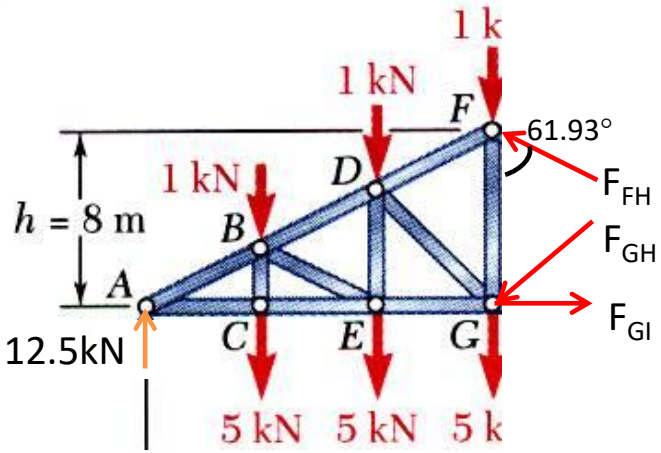
F_{GH} = -1.38kN (Compression)

So;

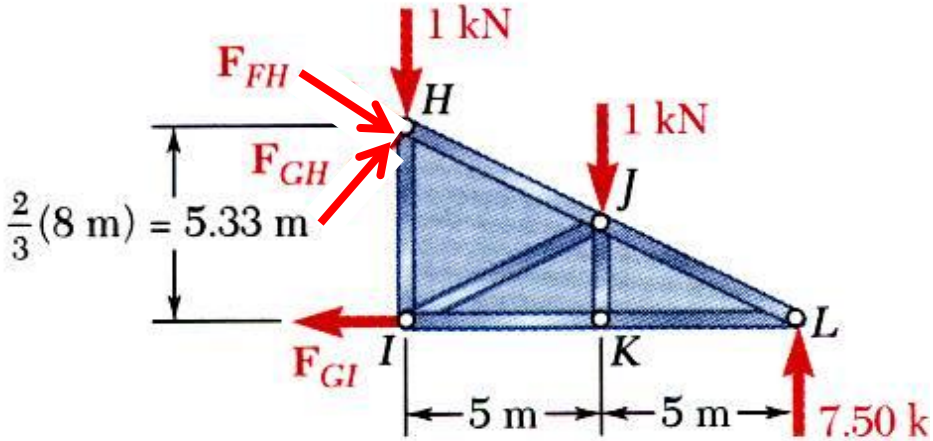
F_{GH} = 13.13kN (Tension)

F_{FH} = 13.81kN (Compression)

F_{GH} = 1.37kN (Compression)



OR



Ex 2: Determine the forces in member *HI*, *HB* and *BC* and determine whether the member in tension or compression

