SEEE1022 INTRODUCTION TO SCIENTIFIC PROGRAMMING

CH11
Data Interpolation

Dr. Mohd Saiful Azimi Mahmud (azimi@utm.my)
P19a-04-03-30, School of Electrical Engineering, UTM

OBJECTIVES

After studying this chapter you should be able to:

1. Perform 1-D interpolation on a dataset using function interp1().
2. Write MATLAB code to find unknown values between some known values.
3. Write MATLAB code to plot interpolated data.
4. Understand the basis usage of interpolation on engineering applications.

INTERPOLATION

WHAT IS INTERPOLATION?

- Interpolation is a mathematical process of estimating unknown values between known points.
- For example, MATLAB plot function connect data points by drawing a straight line between two points. The straight line values is the result of interpolating the two points.

- **Interpolation vs curve fitting**

	Interpolation	Curve Fitting
1	Compute missing values between two points.	Convert data points into mathematical equation.
2	Original data points are preserved.	Fitted curve does not need the original data points to be on the graph.

1-D INTERPOLATION

- **Syntax**

```
vq = interp1(x,v,xq,method)
```

Description

x	:	Sample points.
v	:	Corresponding values of $v(x)$.
xq	:	Interpolated query points.
method	:	Interpolation method (default method is linear if not stated).
vq	:	The returned interpolated values.

INTERPOLATION METHOD

- Below are the variation of the interpolation methods. Each method will return different interpolated values.
- Choosing the right interpolation method depends on the characteristic of the data to be interpolated. Later we will discuss few examples with different interpolation method.

Method	Description	Required Points
Nearest	Nearest neighbour interpolation. This method sets the value of an interpolated point to the value of the nearest existing data point.	At least 2 points
Linear	Linear interpolation. This method uses straight lines to connect the points.	At least 2 points
Spline	Piecewise cubic spline. This is identical to the spline function.	At least 4 points
pchip	Piecewise cubic Hermite interpolation.	At least 4 points

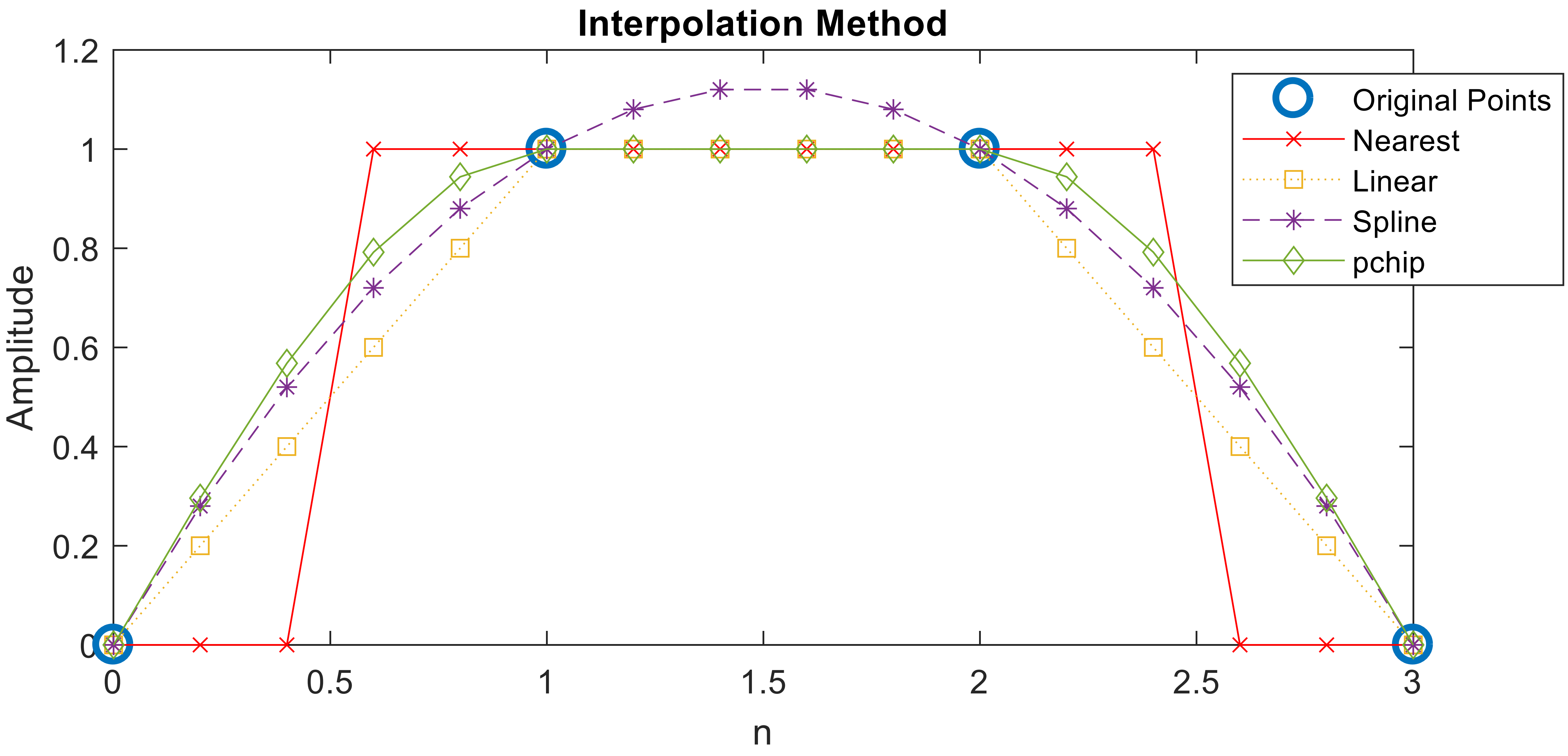
INTERPOLATION METHOD

EXAMPLE 1

Below is a MATLAB script on visualizing difference between the interpolation methods.

```
x = [0 1 1 0];
n1 = 0:3;
n2 = 0:0.2:3;
xnearest = interp1(n1,x,n2,'nearest');
xlinear = interp1(n1,x,n2,'linear');
xspline = interp1(n1,x,n2,'spline');
xpchip = interp1(n1,x,n2,'pchip');

plot(n1,x,' o',n2,xnearest,'-x' ,...
               n2,xlinear ,':s' ,...
               n2,xspline ,'--*',...
               n2,xpchip  ,'-d')
xlabel('n')
ylabel('Amplitude')
title('Interpolation Method')
legend('Original Points','Nearest','Linear','Spline','pchip')
```

INTERPOLATION METHOD



INTERPOLATION APPLICATION

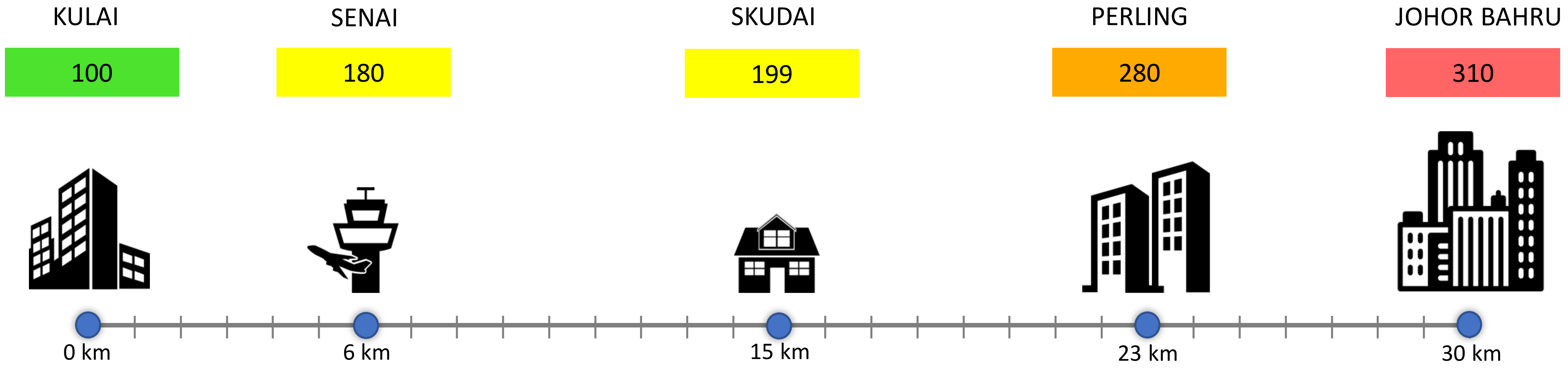
In engineering, especially electrical engineering, interpolation is very useful for the following:

1. Evaluate unknown values between known points.
2. Increase details of a signal.

EVALUATE UNKNOWN VALUES

EVALUATE UNKNOWN VALUES

AIR POLLUTION INDEX



EXAMPLE 2

Above figure shows an air pollution index (API) reading during a haze crisis. Practically, the API sensors were installed at selected discrete location. Assuming the API is linear between two sensors, write a MATLAB code to compute the API values for every kilometre from Kulai to Johor Bahru. Then, find the API value at kilometre 10.

EVALUATE UNKNOWN VALUES

AIR POLLUTION INDEX

EXAMPLE 2

To have an elegant solution, we will create a MATLAB function where its input is a vector stating which kilometres are the API reading is needed. Then, the result will be shown at command window and a plotted graph.

```
function APIValue(km)
d = [0 6 15 23 30];
API = [100 180 199 280 310];
dkm = 0:30;
APIkm = interp1(d,API,dkm);
y = round(APIkm(km+1),0);

fprintf('API value at km%2d is %d\n',[km;y])

figure, plot(d,API,' o',dkm,APIkm,':x')
xtickformat('km %d')
xlabel('Location')
ylabel('API Value')
title('API Calculation from 5 Stations’)
grid on
text(d,API,{'Kulai','Senai','Skudai','Perling','JB'})
```

EVALUATE UNKNOWN VALUES

AIR POLLUTION INDEX

EXAMPLE 2

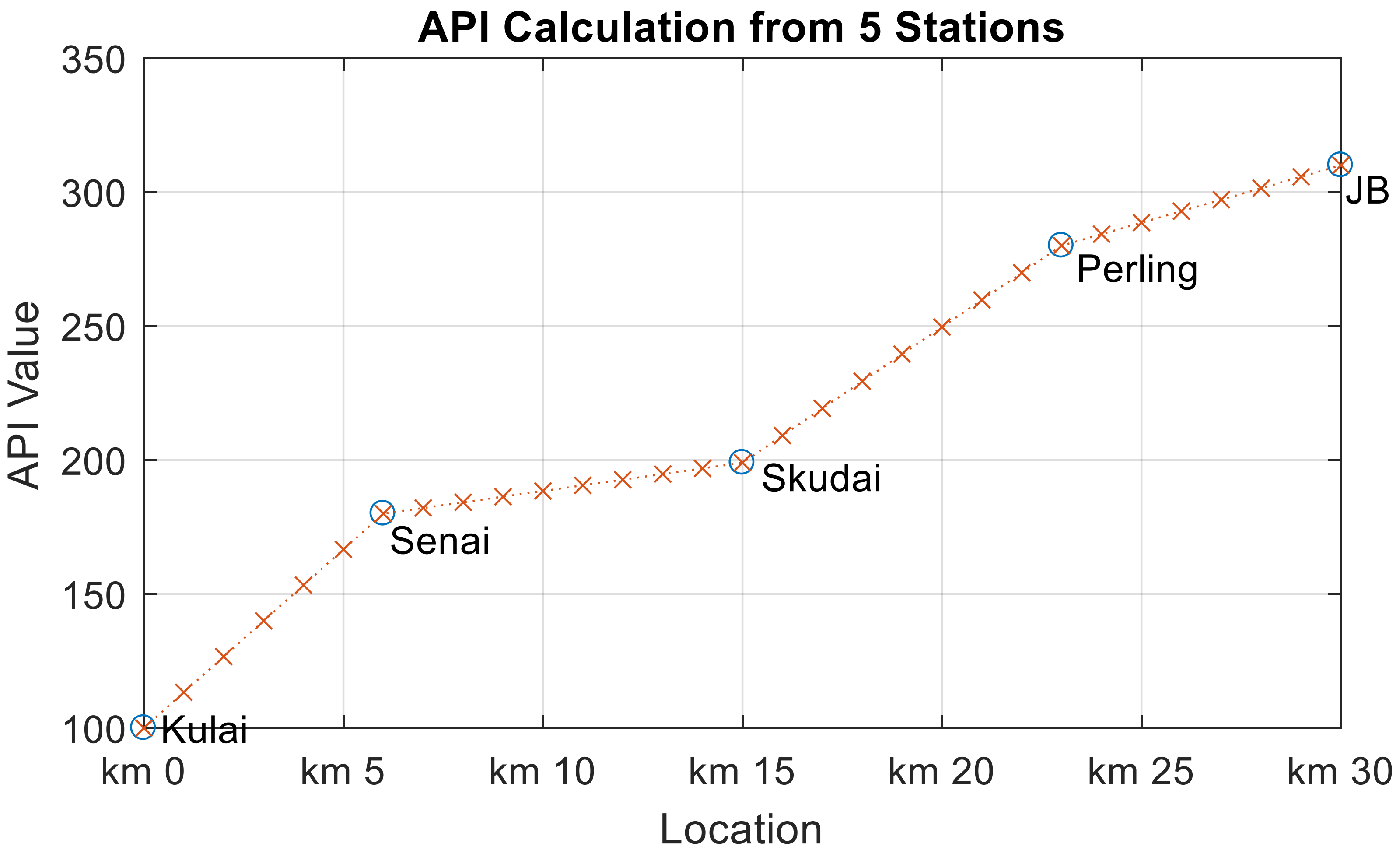
Below are the results of several input vectors.

```
>> APIValue(10)
API value at km10 is 188

>> APIValue(2:5)
API value at km 2 is 127
API value at km 3 is 140
API value at km 4 is 153
API value at km 5 is 167

>> APIValue([3 8 11 26])
API value at km 3 is 140
API value at km 8 is 184
API value at km11 is 191
API value at km26 is 293
```

EVALUATE UNKNOWN VALUES

AIR POLLUTION INDEX

EXAMPLE 2



EVALUATE UNKNOWN VALUES

VLE DIAGRAM

EXAMPLE 3

- In chemical engineering, vapor-liquid equilibrium (VLE) diagram is use to analyze whether the vapor and liquid are pure, which they consist of only one molecular component, or if they are a mixture.
- Constructing the VLE diagram is done by taking mole fraction values for both vapor and liquid at several temperature values. Then, the VLE diagram is plotted on the vapor mole fraction (y-axis) vs the liquid mole fraction (x-axis).
- Below is a table of vapor mole fraction and liquid mole fraction values for 11 different temperatures, recorded at 1 atm for benzene toluene.
- From the table, determine y at x=0.45 using interpolation method.

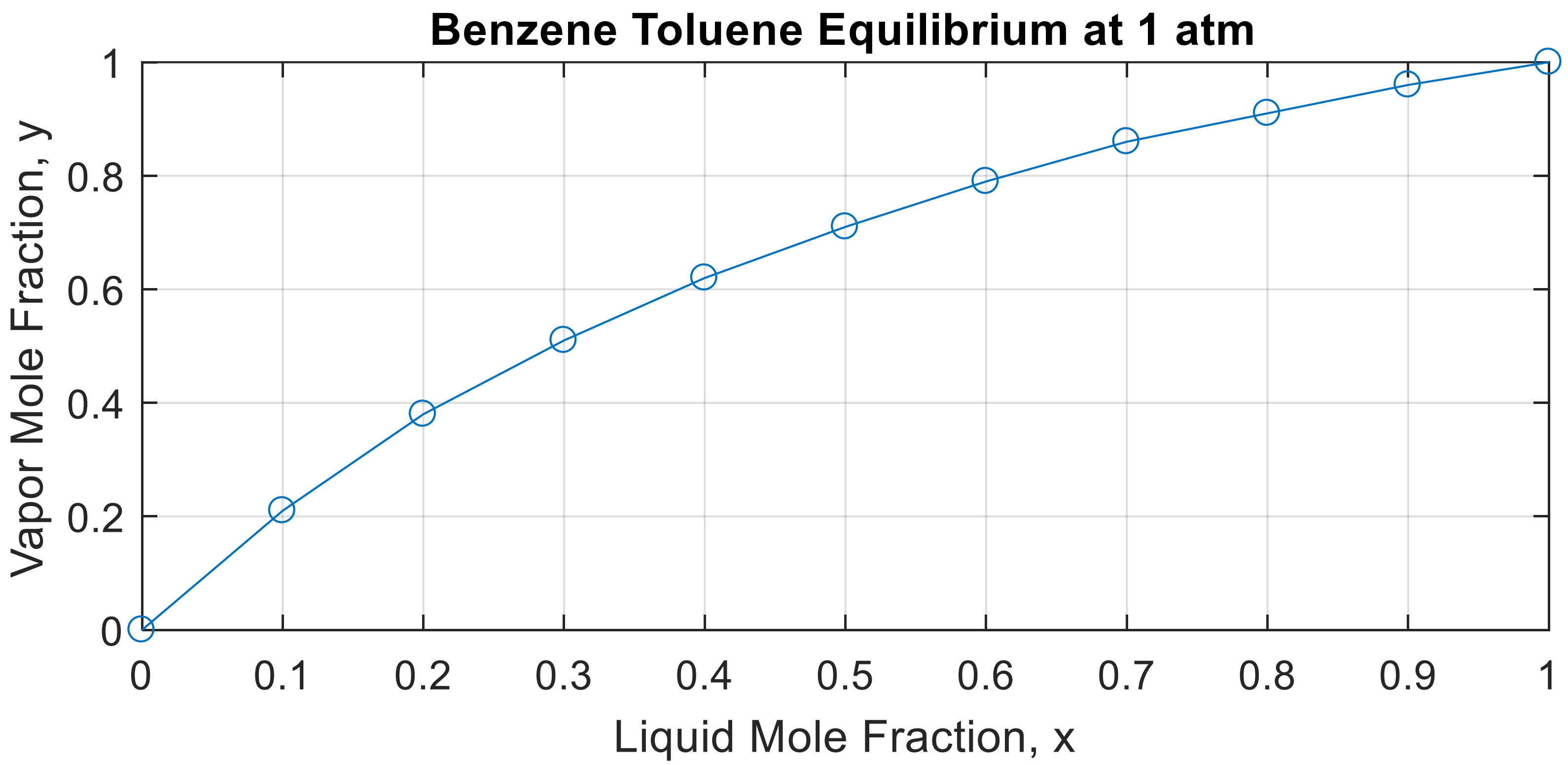
Temp.(℃)	110	106	102	99	95	92	89	87	84	82	80
Liquid mole fraction, x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Vapor mole fraction, y	0.00	0.21	0.38	0.51	0.62	0.71	0.79	0.86	0.91	0.96	1.0

EVALUATE UNKNOWN VALUES

VLE DIAGRAM

EXAMPLE 3

By plotting the VLE diagram as below, it shows that the graph is not linear. The graph has some curvature, stating that the best interpolation method should be spline or pchip.



EVALUATE UNKNOWN VALUES

VLE DIAGRAM

EXAMPLE 3

Below is the MATLAB script to plot the VLE diagram and to find y values for spline and pchip interpolation method.

```
x = [0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0];
y = [0.00 0.21 0.38 0.51 0.62 0.71 0.79 0.86 0.91 0.96 1.00];

yspline = interp1(x,y,0.45,'spline');
ypchip = interp1(x,y,0.45,'pchip');
fprintf('spline: y = %.4f\n pchip : y = %.4f\n',...
        yspline,ypchip)

plot(x,y,'-o')
xlabel('Liquid Mole Fraction, x')
ylabel('Vapor Mole Fraction, y')
title('Benzene Toluene Equilibrium at 1 atm')
grid on
```

```
spline: y = 0.6670
pchip : y = 0.6668
```

INCREASE DETAILS OF A SIGNAL

INCREASE DETAILS OF A SIGNAL

ELECTROCARDIOGRAM (ECG)

EXAMPLE 4

- An electrocardiogram (ECG) records the activity of a heart.
- For wireless mobile ECG devices, low energy consumption is significant, requiring low sampling rate of the heart activity.
- The good thing is, the low sampling rate ECG data can be made more accurate by applying interpolation.
- The **heart.mat** file is a sample of an ECG signal recorded at 60 samples per second or 1 sample at every 0.0167 second. Increase the detail of the signal to 16 times of the original no of samples, interpolation will add 15 new points between each of the original sample.
- Interpolation method suitable to increase the detail of the ECG signal is spline method.

INCREASE DETAILS OF A SIGNAL

ELECTROCARDIOGRAM (ECG)

EXAMPLE 4

- Below is the MATLAB code interpolating the low sampling rate ECG signal. In this case, spline interpolation method is used.
- It can be seen from the plot that peaks which were missing in the low sampling rate ECG signal are recovered with the spline interpolation process.

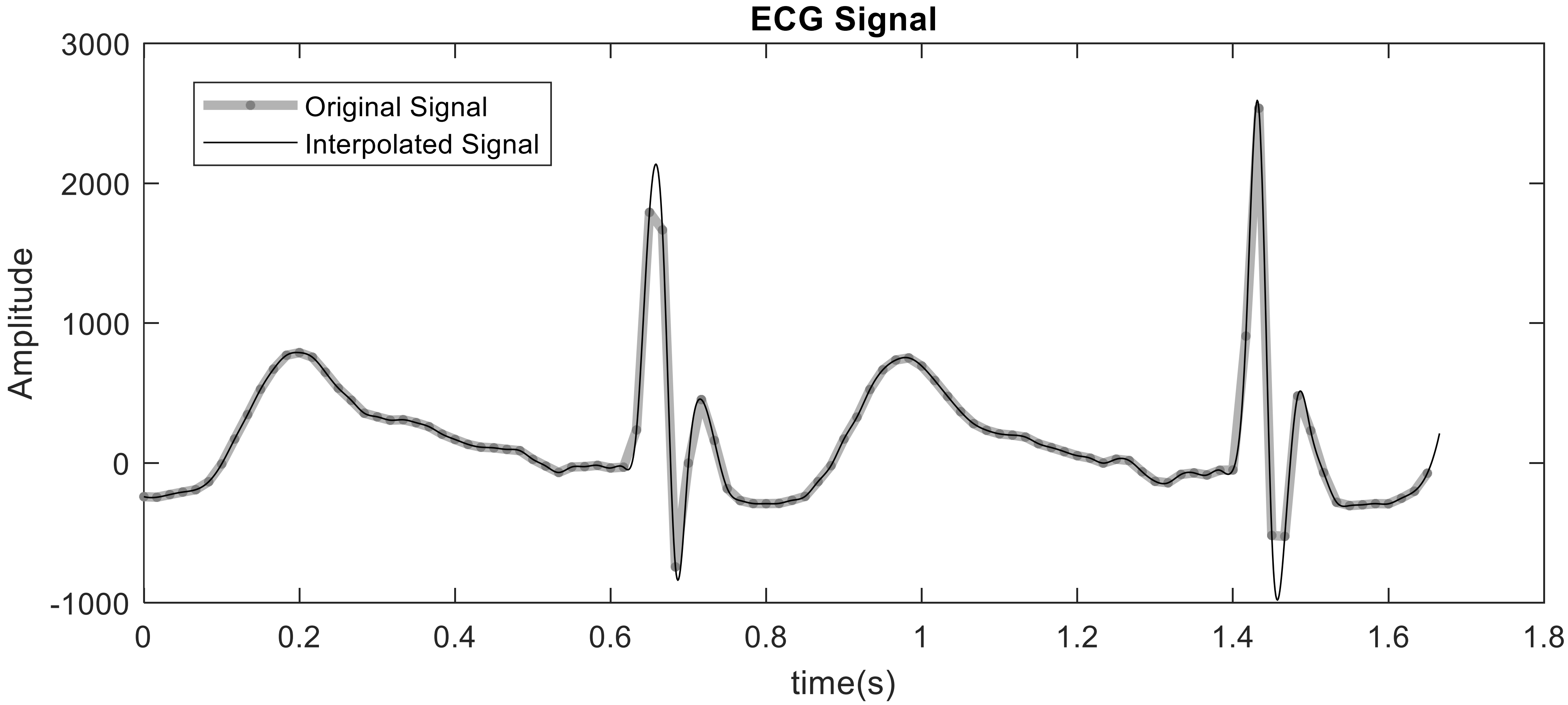
```
load heart
ts = 0.0167;
N = length(x)-1;
t = 0:ts:N*ts;
t2 = 0:ts/16:N*ts;
xinterp = interp1(t,x,t2,'spline');

p = plot(t,x,'-*',t2,xinterp,'k');
p(1).LineWidth = 3;
p(1).Color = [0.7 0.7 0.7];
p(1).MarkerEdgeColor = [0.5 0.5 0.5];
p(1).MarkerSize = 1;
xlabel('time(s)')
ylabel('Amplitude')
title('ECG Signal')
legend('Original Signal','Interpolated Signal')
```

Look for topic '**Modify Lines After Creation**' within the `plot()` function documentation for plotting method used in this example.

INCREASE DETAILS OF A SIGNAL

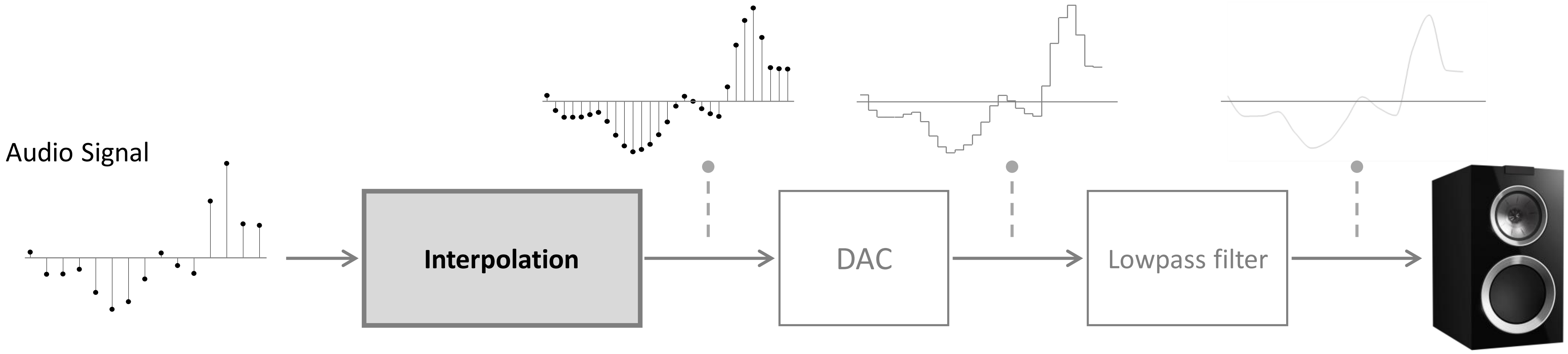
ELECTROCARDIOGRAM (ECG)

EXAMPLE 4



INCREASE DETAILS OF A SIGNAL

DIGITAL TO ANALOG CONVERTER



EXAMPLE 5

- Digital to analog converter is a basic component to an electronic system. For example, a digital audio song played on a computer will be first converted to analog signal before the signal is send to the speaker.
- To improve the DAC output signal, a lowpass filter is applied, and to simplified the circuit of the lowpass filter, signal interpolation is performed before the DAC. This interpolation process is also called upsampling.

INCREASE DETAILS OF A SIGNAL

DIGITAL TO ANALOG CONVERTER

EXAMPLE 5

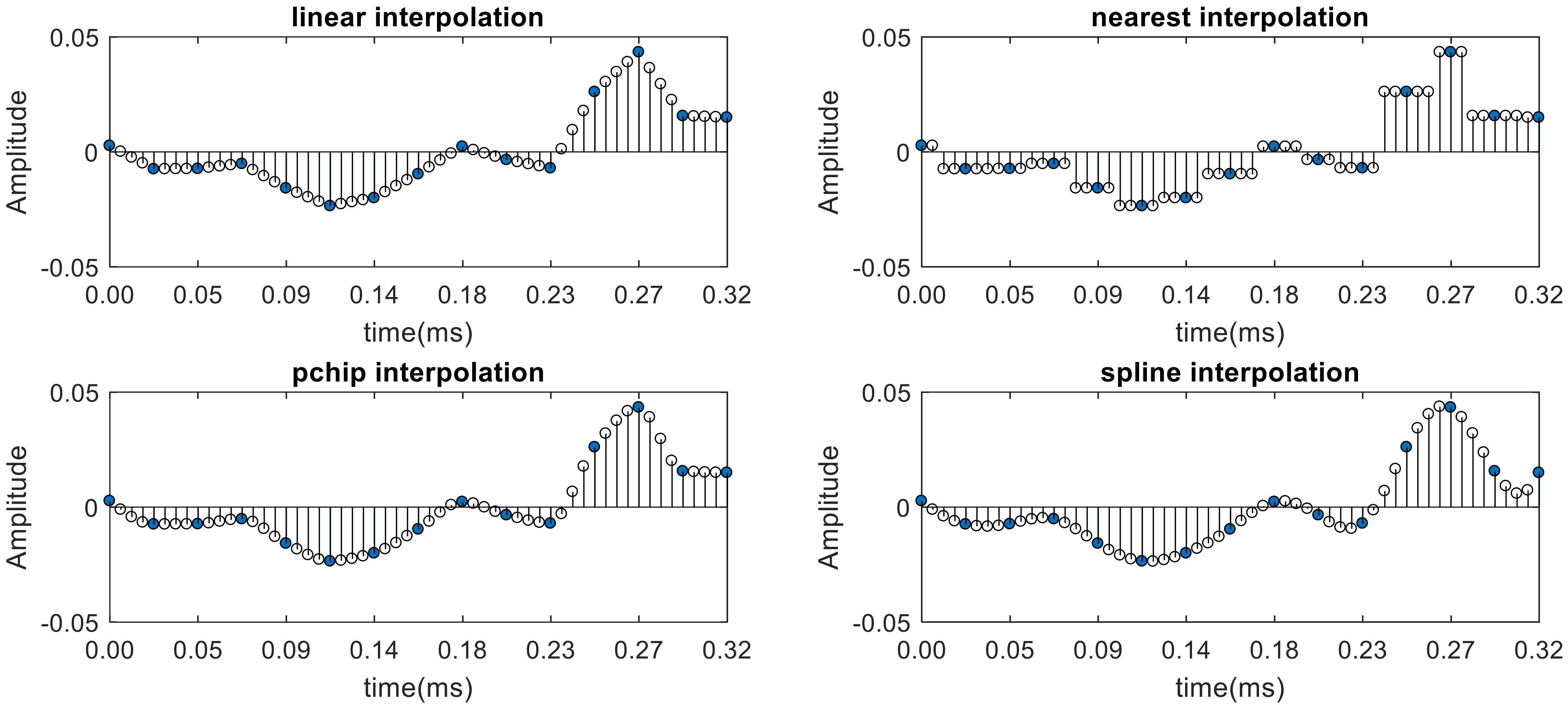
Below the MATLAB script that looks at four interpolation method

```
load signal
fs = 44100;
ts = 1/fs;
N = length(x)-1;
t = 0:ts:N*ts;
t2 = 0:ts/4:N*ts;

method = {'linear','nearest','pchip','spline'};
for i = 1:4
    m = method{i};
    y = interp1(t,x,t2,m);
    subplot(2,2,i), stem(t*1000,x,'filled','MarkerSize',4), hold
    subplot(2,2,i), stem(t2*1000,y,'k','MarkerSize',4)
    title([m ' interpolation'])
    xlabel('time(ms)')
    ylabel('Amplitude')
    xticks(t(1:2:end)*1000)
    xtickformat('%.2f')
    xlim([0 t(end)*1000])
end
```

INCREASE DETAILS OF A SIGNAL

DIGITAL TO ANALOG CONVERTER

EXAMPLE 5



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