SCHOOL OF ELECTRICAL ENGINEERING
FACULTY OF ENGINEERING

SKEE 1223
DIGITAL ELECTRONICS

CHAPTER 4: COMBINATORIAL LOGIC NETWORKS

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STANDARD FORMS OF BOOLEAN EXPRESSIONS

STANDARD FORMS OF BOOLEAN EXPRESSIONS

INTRODUCTION

- All Boolean expression can be converted into two standard form:
  1. The **Sum-of Product form (SOP)**.
  2. The **Product-of-Sum form (POS)**.
- These standardization makes evaluation, simplification and implementation of Boolean expression more systematic and easier.

STANDARD FORMS OF BOOLEAN EXPRESSIONS

SUM-OF-PRODUCT (SOP): INTRODUCTION

- **SOP** is defined as two or more product terms are summed by Boolean addition.

Example

$$AB + ABC$$

Product term/minterm: AB

Sum of product: $AB + ABC$

$$A\bar{B} + \bar{A}B\bar{C} + AC$$

$$ABC + CDE + \bar{B}C\bar{D}$$

STANDARD FORMS OF BOOLEAN EXPRESSIONS

SUM-OF-PRODUCT (SOP): INTRODUCTION

- **Domain of SOP expression:** The set of variables contain in the expression either in complemented and uncomplemented form.

Example

The domain of expression $A\bar{B} + \bar{A}BC + AC = A, B$ and C.

The domain of expression $AB\bar{C} + C\bar{D}E + \bar{B}C\bar{D} = A, B, C, D$ and E.

- In SOP, a single overbar cannot extend more than one variable, example:

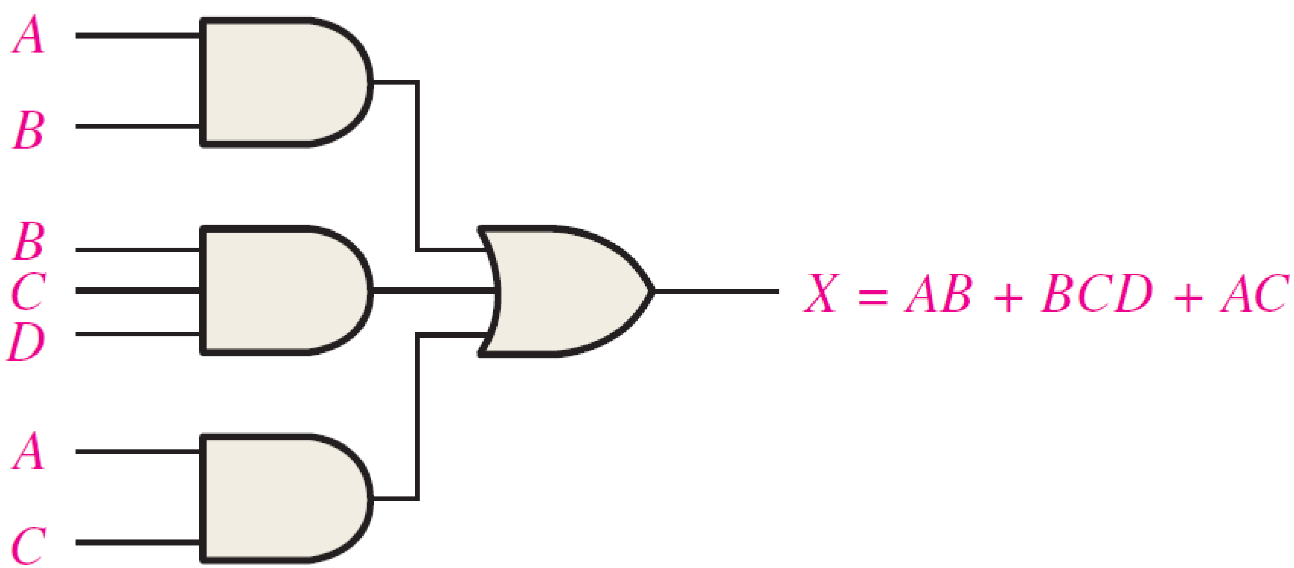
$AB + A\overline{BC}$

Not SOP because $\overline{BC}$

STANDARD FORMS OF BOOLEAN EXPRESSIONS

SUM-OF-PRODUCT (SOP): AND/OR IMPLEMENTATION OF SOP EXPRESSION

- Implementing an **SOP expression** simply requires **OR**ing the outputs of **AND** gates.
- A **product term (minterm)** is produced by AND operation.
- The **SOP** terms is produced by OR operation.



STANDARD FORMS OF BOOLEAN EXPRESSIONS

SUM-OF-PRODUCT (SOP): STANDARD SOP FORM

- A **standard SOP form** must contains **all of the variables in the domain** of the expression for each product term, example:

$$\bar{A}B\bar{C} + A\bar{B}C + ABC$$

- Standard SOP forms are essential in constructing the truth table.
- In the following SOP form,

$$A\bar{B}C + \bar{A}\bar{B} + ABC\bar{D}$$

- How many minterms are there? 3
- Is it a standard SOP form? No
- How do we convert a Boolean expression into standard SOP form?

STANDARD FORMS OF BOOLEAN EXPRESSIONS

SUM-OF-PRODUCT (SOP): STANDARD SOP FORM

- To convert SOP to its standard form, we use Boolean expression:

$$A + \bar{A} = 1 \qquad \textbf{\textit{RULE 6}}$$

$$A(B + C) = AB + AC \qquad \textbf{\textit{DISTRIBUTIVE LAW}}$$

Example

Convert the following Boolean expression into SOP standard form.

$$A\bar{B}C + \bar{A}\bar{B} + AB\bar{C}D$$

Solution

The domain of this SOP are $\boldsymbol{A, B, C}$ and $\boldsymbol{D}$. Take one term at a time, the first term, $A\bar{B}C$ is missing D or $\bar{D}$. So, multiply $\mathbf{1 = D + \bar{D}}$ as follow:

$$A\bar{B}C = A\bar{B}C(\boldsymbol{D + \bar{D}}) = A\bar{B}CD + A\bar{B}C\bar{D}$$

STANDARD FORMS OF BOOLEAN EXPRESSIONS

SUM-OF-PRODUCT (SOP): STANDARD SOP FORM

The **second term**, $\bar{A}\bar{B}$ is missing C or $\bar{C}$ and D or $\bar{D}$. So, multiply $\mathbf{1 = C + \bar{C}}$ and $\mathbf{1 = D + \bar{D}}$ as follow:

$$\begin{aligned}\bar{A}\bar{B} &= \bar{A}\bar{B}(\boldsymbol{C + \bar{C}})(\boldsymbol{D + \bar{D}}) \\ &= (\bar{A}\bar{B}\boldsymbol{C} + \bar{A}\bar{B}\boldsymbol{\bar{C}})(\boldsymbol{D + \bar{D}}) \\ &= \bar{A}\bar{B}\boldsymbol{CD} + \bar{A}\bar{B}\boldsymbol{C\bar{D}} + \bar{A}\bar{B}\boldsymbol{\bar{C}D} + \bar{A}\bar{B}\boldsymbol{\bar{C}\bar{D}}\end{aligned}$$

The **third term**, $\mathrm{AB}\bar{C}D$ already in standard form (nothing missing).

Thus, the **standard SOP form**:

$$A\bar{B}C\boldsymbol{D} + A\bar{B}C\boldsymbol{\bar{D}} + \bar{A}\bar{B}\boldsymbol{CD} + \bar{A}\bar{B}\boldsymbol{C\bar{D}} + \bar{A}\bar{B}\boldsymbol{\bar{C}D} + \bar{A}\bar{B}\boldsymbol{\bar{C}\bar{D}} + AB\bar{C}D$$

STANDARD FORMS OF BOOLEAN EXPRESSIONS

PRODUCT OF SUM (POS): INTRODUCTION

- **POS** is defined as two or more sum terms are multiplied.

Example

$$(\bar{A} + B)(A + B + \bar{C})$$

Sum term / maxterm: $(\bar{A} + B)$

Product of sum: $(\bar{A} + B)(A + B + \bar{C})$

$$(\bar{A} + \bar{B} + \bar{C} + \bar{D})(C + D + \bar{E})(\bar{B} + C + D)$$

- In POS, a single overbar cannot extend more than one variable, example:

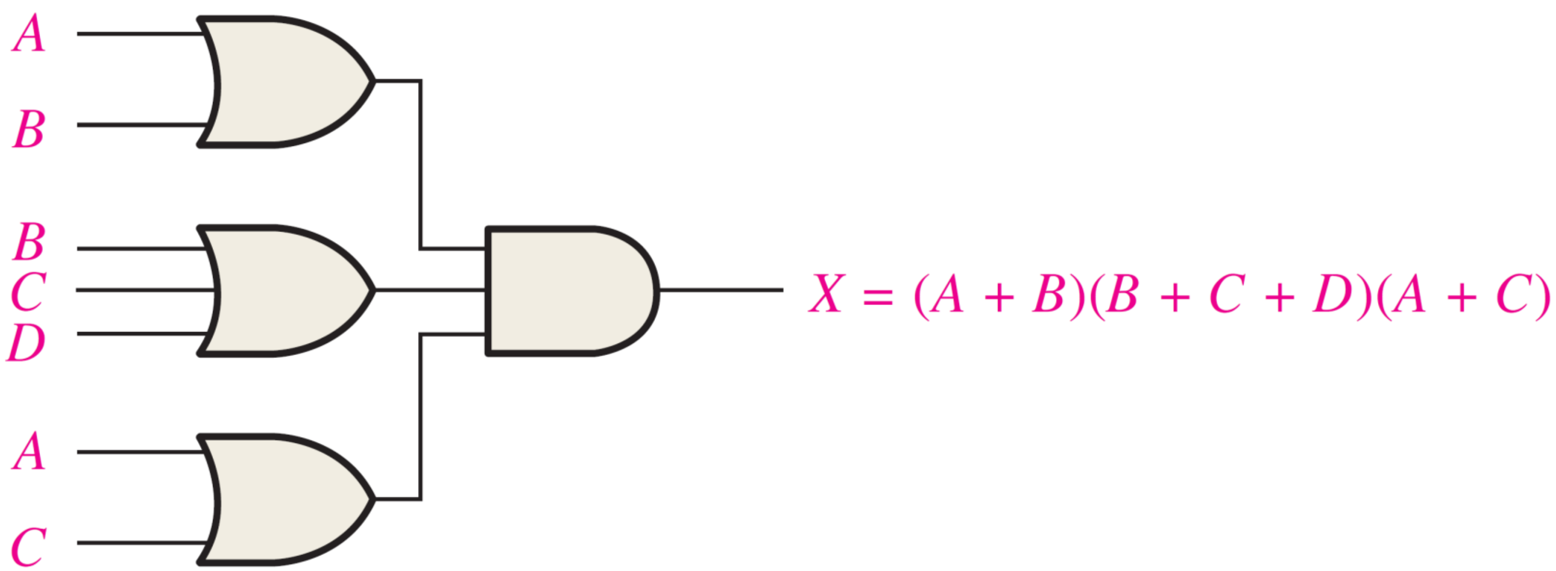
$$(\bar{A} + \bar{B})(A + \overline{B + C})$$

Not POS because $\overline{B + C}$

STANDARD FORMS OF BOOLEAN EXPRESSIONS

PRODUCT OF SUM (POS): AND/OR IMPLEMENTATION OF POS EXPRESSION

- Implementing a **POS expression** simply requires **AND**ing the outputs of **OR** gates.
- A **sum term (maxterm)** is produced by **OR** operation.
- The **product of sum** term is produced by **AND** operation.



STANDARD FORMS OF BOOLEAN EXPRESSIONS

PRODUCT OF SUM (POS): STANDARD POS FORM

- A **standard POS form** must contains **all of the variables in the domain** of the expression for each sum term, example:

$$(\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+\bar{C}+D)(A+\bar{B}+C+D)$$

- Standard POS forms are essential in constructing the truth table.
- In the following POS form,

$$(A+\bar{B}+C)(\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$$

- How many maxterms are there? 3
- Is it a standard POS form? No
- How do we convert a Boolean expression into standard POS form?

STANDARD FORMS OF BOOLEAN EXPRESSIONS

PRODUCT-OF-SUM (POS): STANDARD POS FORM

- To convert POS to its standard form, we use Boolean expression:

$$A \cdot \bar{A} = 0 \qquad \textbf{\textit{RULE 8}}$$

$$A + BC = (A + B)(A + C) \quad \textbf{\textit{DISTRIBUTIVE LAW}}$$

Example

Convert the following Boolean expression into POS standard form.

$$(A + \bar{B} + C)(\bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)$$

Solution

The domain of this POS are $\boldsymbol{A, B, C}$ and $\boldsymbol{D}$. Take one term at a time, **the first term**, $(A + \bar{B} + C\ )$ is missing D or $\bar{D}$. So, add $\mathbf{0 = \boldsymbol{D \cdot \bar{D}}}$ as follow:

$$A + \bar{B} + C = A + \bar{B} + C + \boldsymbol{D \cdot \bar{D}} = (A + \bar{B} + C + \boldsymbol{D})(A + \bar{B} + C + \boldsymbol{\bar{D}})$$

STANDARD FORMS OF BOOLEAN EXPRESSIONS

PRODUCT-OF-SUM (POS): STANDARD POS FORM

The **second term**, $(\bar{B}+C+\bar{D})$ is missing A or $\bar{A}$. So, add $\mathbf{0} = \boldsymbol{A} \cdot \bar{\boldsymbol{A}}$ as follow:

$$\begin{aligned} \bar{B}+C+\bar{D} &= \bar{B}+C+\bar{D}+\boldsymbol{A}\cdot\bar{\boldsymbol{A}} \\ &= (\boldsymbol{A}+\bar{B}+C+\bar{D})(\bar{\boldsymbol{A}}+\bar{B}+C+\bar{D}) \end{aligned}$$

The **third term**, $(A+\bar{B}+\bar{C}+D)$ already in standard form (nothing missing).

Thus, the **standard POS form**:

$$(A+\bar{B}+C+\boldsymbol{D})(A+\bar{B}+C+\bar{\boldsymbol{D}})(\boldsymbol{A}+\bar{B}+C+\bar{D}) \\ (\bar{\boldsymbol{A}}+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)$$

STANDARD FORMS OF BOOLEAN EXPRESSIONS

ASSESSMENT

1. Convert the following Boolean expression into **standard SOP form**.

$$A(B + CD)$$

2. Convert the following Boolean expression into **standard POS form**.

$$(A + \bar{B})(B + C)$$

BOOLEAN EXPRESSION AND TRUTH TABLE

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: INTRODUCTION

- A common way of representing a logical operation of a circuit.
- If $\boldsymbol{n}$ is the number of inputs, combination in truth table equal to $\mathbf{2}^{\boldsymbol{n}}$.

Example

Inputs are A, B and C. Thus, there will be $2^3 = 8$ combinations.

Input			Output
A	B	C	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: SOP TO TRUTH TABLE

- Develop a truth table for the standard SOP expression:

$$\bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$$

- Domain = A, B and C. So, combination $= 2^3 = 8$.
- Determine the inputs that makes the product term $= 1$.
- Currently, we have three product terms:

$\bar{A}\bar{B}C = 1?$ → 001

$A\bar{B}\bar{C} = 1?$ → 100

$ABC = 1?$ → 111

- Finally, fills the truth table.

Input			Output	Product Terms
A	B	C	F	
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	0	
0	1	1	0	
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: POS TO TRUTH TABLE

- Develop a truth table for the standard POS expression:

$$(A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$$

- Domain $= A, B$ and C. So, combination $= 2^3 = 8$.
- Determine the inputs that makes the sum term $= 0$.
- Currently, we have five product terms:

$(A+B+C)=0? \rightarrow 000$

$(A+\bar{B}+C)=0? \rightarrow 010$

$(A+\bar{B}+\bar{C})=0? \rightarrow 011$

$(\bar{A}+B+\bar{C})=0? \rightarrow 101$

$(\bar{A}+\bar{B}+C)=0? \rightarrow 110$

- Finally, fills the truth table.

Input			Output	Sum Terms
A	B	C	F	
0	0	0	0	$(A+B+C)$
0	0	1	1	
0	1	0	0	$(A+\bar{B}+C)$
0	1	1	0	$(A+\bar{B}+\bar{C})$
1	0	0	1	
1	0	1	0	$(\bar{A}+B+\bar{C})$
1	1	0	0	$(\bar{A}+\bar{B}+C)$
1	1	1	1	

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: BOOLEAN EXPRESSION AND TRUTH TABLE

Step 1

Convert expression to **Standard SOP/POS**.

Step 2

Determine domain and combinations of binary values at input.

Step 3

Find the inputs values that make the term:

1. Equal to **1** for **SOP**, e.g. $ABC = 111$.
2. Equal to **0** for **POS**, e.g. $A + B + C = 000$.

Step 3

Fill the truth table (the remaining blanks with inverse values).

BOOLEAN EXPRESSION AND TRUTH TABLE

ASSESSMENT

Make a truth table for the following functions:

1. $F = AB + A\bar{C}$

2. $F = (A + C)(B + \bar{C})$

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: TRUTH TABLE TO SOP FORM

- Can write **standard SOP expression** simply from truth table.

Input			Output	Product term/ minterm
A	B	C	F	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	1	$\bar{A}BC$
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	1	$A\bar{B}C$
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

Note that each term has **ALL variables. If a product term has **ALL** variable present, it is a **MINTERM**.

$$F(A,B,C) = \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: TRUTH TABLE TO POS FORM

- Can write **standard POS expression** simply from truth table.

Input			Output	Sum term/ maxterm
A	B	C	F	
0	0	0	0	$A + B + C$
0	0	1	0	$A + B + \bar{C}$
0	1	0	0	$A + \bar{B} + C$
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	1	

Note that each term has **ALL variables. If a sum term has **ALL** variable present, it is a **MAXTERM**.

$$\boldsymbol{F(A,B,C) = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)}$$

BOOLEAN EXPRESSION AND TRUTH TABLE

ACTIVITY 1

Derive the SOP expression for the logic function specified in the truth table.

Input			Output	Sum term/ maxterm
A	B	C	L	
0	0	0	1	
0	0	1	1	
0	1	0	0	
0	1	1	1	
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: MINTERM AND MAXTERM

- From previous slides, we saw that:
  - ❖ **SOP Form:** $F(A,B,C) = \overline{A}BC + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC$

    If a product term has all variables present, it is a **MINTERM**.

  - ❖ **POS Form:** $F(A,B,C) = (A+B+C)(A+B+\overline{C})(A+\overline{B}+C)$

    If a sum term has all variables present, it is a **MAXTERM**.

- All Boolean function can be written in terms of either Minterm or Maxterm notations.

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: MINTERM AND MAXTERM NOTATION

- How to write either in Minterm or Maxterm?
- Each line in a truth table represents both a Minterm and Maxterm.

Row	Input			Product term/ minterm	Sum term/ maxterm
	A	B	C		
0	0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A + B + C = m_0$
1	0	0	1	$\bar{A}\bar{B}C = m_1$	$A + B + \bar{C} = m_1$
2	0	1	0	$\bar{A}B\bar{C} = m_2$	$A + \bar{B} + C = m_2$
3	0	1	1	$\bar{A}BC = m_3$	$A + \bar{B} + \bar{C} = m_3$
4	1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A} + B + C = m_4$
5	1	0	1	$A\bar{B}C = m_5$	$\bar{A} + B + \bar{C} = m_5$
6	1	1	0	$AB\bar{C} = m_6$	$\bar{A} + \bar{B} + C = m_6$
7	1	1	1	$ABC = m_7$	$\bar{A} + \bar{B} + \bar{C} = m_7$

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: USING MINTERM AND MAXTERM NOTATION

- A Boolean function can be written in terms of Minterm or Maxterm notation as shorthand method of specifying the function.

Input			Output	minterm/ maxterm
A	B	C	F	
0	0	0	0	$A+B+C$
0	0	1	0	$A+B+\bar{C}$
0	1	0	0	$A+\bar{B}+C$
0	1	1	1	$\bar{A}BC$
1	0	0	1	$A\bar{B}\bar{C}$
1	0	1	1	$A\bar{B}C$
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

*Minterms correspond to **‘1’** of **F**, Maxterms correspond to **‘0’** of **F** in truth table.

Minterm notation

$$F(A,B,C) = \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$
$$= m_3 + m_4 + m_5 + m_6 + m_7$$
$$= \sum m(3,4,5,6,7)$$

Maxterm notation

$$F(A,B,C) = (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)$$
$$= m_0 \cdot m_1 \cdot m_2$$
$$= \prod M(0,1,2)$$

BOOLEAN EXPRESSION AND TRUTH TABLE

ACTIVITY 2

Express the output function F1 in sum of minterms form.

Input			Output
x	y	z	F1
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

BOOLEAN EXPRESSION AND TRUTH TABLE

TRUTH TABLE: MINTERM AND MAXTERM TO TRUTH TABLE

- The **Minterms** correspond to **'1'** of **F**, Maxterms correspond to **'0'** of **F** in truth table.
- Given minterm and maxterm as follows. Find the truth table.

Minterm notation

$$F(A,B,C) = \sum m(1,2,6)$$
$$= m_1 + m_2 + m_6$$
$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C}$$

Maxterm notation

$$F(A,B,C) = \prod M(0,3,4,5,7)$$
$$= M_0 \cdot M_3 \cdot M_4 \cdot M_5 \cdot M_7$$
$$= (A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$$

Minterm/ Maxterm notation	Input			Output
	A	B	C	F
M_0	0	0	0	0
m_1	0	0	1	1
m_2	0	1	0	1
M_3	0	1	1	0
M_4	1	0	0	0
M_5	1	0	1	0
m_6	1	1	0	1
M_7	1	1	1	0

BOOLEAN EXPRESSION AND TRUTH TABLE

ACTIVITY 3

Express the output function F1 in product of maxterms form.

Input			Output
x	y	z	F1
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

BOOLEAN EXPRESSION AND TRUTH TABLE

ASSESSMENT

Derive the simplest SOP expression from the following functions.

i. $F(A,B,C) = \sum m\,(3,5,7)$

ii. $F = (A + C)(A\bar{B} + AC)(\bar{A}\bar{C} + \bar{B})$

iii. $F = \left(B + \bar{C}\right)(\bar{B} + C) + (\bar{A} + B + \bar{C})$

iv. $F(A,B,C,D) = \prod M\,(2,5,6,7,10,14)$

Answer

i. $F = AC + BC$

K-MAP

K-MAP

INTRODUCTION

- **K-map (Karnaugh Map)** is similar to truth table because it presents all of the **positive values** of input variables and the resulting **output** of each value.
- K-map represents by **an array of cells** in which each cell represents a binary value of the input valuable.
- It provides a systematic method for **simplifying Boolean expressions** and could produce the simplest SOP or POS expression possible.
- K-map can be used for expression with two, three, four and five variables.
- Number of cells $= 2^n$.
- For three variables (A, B, C) the number of cells $= 2^3 = 8$.
- For four variables (A, B, C, D) the number of cells $= 2^4 = 16$.

K-MAP
2-VARIABLE

- 2-Variable K-Map consists of an array of **four cells**.

Input		Output
A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

A \ B	0	1
0	0	1
1	1	0

K-MAP

4-VARIABLE

- 4-Variable K-Map consists of an array of **sixteen cells**.

Input				Output
A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0

Input				Output
A	B	C	D	F
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

AB \ CD	00	01	11	10
00	0	1	1	0
01	1	0	0	0
11	1	1	1	0
10	0	0	0	1

K-MAP
EXAMPLE

Given the following **standard SOP form**, complete the truth table and K-map.

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$$

$F(A,B,C)$ truth table

Input			Output	
A	B	C	F	
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

A \ BC	00	01	11	10
0	0	1	0	1
1	0	0	1	1

K-MAP
EXAMPLE

Given the following **SOP form**, complete the truth table and K-map.

$$F = \bar{B}\bar{C} + A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD$$

Find the **standard SOP form** first.

$$\begin{aligned} F &= \bar{B}\bar{C}(A + \bar{A})(D + \bar{D}) + A\bar{B}(C + \bar{C})(D + \bar{D}) + AB\bar{C}(D + \bar{D}) \\ &\quad + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD \\ &= (A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C})(D + \bar{D}) + (A\bar{B}C + A\bar{B}\bar{C})(D + \bar{D}) + AB\bar{C}D \\ &\quad + AB\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD \\ &= A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} \\ &\quad + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D \\ &\quad + A\bar{B}CD \\ &= A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} \\ &\quad + AB\bar{C}D + AB\bar{C}\bar{D} \end{aligned}$$

K-MAP

EXAMPLE (CONT.)

Transfer the **standard SOP form**, into the truth table and complete the K-map.

$$F = A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D}$$

$F(A, B, C, D)$ truth table

Input				Output
A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0

Input				Output
A	B	C	D	F
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

4-variable K-map

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	1	1	0	0
10	1	1	1	1

K-MAP

EXAMPLE (CONT.)

Transfer the **SOP form** directly into the K-map then complete the truth table.

$$F = \bar{B}\bar{C} + A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}CD$$

$F(A, B, C, D)$ truth table

Input				Output	Input				Output
A	B	C	D	F	A	B	C	D	F
0	0	0	0	1	1	0	0	0	1
0	0	0	1	1	1	0	0	1	1
0	0	1	0	0	1	0	1	0	1
0	0	1	1	0	1	0	1	1	1
0	1	0	0	0	1	1	0	0	1
0	1	0	1	0	1	1	0	1	1
0	1	1	0	0	1	1	1	0	0
0	1	1	1	0	1	1	1	1	0

4-variable K-map

AB \ CD	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	1	1	0	0
10	1	1	1	1

K-MAP

ASSESSMENT

Given the following standard SOP form, complete the truth table and K-map.

$$F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$$

$F(A, B, C)$ truth table

Input			Output	
A	B	C	F	
0	0	0	0	
0	0	1	1	$\bar{A}\bar{B}C$
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

A \ BC	00	01	11	10
0	0	1	0	1
1	0	0	1	1

K-MAP

ASSESSMENT

Given the following min and max terms, complete the K-map.

1. $F(A, B, C, D) = \sum m \ (1, 3, 5, 9, 12, 13, 14)$

2. $F(W, X, Y, Z) = \prod M \ (3, 4, 5, 6, 7, 9, 11)$

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	0	0
11	1	1	0	1
10	0	1	0	0

WX \ YZ	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	1	1	1
10	1	0	0	1

K-MAP
ASSESSMENT

Given the following expressions, complete the K-map.

1. $F(x, y, z) = \bar{x}\bar{z} + \bar{y}\bar{z} + y\bar{z} + xy$

2. $F(a, b, c, d) = (a + \bar{b} + c)(a + \bar{b})(a + \bar{c} + \bar{d})(\bar{a} + b + c + \bar{d})(b + \bar{c} + \bar{d})$

z \ xy	00	01	11	10
0	1	1	1	1
1	0	0	1	0

cd \ ab	00	01	11	10
00	1	0	1	1
01	1	0	1	0
11	0	0	1	0
10	1	0	1	1

K-MAP SIMPLIFICATION

K-MAP SIMPLIFICATION

GROUPING

- After SOP expression has been mapped, minimum expression is obtained by grouping the 1s and determining the minimum SOP expression from the K-map.

- When grouping the 1s, the goals are:
  1. To maximize the size of the groups.
  2. To minimize the number of groups.

K-MAP SIMPLIFICATION
GROUPING

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Rules for grouping of 1s

- A group must **contain either 1, 2, 4, 8 or 16 cells**. For x-variable K-map, 2^x cells is maximum.
- **Each cell in a group must be adjacent** to one or more cells in that same group, but all cells in the group don't have to be adjacent to each other.
- Always include the **largest possible number** of 1s in a group.
- **Each 1 on the map must be included in at least one group**. The 1s already in a group can be included in another group as long as the overlapping groups include common 1s.

K-MAP SIMPLIFICATION

GROUPING: EXAMPLE

Example

Group the 1s in each of the K-map

AB \ C	0	1
00	1	
01		1
11	1	1
10		

AB \ C	0	1
00	1	1
01	1	
11		1
10	1	1

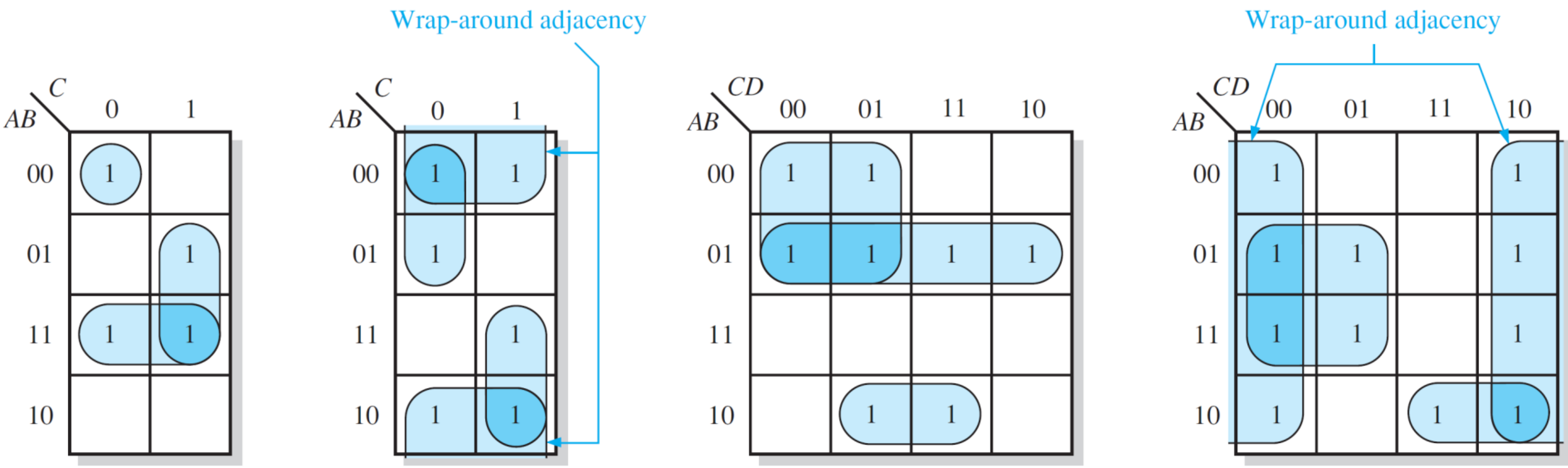
AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

K-MAP SIMPLIFICATION
GROUPING: EXAMPLE

Solution

Group the 1s in each of the K-map



K-MAP SIMPLIFICATION

DETERMINE THE MINIMUM SOP FROM K-MAP

The following rules are applied to find the minimum product terms and the minimum SOP expression:

STEP 1: Group the cells that have 1s.

STEP 2: Determine the minimum product term for each group.

- For a 3-variable K-map:
  1. A 1-cell group yields a 3-variable product term.
  2. A 2-cell group yields a 2-variable product term.
  3. A 4-cell group yields a 1-variable product term.
  4. An 8-cell group yields a value of 1 for the expression.
- For a 4-variable K-map:
  1. A 1-cell group yields a 4-variable product term.
  2. A 2-cell group yields a 3-variable product term.
  3. A 4-cell group yields a 2-variable product term.
  4. An 8-cell group yields a 1-variable product term.
  5. A 16-cell group yields a value of 1 for the expression.

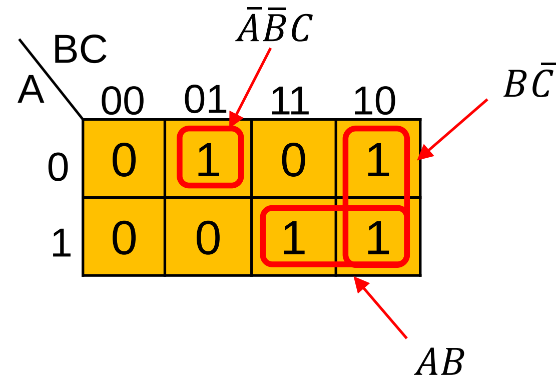
STEP 3: When all the minimum product term are derived from the K-map, they are summed to form SOP expression.

K-MAP SIMPLIFICATION

DETERMINE THE MINIMUM SOP FROM K-MAP

Example

Group the 1s and find the minimum SOP expression in the K-map below:



$$F = AB + \bar{A}\bar{B}C + B\bar{C}$$

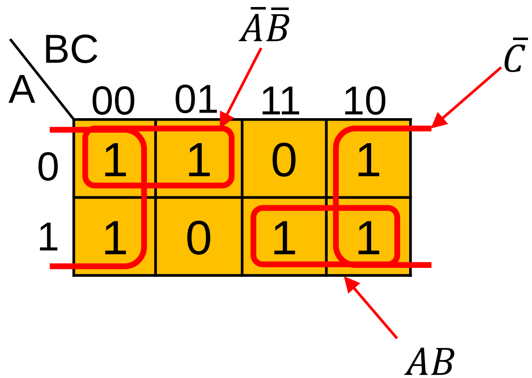
Expression is minimized when taking large cell possible

K-MAP SIMPLIFICATION

DETERMINE THE MINIMUM SOP FROM K-MAP

Example

Group the 1s and find the minimum SOP expression in the K-map below:



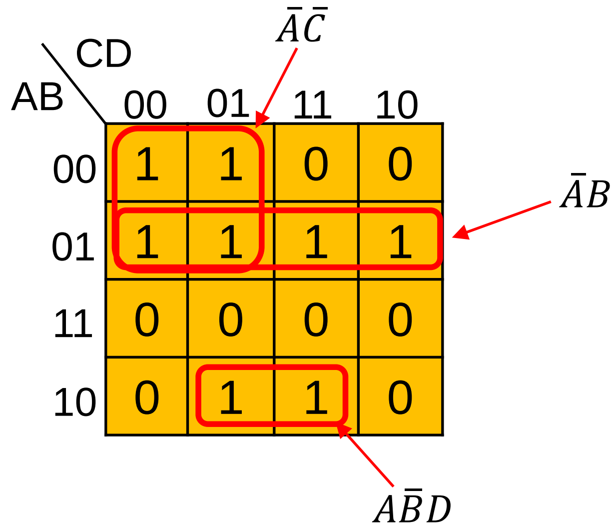
$$F = AB + \bar{A}\bar{B} + \bar{C}$$

K-MAP SIMPLIFICATION

DETERMINE THE MINIMUM SOP FROM K-MAP

Example

Group the 1s and find the minimum SOP expression in the K-map below:



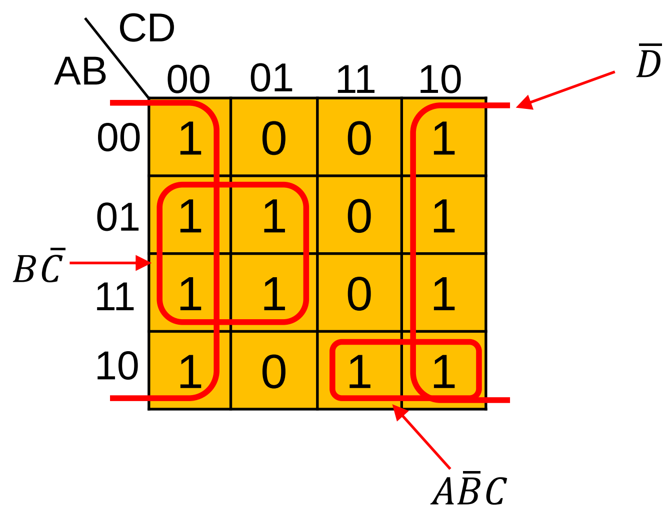
$$F = \bar{A}B + \bar{A}\bar{C} + A\bar{B}D$$

K-MAP SIMPLIFICATION

DETERMINE THE MINIMUM SOP FROM K-MAP

Example

Group the 1s and find the minimum SOP expression in the K-map below:



$$F = \overline{D} + B\overline{C} + A\overline{B}C$$

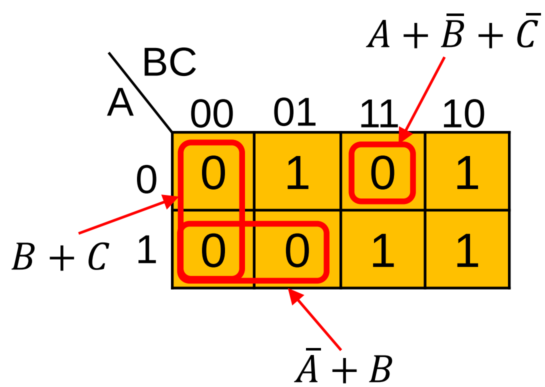
K-MAP SIMPLIFICATION

DETERMINE THE MINIMUM POS FROM K-MAP

K-map can also be used to obtain **POS expression** by grouping **0s**.

Example

Group the 0s and find the minimum POS expression in the K-map below:

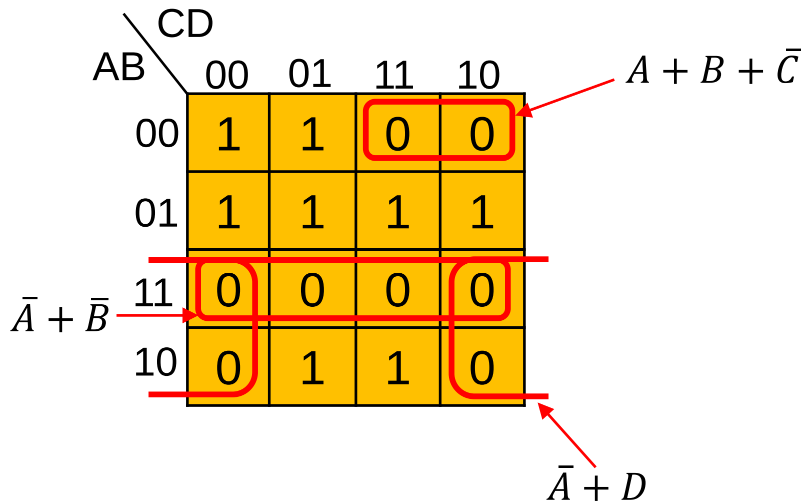


$$F = (B + C)(\bar{A} + B)(A + \bar{B} + \bar{C})$$

K-MAP SIMPLIFICATION
DETERMINE THE MINIMUM POS FROM K-MAP

Example

Find the minimum POS expression in the K-Map below:



$$F = (\bar{A} + D) + (\bar{A} + \bar{B}) + (A + B + \bar{C})$$

K-MAP SIMPLIFICATION

HOMEWORK 1

A network switch has two control inputs $(C1, C2)$, two data inputs $(X1, X2)$, and one output (Z).

- $If\ C1 = C2 = 0, Z = 0$
- $If\ C1 = C2 = 1, Z = 1$
- $If\ C1 = 1\ and\ C2 = 0, Z = X1$
- $If\ C1 = 0\ and\ C2 = 1, Z = X2$

Get the minimum SOP expression.

K-MAP SIMPLIFICATION
HOMEWORK 2

A logic circuit controls the path of signal A based on the following conditions:

- The output X will be contrary to A when the control input B equals to the control input C.
- X will always be LOW when B and C are different.

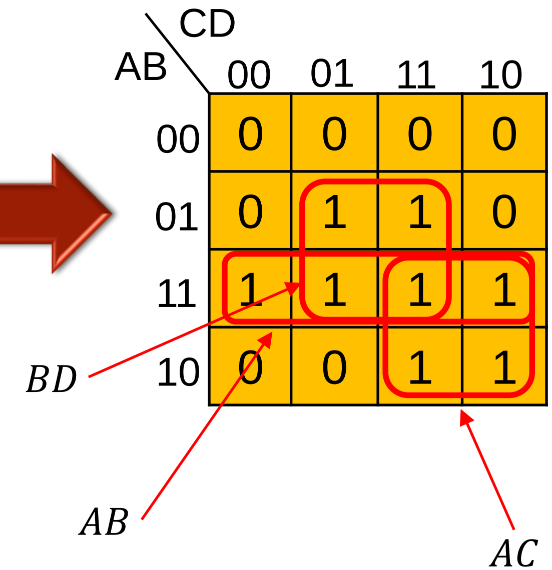
Design the circuit using NOR gates only.

K-MAP SIMPLIFICATION
HOMEWORK 1 (ANSWER)

Input				Output
C1	C2	X1	X2	Z
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1

Input				Output
C1	C2	X1	X2	Z
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

4-variable K-map



$$Z = BD + AB + AC$$

K-MAP SIMPLIFICATION
HOMEWORK 2 (ANSWER)

$X(A,B,C)$ truth table

Input			Output
A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

A \ BC	00	01	11	10
0	1	0	1	0
1	0	0	0	0

$$X = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}BC$$

K-MAP SIMPLIFICATION

DON'T CARE CONDITION

- **Don't care** is the condition when the output can be either '0' or '1', which is denoted by ***'X'*** in the truth table or K-map.
- For both SOP and POS minimum expression, ***'X'*** can be included or ignored.

Example

Find the minimum SOP expression in the K-Map below:

AB \ CD	00	01	11	10
00	0	1	1	0
01	0	1	1	X
11	1	1	X	1
10	0	0	0	0

$\bar{A}D$

AB

$$\boldsymbol{F = \bar{A}D + AB}$$

If the X is replaced by 0, find the minimum SOP expression.

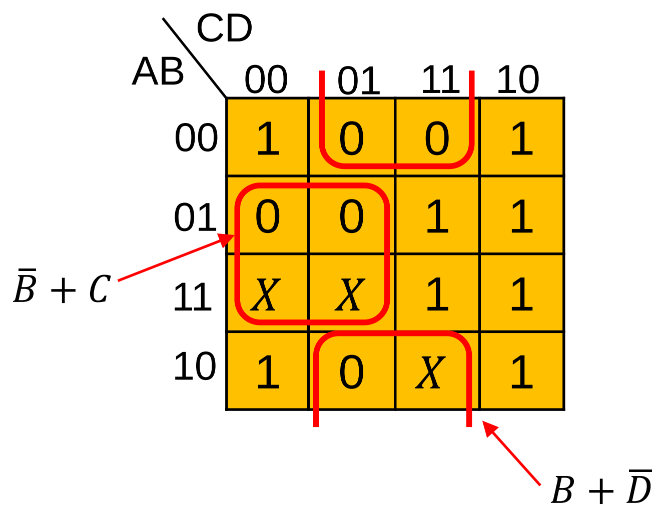
$$\boldsymbol{F = \bar{A}D + AB\bar{C} + AB\bar{D}}$$

K-MAP SIMPLIFICATION

DON'T CARE CONDITION

Example

Find the minimum POS expression in the K-map below:



$$F = (\bar{B} + C) + (B + \bar{D})$$

K-MAP SIMPLIFICATION

HOMEWORK 3

Simplify the following equations using K-map.

i. $F(A,B,C,D) = \sum m(1,3,6,7) + d(4,9,11)$

ii. $F(W,X,Y,Z) = \sum m(2,9,10,12,13) + d(1,5,14)$

Answer:

i. $F = \bar{B}D + \bar{A}BC = \bar{A}(B+D)(\bar{B}+C)$

ii. $F = WX\bar{Y} + \bar{X}Y\bar{Z} + \bar{Y}Z = (X+Y+Z)(W+\bar{X})(\bar{Y}+\bar{Z})$

GATES TRANSFORMATION

GATES TRANSFORMATION

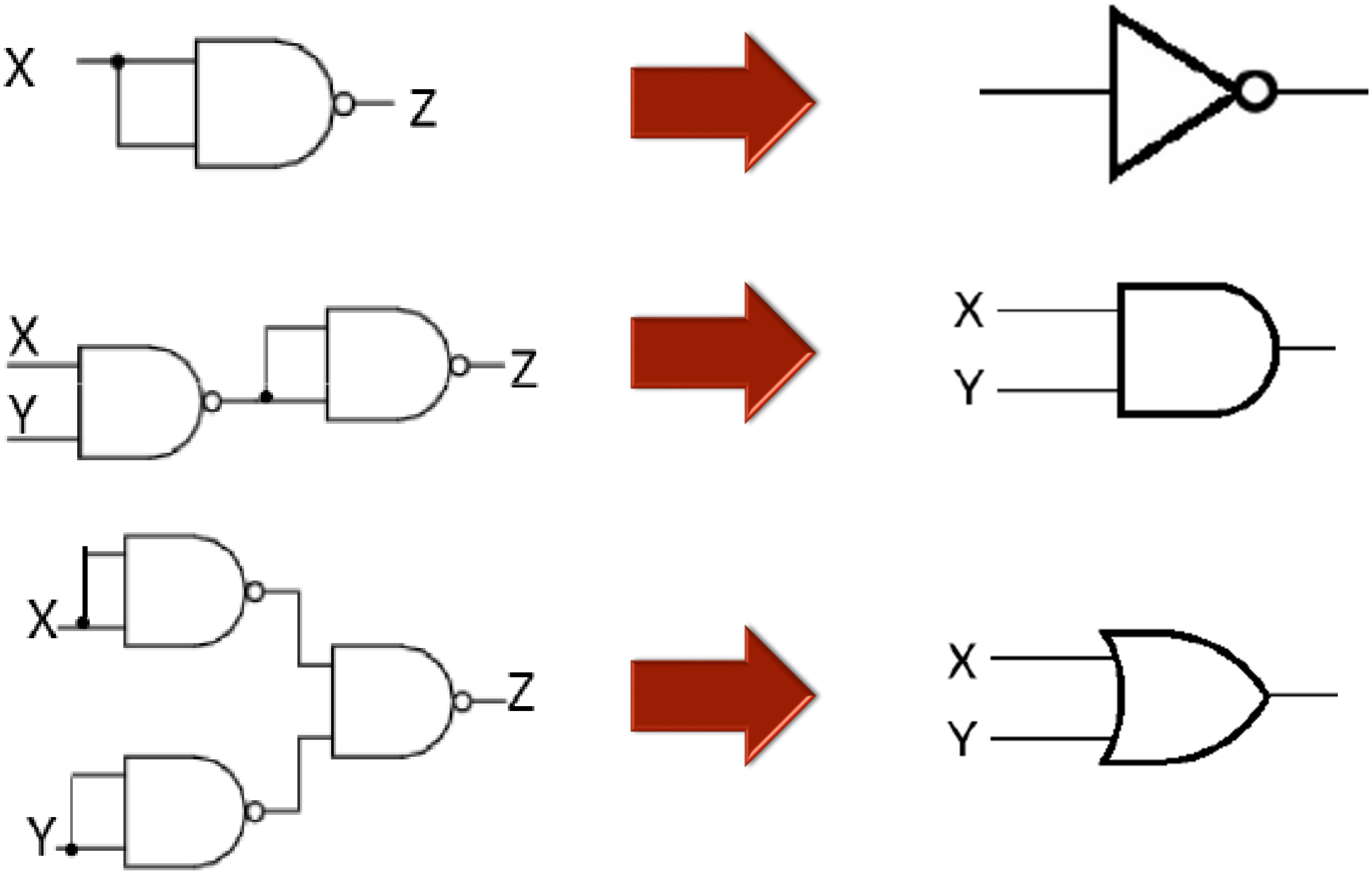
INTRODUCTION

Gate transformation is a process that involves a conversion of logic circuit from:

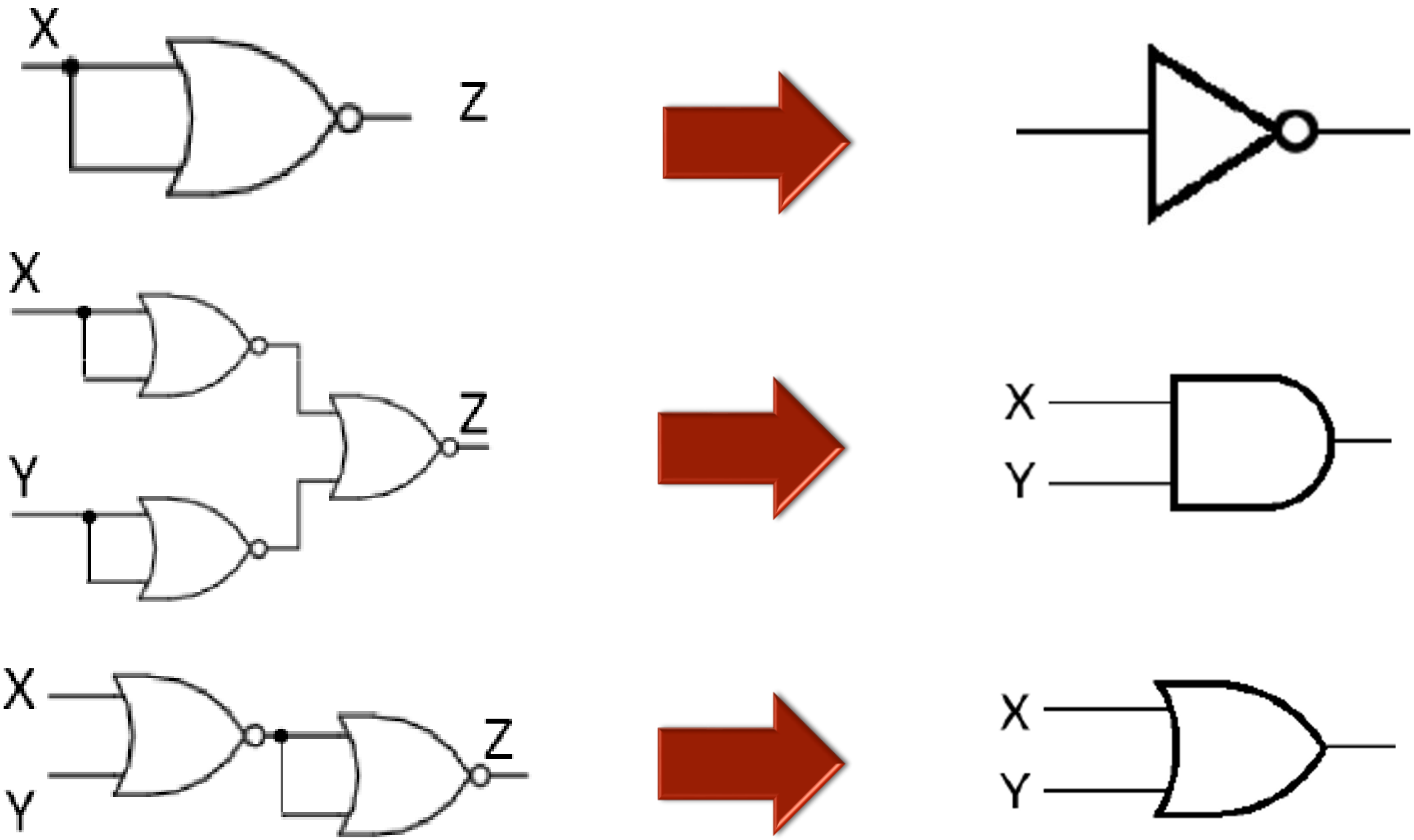
- OR-AND to NAND-NAND
- AND-OR to NOR-NOR

}DeMorgan Equivalent Circuit

GATES TRANSFORMATION
NAND GATE IS UNIVERSAL GATE



GATES TRANSFORMATION
NOR GATE IS UNIVERSAL GATE



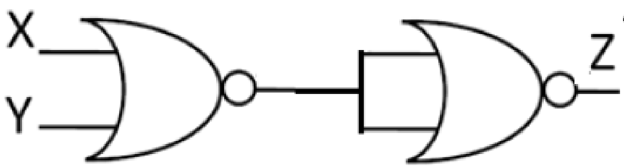
GATES TRANSFORMATION
DON'T CARE CONDITION

Example

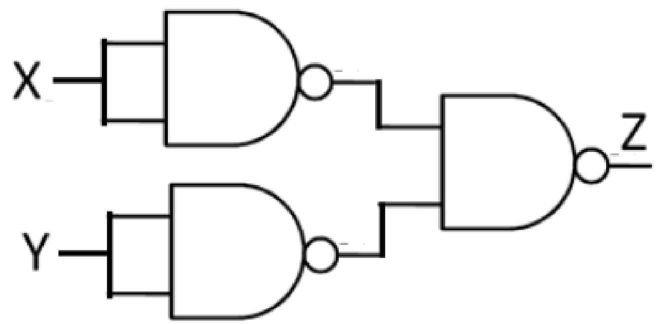
Implement $F(X,Y) = X + Y$ using only:

a) NOR gate.

b) NAND gate.



(a) NOR gate



(b) NAND gate

GATES TRANSFORMATION
DON'T CARE CONDITION

Example

$$F(A,B,C) = A\bar{B} + BC + \bar{A}\bar{C}$$

This SOP function can be implemented using:

a) AND-OR gate.
b) NAND-NAND gate.

$$F(A,B,C) = (A + \bar{B} + C)(A + B + \bar{C})$$

This POS function can be implemented using:

a) OR-AND gate.
b) NOR-NOR gate.

GATES TRANSFORMATION

ASSESSMENT

1. Show how a NOT function can be obtained by using the following gates only:
   a) 3-input NAND gate.
   b) 2-input NOR gate.

2. Show how a 3-input OR function can be implemented using a minimum number of:
   a) 2-input OR gate.
   b) 2-input NAND gate.

3. What is the minimum number of 2-input NAND gate required to implement the following functions:
   a) $F(A, B, C) = ABC$
   b) $F(A, B, C) = AB + C$