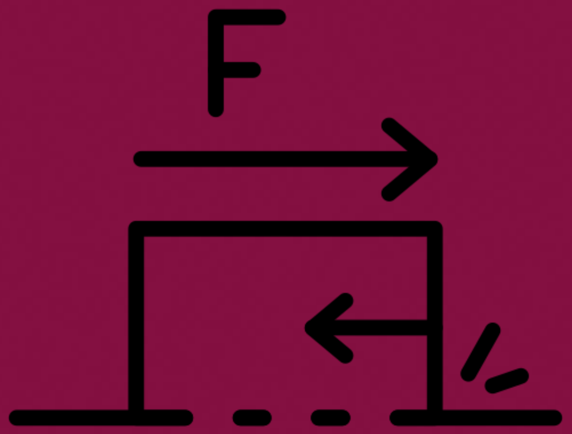
SEEM1113 ENGINEERING MECHANICS

CH4
Force System Resultants

Dr. Mohd Saiful Azimi Mahmud (azimi@utm.my)
P19a-04-03-30, School of Electrical Engineering, UTM



OBJECTIVES

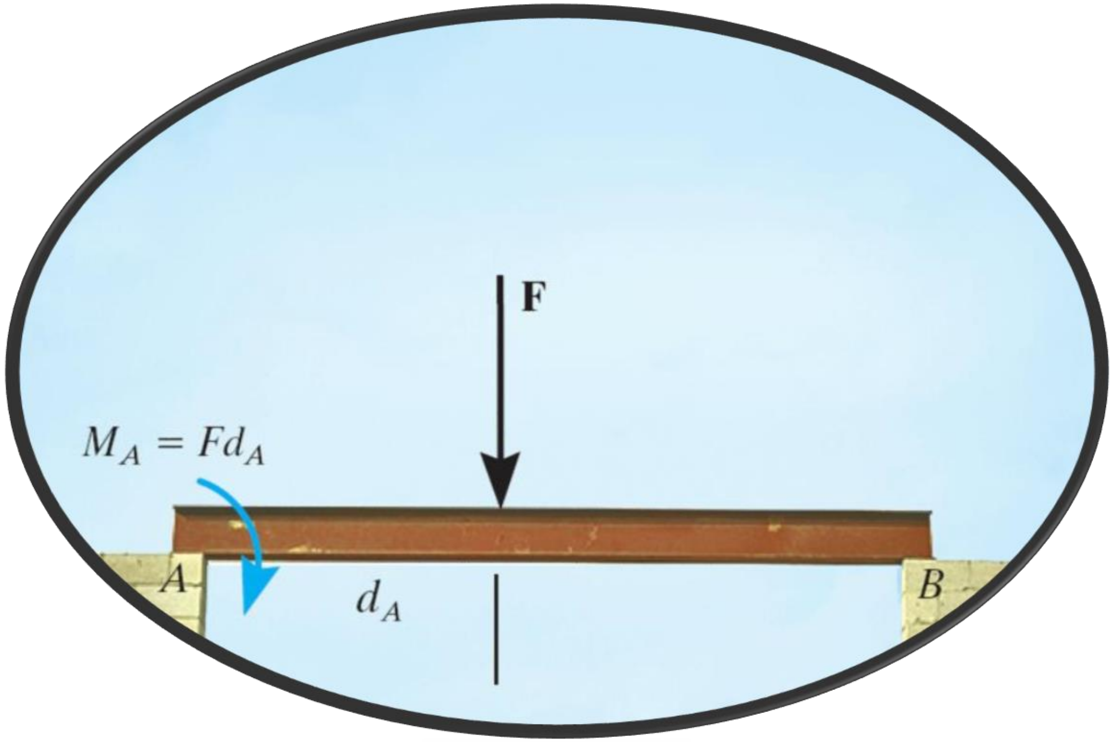
At the end of this lesson, you should be able to:

1. Describe the concept of **moment of a force (MoF).**
2. Define the magnitude, direction & resultant moment of MoF
3. Calculate the **MoF** in 2 or 3 dimensions.

Scalar Formulation

SCALAR FORMULATION

APPLICATION

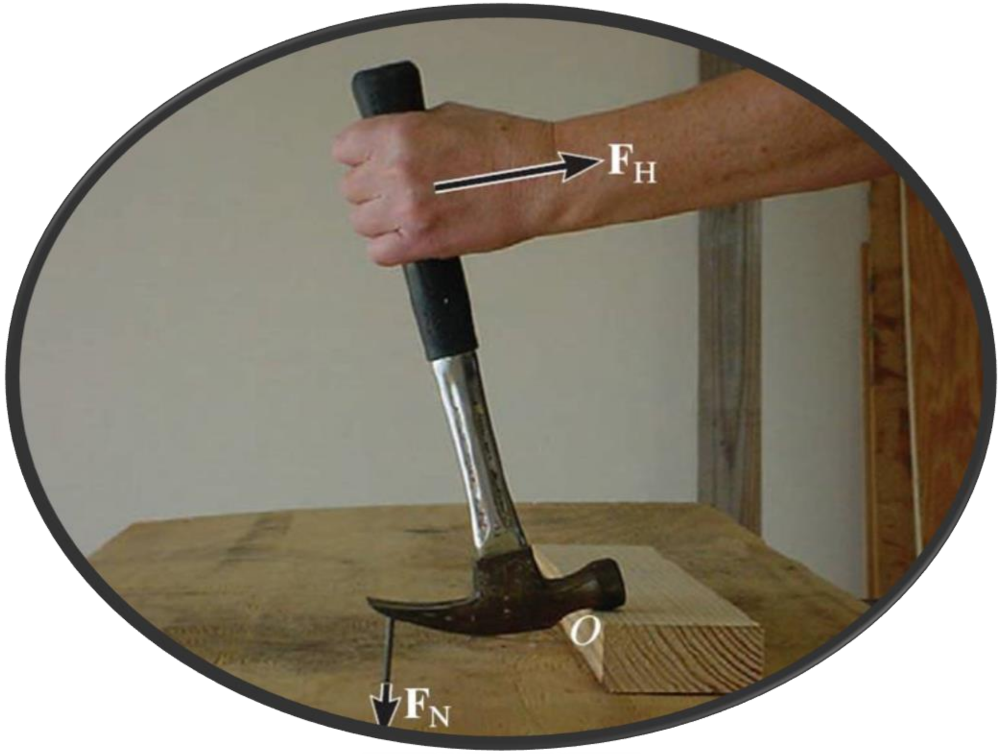


Beams are often used to bridge gaps in walls. We have to know what the effect of the force on the beam will have on the beam supports.

What do you think those impacts are at points A and B?

SCALAR FORMULATION

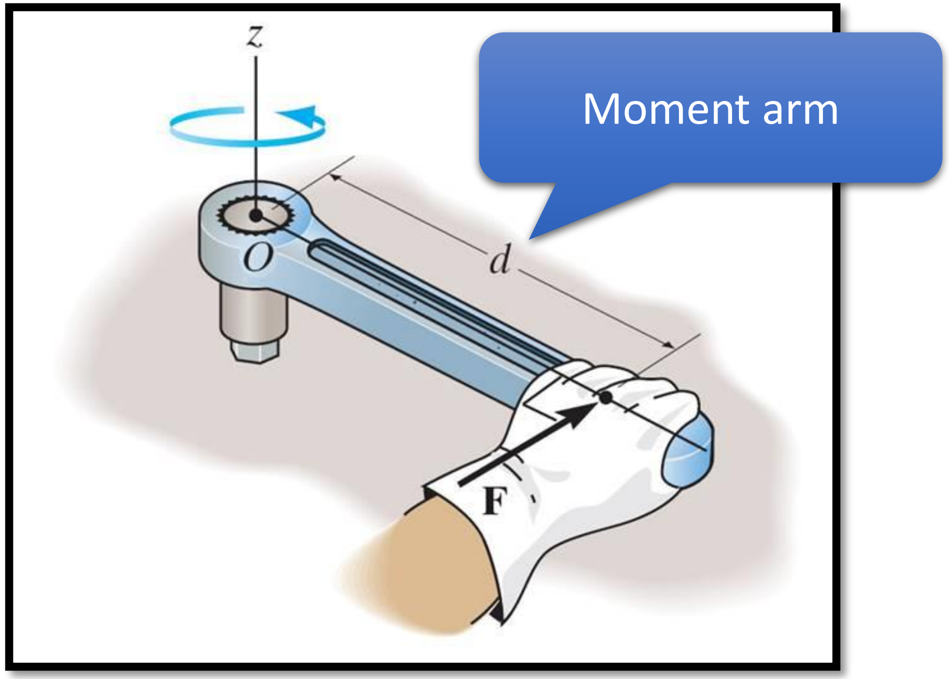
APPLICATION



Carpenters often use a hammer in this way to pull a stubborn nail, $\mathbf{F_N}$. Through what sort of action does the force $\mathbf{F_H}$ at the handle pull the nail? How can you mathematically model the effect of force $\mathbf{F_H}$ at point O?

SCALAR FORMULATION

GENERAL PRINCIPLE



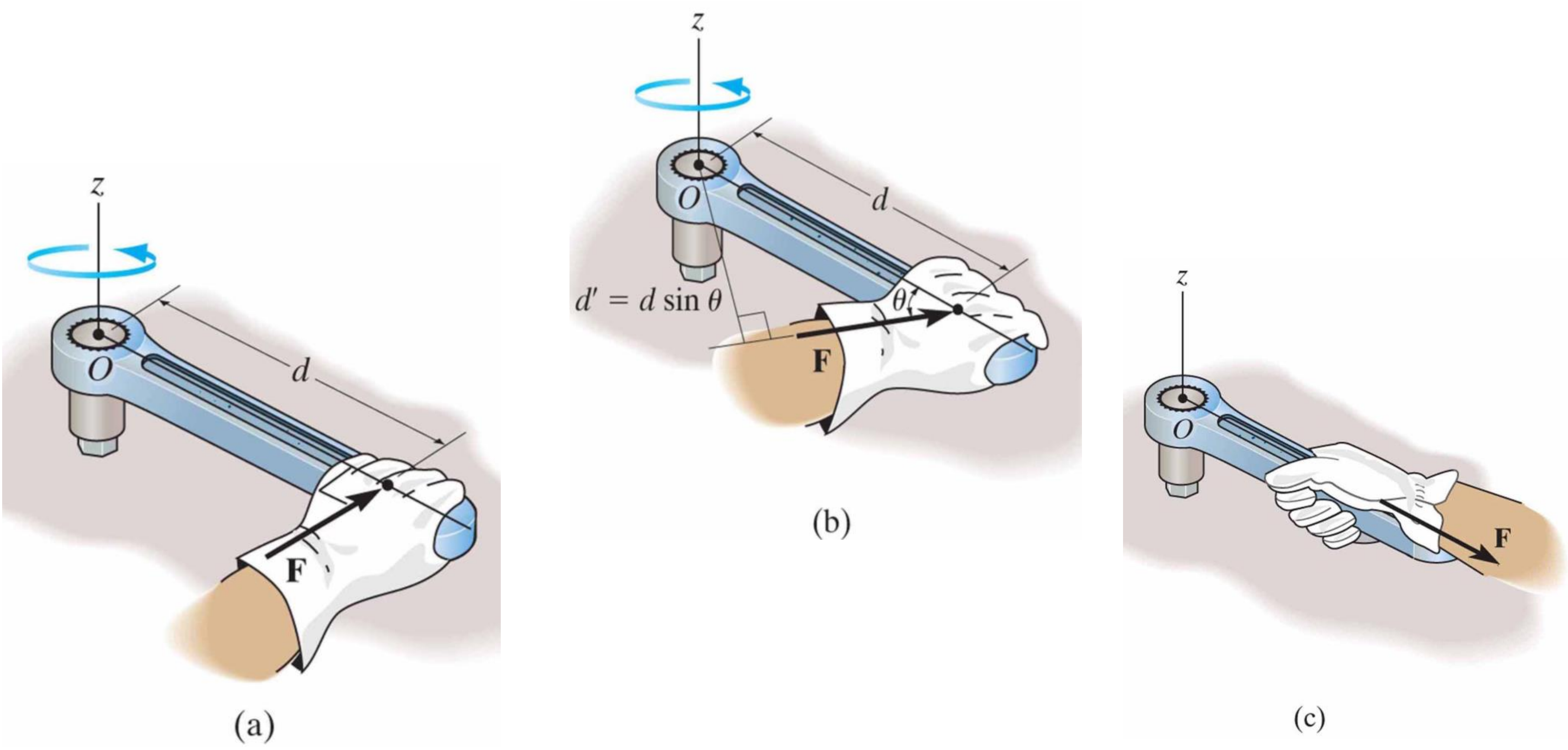
The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

The moment is a vector (has magnitude and direction)

SCALAR FORMULATION

GENERAL PRINCIPLE

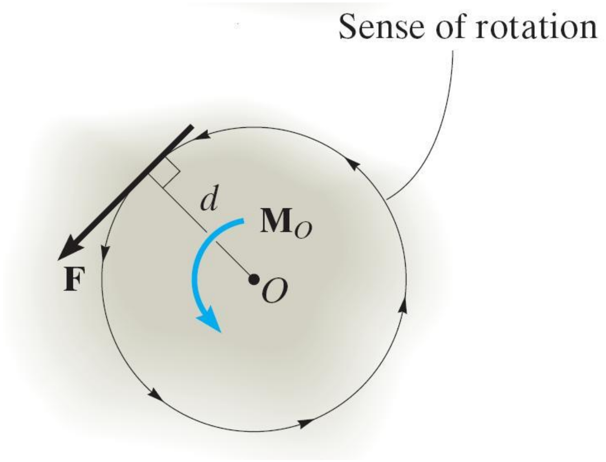
Which action give the highest moment?



SCALAR FORMULATION

GENERAL PRINCIPLE

In the 2-D case, the **magnitude** of the moment is



We can determine the direction of rotation by imagining the body pinned at O and deciding which way the body would rotate because of the force.

d is the **perpendicular** distance from point O to the line of action of the force.

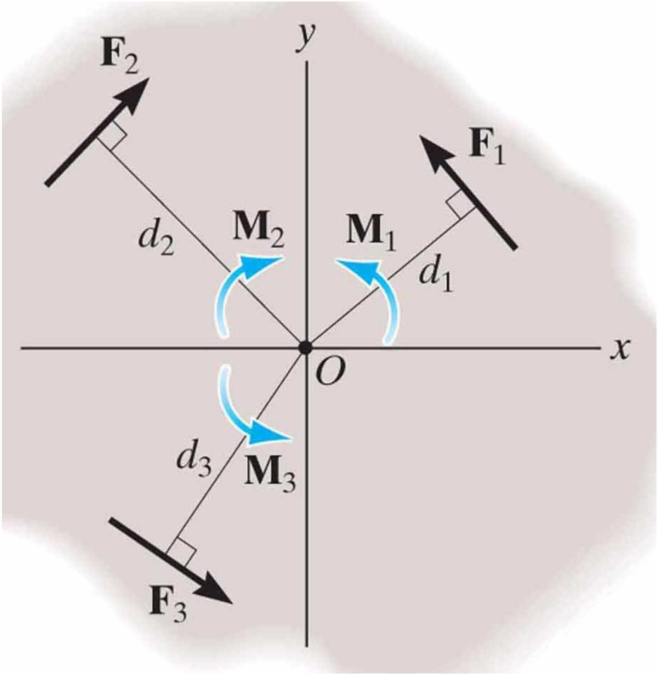
*In 2-D, the direction of MO is either **clockwise** or **counter-clockwise**, depending on the tendency for rotation.*

SCALAR FORMULATION

GENERAL PRINCIPLE

Resultant moment:

$$(M_R)_o = \sum Fd = F_1 d_1 - F_2 d_2 + F_3 d_3$$



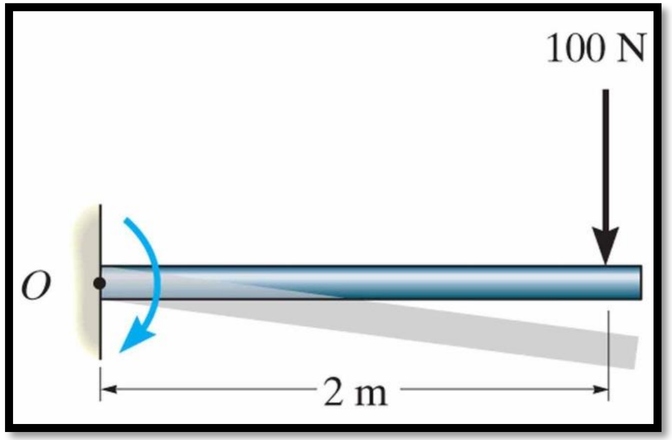
As a convention we will consider positive moment as counter-clockwise.

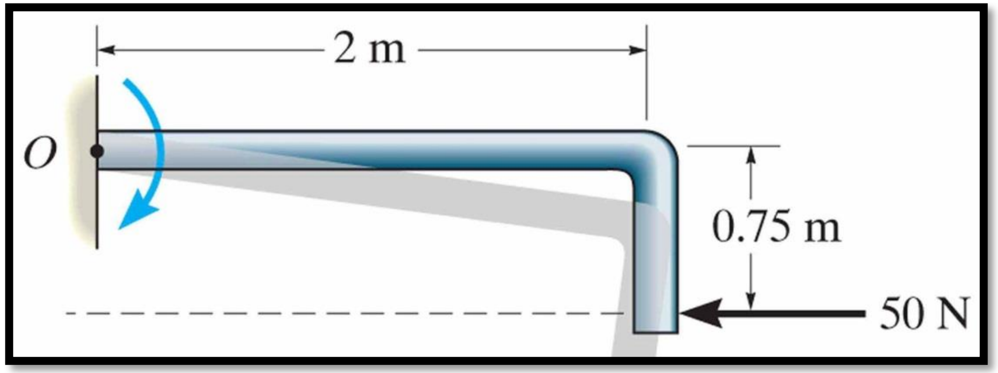
SCALAR FORMULATION

GENERAL PRINCIPLE

Example 1

Determine the moment of a force about point O:



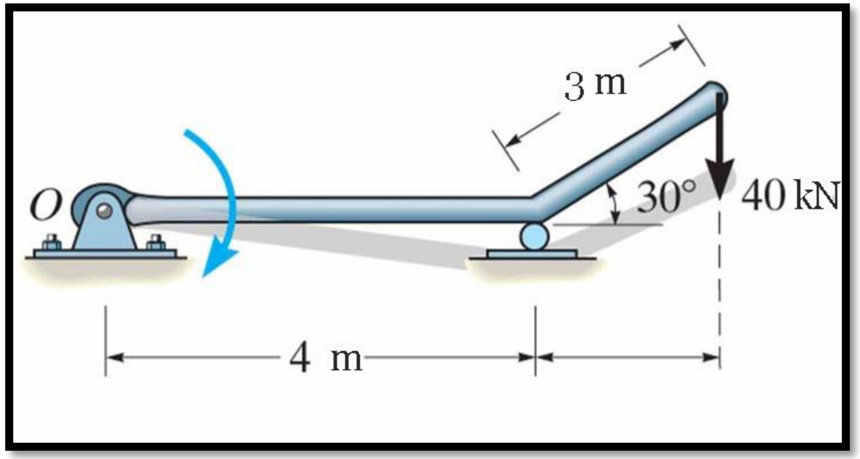


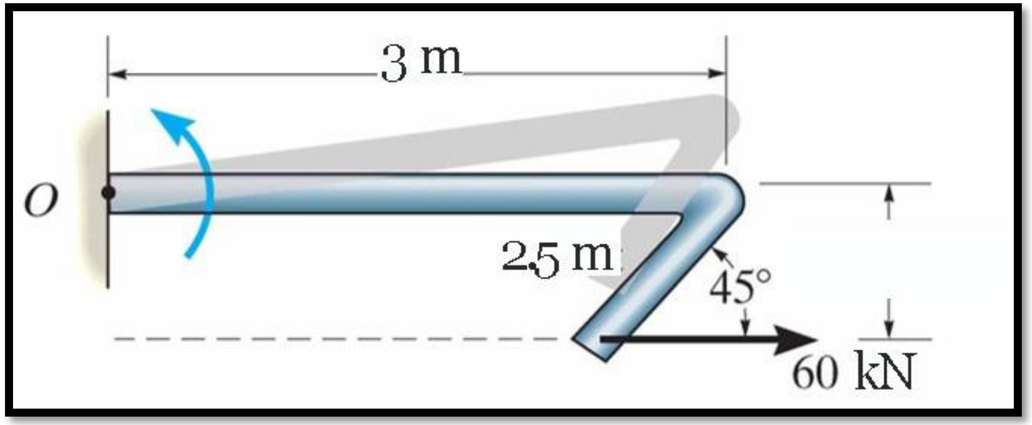
SCALAR FORMULATION

GENERAL PRINCIPLE

Example 1

Determine the moment of a force about point O:



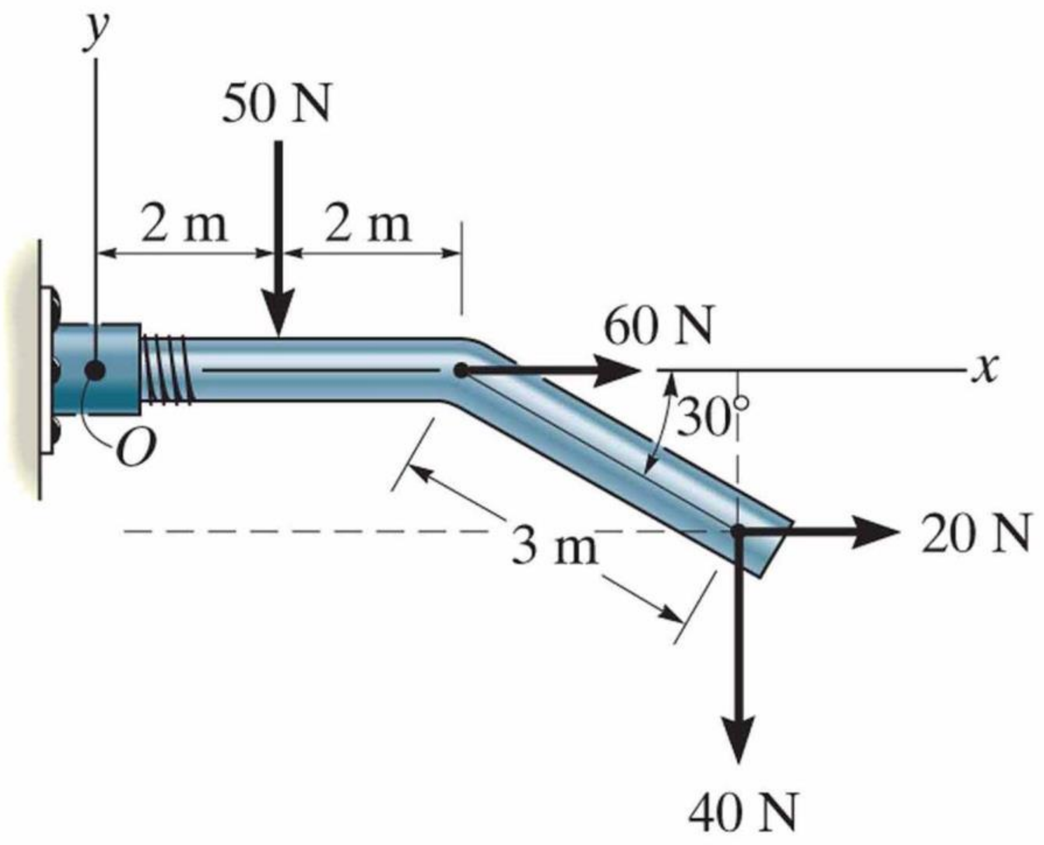


SCALAR FORMULATION

GENERAL PRINCIPLE

Example 2

Determine the resultant moment of the four forces acting on the rod shown in the figure about point O:

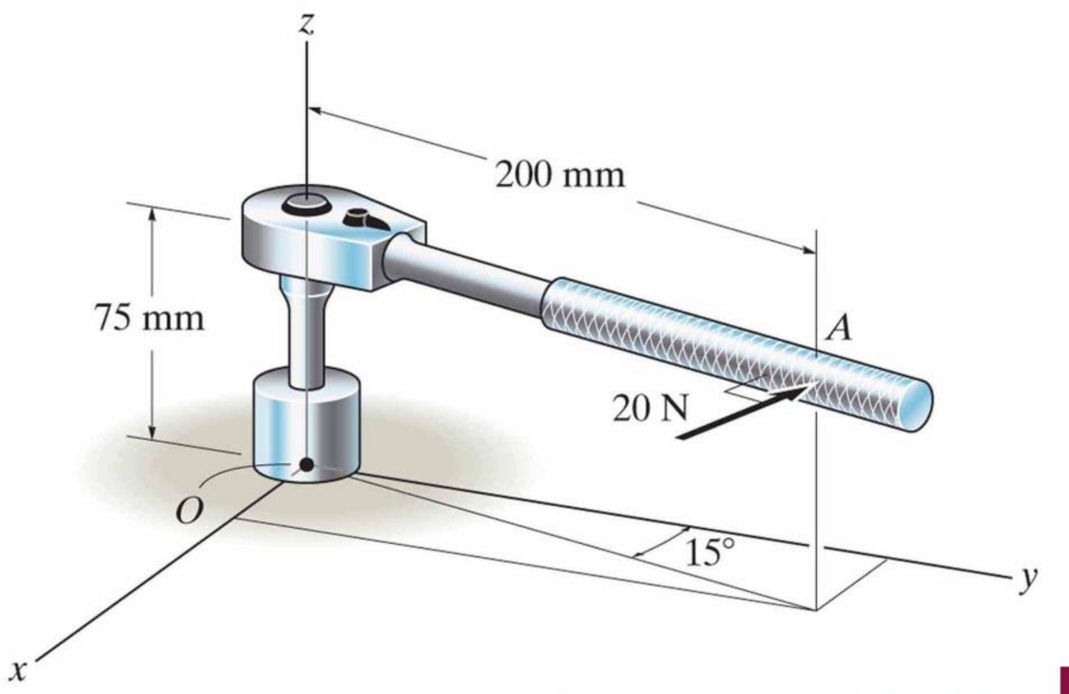


Vector Formulation

VECTOR FORMULATION

CROSS PRODUCT

- In 3-D, moment of force will be formulated in Cartesian vectors.
- Finding the perpendicular distances can be hard, especially when you are working with forces in three dimensions.
- A general method of finding the moment of a force uses a vector operation called the cross product of two vectors.



VECTOR FORMULATION

CROSS PRODUCT

- Cross product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ results in another vector, $\boldsymbol{C}$, written as:

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$$

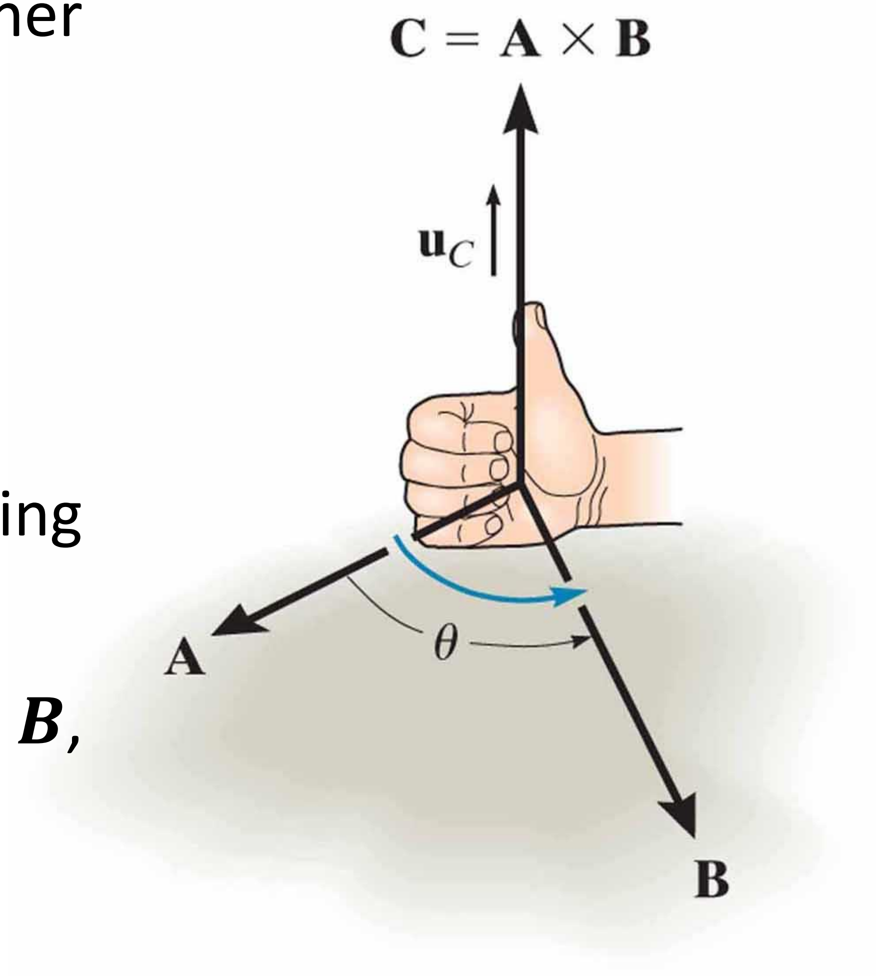
- The magnitude of $\boldsymbol{C}$:

$$C = ABsin\theta$$

- Direction of $\boldsymbol{C}$ is perpendicular to the plane containing $\boldsymbol{A}$ and $\boldsymbol{B}$ such that $\boldsymbol{C}$ is specified by the RHR.

- Curling the fingers of the right hand from $\boldsymbol{A}$ (cross) to $\boldsymbol{B}$, the thumb points in the direction of $\boldsymbol{C}$. Thus:

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = (ABsin\theta)\boldsymbol{u_c}$$



VECTOR FORMULATION

CROSS PRODUCT: LAWS OF OPERATION

- Commutative law is not valid:

$$\boldsymbol{A} \times \boldsymbol{B} \neq \boldsymbol{B} \times \boldsymbol{A}$$

But

$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

- Associative law

$$a(\boldsymbol{A} \times \boldsymbol{B}) = (a\boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (a\boldsymbol{B}) = (\boldsymbol{A} \times \boldsymbol{B})a$$

- Distributive law

$$\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = (\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})$$

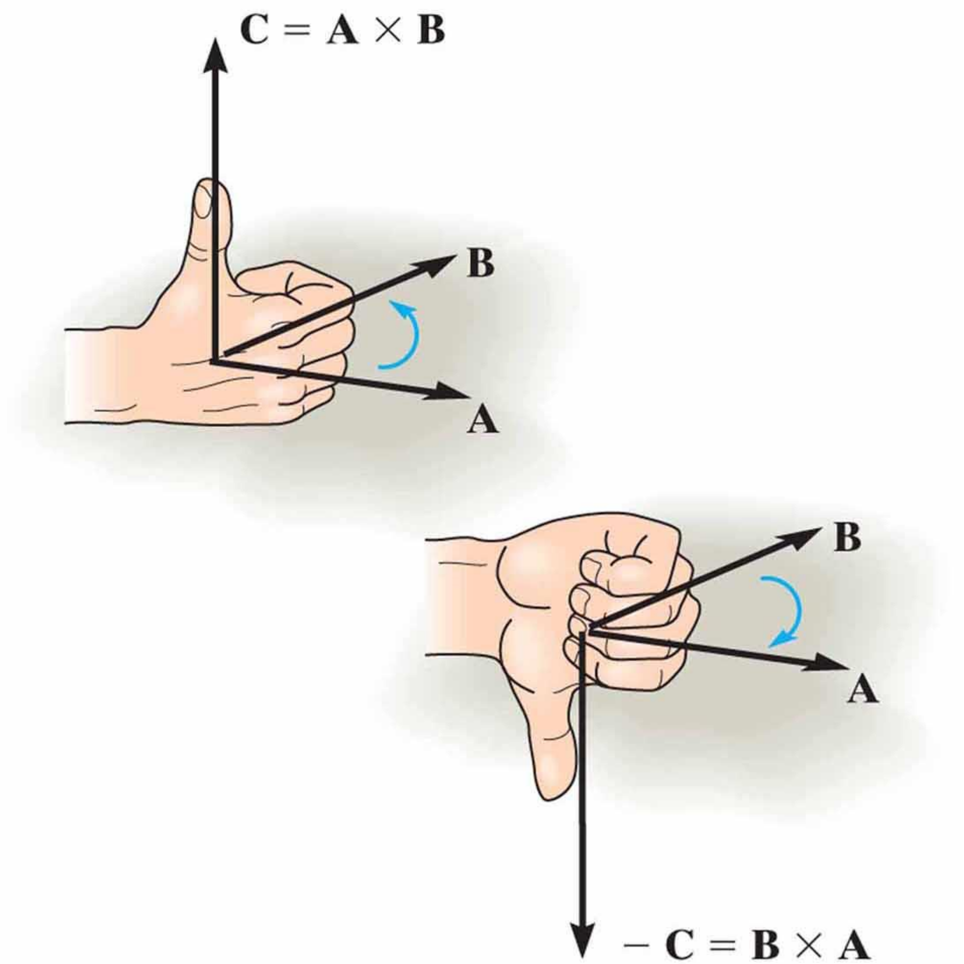
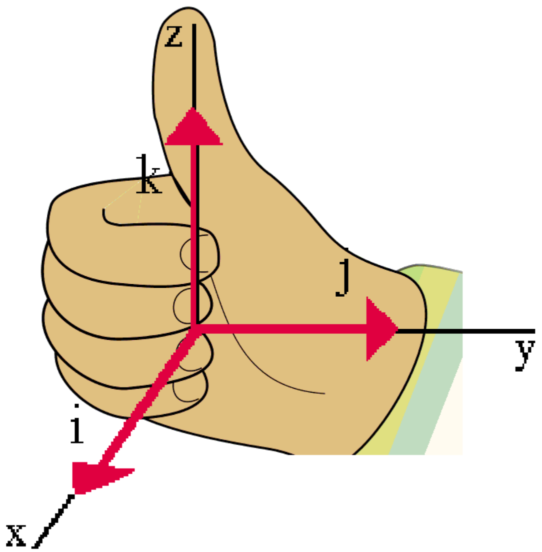


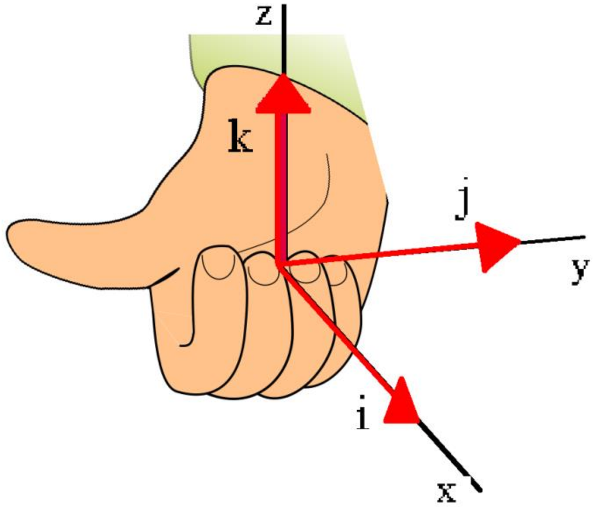
fig04_07.jpg

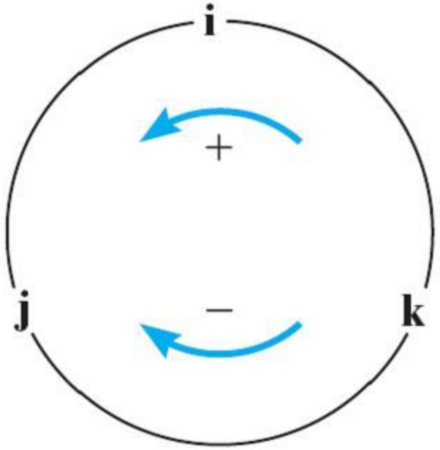
VECTOR FORMULATION

CROSS PRODUCT

- To find the cross product of any pair of cartesian unit vector, equation $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = (ABsin\theta)\boldsymbol{u_c}$ can be used.
- For example: $i \times j = (i)(j)\,(\sin 90°) = (1)(1)(1) = 1$
- The direction is determined by using the right hand rule.
- Therefore, $i \times j = (1)k$







VECTOR FORMULATION

CROSS PRODUCT

- Consider a cross product of two vectors, ***A*** and ***B***

$$\boldsymbol{A}\times\boldsymbol{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$

$$= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$
$$= A_xB_x(\mathbf{i}\times\mathbf{i}) + A_xB_y(\mathbf{i}\times\mathbf{j}) + A_xB_z(\mathbf{i}\times\mathbf{k}) + A_yB_x(\mathbf{j}\times\mathbf{i}) + A_yB_y(\mathbf{j}\times\mathbf{j}) + A_yB_z(\mathbf{j}\times\mathbf{k})$$
$$+A_zB_x(\mathbf{k}\times\mathbf{i}) + A_zB_y(\mathbf{k}\times\mathbf{j}) + A_zB_z(\mathbf{k}\times\mathbf{k})$$

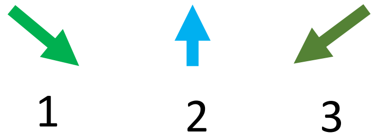
- Carrying out the cross-product operations and combining terms yield:

$$\boldsymbol{A}\times\boldsymbol{B} = (A_yB_z - A_zB_y)\mathbf{i} + (A_zB_x - A_xB_z)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$
$$= (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

- Can be summarized as:

$$\boldsymbol{A}\times\boldsymbol{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} - \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$(A_yB_z - A_zB_y)\mathbf{i} \quad (A_xB_z - A_zB_x)\mathbf{j} \quad (A_xB_y - A_yB_x)\mathbf{k}$$



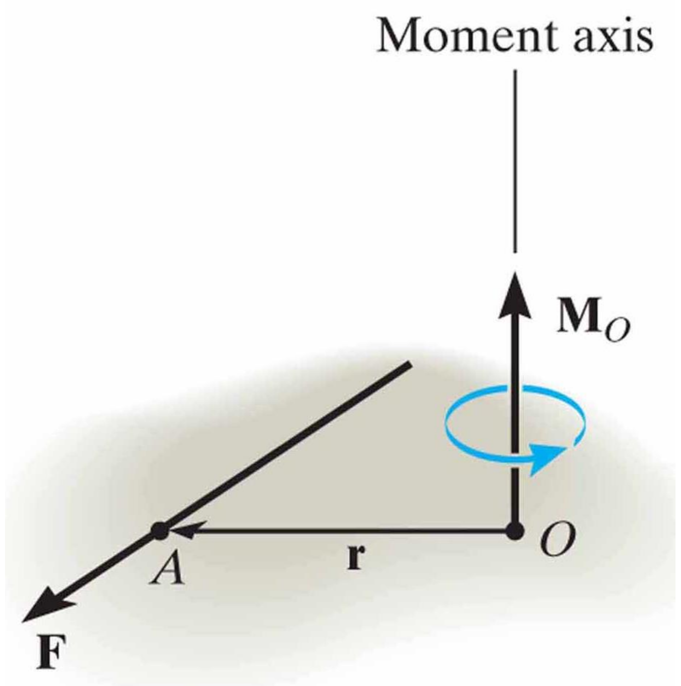
MOMENT OF A FORCE: VECTOR FORMULATION

- Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the **vector cross product**.

- Moment of force ***F*** about point O can be expressed using cross product

$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}$$

Where $\boldsymbol{r}$ is the **position vector from point O to any point on the line of action of** $\boldsymbol{F}$.



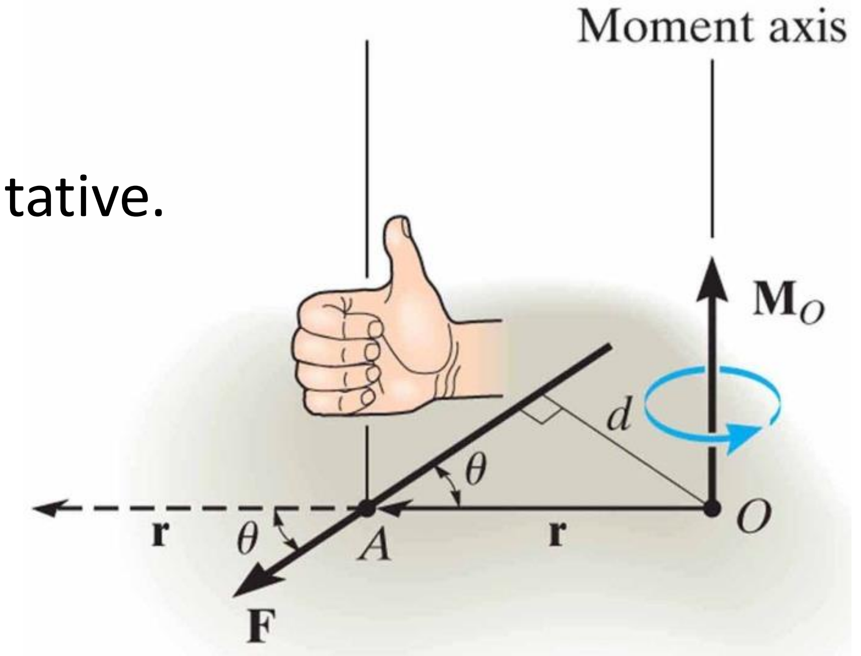
MOMENT OF A FORCE: VECTOR FORMULATION

- The moment, $\boldsymbol{M}_O$ determined using the cross product which has the proper magnitude and direction.

- Magnitude:

$$M_O = rFsin\theta = F(rsin\theta) = Fd$$

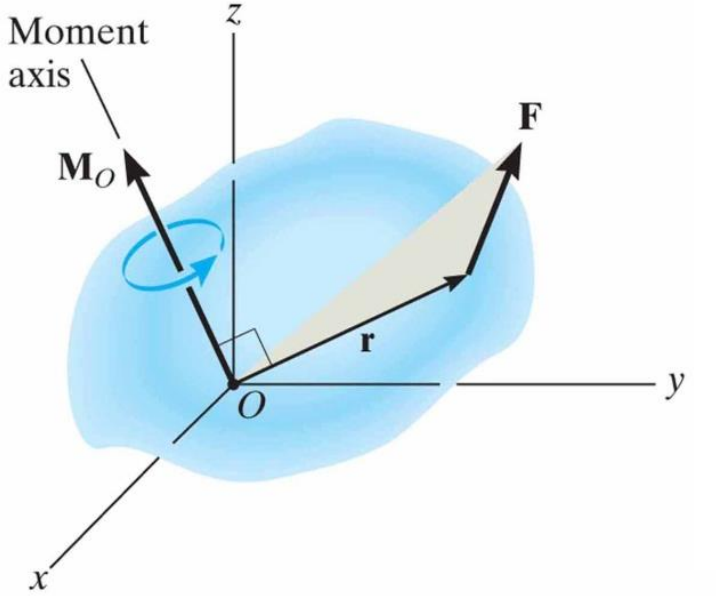
- Direction of $\boldsymbol{M}_O$ is determined by right-hand rule.
- Maintain proper order of $\boldsymbol{r}$ and $\boldsymbol{F}$ since cross is not commutative.



MOMENT OF A FORCE: VECTOR FORMULATION

- For a force expressed in a Cartesian form:

$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



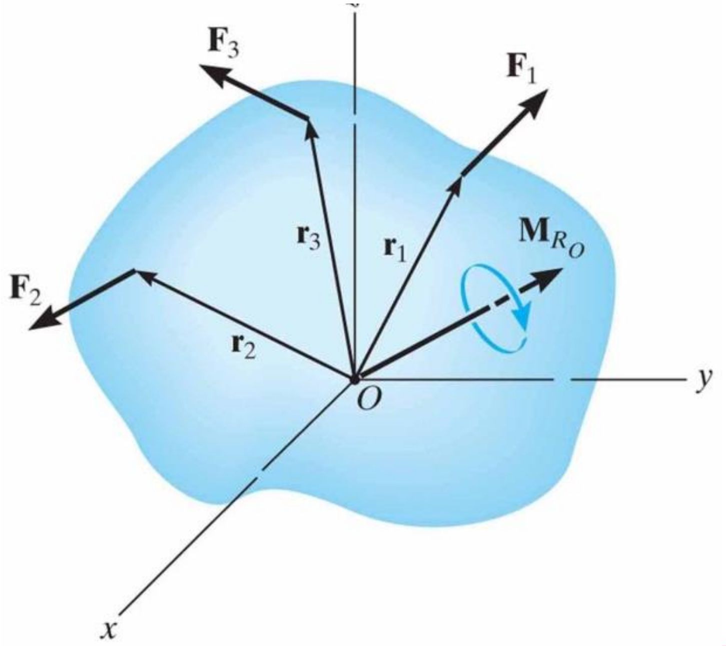
- Hence:

$$\boldsymbol{M}_O = (r_y F_z - r_z F_y)\boldsymbol{i} - (r_x F_z - r_z F_x)\boldsymbol{j} + (r_x F_y - r_y F_x)\boldsymbol{k}$$

Resultant Moment of a System of Forces

Resultant moment of forces about point O can be determined by vector addition

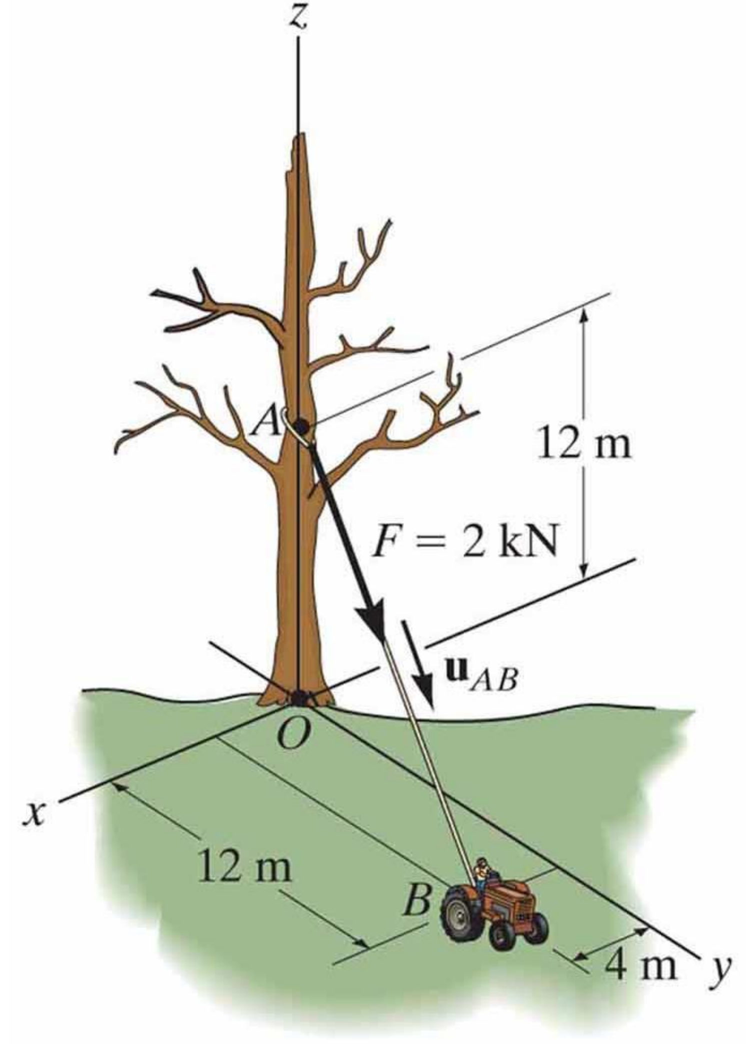
$$(\boldsymbol{M_R})_O = \sum(\boldsymbol{r} \times \boldsymbol{F})$$



MOMENT OF A FORCE: VECTOR FORMULATION

Example 3

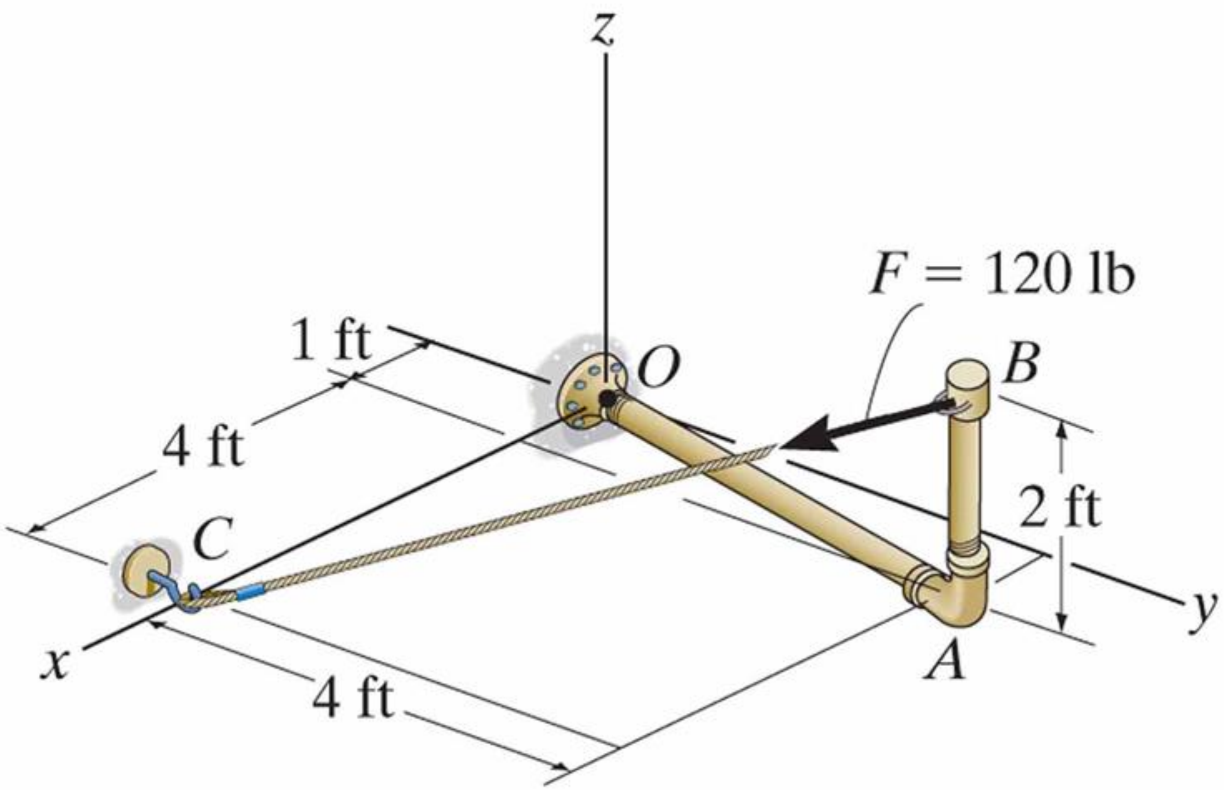
Determine the moment produced by the force ***F*** in the figure below about point *O*. Express the result as a Cartesian vector.



MOMENT OF A FORCE: VECTOR FORMULATION

Example 4

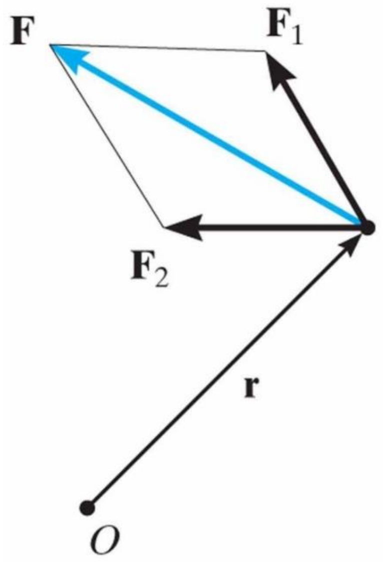
Determine the moment of force ***F*** about point *O*. Express the result as a Cartesian vector.

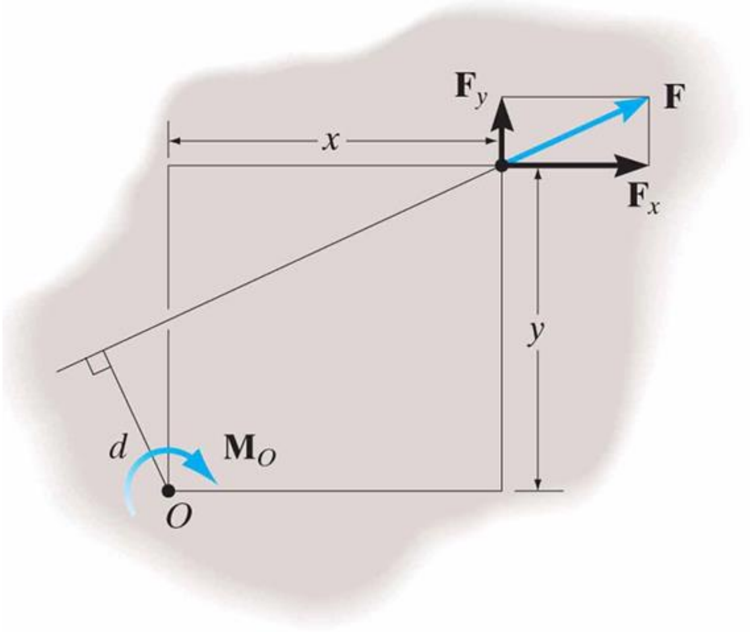


VECTOR FORMULATION

MOMENT OF A FORCE: PRINCIPLE OF MOMENT

Varignon's theorem: ***Moment of a force about a point is equal to the sum of the moments of the forces' components about the point.***





Vector equation

$$M_O = r \times F = r \times (F_1 + F_2)$$
$$= r \times F_1 + r \times F_2$$

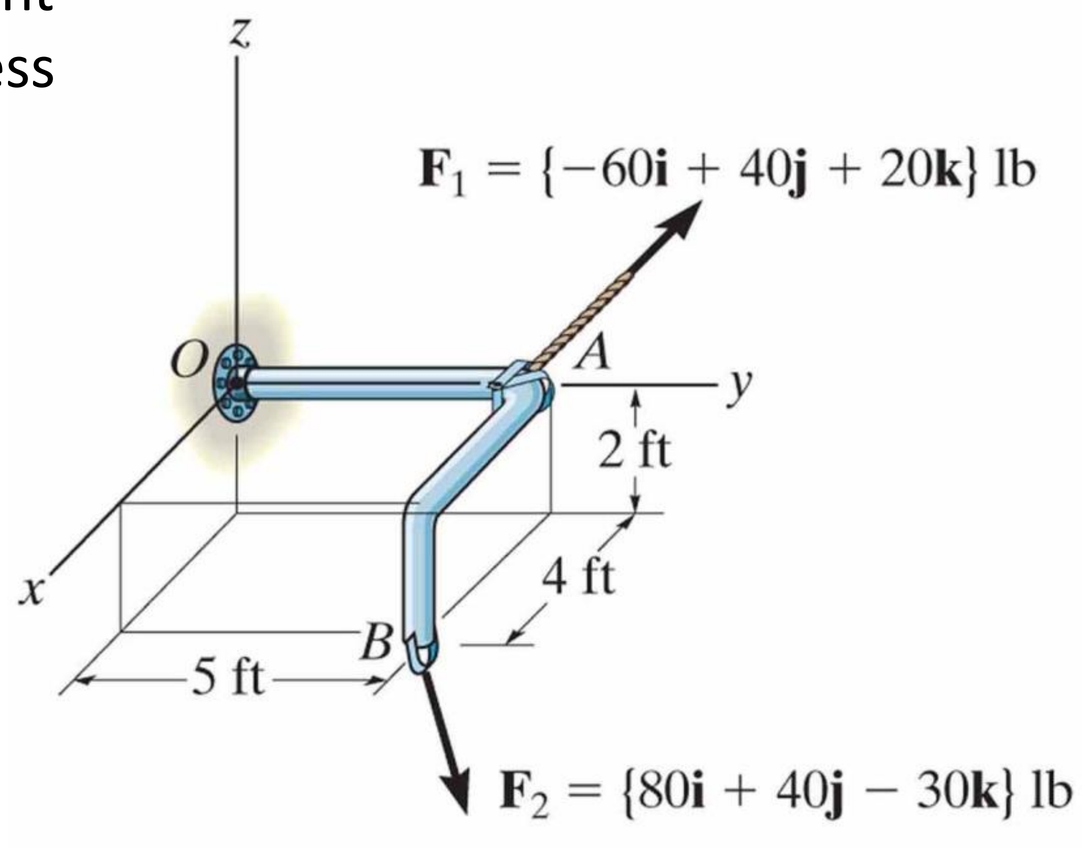
Scalar equation (simpler than finding d):

$$M_O = F_x y - F_y x$$

MOMENT OF A FORCE: PRINCIPLE OF MOMENT

Example 5

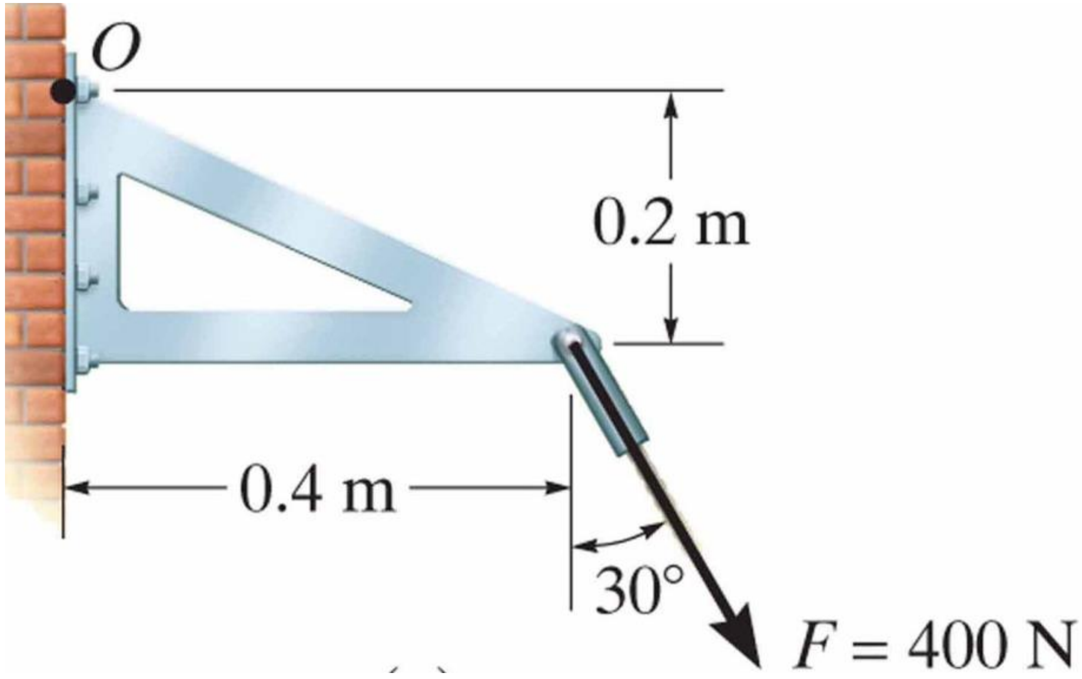
Two forces act on the rod. Determine the resultant moment they create about the flange at *O*. Express the result as a Cartesian vector.



MOMENT OF A FORCE: PRINCIPLE OF MOMENT

Example 6

Force F acts at the end of the angle bracket shown in the figure below. Determine the moment of the force about point O.



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