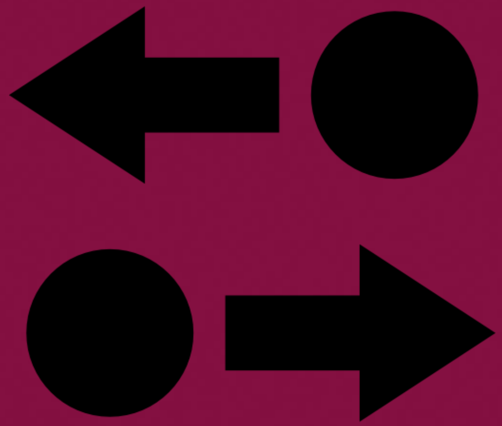
SEEM1113 ENGINEERING MECHANICS

CH5
Kinematics of Particles



Dr. Mohd Saiful Azimi Mahmud (azimi@utm.my)
P19a-04-03-30, School of Electrical Engineering, UTM

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OBJECTIVES

At the end of this lesson, you should be able to:

1. To understand the concept of **position**, **displacement**, **velocity** and **acceleration**.
2. To study particle **motion along a straight line**.
3. Describe the motion of a particle travelling along a **curved path**.
4. Relate kinematics quantities in terms of the **rectangular components** of the vectors.
5. Analyze the **free-flight motion** of a projectile

RECTILINEAR KINEMATICS

RECTILINEAR KINEMATICS

INTRODUCTION

Mechanics: The study of how bodies react to forces acting on them.

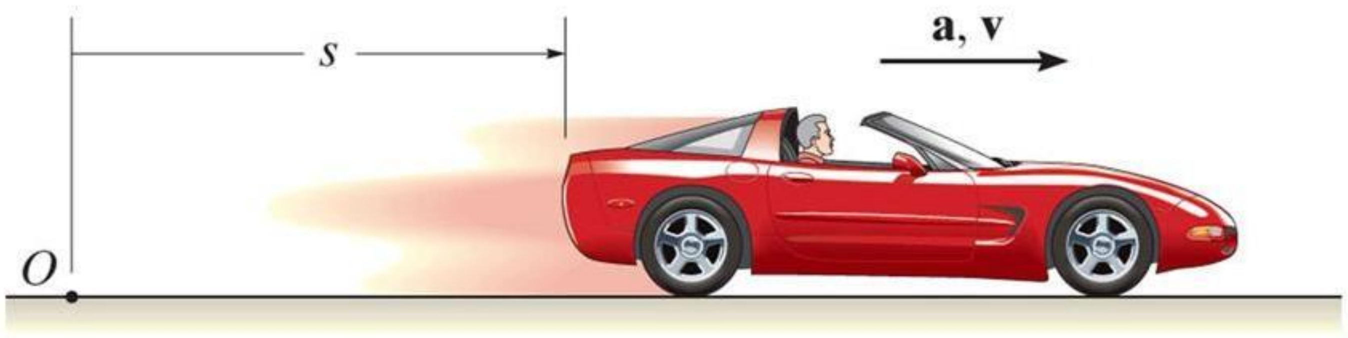
Statics: The study of bodies in equilibrium.

Dynamics:

Kinematics – concerned with the geometric aspects of motion

Kinetics - concerned with the forces causing the motion

APPLICATION



A sports car travels along a straight road.

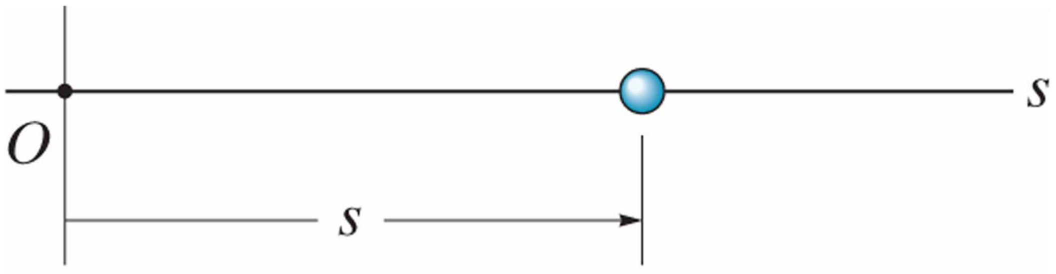
Can we treat the car as a particle?

If the car accelerates at a constant rate, how can we determine its position and velocity at some instant?

RECTILINEAR KINEMATICS

CONTINUOUS MOTION

- **RECTILINEAR KINEMATICS** : characterized by specified time, particle's position, velocity and acceleration.
- **POSITION** : Origin O is fixed point. Position coordinate s is used specify the location at any given instant.
- Measured in meters (m)



Position

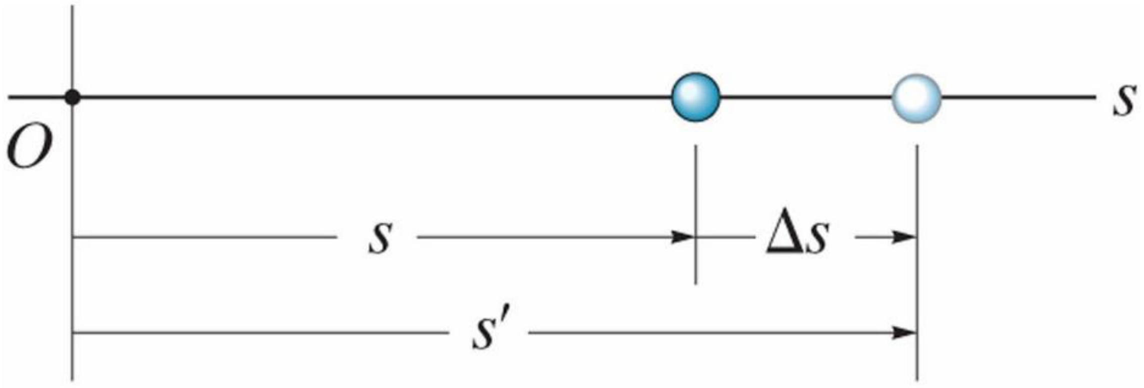
RECTILINEAR KINEMATICS

CONTINUOUS MOTION

- **DISPLACEMENT** : change of position.
- Particle moves from one point to another.

$$\Delta s = s' - s$$

- Can be positive or negative



Displacement

RECTILINEAR KINEMATICS

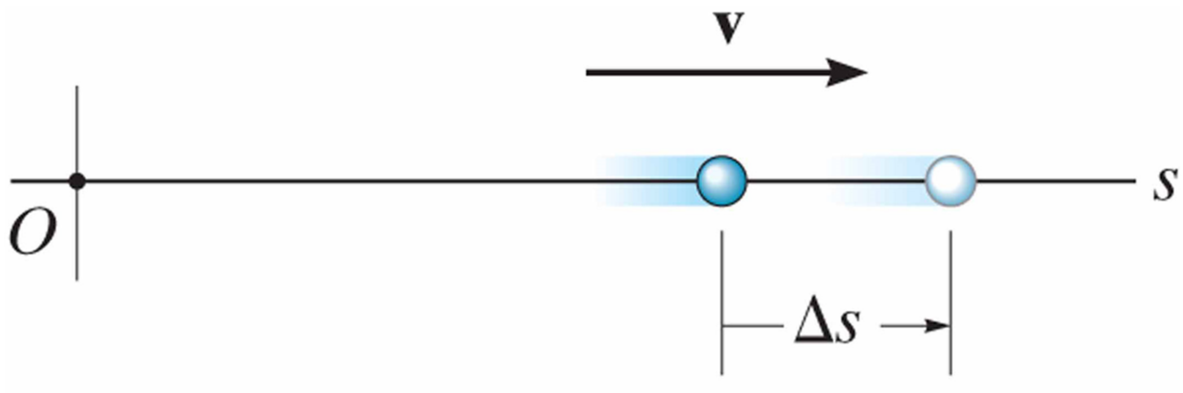
CONTINUOUS MOTION

- **VELOCITY** : particles that moves through displacement Δs during time interval Δt.

- Average velocity : $v_{avg} = \dfrac{\Delta s}{\Delta t}$

- Instantaneous velocity is a vector as:

- $\overset{+}{\rightarrow}$ $\quad v = \dfrac{ds}{dt}$

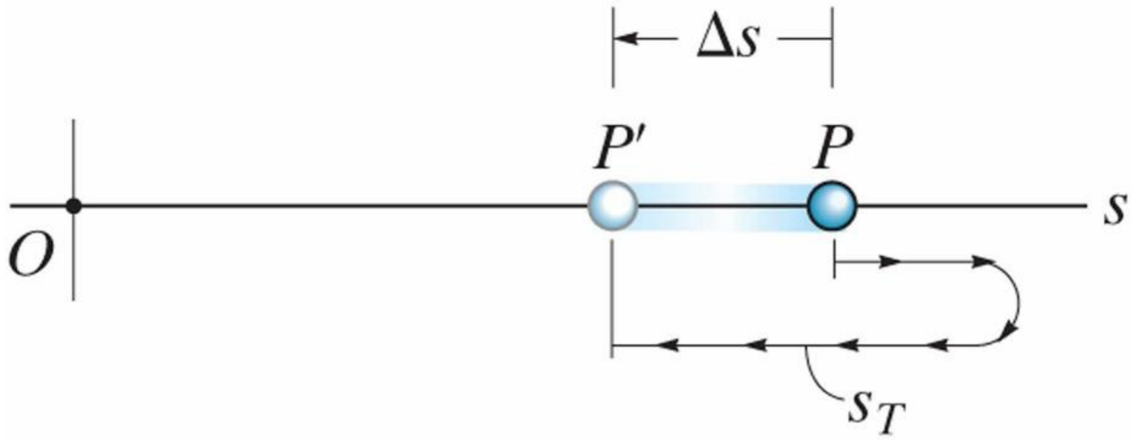


Velocity

CONTINUOUS MOTION

- Average speed : $(v_{sp})_{avg} = \frac{s_T}{\Delta t}$

- Average velocity : $v_{avg} = \frac{-\Delta s_T}{\Delta t}$



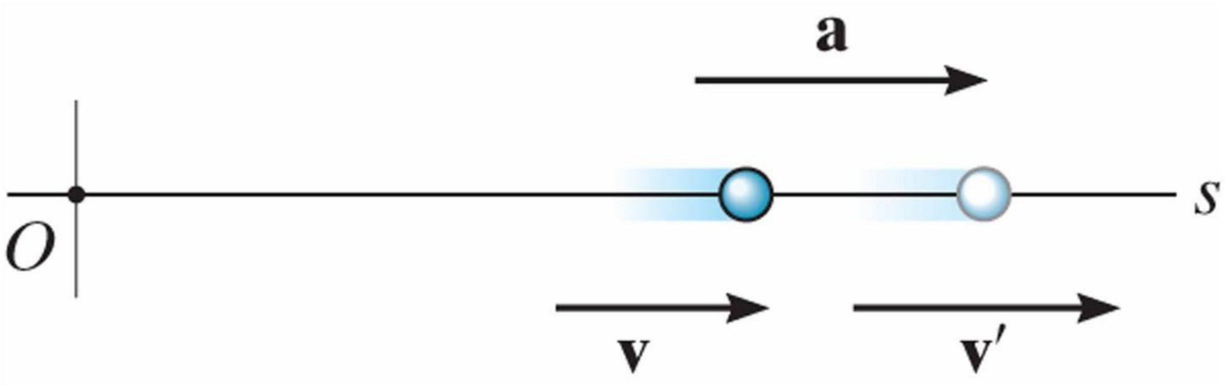
Average velocity and
Average speed

RECTILINEAR KINEMATICS

CONTINUOUS MOTION

- **ACCELERATION** : average acceleration of particle during the time interval Δt is defined as:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$



CONTINUOUS MOTION

- **CONSTANT ACCELERATION** :

$$a = a_{\dot{c}}, a_c = \frac{dv}{dt}, v = \frac{ds}{dt}$$

- **VELOCITY AS A FUNCTION OF TIME :**

$$\int_{v_0}^{v} dv = \int_{0}^{t} a_{c\,dt}$$

$$v = v_0 + a_c t$$

- **POSITION AS A FUNCTION OF TIME:**

$$\int_{s_0}^{s} ds = \int_{0}^{t} (v_0 t + a_c t) dt$$

$$s = s_0 + v_0 t + 1/2 a_c t^2$$

CONTINUOUS MOTION

- **VELOCITY AS A FUNCTION OF POSITION :**

$$\int_{dv}^{v} v\,dv = \int_{s_0}^{s} a_c\,ds$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

RECTILINEAR KINEMATICS

CONTINUOUS MOTION

Example 1

The car in the photo below moves in straight line such that for a short time its velocity is defined by $v = \left(3t^2 + 2t\right) m/s$, where t is in seconds. Determine its position and acceleration when t=3s , t=0 and s=0

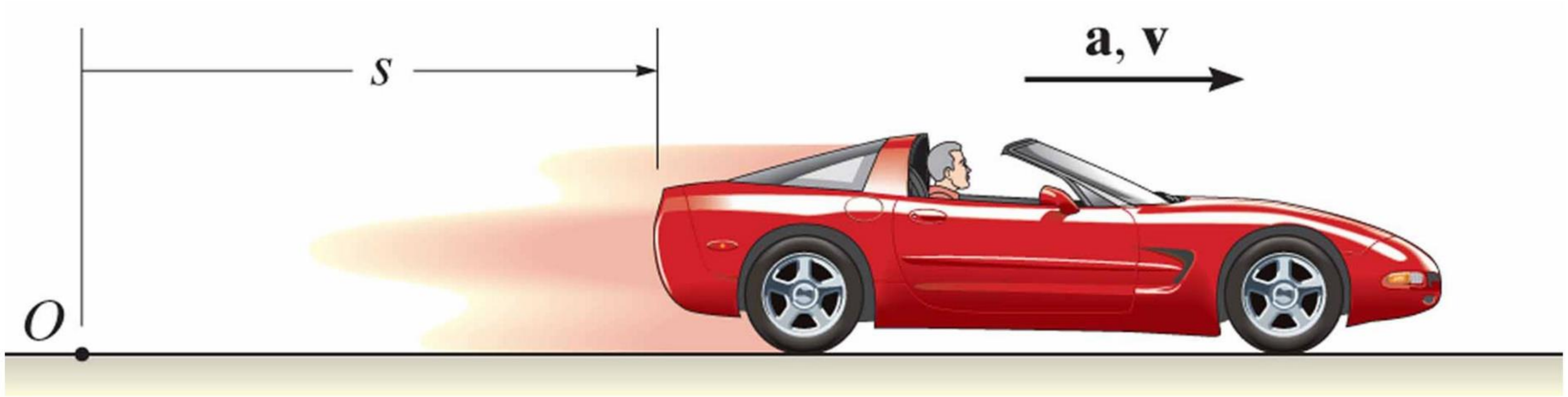


fig12_02.jpg

RECTILINEAR KINEMATICS

CONTINUOUS MOTION

Example 2

A small projectile is fired vertically downward into a fluid medium with an initial velocity of $60\ m/s$. Due to the drag resistance of the fluid, the projectile experiences a deceleration of $a = (-0.4v^3) m/s$, where v is in m/s. Determine the projectile's velocity and position 4s after it is fired.

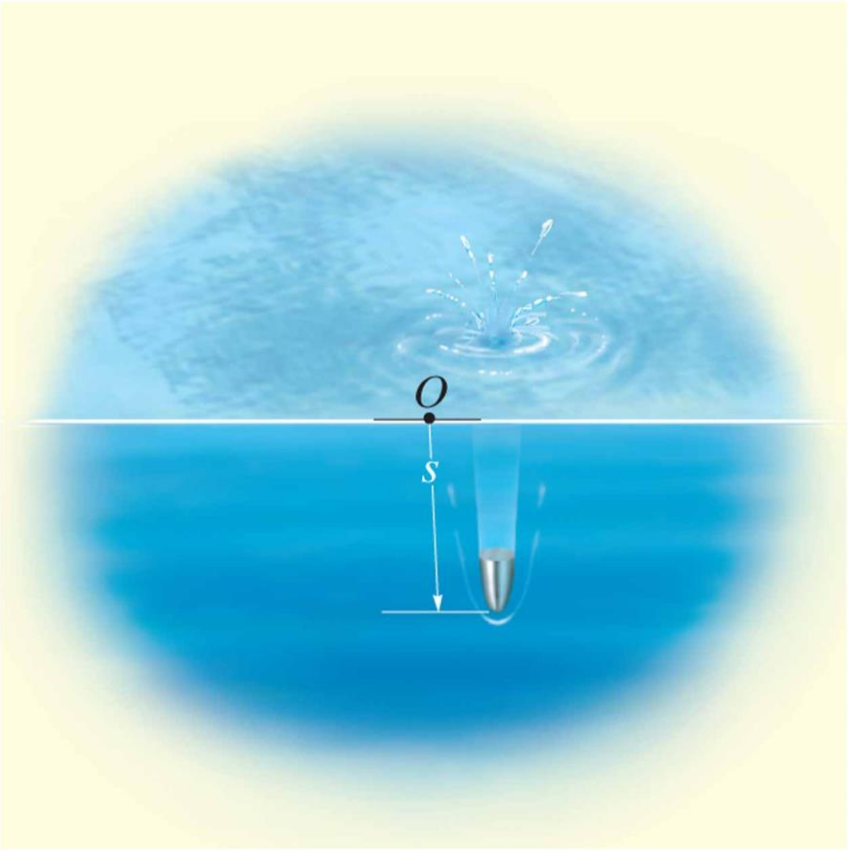


fig12_03.jpg

RECTILINEAR KINEMATICS

CONTINUOUS MOTION

Example 3

During a test a rocket travels upward at 75 m/s, and when it is at 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

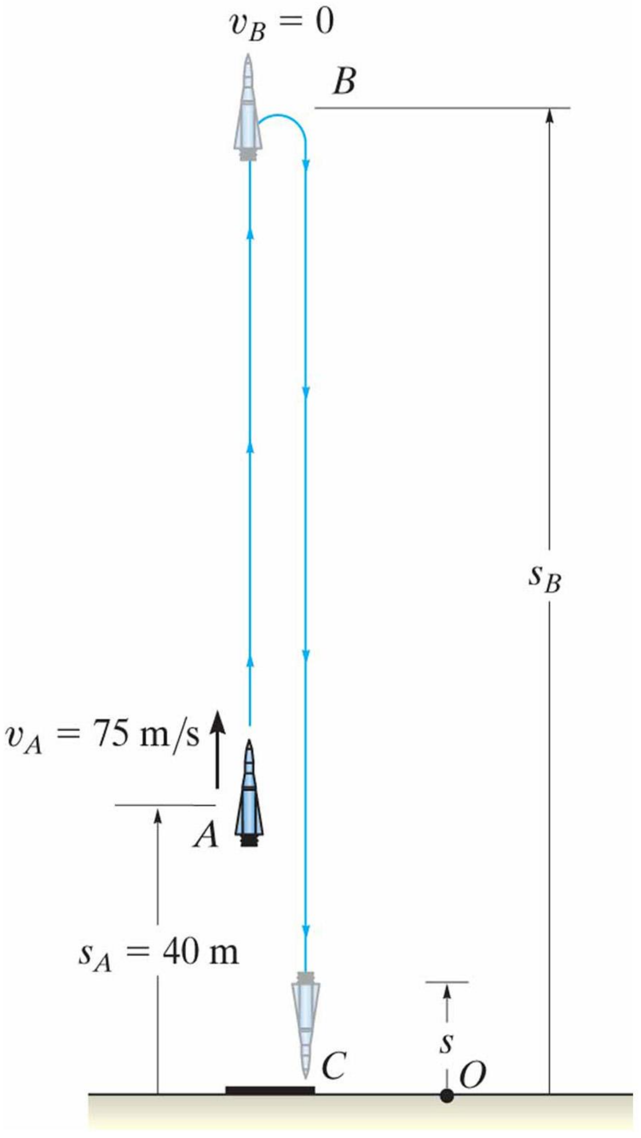


fig12_04.jpg

CONTINUOUS MOTION

Example 4

A metallic particle is subjected to the influence of a magnetic field as it travels downward through a fluid that extends from plate A to plate B shown in the figure. If the particle is released from rest at the midpoint C, $s = 100\ mm$, and the acceleration is $a = (4s) m/s^2$, where s is in meters, determine the velocity of the particle when it reaches plate B, $s = 200\ mm$, and the time it takes to travel from C to B.

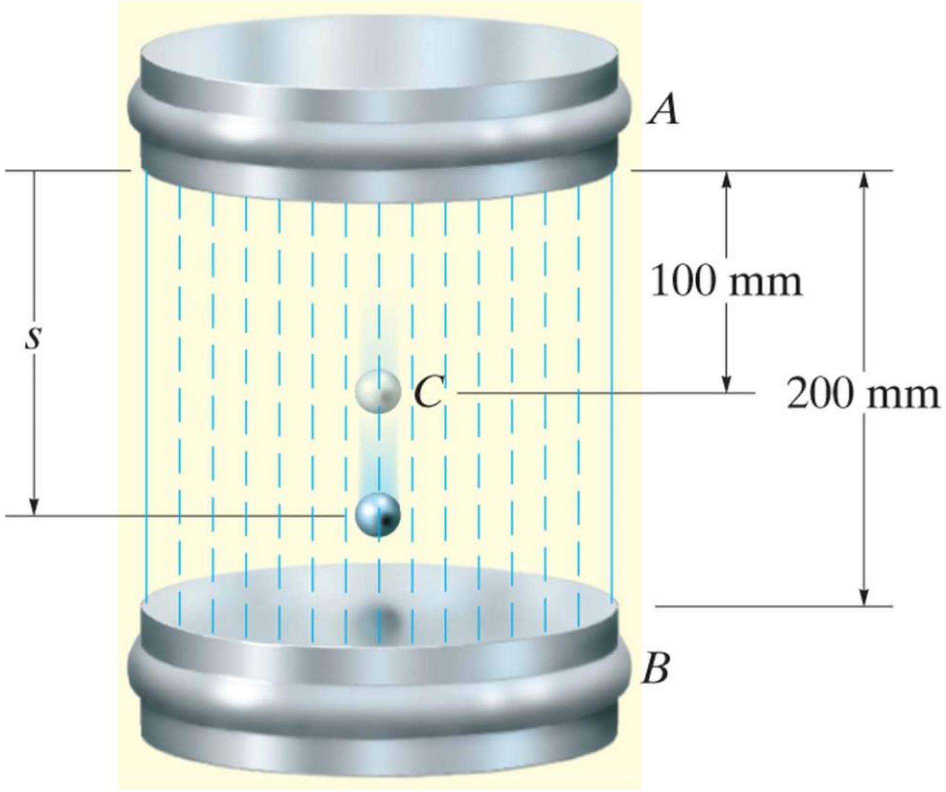
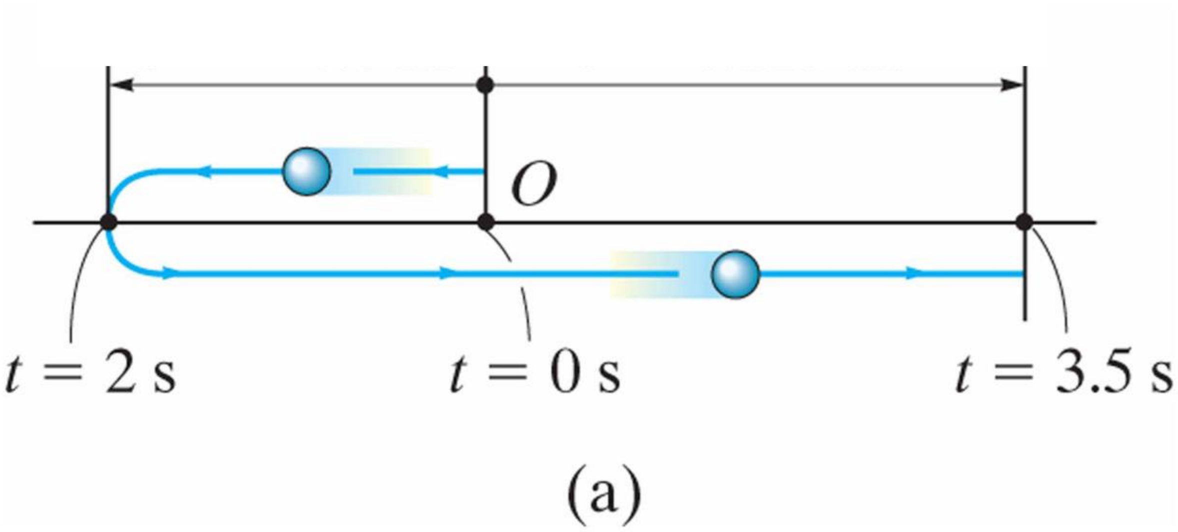


fig12_05.jpg

CONTINUOUS MOTION

Example 5

A particle moves along a horizontal path with a velocity of $v = (3t^2 - 6t)m/s$, where t is the time in seconds. If it is initially located at the origin O, determine the distance traveled in 3.5 s, and the particle's average velocity and average speed during the time interval.



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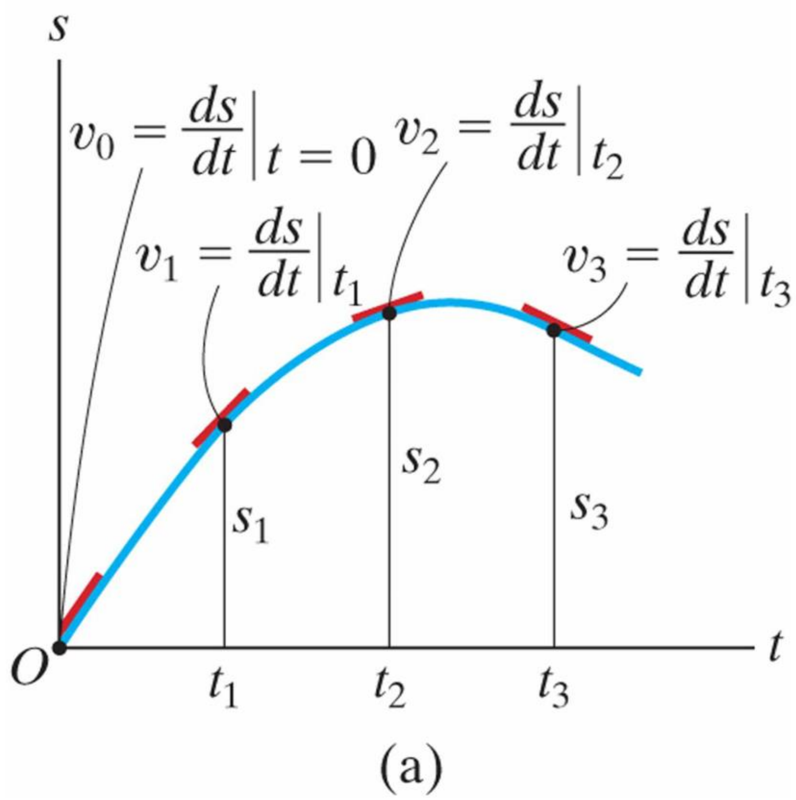
RECTILINEAR KINEMATICS

ERRATIC MOTION

- **Erratic** is changing in motion.
- When a particle is in **erratic**, the position, velocity and acceleration cannot be described by using a single continuous mathematical function along the entire path.
- Therefore, a series of functions will be required to specify the motion at different intervals.
- It is convenient to represent the motion in a graph that can be used to construct the subsequent graph that relates two variables.



(a)

fig12_07a.jpg

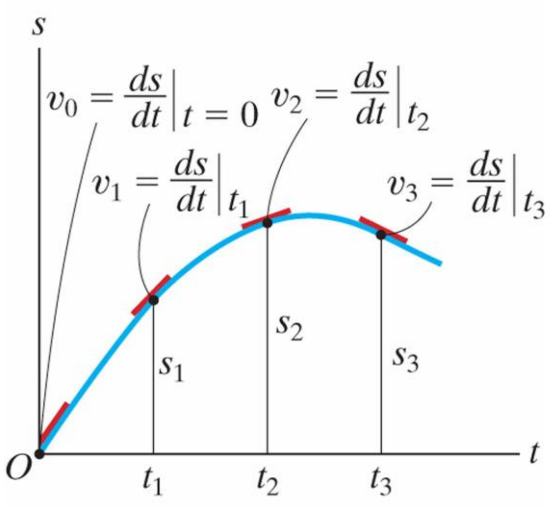
$$v = \frac{ds}{dt}$$

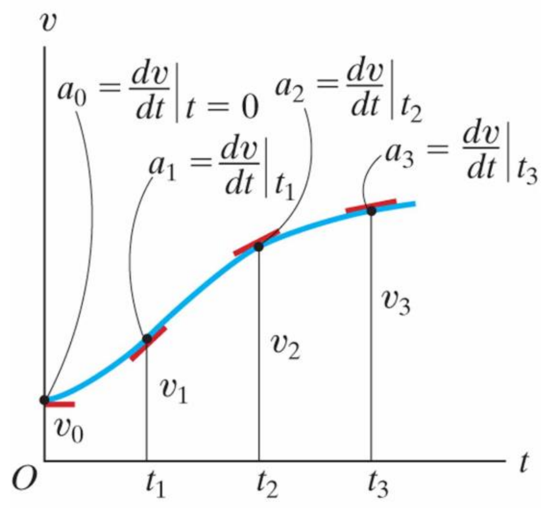
$$a = \frac{dv}{dt}$$

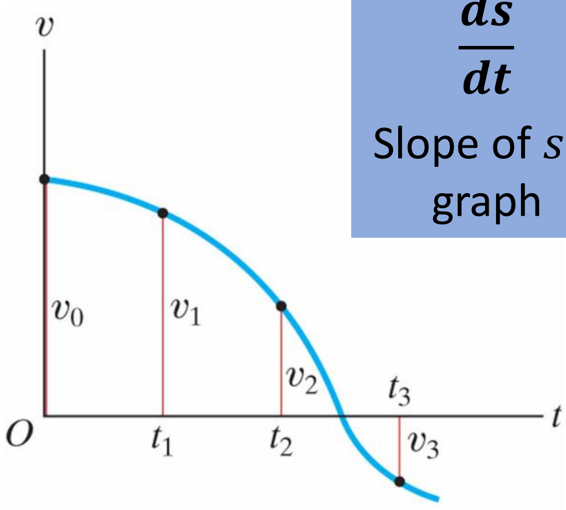
$$a\,ds = v\,dv$$

RECTILINEAR KINEMATICS

THE s-t, v-t AND a-t GRAPHS

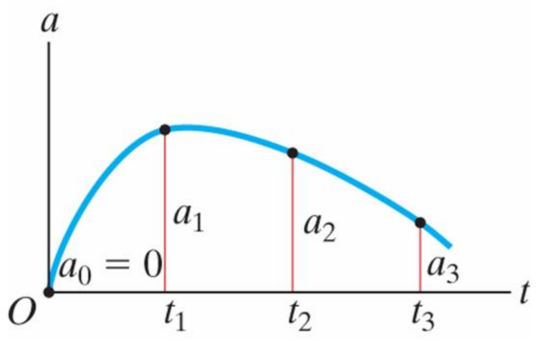






$$\frac{ds}{dt} = v$$

Slope of s-t graph = velocity

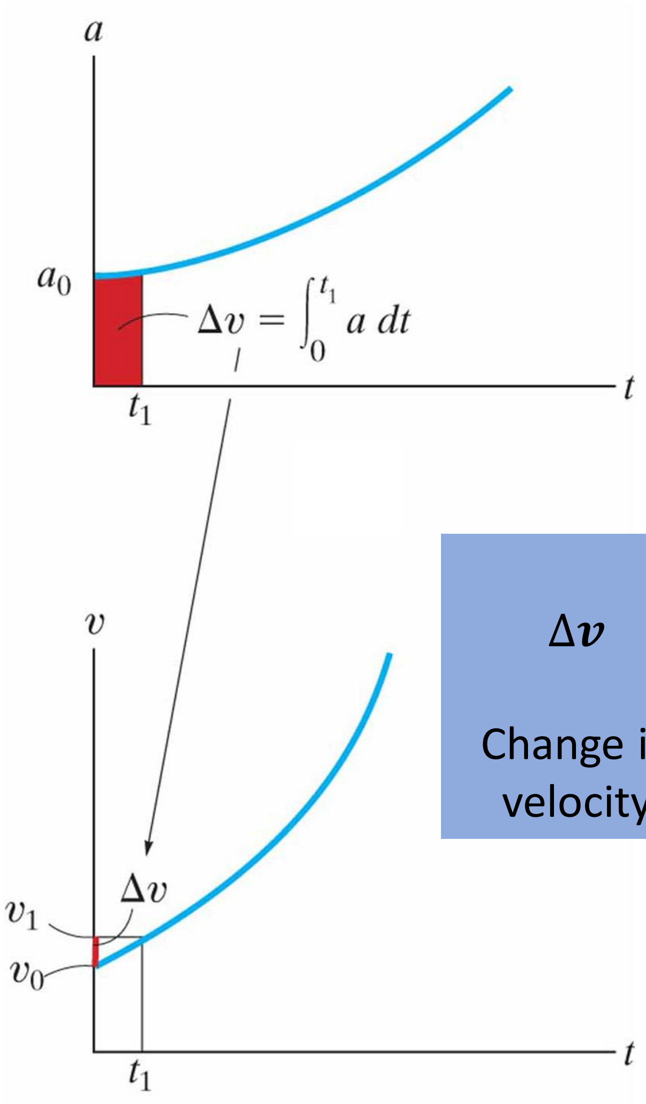


$$\frac{dv}{dt} = a$$

Slope of v-t graph = acceleration

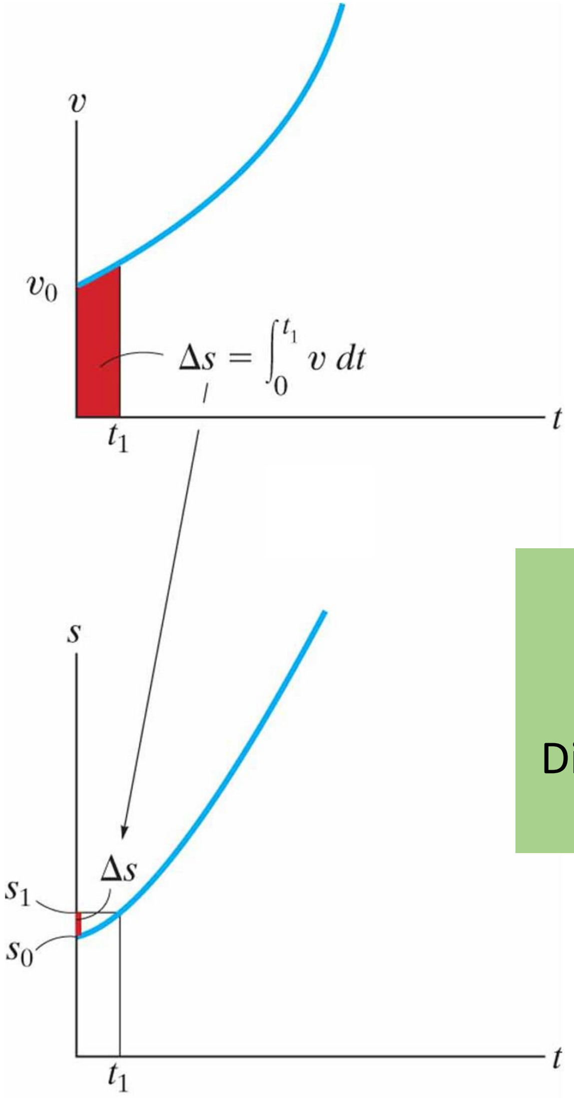
RECTILINEAR KINEMATICS

THE s-t, v-t AND a-t GRAPHS



$\Delta \boldsymbol{v} = \int \boldsymbol{a}\, \boldsymbol{dt}$

Change in velocity = Area under a-t graph

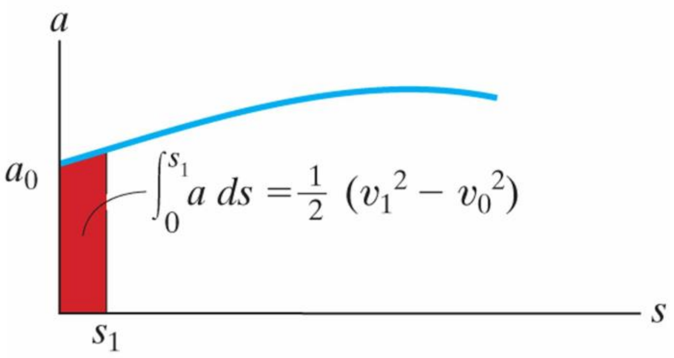


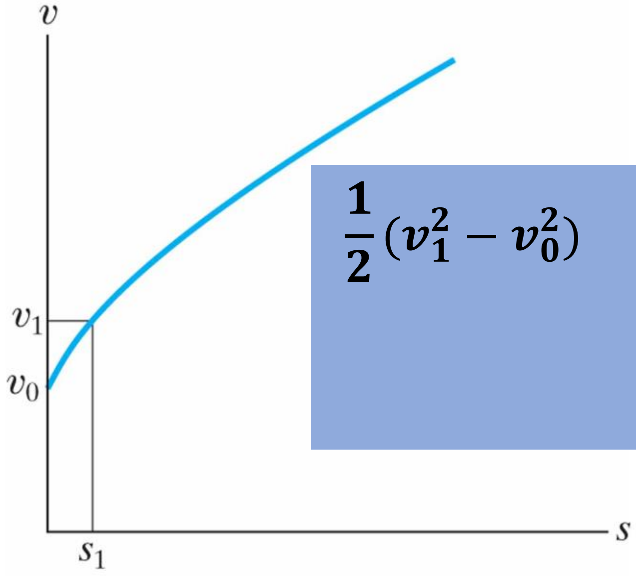
$\Delta \boldsymbol{s} = \int \boldsymbol{v}\, \boldsymbol{dt}$

Displacement = Area under v-t graph

RECTILINEAR KINEMATICS

THE v-s AND a-s GRAPHS

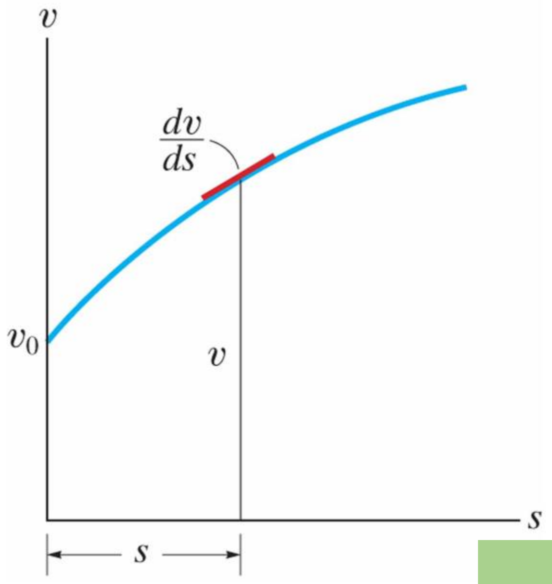


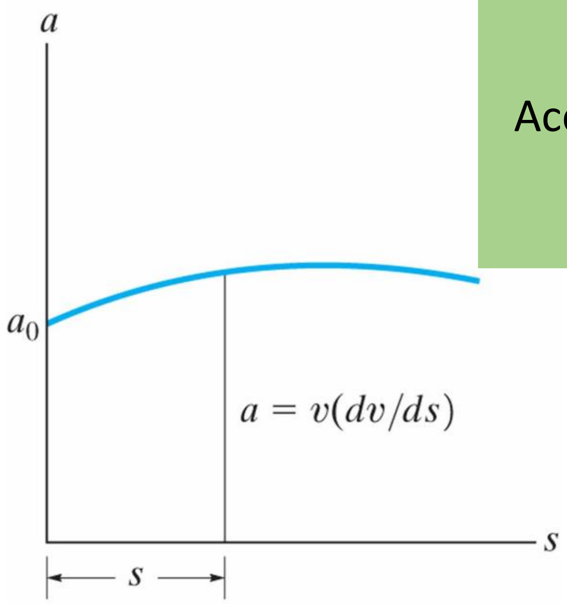


$$a\,ds = v\,dv$$

$$\frac{1}{2}(v_1^2 - v_0^2) = \int_{s_0}^{s_1} a\,ds$$

Area under a-s graph



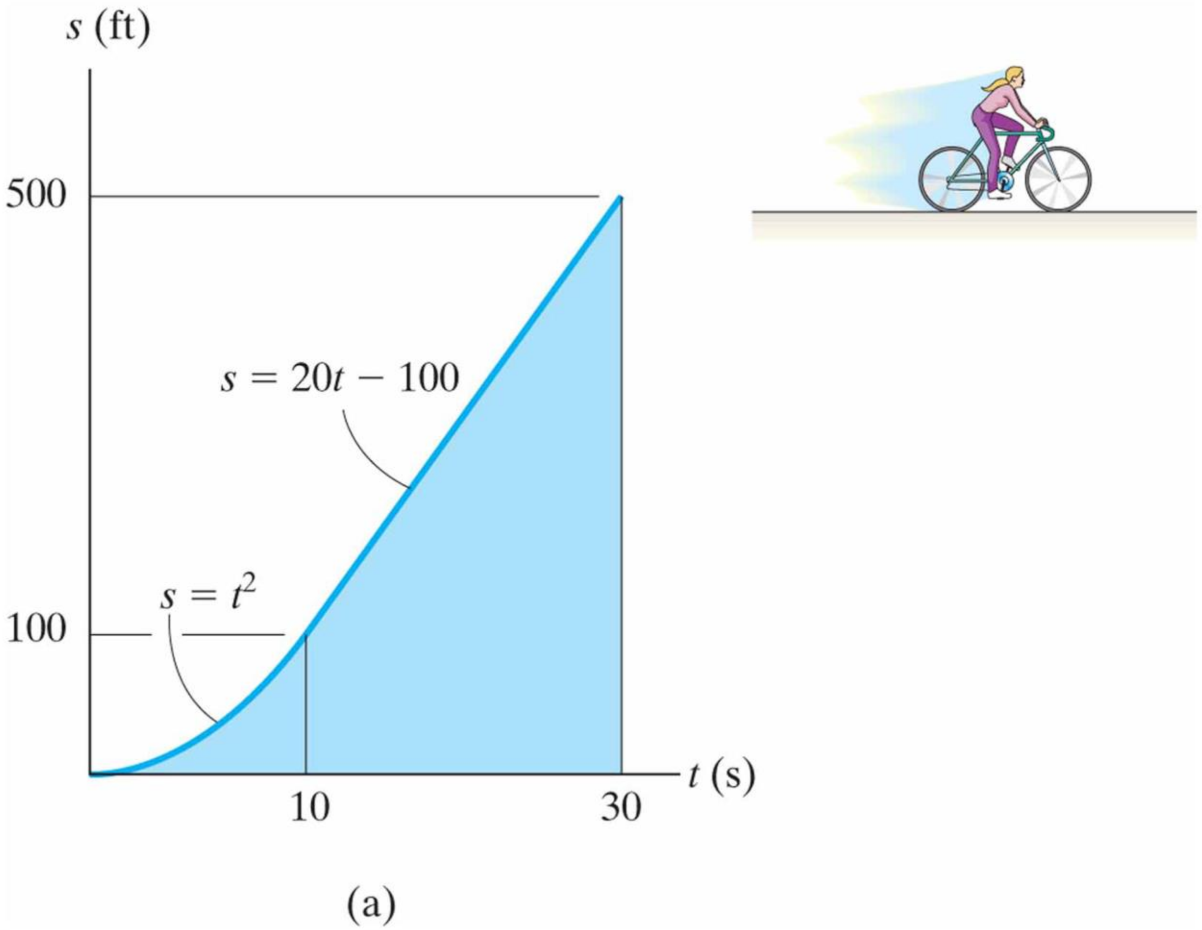


a	=	$v\left(\frac{dv}{ds}\right)$
Acceleration	=	Velocity times slope of v-s graph

ERRATIC MOTION

Example 6

A bicycle moves along a straight road such that its position is described by the graph shown. Construct the v-t and a-t graphs for $0 \leq t \leq 30\ s$.



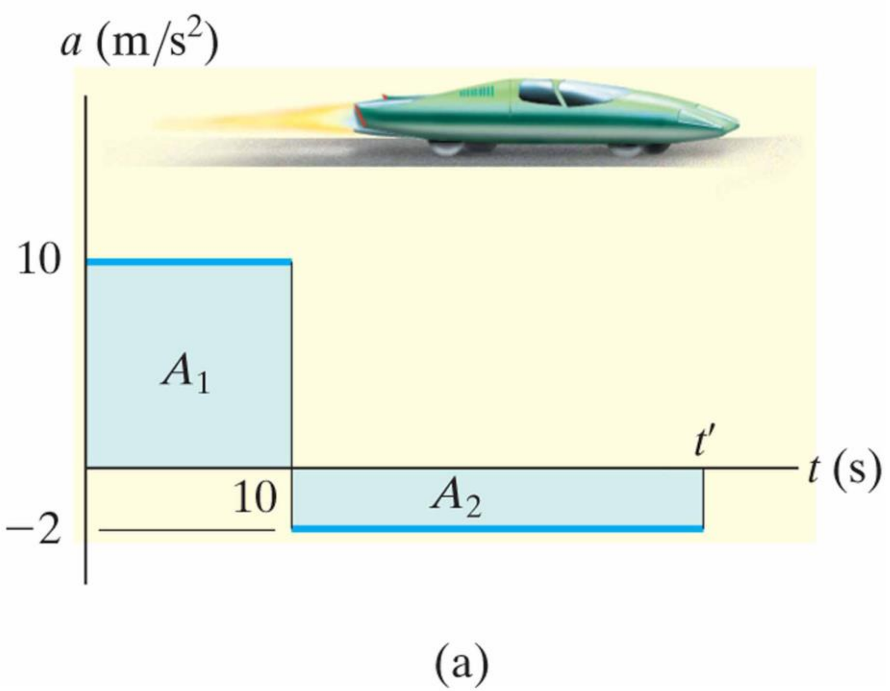
(a)

fig12_13a.jpg

ERRATIC MOTION

Example 7

The car in the figure starts from rest and travels along a straight track such that it accelerates at 10 m/s^2 for 10 s, and then decelerates at 2 m/s^2. Draw the v-t and s-t graphs and determine the time t' needed to stop the car. How far has the car traveled?



(a)

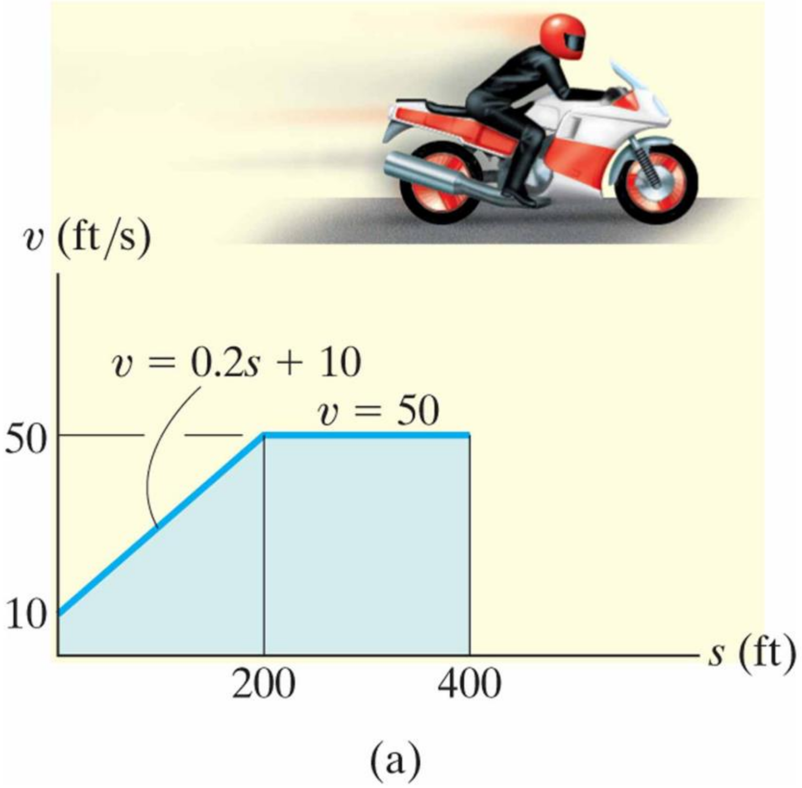
fig12_14a.jpg

RECTILINEAR KINEMATICS

ERRATIC MOTION

Example 8

The v-s graph describing the motion of a motorcycle is shown in the figure. Construct the a-s graph of the motion and determine the time needed for the motorcycle to reach the position $s = 120\ m$.



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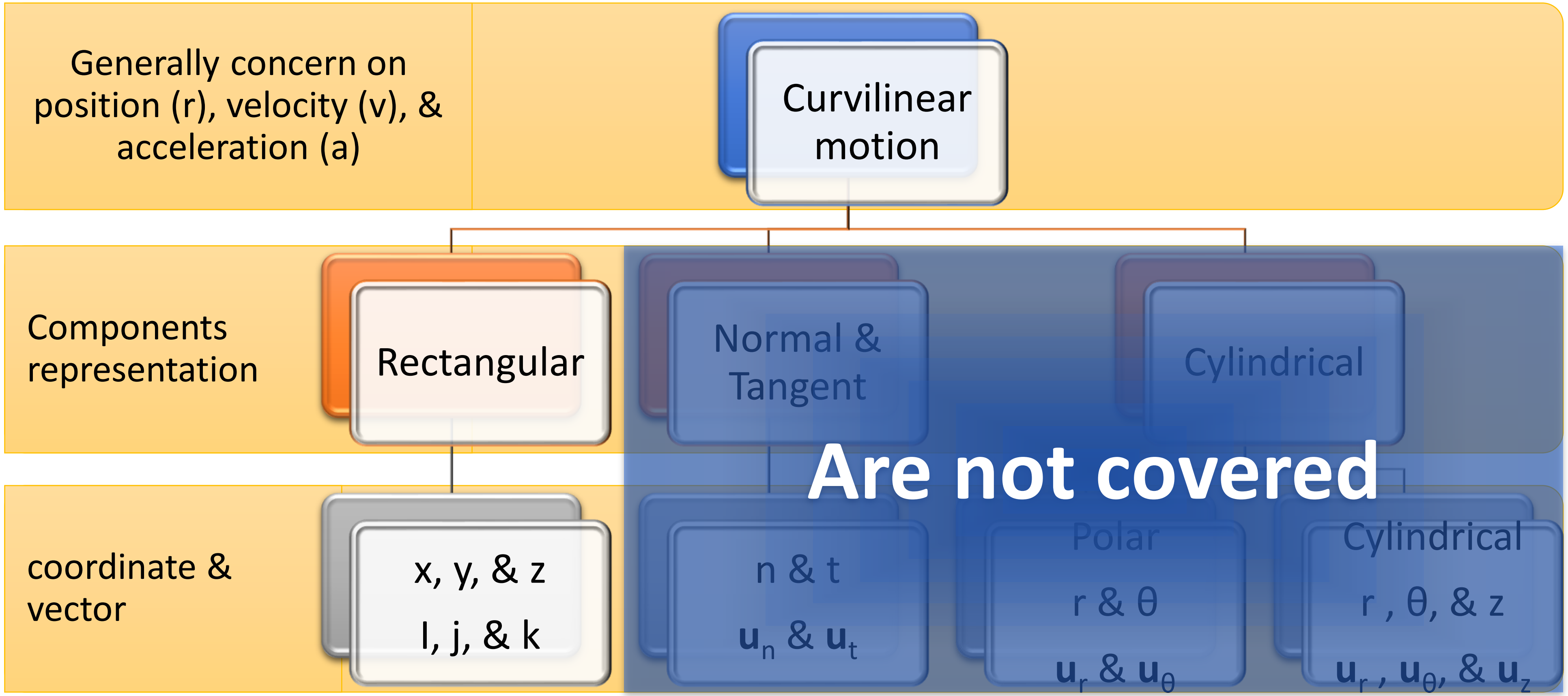
Curvilinear Motion

CURVILINEAR MOTION : IN A GLANCE



CURVILINEAR MOTION

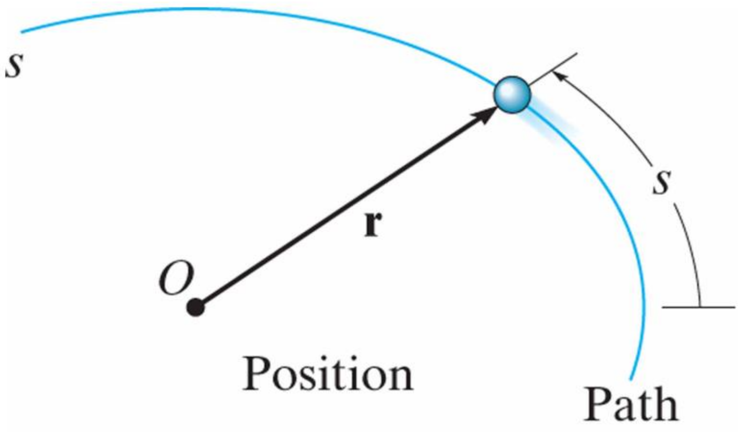
APPLICATIONS

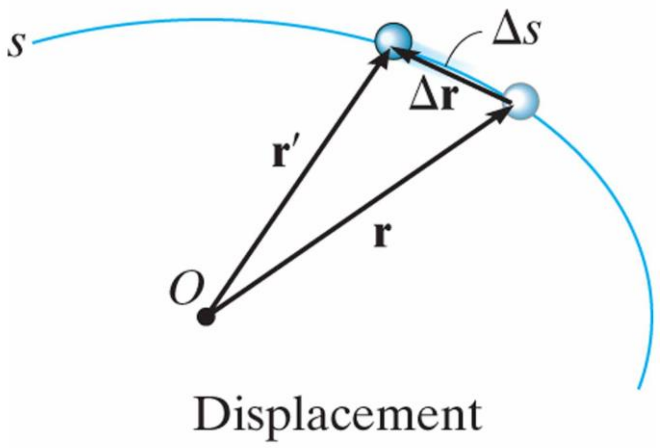
The path of motion of a plane can be tracked with radar and its x, y, and z coordinates (relative to a point on earth) recorded as a function of time.

How can we determine the velocity or acceleration of the plane at any instant?

GENERAL CURVILINEAR MOTION

- A particle moving along a curved path undergoes curvilinear motion. Since the motion is often three-dimensional, **vectors** are used to describe the motion.
- A particle moves along a curve defined by the path function, s.
- The position of the particle at any instant is designated by the vector $\boldsymbol{r} = \boldsymbol{r}(t)$. Both the magnitude and direction of $\boldsymbol{r}$ may vary with time.



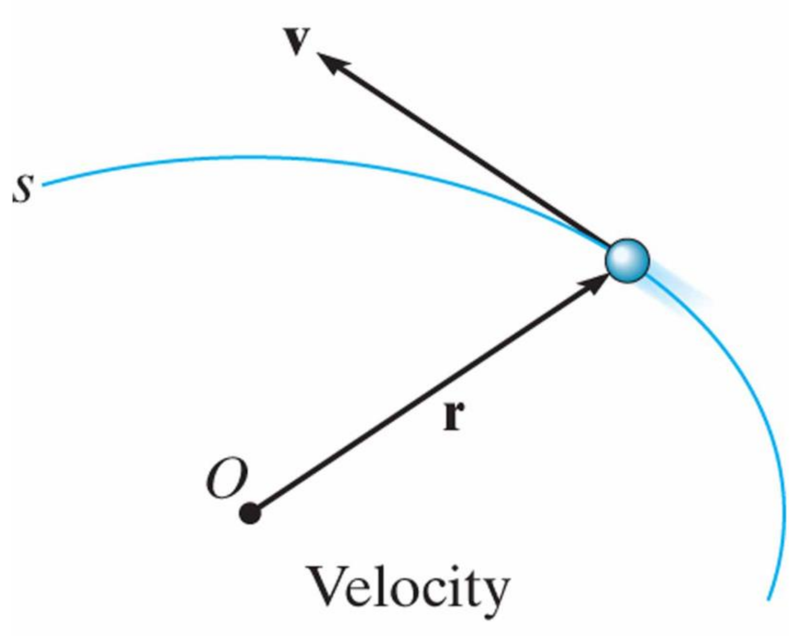


If the particle moves a distance Δs along the curve during time interval Δt, the displacement is determined by vector subtraction: $\Delta\boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$

CURVILINEAR MOTION

VELOCITY

- **Velocity** represents the rate of change in the position of a particle.
- The **average velocity** of the particle during the time increment Dt is $\boldsymbol{v}_{avg} = \Delta\boldsymbol{r}/\Delta t$.
- The instantaneous velocity is the time-derivative of position $\boldsymbol{v} = d\boldsymbol{r}/dt$.
- The velocity vector, $\boldsymbol{v}$, is always tangent to the path of motion.
- The magnitude of $\boldsymbol{v}$ is called the **speed**.



- Since the arc length $\Delta\boldsymbol{s}$ approaches the magnitude of $\Delta\boldsymbol{r}$ as $t \rightarrow 0$, the speed can be obtained by differentiating the path function ($v = ds/dt$).
- Note that this is **not a vector!**

CURVILINEAR MOTION

ACCELERATION

- **Acceleration** represents the rate of change in the velocity of a particle.

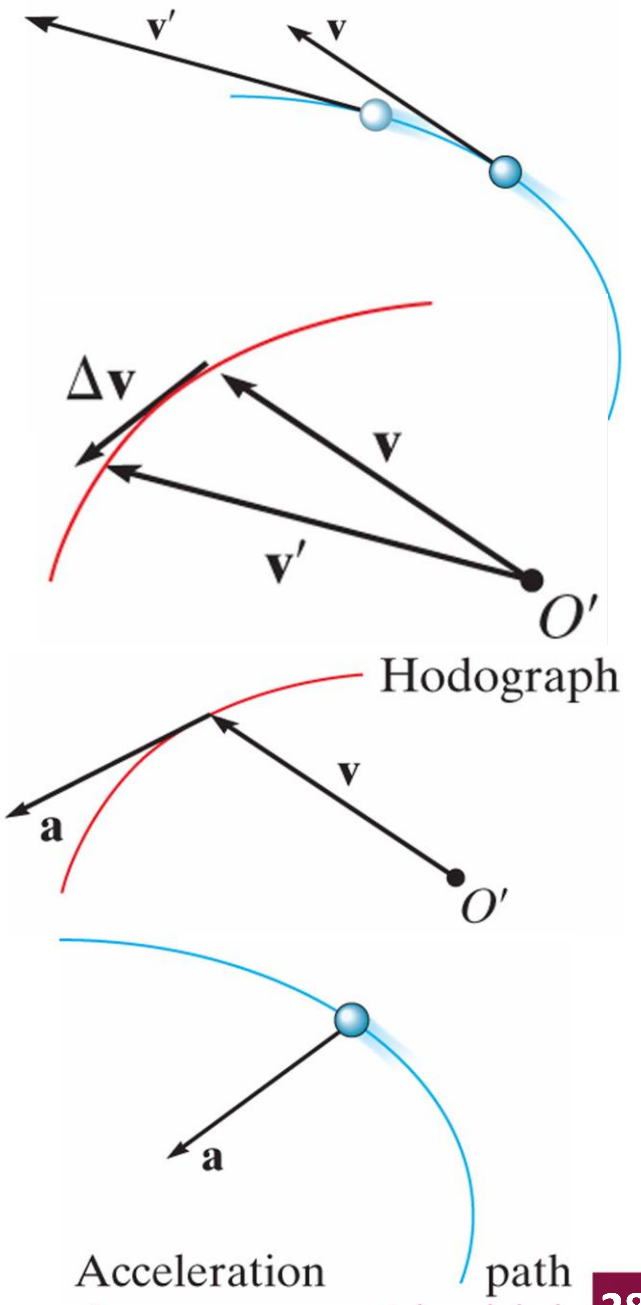
- If a particle's velocity changes from $\boldsymbol{v}$ to $\boldsymbol{v'}$ over a time increment Δt, the **average acceleration** during that increment is:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v'}{\Delta t}$$

- The **instantaneous acceleration** is the time-derivative of velocity:

$$a = \frac{dv}{dt} = \frac{d^2 r}{dt}$$

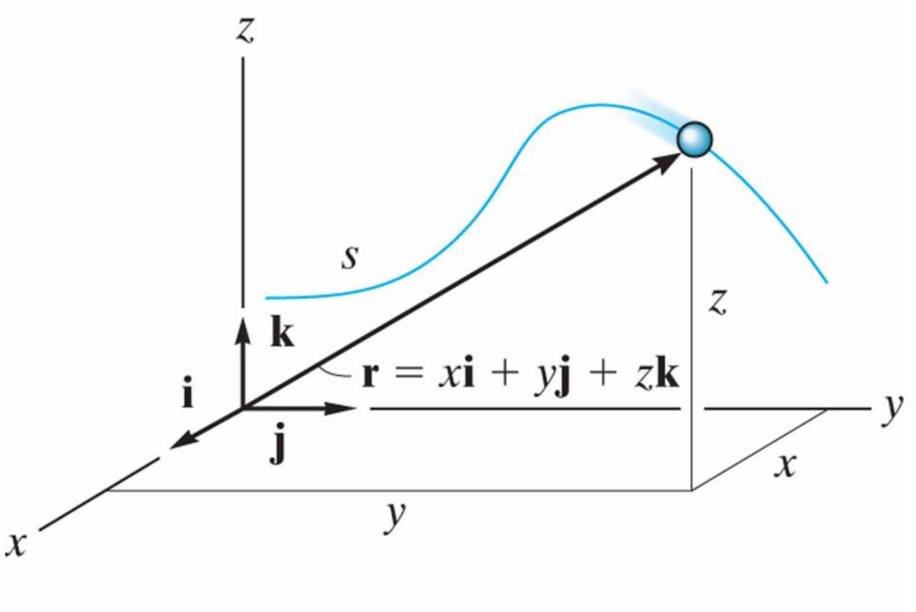
- A plot of the locus of points defined by the arrowhead of the velocity vector is called a hodograph. The acceleration vector is tangent to the hodograph, but not, in general, tangent to the path function.



CURVILINEAR MOTION

RECTANGULAR COMPONENTS

- It is often convenient to describe the motion of a particle in terms of its x, y, z or rectangular components, relative to a fixed frame of reference.



Position

- The position of the particle can be defined at any instant by the **position vector**
  $\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$.
- The x, y, z components may all be **functions of time**, i.e.,
  $x = x(t), y = y(t), and\ z = z(t)$.

- The **magnitude** of the position vector is: $r = \sqrt{(x^2 + y^2 + z^2)}$
- The **direction** of $\boldsymbol{r}$ is defined by the unit vector: $\boldsymbol{u_r} = (\boldsymbol{r}/r)$

CURVILINEAR MOTION

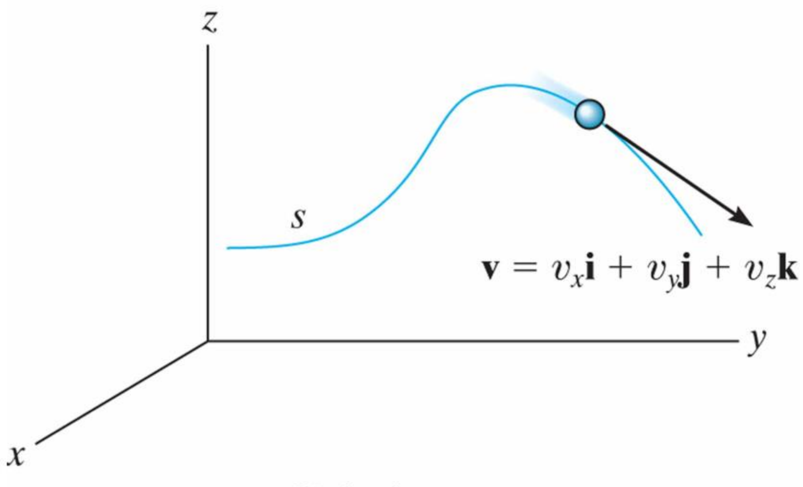
RECTANGULAR COMPONENTS : VELOCITY

- The velocity vector is the time derivative of the position vector:

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \frac{d(x\boldsymbol{i})}{dt} + \frac{d(y\boldsymbol{j})}{dt} + \frac{d(z\boldsymbol{k})}{dt}$$

- Since the unit vectors $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are constant in magnitude and direction, this equation reduces to $v = v_x \boldsymbol{i} + v_y \boldsymbol{j} + v_z \boldsymbol{k}$

where $v_x = \dot{x} = \frac{dx}{dt}$, $v_y = \dot{y} = \frac{dy}{dt}$, $v_z = \dot{z} = \frac{dz}{dt}$



Velocity

The magnitude of the velocity vector is

$$v = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$$

The direction of $\boldsymbol{v}$ is tangent to the path of motion.

CURVILINEAR MOTION

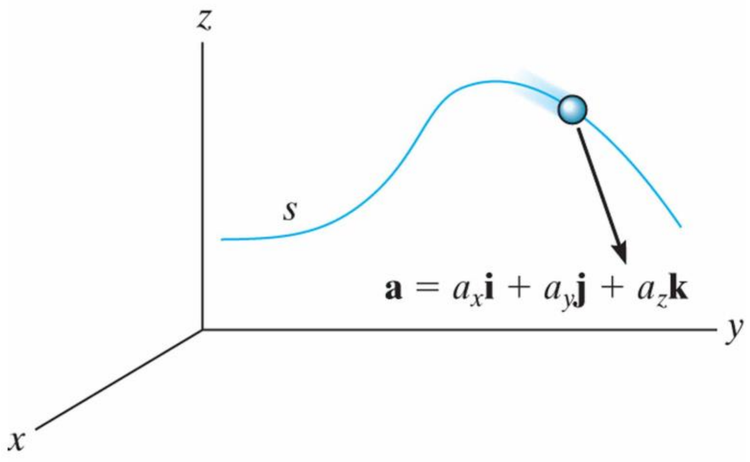
RECTANGULAR COMPONENTS : ACCELERATION

- The acceleration vector is the time derivative of the velocity vector (second derivative of the position vector):

$$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{d^2\boldsymbol{r}}{dt^2} = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k}$$

where $a_x = \dot{v}_x = \ddot{x} = \frac{dv_x}{dt}, a_y = \dot{v}_y = \ddot{y} = \frac{dv_y}{dt}, a_z = \dot{v}_z = \ddot{z} = \frac{dv_z}{dt}$

- The magnitude of the acceleration vector is : $a = \sqrt{(a_x)^2 + (a_y)^2 + (a_z)^2}$



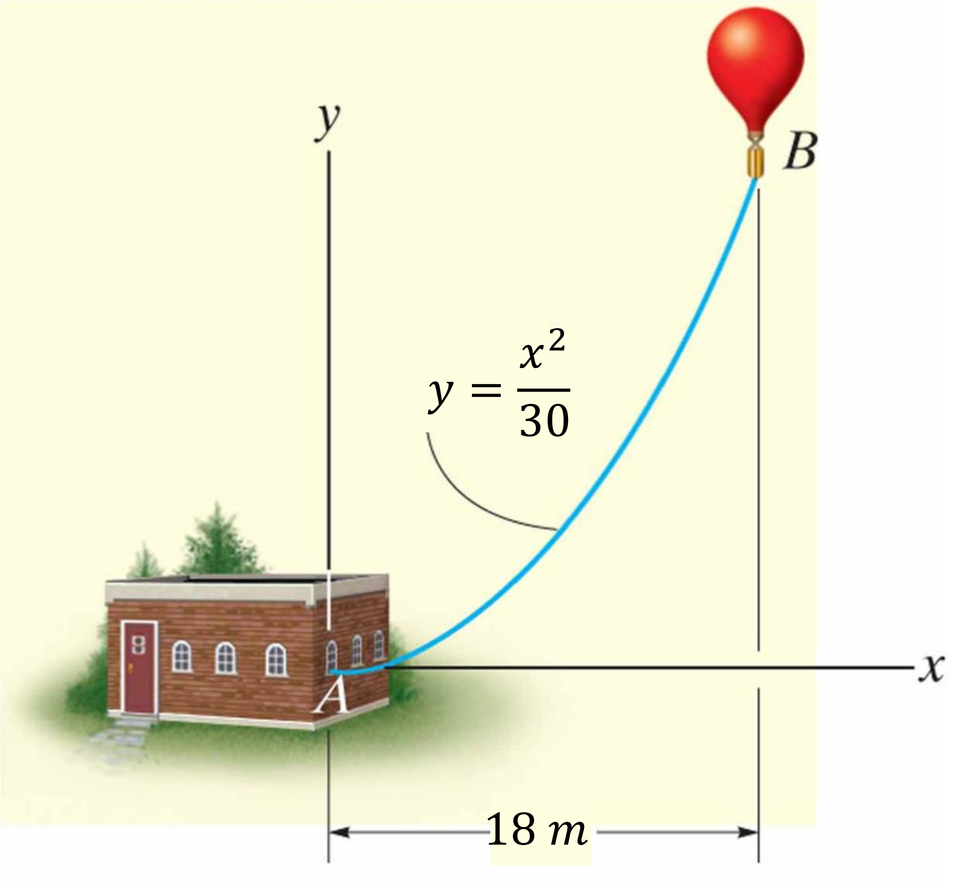
Acceleration

The direction of $\boldsymbol{a}$ is usually not tangent to the path of the particle.

MOMENT OF A FORCE: PRINCIPLE OF MOMENT

Example 9

At any instant the horizontal position of the weather balloon in the figure is defined as $x = (9t)$ m, where t is in seconds. If the equation of the path is $y = \frac{x^2}{30}$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

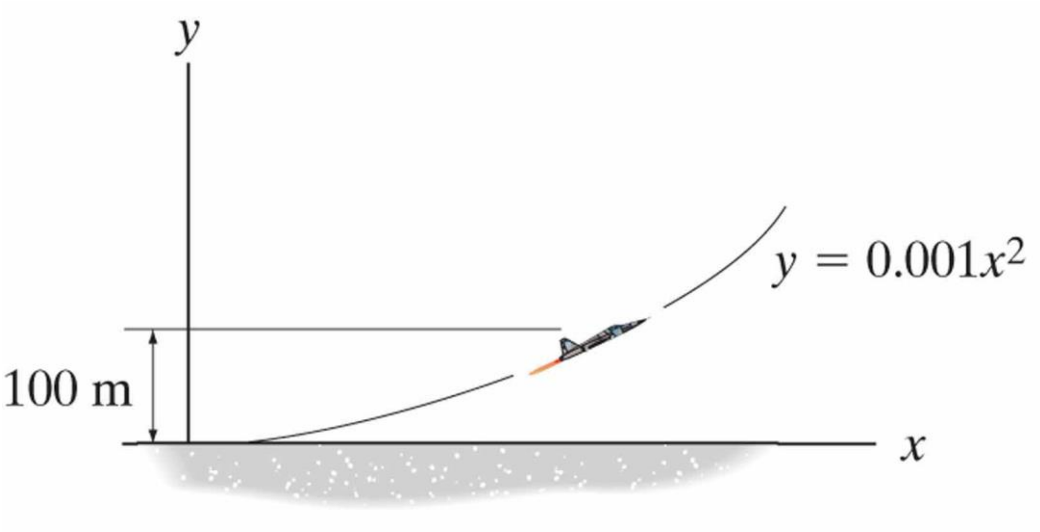


CURVILINEAR MOTION

MOMENT OF A FORCE: PRINCIPLE OF MOMENT

Example 10

For a short time, the path of the plane in the figure is described by $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.



Motion of a Projectile

MOTION OF A PROJECTILE

APPLICATIONS

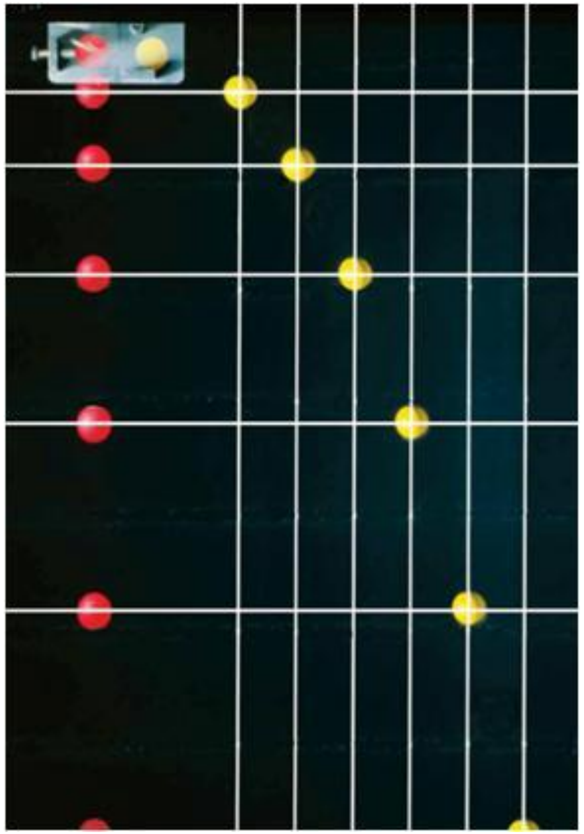
A basketball is shot at a certain angle. What parameters should the shooter consider in order for the basketball to pass through the basket?

Distance, speed, the basket location, … anything else ?

MOTION OF A PROJECTILE

INTRODUCTION

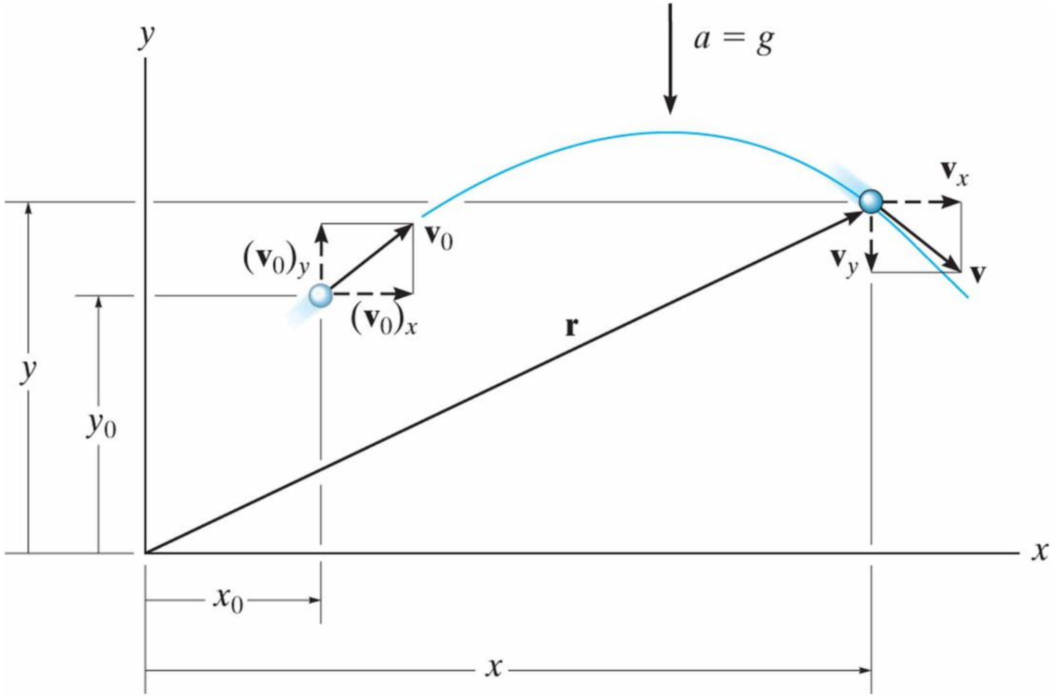
- Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., from gravity).



- For illustration, consider the two balls on the left.
- The red ball falls from rest, whereas the yellow ball is given a horizontal velocity.
- Each picture in this sequence is taken after the same time interval.
- Notice both balls are subjected to the same downward acceleration since they remain at the same elevation at any instant.
- Also, note that the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction is constant.

MOTION OF A PROJECTILE

KINEMATIC EQUATIONS : HORIZONTAL MOTION



Since $a_x = 0$, the velocity in the horizontal direction remains constant ($\boldsymbol{v_x} = \boldsymbol{v_{ox}}$) and the position in the x direction can be determined by:

$$x = xo + (vox)\,t$$

Why is a_x equal to zero (assuming movement through the air)?

MOTION OF A PROJECTILE

KINEMATIC EQUATIONS : VERTICAL MOTION

- Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equations yields:

$$v_y = v_{oy} - g\,t$$

$$y = y_o + (v_{oy})\,t - \frac{1}{2} g\,t^2$$

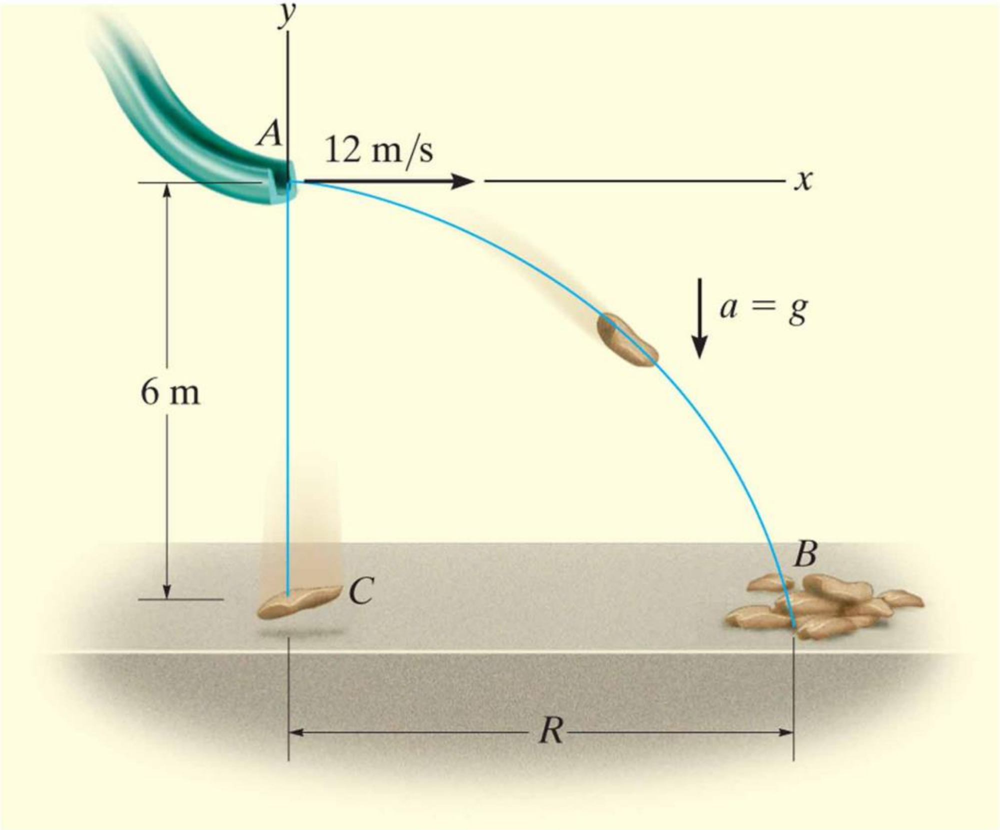
$$v_y^{\,2} = v_{oy}^{\,2} - 2\,g\,(y - y_o)$$

For any given problem, only two of these three equations can be used. Why?

MOMENT OF A FORCE: PRINCIPLE OF MOMENT

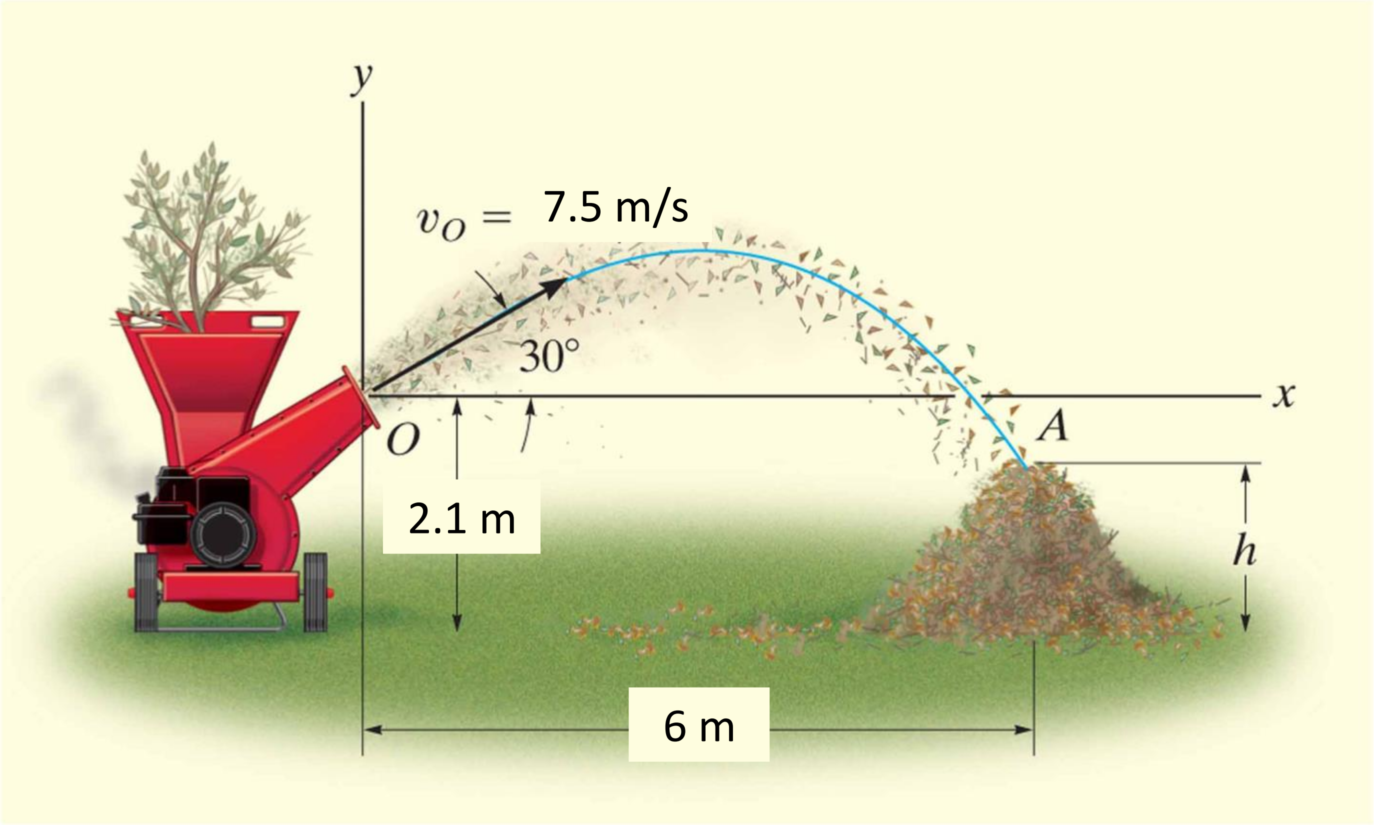
Example 11

A sack slides off the ramp, shown in the figure , with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.



Example 12

The chipping machine is designed to eject wood chips at $v_o = 7.5$ m/s as shown in the figure. If the tube is oriented at $30°$ from the horizontal, determine how high, h, the chips strike the pile if at this instant they land on the pile 6 m from the tube.

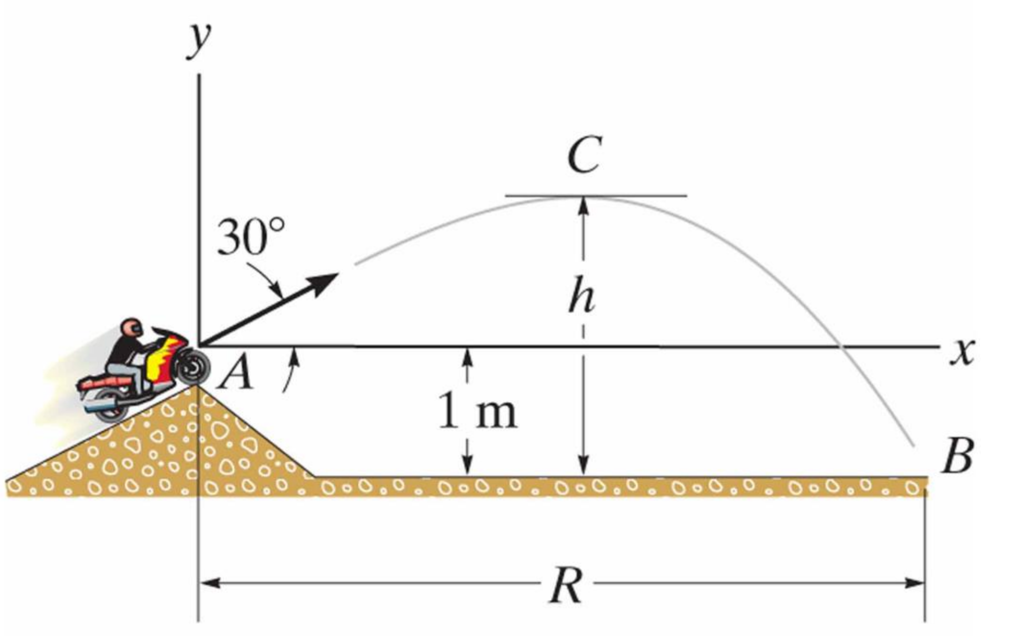


CURVILINEAR MOTION

MOMENT OF A FORCE: PRINCIPLE OF MOMENT

Example 13

The track for this racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race, it was observed that the rider shown in the figure remained in mid air for 1.5 s. Determine the speed at which he was travelling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and the rider.



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