SEEM1113 ENGINEERING MECHANICS

CH6
Kinetics of a Particle

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OBJECTIVES

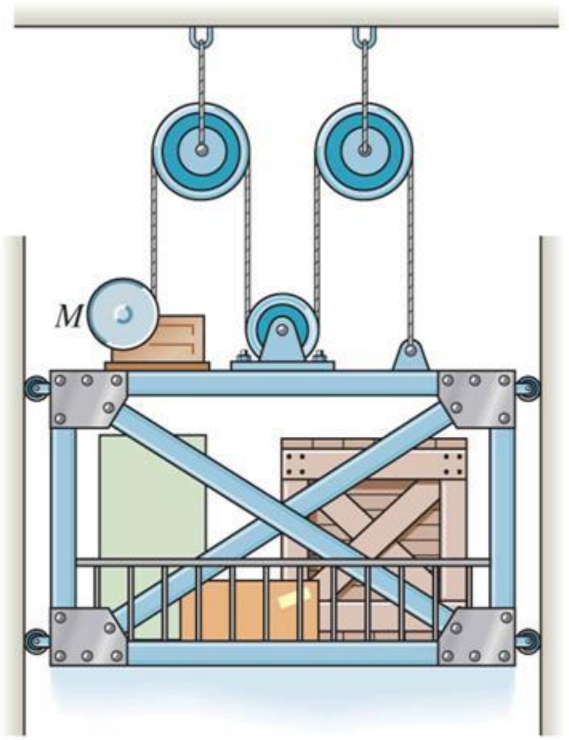
At the end of this lesson, you should be able to:

1. Write the equation of motion for an accelerating body.
2. Draw the free-body and kinetic diagrams for an accelerating body.
3. Apply Newton’s second law to determine forces and accelerations for particles in rectilinear motion.

FORCE AND ACCELERATION

FORCE AND ACCELERATION

APPLICATION



A freight elevator is lifted using a motor attached to a cable and pulley system as shown.

How can we determine the tension force in the cable required to lift the elevator and load at a given acceleration? This is needed to decide what size cable should be used.

Is the tension force in the cable greater than the weight of the elevator and its load?

FORCE AND ACCELERATION

NEWTON'S LAW OF MOTION

- The motion of a particle is governed by **Newton's three laws of motion**.
- **First Law:** A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.
- **Second Law:** If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force. This acceleration has a magnitude proportional to the resultant force.
- **Third Law:** Mutual forces of action and reaction between two particles are equal, opposite, and collinear.

FORCE AND ACCELERATION

NEWTON'S LAW OF MOTION

- Mathematically, Newton's second law of motion can be written

$$\bar{\boldsymbol{F}} = m\bar{\boldsymbol{a}}$$

where $\bar{\boldsymbol{F}}$ is the resultant unbalanced force acting on the particle, and $\bar{\boldsymbol{a}}$ is the acceleration of the particle. The positive scalar m is called the mass of the particle.

Newton's second law cannot be used when the particle's speed approaches the speed of light, or if the size of the particle is extremely small (~ size of an atom).

FORCE AND ACCELERATION

MASS AND WEIGHT

- It is important to understand the difference between the mass and weight of a body!

- Mass is an **absolute property of a body**. It is independent of the gravitational field in which it is measured. The mass provides a measure of the resistance of a body to a change in velocity, as defined by Newton's second law of motion

$$(m = \frac{F}{a})$$

- The weight of a body is **not absolute**, since it depends on the gravitational field in which it is measured. Weight is defined as

$$W = mg$$

where g is the acceleration due to gravity.

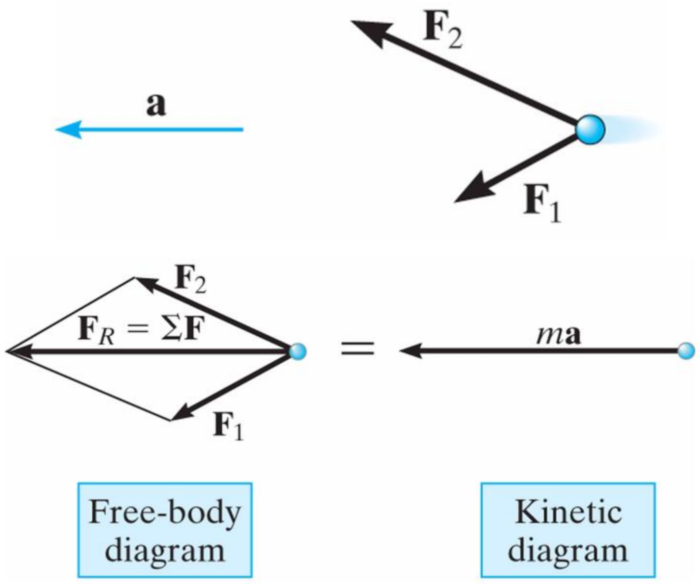
FORCE AND ACCELERATION

EQUATION OF MOTION

- If more than one force acts on the particle, the equation of motion can be written

$$\sum \bar{\boldsymbol{F}} = \overline{\boldsymbol{F}_R} = m\bar{\boldsymbol{a}}$$

where F_R is the resultant force, which is a vector summation of all the forces.



An illustration on this condition

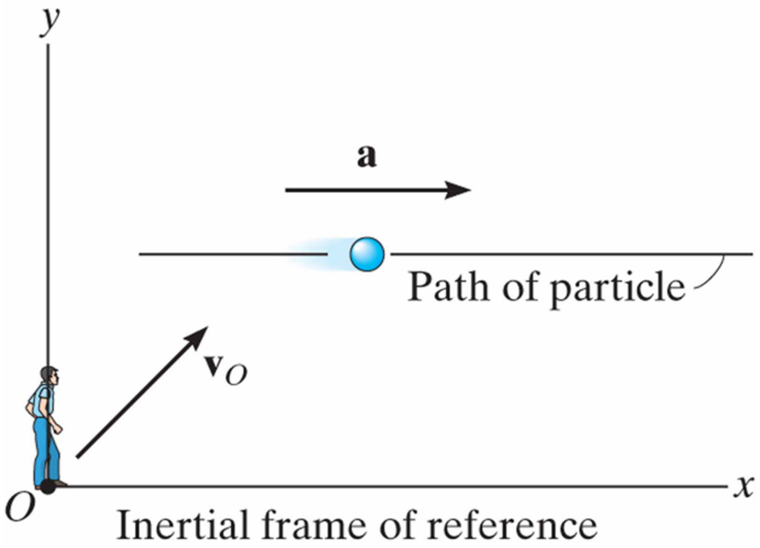
First, draw the particle's free-body diagram, showing all forces acting on the particle.

Next, draw the kinetic diagram, showing the inertial force m**a** acting in the same direction as the resultant force $\boldsymbol{F}_R$.

FORCE AND ACCELERATION

INERTIAL FRAME OF REFERENCE

This equation of motion is only valid if the acceleration is measured in a Newtonian or inertial frame of reference. **What does this mean?**



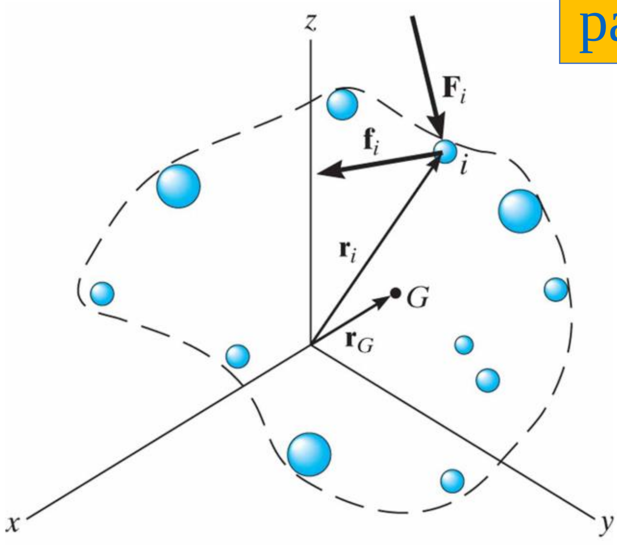
For problems concerned with motions **at or near the earth's surface**, we typically assume our "inertial frame" to be fixed to the earth. We neglect any acceleration effects from the earth's rotation.

For problems involving **satellites or rockets**, the inertial frame of reference is often fixed to the stars.

FORCE AND ACCELERATION

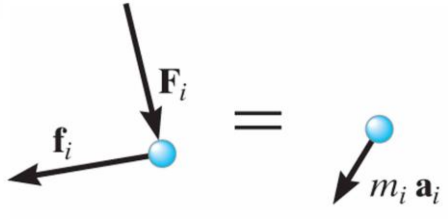
EQUATION OF MOTION FOR A SYSTEM OF PARTICLES

The equation of motion **can be extended** to include systems of particles. This includes the motion of solids, liquids, or gas systems.



Inertial coordinate system

As in statics, there are internal forces and external forces acting on the system. What is the difference between them?



Free-body diagram

Kinetic diagram

Using the definitions of $m = \sum m_i$ as the total mass of all particles and $\boldsymbol{a}_G$ as the acceleration of the center of mass G of the particles, then $m\boldsymbol{a}_G = \sum m_i \boldsymbol{a}_i$.

FORCE AND ACCELERATION

KEY POINTS

1. Newton's second law is a "**law of nature**"-- experimentally proven, not the result of an analytical proof.

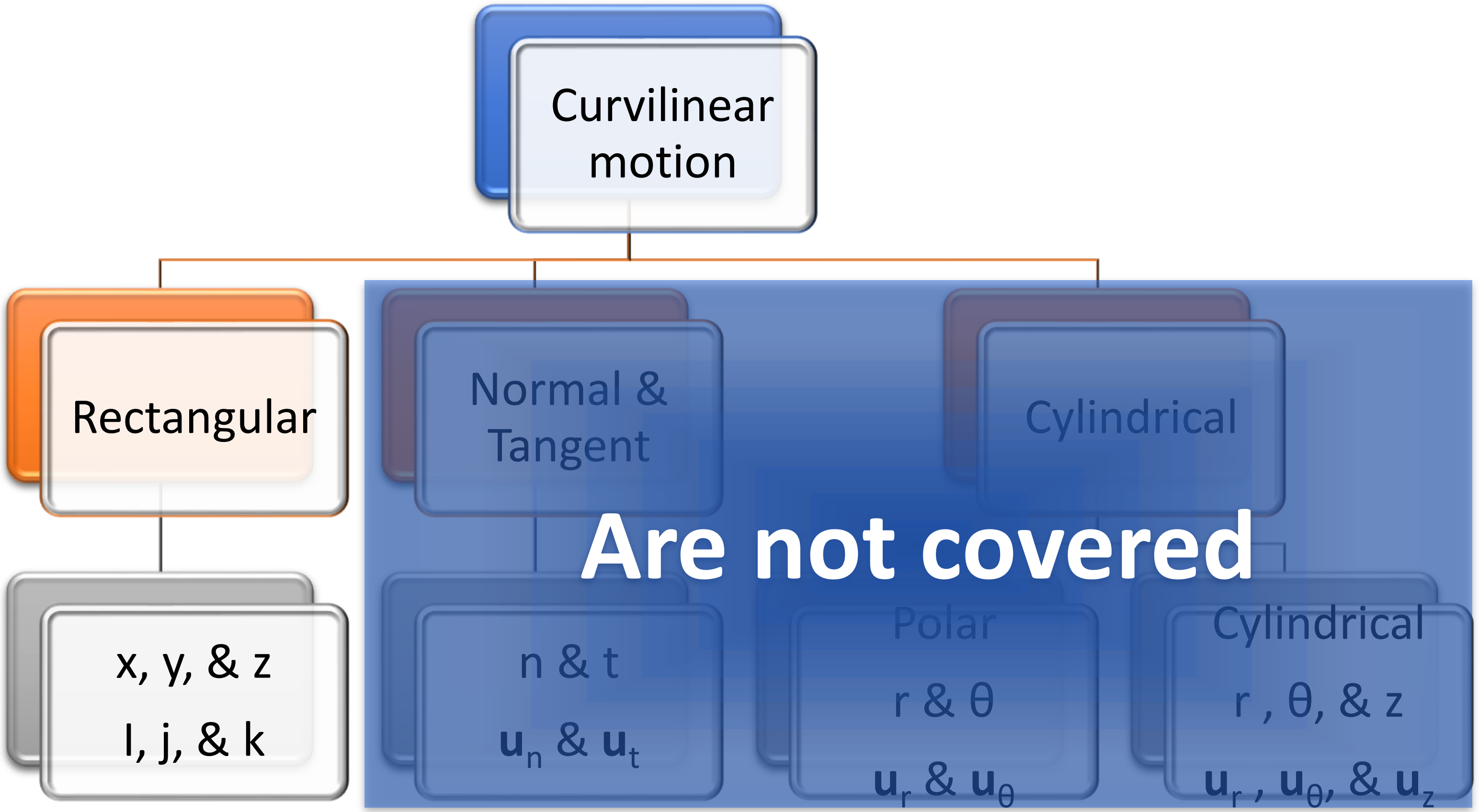
2. Mass (property of an object) is a **measure of the resistance** to a change in velocity of the object.

3. Weight (a force) **depends on the local gravitational field**. Calculating the weight of an object is an application of $F = ma$, i.e., $W = mg$.

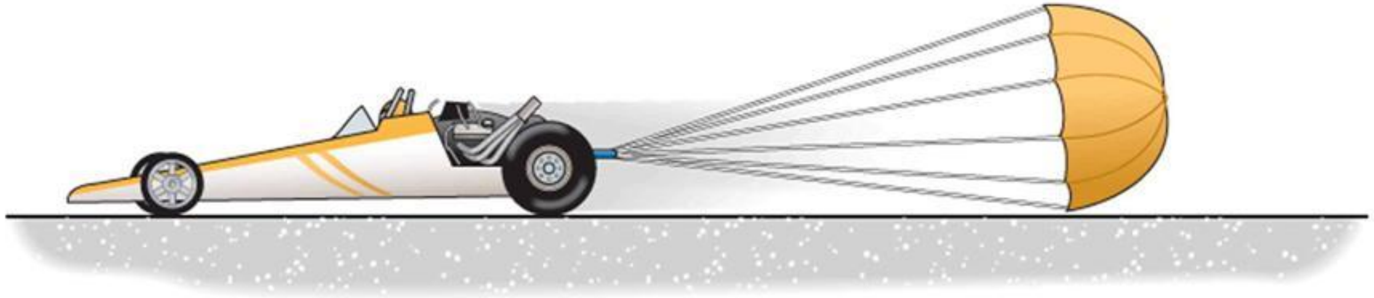
4. **Unbalanced forces** cause the acceleration of objects. This condition is fundamental to all dynamics problems!

FORCE AND ACCELERATION

EQUATION OF MOTION FOR A SYSTEM OF PARTICLES



EQUATION OF MOTION FOR A SYSTEM OF PARTICLES APPLICATION



Objects that move in air (or other fluid) have a drag force acting on them. This drag force is a function of velocity.

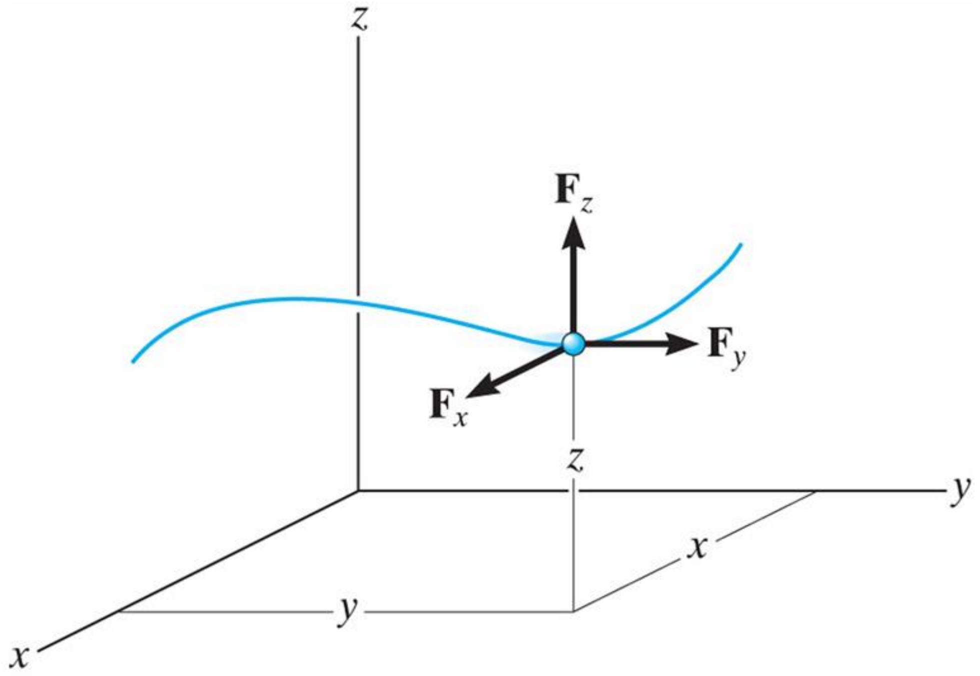
If the dragster is traveling with a known velocity and the magnitude of the opposing drag force at any instant is given as a function of velocity, can we determine the time and distance required for dragster to come to a stop if its engine is shut off? How ?

FORCE AND ACCELERATION

RECTANGULAR COORDINATES

- The equation of motion, $F = ma$

- Three scalar equations can be written from this vector equation. The equation of motion, being a vector equation, may be expressed in terms of its three components in the Cartesian (rectangular) coordinate system as



$$\sum \bar{F} = m\bar{a}$$

$$\sum F_x \boldsymbol{i} + \sum F_y \boldsymbol{j} + \sum F_z \boldsymbol{k} = m(a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k})$$

or, as **scalar equations**, $\sum F_x = ma_x$, $\sum F_y = ma_y$, and $\sum F_z = ma_z$.

FORCE AND ACCELERATION

PROCEDURE FOR ANALYSIS (FREE BODY DIAGRAM)

1. Select the inertial coordinate system. Most often, rectangular or x, y, z coordinates are chosen to analyze problems from which the particle has rectilinear motion.
2. Once the coordinate has been established, draw the particle's free-body diagram. Drawing this diagram is very important since it provides a graphical representation that accounts for all the forces $(\sum \boldsymbol{F})$ which act on the particle, and thereby makes it possible to resolve these forces into their x, y, z components.
3. The direction and sense of the particle's acceleration $\boldsymbol{a}$ should also be established. If the sense is unknown, for mathematical convenience assume that the sense of each acceleration component acts in the same direction as its positive inertial coordinate axis.
4. The acceleration may be represented as the $m\boldsymbol{a}$ vector on the kinetic diagram.
5. Identify the unknowns in the problem

FORCE AND ACCELERATION

PROCEDURE FOR ANALYSIS (EQUATIONS OF MOTION)

1. If the forces can be resolved directly from the free-body diagram, apply the equations of motion in their scalar component form.

2. If the geometry of the problem appears complicated, which often occurs in three dimensions, Cartesian vector analysis can be used for the solution.

Friction : If a moving particle contacts a rough surface, it may be necessary to use the frictional equation, which relates the frictional and normal forces F_f and N acting at the surface of contact by the using coefficient of kinetic friction, $F_f = \mu_k N$. Remember that F_f always acts on the free-body diagram such that it opposes the motion of the particle relative to the surface it contacts. If the particle is on the verge of relative motion, then the coefficient of static friction should be used.

Spring : If the particle is connected to an elastic spring having negligible mass, the spring force F_s can be related to the deformation of the spring by the equation $F_s = ks$. Here k is the spring's stiffness measured as a force per unit length, and s is the stretch or compression defined as the difference between the deformed length l and the undeformed length l_0, $s = l - l_0$.

FORCE AND ACCELERATION

PROCEDURE FOR ANALYSIS (KINEMATICS)

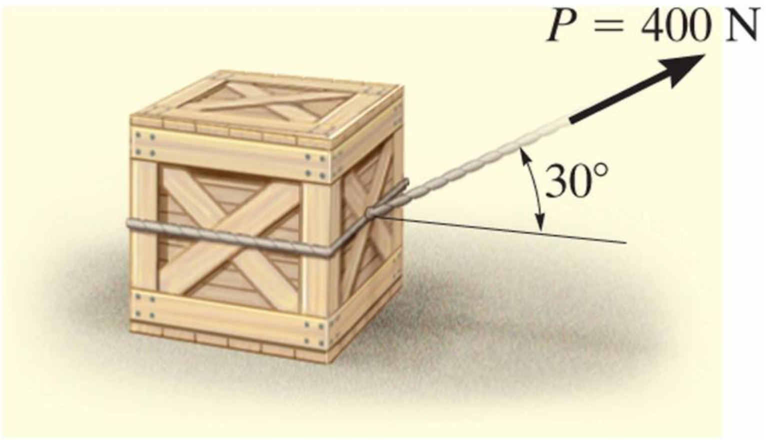
1. If the velocity or position of the particle is to be found, it will be necessary to apply the necessary kinematic equations once the particle's acceleration is determined from $\sum F = ma$.
2. If acceleration is a function of time, use $a = dv/dt$ and $v = ds/dt$ which, when integrated, yield the particle's velocity and position, respectively.
3. If acceleration is a function of displacement, integrate $a\,ds = v\,dv$ to obtain the velocity as a function of position.
4. If acceleration is constant, use $v = v_0 + a_c t$, $s = s_0 + v_0 t + \frac{1}{2} a_c t^2, v^2 = v_0^2 + 2a_c(s - s_0)$ to determine the velocity or position of the particle.

FORCE AND ACCELERATION

CONTINUOUS MOTION

Example 1

The 50-kg crate shown rests on a horizontal surface for which the coefficient of kinetic friction $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from the rest.

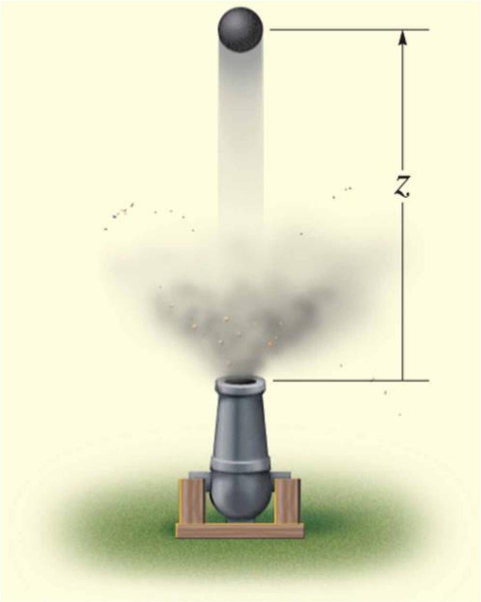


FORCE AND ACCELERATION

CONTINUOUS MOTION

Example 2

A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s. Determine the maximum height to which it will travel if (a) atmospheric resistance is neglected; and (b) atmospheric resistance is measured as $F_D = (0.01v^2)$ N, where v is the speed of the projectile at any instant, measured in m/s.

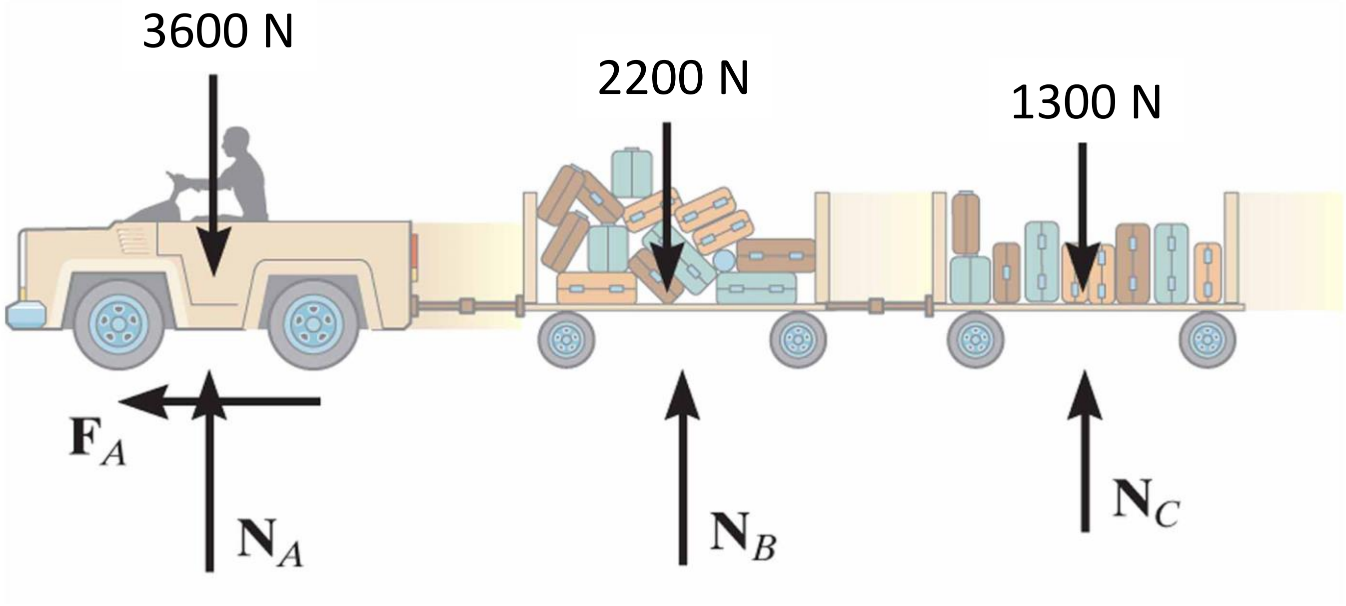


FORCE AND ACCELERATION

CONTINUOUS MOTION

Example 3

A baggage truck A shown in the photo has a weight of 3600-N and tows a 2200-N cart B and a 1300-N cart C. For a short time, the driving frictional force developed at the wheels of the truck is $F_A = (160t)$ N, where t is in seconds. If the truck starts from rest, determine its speed in 2 seconds. Also, what is the horizontal force acting on the coupling between the truck and cart B at this instant? Neglect the size of the truck and carts.

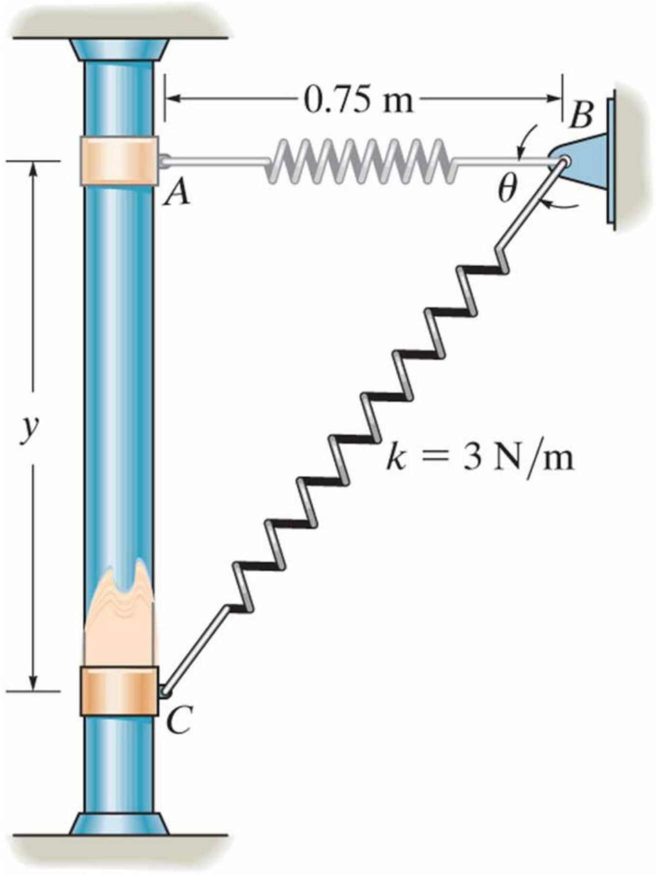


FORCE AND ACCELERATION

CONTINUOUS MOTION

Example 4

A smooth 2-kg collar C, is attached to a spring having a stiffness $k = 3$ N/m and an unstretched length of 0.75 m. If the collar is released from rest at A, determine its acceleration and the normal force of the rod on the collar at the instant $y = 1$ m.



WORK AND ENERGY

OBJECTIVES

At the end of this lesson, you should be able to:

1. Calculate the work of a force.
2. Apply the principle of work and energy to a particle or system of particles.
3. Determine the power generated by a machine, engine, or motor.
4. Calculate the mechanical efficiency of a machine.

WORK AND ENERGY

APPLICATIONS

Crash barrels are often used along roadways for crash protection.

The barrels absorb the car's kinetic energy by deforming.

If we know the velocity of an oncoming car and the amount of energy that can be absorbed by each barrel, how can we design a crash cushion?

WORK AND ENERGY

INTRODUCTION

- Another equation for working kinetics problems involving particles can be derived by integrating the equation of motion $(F = ma)$ with respect to displacement.

- By substituting $a_t = v\left(\frac{dv}{ds}\right)$ into $F_t = ma_t$, the result is integrated to yield an equation known as the principle of work and energy.

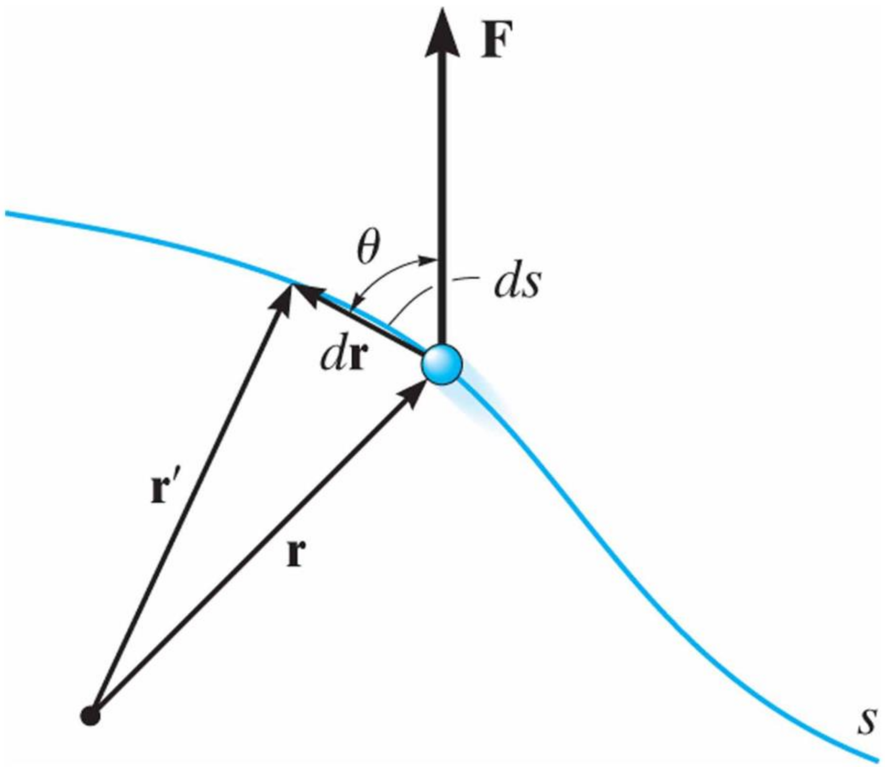
- This principle is useful for solving problems that involve force, velocity, and displacement. It can also be used to explore the concept of power.

- To use this principle, we must first understand how to calculate the work of a force.

WORK AND ENERGY

WORK OF A FORCE

- A force does work on a particle when the particle undergoes a displacement along the line of action of the force.



- Work is defined as the product of force (F) and displacement components (ds) acting in the same direction.
- So, if the angle between the force and displacement vector is θ, the increment of work dU done by the force is:

$$dU = F ds \cos\theta$$

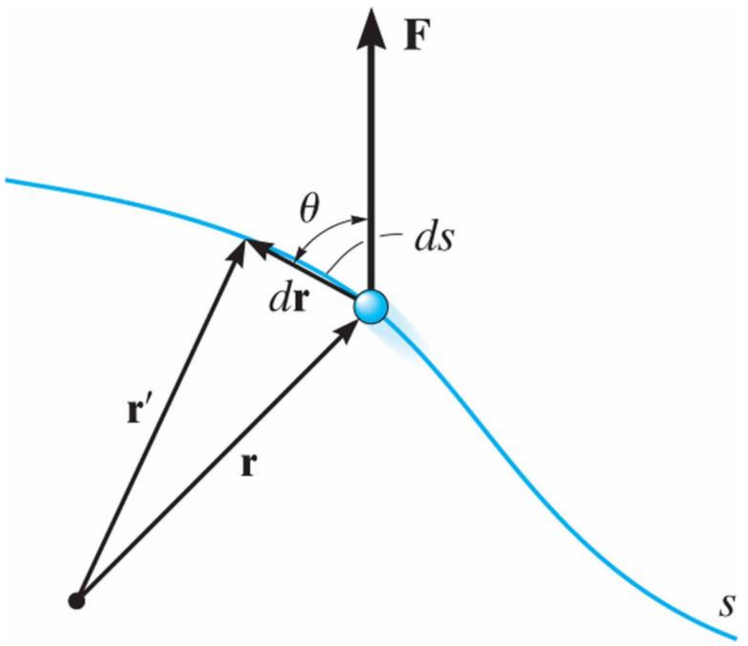
By using the definition of the dot product and integrating, the total work can be written as:

$$U_{1-2} = \int_{r_1}^{r_2} F \cdot dr$$

WORK OF A VARIABLE FORCE

- If F is a function of position (a common case) this becomes

$$U_{1-2} = \int_{S_1}^{S_2} F\cos\theta \; ds$$



- If both F and θ are constant ($F = F_c$), this equation further simplifies to

$$U_{1-2} = F_c\cos\theta\,(s_2 - s_1)$$

Work is:

- **Positive** if the force and the movement are in the same direction.
- If they are opposing, then the work is **negative**.
- If the force and the displacement directions are perpendicular, the work is **zero**.

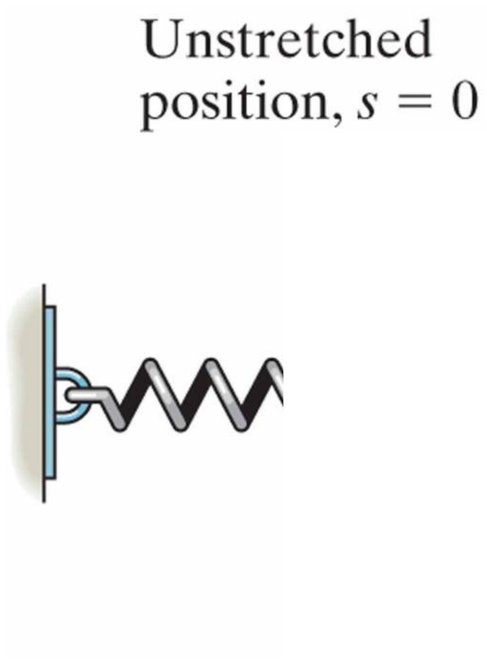
WORK OF A WEIGHT

- The work done by the gravitational force acting on a particle (or weight of an object) can be calculated by using

$$U_{1-2} = \int_{Z_1}^{Z_2} -W\,dz = -W(z_2 - z_1) = -W\Delta z$$

- The work of a weight is the product of the magnitude of the particle's weight and its vertical displacement.
- If Δz is upward, the work is negative since the weight force always acts downward.

WORK OF A SPRING FORCE

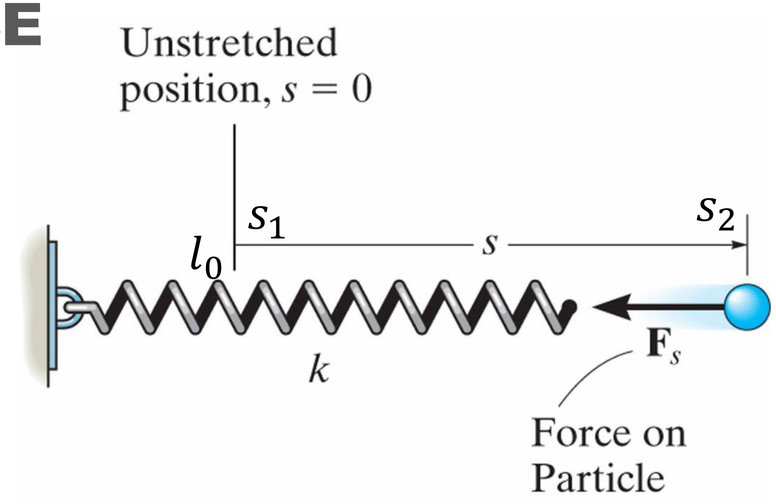


- When stretched, a linear elastic spring develops a force of magnitude

$$F_s = ks$$

where k is the spring stiffness and s is the displacement from the unstretched position.

WORK OF A SPRING FORCE



If $l_o = 0$, The work of the spring force moving from position s_1 to position s_2 is

$$U_{1-2} = \int_{s_1}^{s_2} F_s ds = \int_{s_1}^{s_2} -ksds = -0.5k[(s_2)^2 - (s_1)^2]$$

If a particle is attached to the spring, the force F_s exerted on the particle is opposite to that exerted on the spring. Thus, the work done on the particle by the spring force will be negative or

$$U_{1-2} = -\ 0.5\,k[(s_2)^2 - (s_1)^2]\ .$$

SPRING FORCES

- It is important to note the following about spring forces.

   1. The equations above are for **linear** springs only! Recall that a linear spring develops a force according to $\boldsymbol{F} = \boldsymbol{ks}$ (essentially the equation of a line).

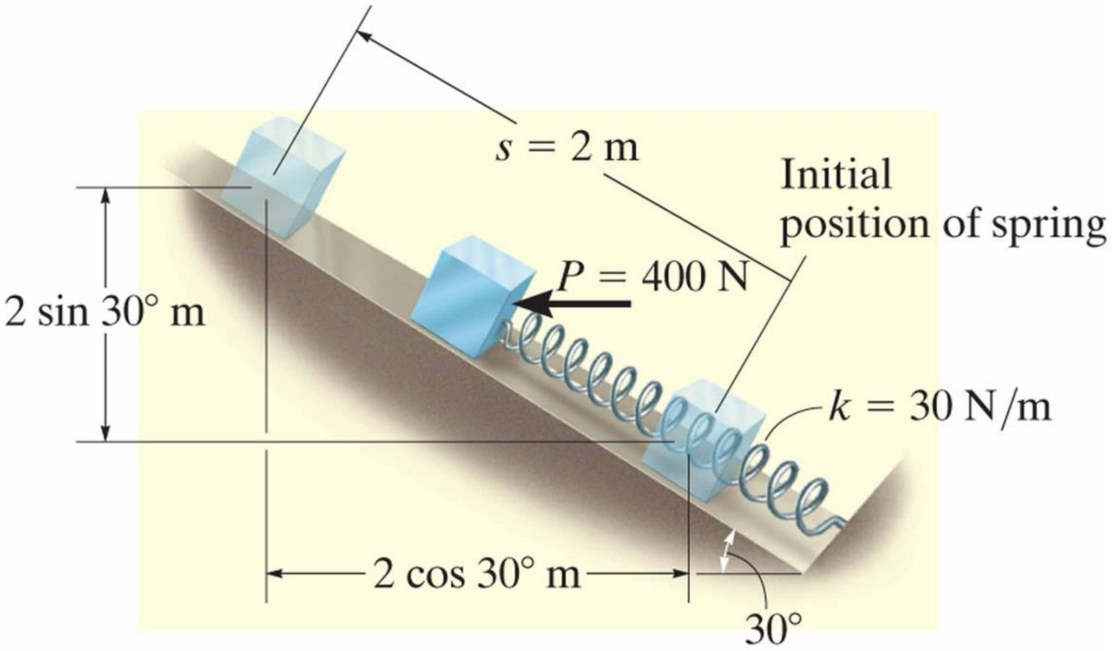
   2. The work of a spring is **not** just spring force times distance at some point, i.e., $(ks_i)(s_i)$. **Beware**, this is a trap that students often fall into!

   3. Always **double check** the sign of the spring work after calculating it. It is positive work if the force put on the object by the spring and the movement are in the same direction.

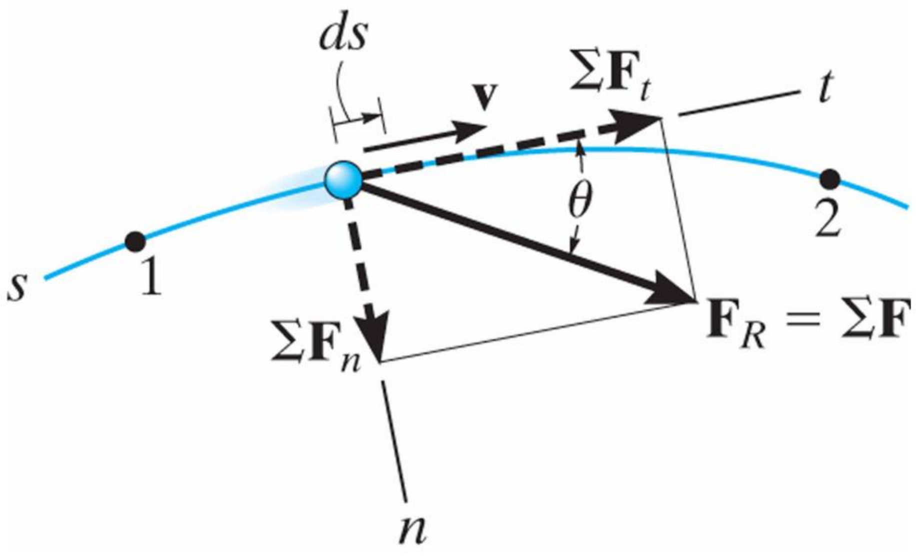
SPRING FORCES

Example 5

The 10-kg block shown in the figure rests on the smooth incline. If the spring is originally stretched at 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.



PRINCIPLE OF WORK AND ENERGY



By integrating the equation of motion, $\sum F_t = ma_t = mv(dv/ds)$, thus

$$\sum \int_{s_1}^{s_2} F_t \, ds = \int_{v_1}^{v_2} mv \, dv$$

$$\sum \int_{s_1}^{s_2} F_t \, ds = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\sum U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

WORK AND ENERGY

PRINCIPLE OF WORK AND ENERGY

The principle of work and energy can be written as

$$\sum U_{1-2} = 0.5\, m\, (v_2)^2 - 0.5\, m\, (v_1)^2 \quad or \quad T_1 + \sum U_{1-2} = T_2$$

- $\sum U_{1-2}$ is the **work done by all the forces** acting on the particle as it moves from point 1 to point 2. Work can be either a **positive or negative** scalar.

T_1 and T_2 are the kinetic energies of the particle at the initial and final position, respectively. Thus, $T_1 = 0.5\, m\, (v_1)^2$ and $T_2 = 0.5\, m\, (v_2)^2$. The kinetic energy is always a positive scalar (velocity is squared!).

- So, the particle’s initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to final position is equal to the particle’s final kinetic energy.

PRINCIPLE OF WORK AND ENERGY

- Note that the principle of work and energy $(T_1 + \sum U_{1-2} = T_2)$ is **not a vector equation**! Each term results in a scalar value.

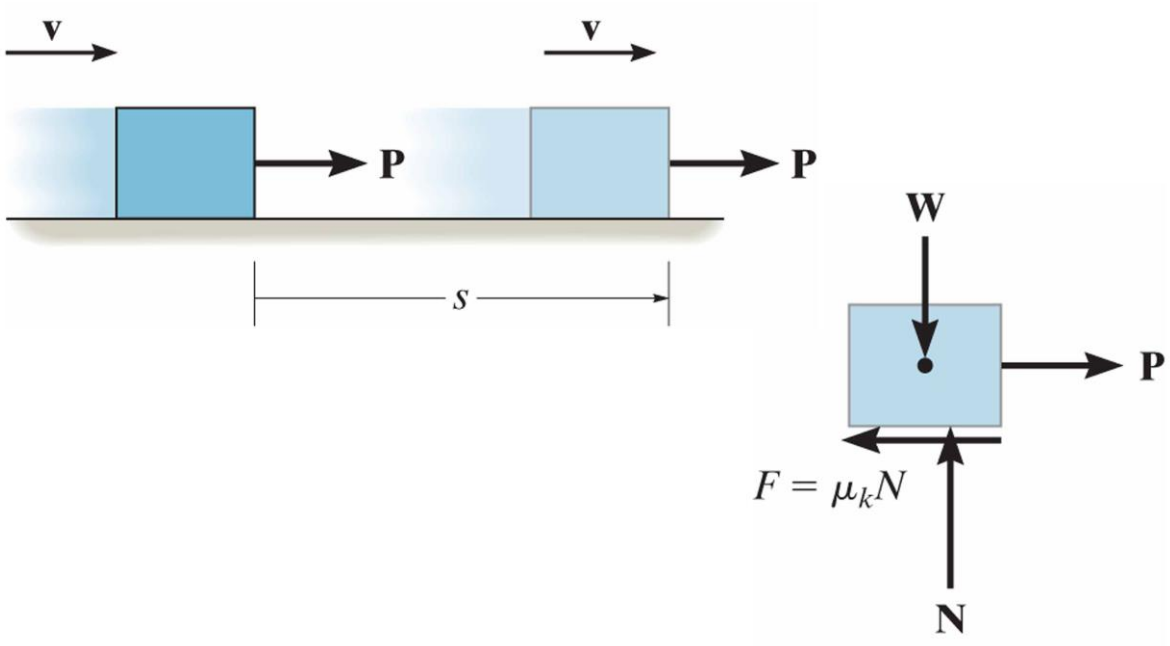
- Both kinetic energy and work have the same units, that of energy! In the SI system, the unit for energy is called a **joule** (J), where 1 J = 1 N·m. In the FPS system, units are ft·lb.

- The principle of work and energy **cannot** be used, in general, to determine forces directed **normal** to the path, since these forces do no work.

- The principle of work and energy can also be applied to a **system of particles** by summing the kinetic energies of all particles in the system and the work due to all forces acting on the system.

WORK OF FRICTION CAUSED BY SLIDING

- The case of a body sliding over a **rough surface** merits special consideration.



Consider a block which is moving over a rough surface. If the applied force ***P*** just balances the resultant **frictional force** μ_k N, a constant velocity v would be maintained.

The principle of work and energy would be applied as:

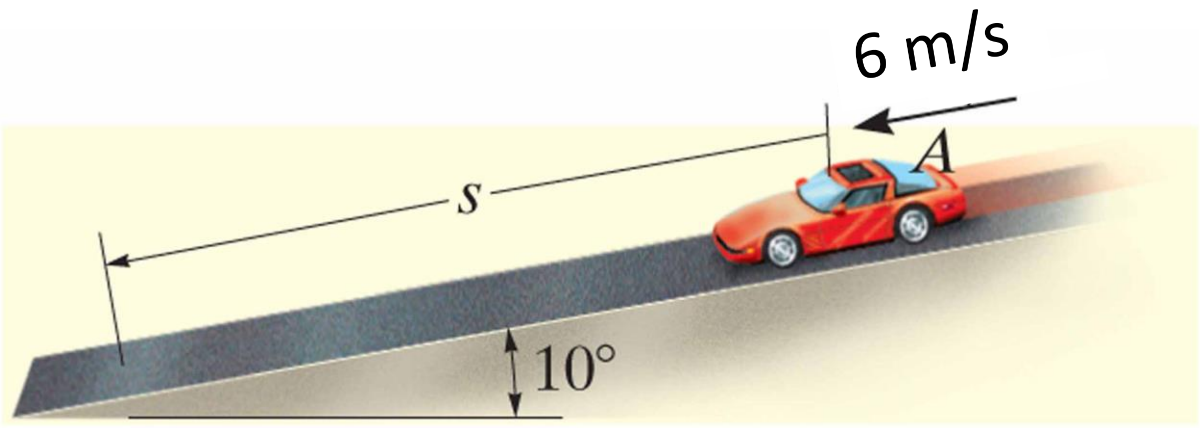
$$0.5m\,(v)^2 + Ps - (\mu_k\,N)\,s = 0.5m\,(v)^2$$

This equation is satisfied if $P = \mu_k$ N. However, we know from experience that friction generates **heat**, a form of energy that does not seem to be accounted for in this equation. It can be shown that the work term (μ_k N)s represents **both** the **external work** of the friction force and the **internal work** that is converted into heat.

SPRING FORCES

Example 6

The 17.5 kN car shown in the figure travels down the 10° inclined road at a speed of 6 m/s. If the driver jams on the brakes, causing his wheels to lock, determine how far s the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.



WORK AND ENERGY

SPRING FORCES

Example 7

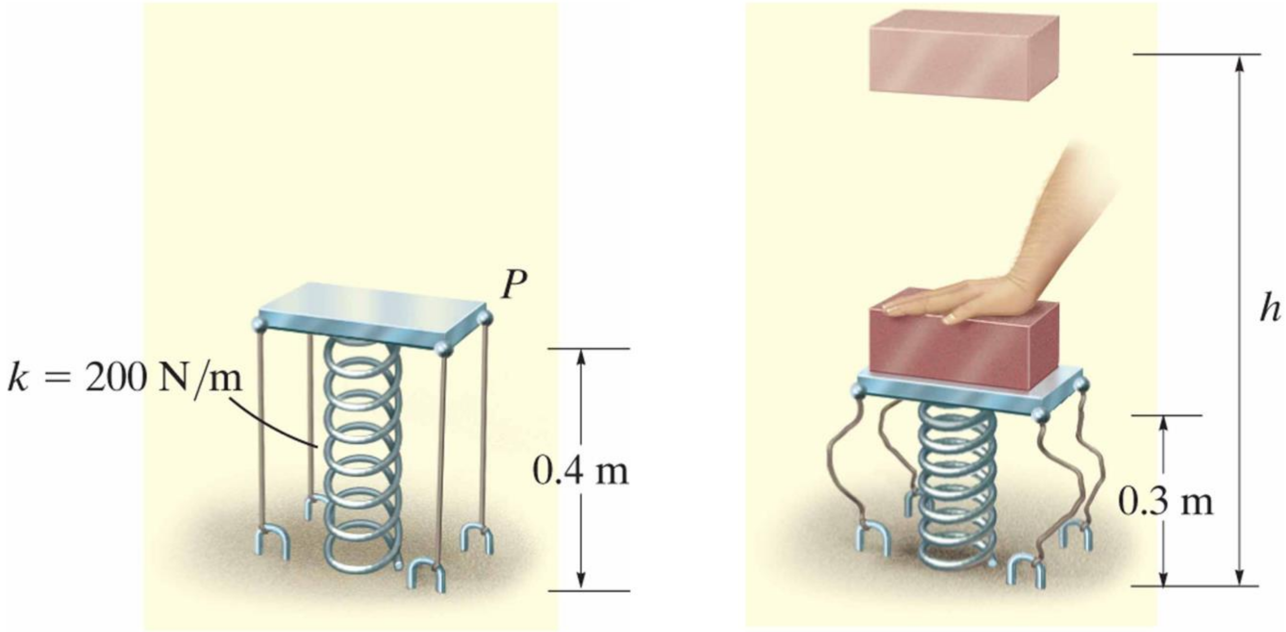
For a short time the crane in the figure lifts the 2.50-Mg beam with a force of $F = (28 + 3s)^2$ kN. Determine the speed of the beam when it has risen $s = 3$ m. Also, how much time does it take to attain this height starting from rest?

WORK AND ENERGY

SPRING FORCES

Example 8

The platform P, shown in the figure, has a negligible mass and is tied down so that the 0.4 m-long cords keep a 1-m-long spring compressed 0.6 m when nothing is on the platform. If a 2-kg block is placed on the platform and released from the rest after the platform is pushed down 0.1 m, determine the maximum height h the block rises in the air, measured from the ground.



WORK AND ENERGY

POWER AND EFFICIENCY (APPLICATION)

Engines and motors are often rated in terms of their power output. The power output of the motor lifting this elevator is related to the vertical force ***F*** acting on the elevator, causing it to move upwards.

Given a desired lift velocity for the elevator (with a known maximum load), how can we determine the power requirement of the motor?

POWER AND EFFICIENCY

- Power is defined as the amount of work performed per unit of time.
- If a machine or engine performs a certain amount of work, dU, within a given time interval, dt, the power generated can be calculated as:

$$P = dU/dt$$

- Since the work can be expressed as $dU = F \cdot dr$, the power can be written:

$$P = \frac{dU}{dt} = \frac{F \cdot dr}{dt} = F \cdot \left(\frac{dr}{dt}\right) = F \cdot v$$

- Thus, power is a scalar defined as the product of the force and velocity components acting in the same direction.

WORK AND ENERGY

POWER

- Using scalar notation, power can be written

$$P = F \cdot v = Fv\, cos\theta$$

  where θ is the angle between the force and velocity vectors.

- So if the velocity of a body acted on by a force F is known, the power can be determined by calculating the dot product or by multiplying force and velocity components.

- The unit of power in the SI system is the Watt (W) where:

$$1\,W = 1\,J/s = 1\,(N \cdot m)/s$$

- In the FPS system, power is usually expressed in units of horsepower (hp) where

$$1\,hp = 550\,(ft \cdot lb)/s = 746\,W\,.$$

WORK AND ENERGY

EFFICIENCY

- The **mechanical efficiency** of a machine is the ratio of the useful power produced (**output power**) to the power supplied to the machine (**input power**) or

$$\varepsilon = (power\ output)/(power\ input)$$

- If energy input and removal occur at the same time, efficiency may also be expressed in terms of the ratio of **output energy** to **input energy** or

$$\varepsilon = (energy\ output)/(energy\ input)$$

- Machines will always have frictional forces. Since frictional forces **dissipate** energy, additional power will be required to overcome these forces. Consequently, the efficiency of a machine is always **less than 1**.

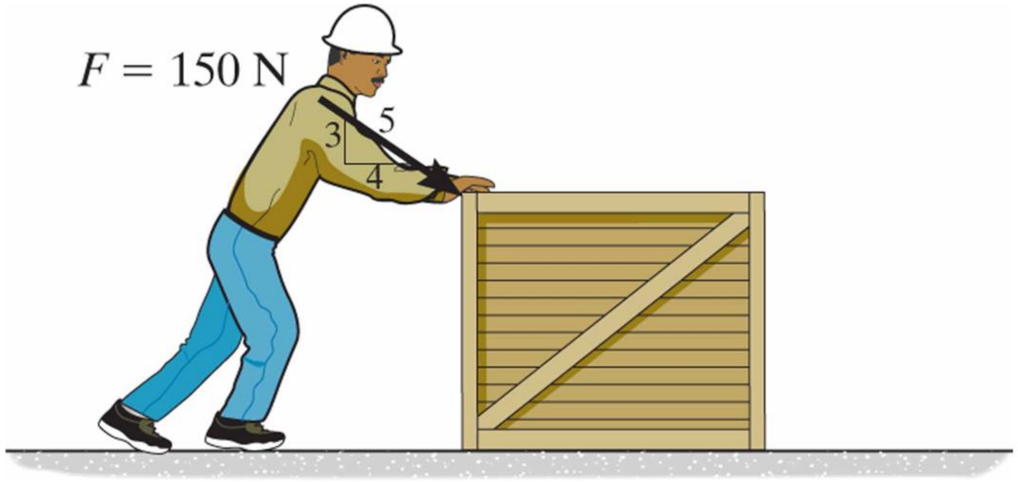
PROCEDURE FOR ANALYSIS

- Find the **resultant external force** acting on the body causing its motion. It may be necessary to draw a free-body diagram.
- Determine the **velocity** of the **point** on the body **at which the force is applied**. Energy methods or the equation of motion and appropriate kinematic relations, may be necessary.
- Multiply the **force magnitude** by the component of **velocity** acting **in the direction of *F*** to determine the power supplied to the body $(P = F\, v \cos\theta)$.
- In some cases, **power** may be found by calculating the **work done per unit of time** $(P = dU/dt)$.
- If the **mechanical efficiency** of a machine is known, either the power input or output can be determined.

SPRING FORCES

Example 9

The man in the figure pushes on the 50-kg crate with a force of $F = 150$ N. Determine the power supplied by the man when $t = 4$ s. The coefficient of kinetic friction between the floor and the crate is $\mu_k = 0.2$. Initially the crate is at rest.



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