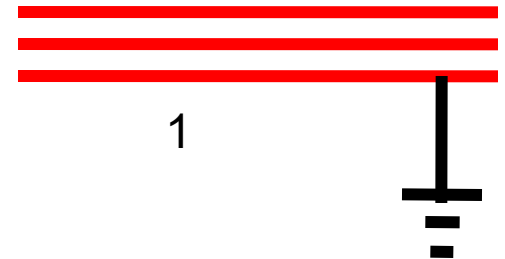
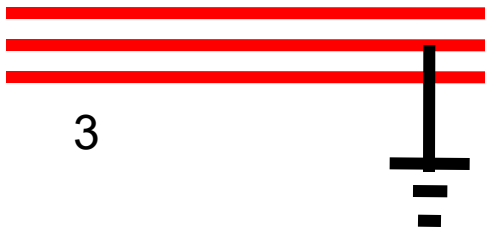


**SKEE 4443**  
**POWER SYSTEM ANALYSIS**

**CHAPTER 4 Cont'd**  
**Unsymmetrical Components**

# Unsymmetrical Components

- Most faults are unsymmetrical faults, which result in unbalanced currents.
- Require symmetrical components to solve for the voltages and currents during the faults.
- Three major types are:
  1. Single line-to-ground
  2. Line-to-line
  3. Double-line-to-ground



$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}}_A \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = A \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

**The symmetrical components of the unbalanced three-phase voltage can be expressed as**

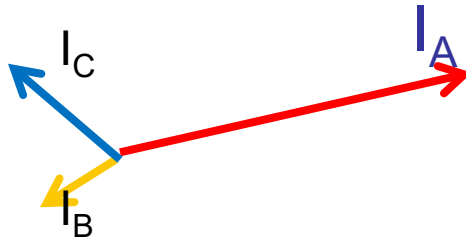
$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = A^{-1} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \quad \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

# Symmetrical components of an unbalanced three-phase current

$$I_C = I_{C1} + I_{C2} + I_{C0}$$

$$I_C = a I_1 + a^2 I_2 + I_0$$



$$I_A = I_{A1} + I_{A2} + I_{A0}$$

$$I_A = I_1 + I_2 + I_0$$

$$I_B = I_{B1} + I_{B2} + I_{B0}$$

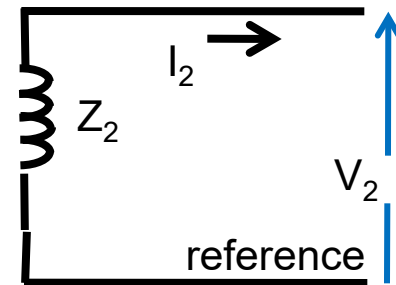
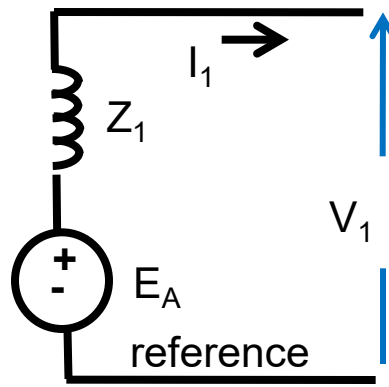
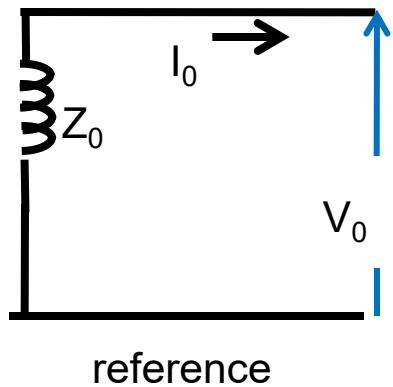
$$I_B = a^2 I_1 + a I_2 + I_0$$

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

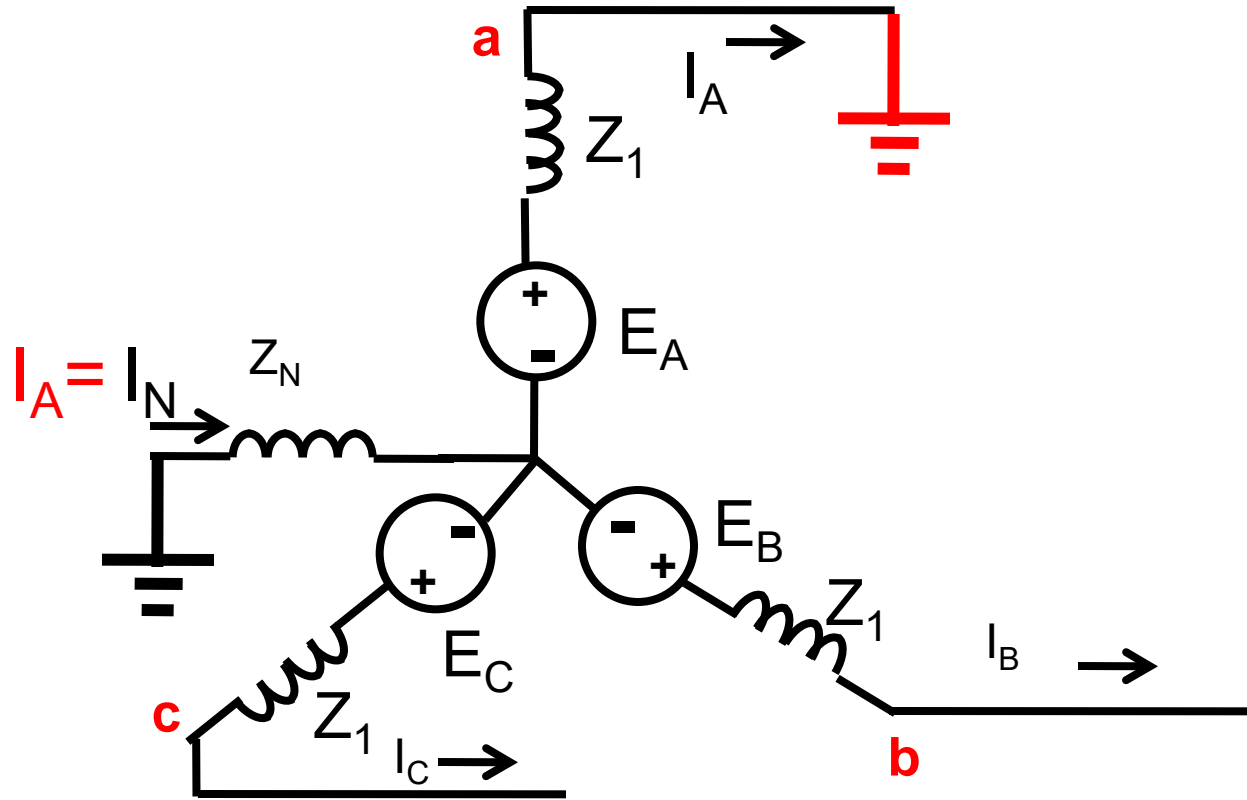
The following equations will be used in the analysis:

$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_A \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix}$$



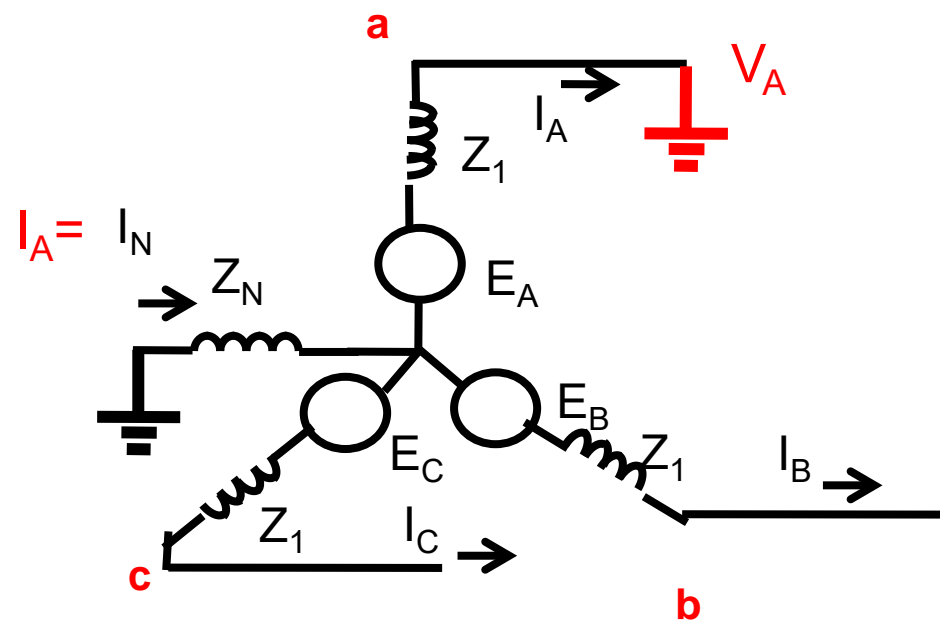
Thevenin equivalents of the sequence networks as seen from the fault point F

# Unloaded generator with single line-to-ground fault



Unloaded generator grounded through a reactor (impedance  $Z_N$ )

When the fault occur



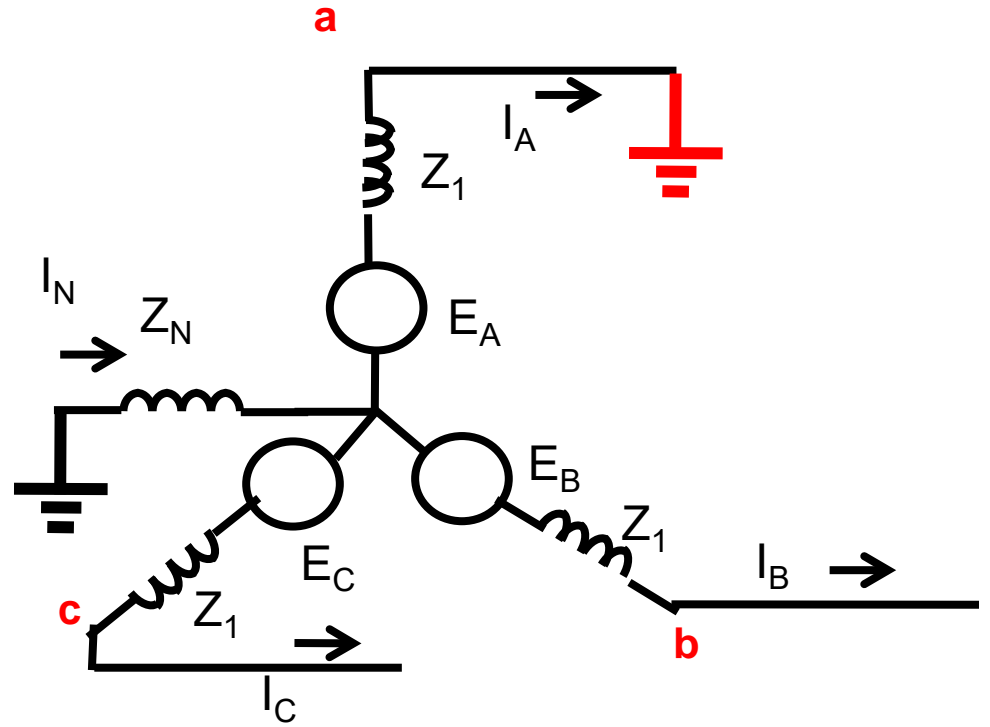
$$I_B = I_C = 0 \qquad V_A = 0$$

Symmetrical components of the fault currents can be found from

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ 0 \\ 0 \end{bmatrix}$$

$$I_0 = I_1 = I_2 = I_A / 3$$



$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_A \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 = I_1 \\ I_1 \\ I_2 = I_1 \end{bmatrix}$$



Hence:

$$\begin{aligned} V_0 &= -\mathbf{I}_1 \mathbf{Z}_0 \\ V_1 &= \mathbf{E}_A - \mathbf{I}_1 \mathbf{Z}_1 \\ V_2 &= -\mathbf{I}_1 \mathbf{Z}_2 \end{aligned} \quad \begin{bmatrix} \mathbf{V}_0 \\ \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{E}_A \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_1 \\ \mathbf{I}_1 \end{bmatrix}$$

And

The diagram shows the derivation of the total voltage  $V_A$  as the sum of three components:  $V_1$ ,  $V_2$ , and  $V_0$ . The equation is written as  $V_A = V_1 + V_2 + V_0$  and  $= E_A - I_1 Z_1 - I_1 Z_2 - I_1 Z_0$ . The terms are highlighted with colored regions:  $V_1$  is in a yellow region with a red dashed border,  $V_2$  is in a green region with a blue solid border, and  $V_0$  is in an orange region with a green dotted border. The second line of the equation is also color-coded to match the regions above it.

$$\begin{aligned} V_A &= V_1 + V_2 + V_0 \\ &= E_A - I_1 Z_1 - I_1 Z_2 - I_1 Z_0 \end{aligned}$$

$$V_A = E_A - I_1 Z_1 - I_1 Z_2 - I_1 Z_0$$

However  $V_A = 0$  hence

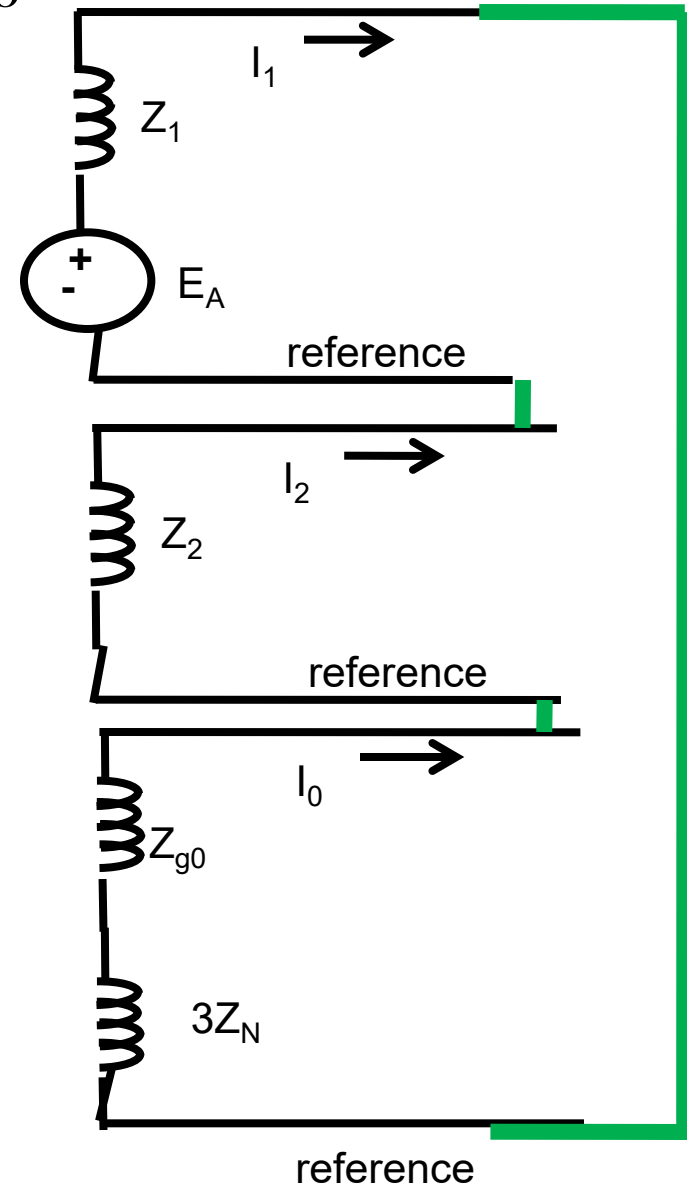
$$I_1 (Z_1 + Z_2 + Z_0) = E_A$$

$$I_1 = \frac{E_A}{(Z_1 + Z_2 + Z_0)}$$

So,

$$I_A = I_0 + I_1 + I_2 = 3 \times I_1$$

This is equivalent to connecting the three sequence networks in series



## Example

- A 150 MVA, 13.8 kV, Y connected synchronous generator has a synchronous, negative and zero sequence reactances 20%, 10%, and 5% respectively. The neutral of the generator is solidly grounded. Generator is connected to short transmission line with positive and negative sequence reactances of 10%, and zero sequence impedance of 30%. The generator was unloaded when a single-line-to-ground fault occurs on phase a at the end of the line. Find the fault currents and the voltage at fault point and at the generator terminals.

## Solution

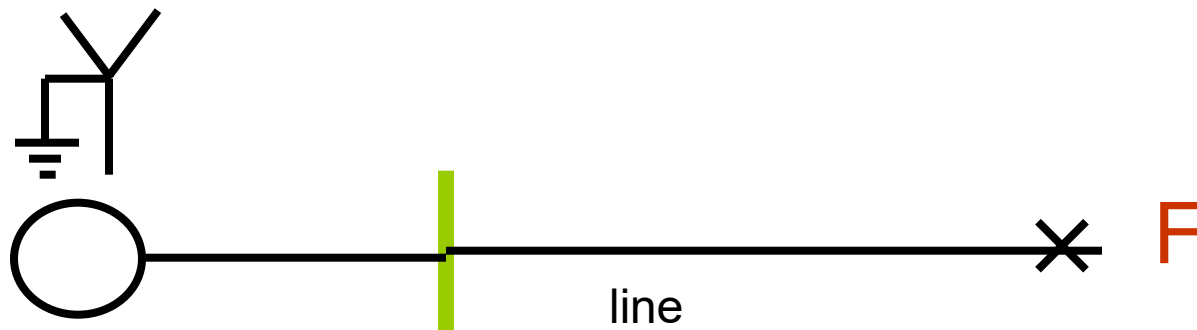
150 MVA,

13.8 kV

$X_s = 0.20$  pu

$X_2 = 0.10$  pu

$X_0 = 0.05$  pu

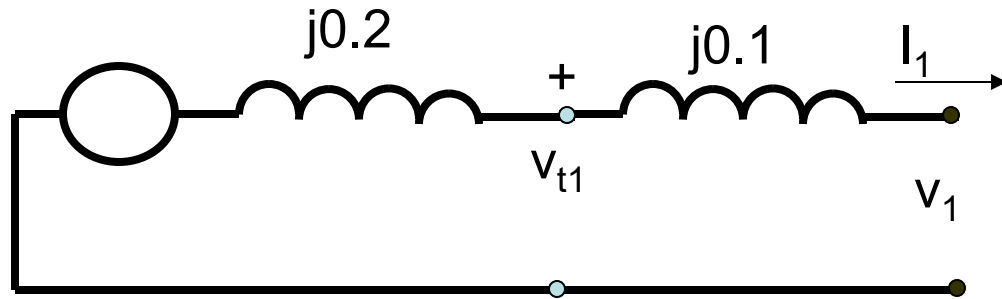


line  
 $X_1 = X_2 = 0.10$  pu

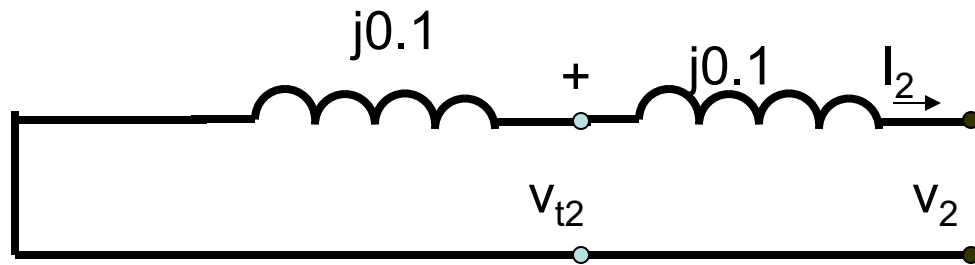
$X_0 = 0.30$  pu

One line diagram

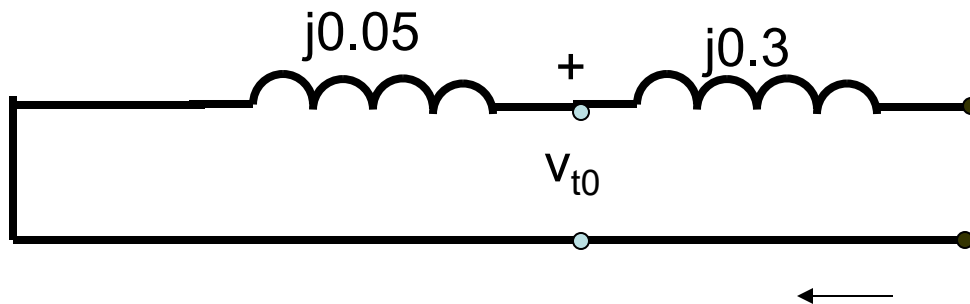
Draw positive sequence network



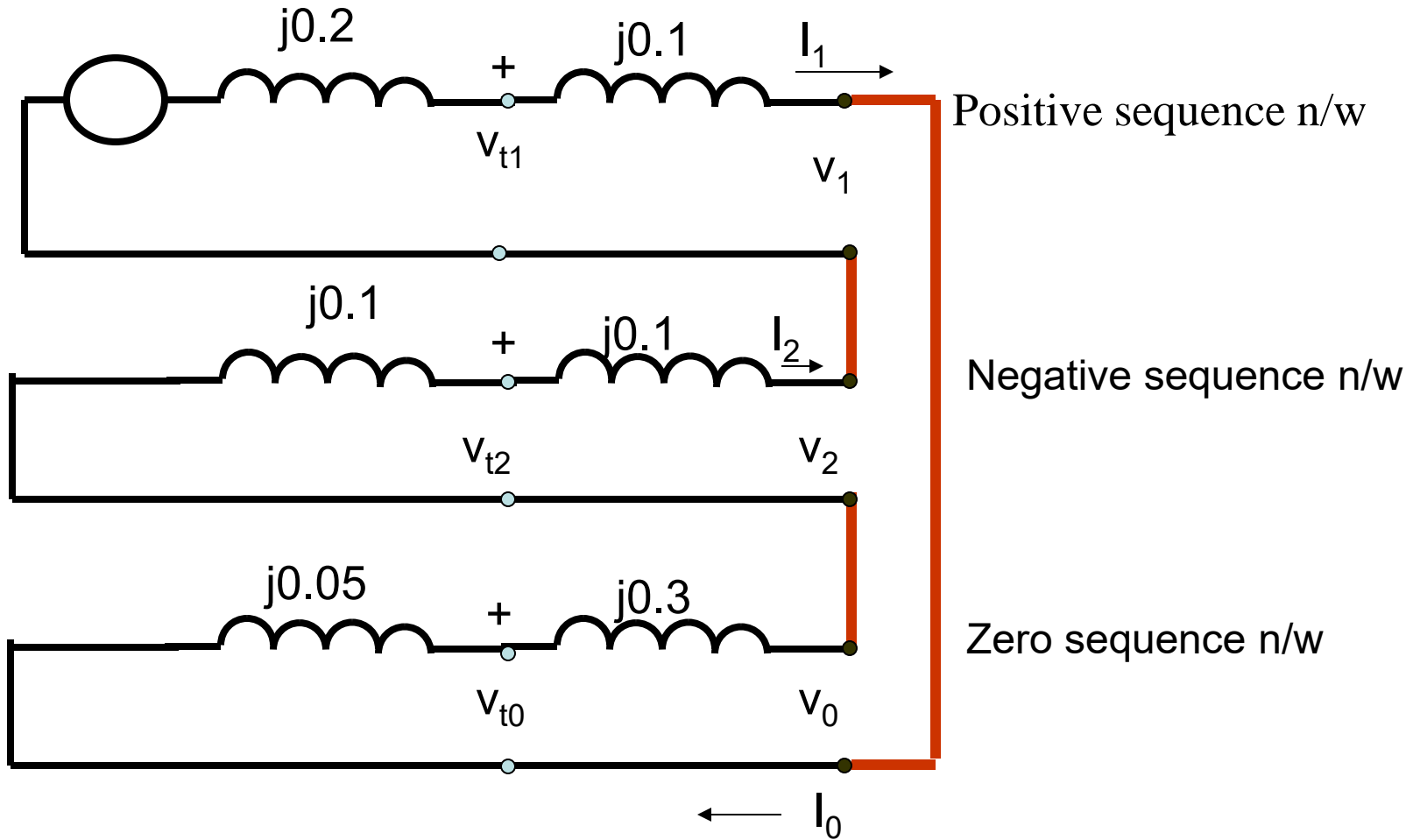
Draw negative sequence network



Draw zero sequence network



# Connection for single-line-to-ground fault



$$I_1 = \frac{E_A}{(Z_1 + Z_2 + Z_0)}$$

$$E_a = 1.0 \angle 0^\circ$$

$$\begin{aligned}
I_1 &= \frac{1\angle 0^\circ}{(Z_1 + Z_2 + Z_0)} \\
&= \frac{1\angle 0^\circ}{j(0.3 + 0.2 + 0.35)} \\
&= \frac{1\angle 0^\circ}{j(0.85)} \\
&= 1.176\angle -90^\circ \\
&= -j1.176
\end{aligned}$$

$$\begin{aligned}
I_1 &= I_2 = I_0 \\
&= 1.176\angle -90^\circ
\end{aligned}$$

$$\begin{aligned}
I_a &= 3I_1 \\
&= 3 \times 1.176\angle -90^\circ \\
&= 3.528\angle -90^\circ \text{ pu}
\end{aligned}$$

## Phase currents

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1.176 \angle -90^\circ \\ 1.176 \angle -90^\circ \\ 1.176 \angle -90^\circ \end{bmatrix}$$
$$= \begin{bmatrix} 3.528 \angle -90^\circ \\ 0 \\ 0 \end{bmatrix} pu$$

$I_{\text{base}}$  at the fault

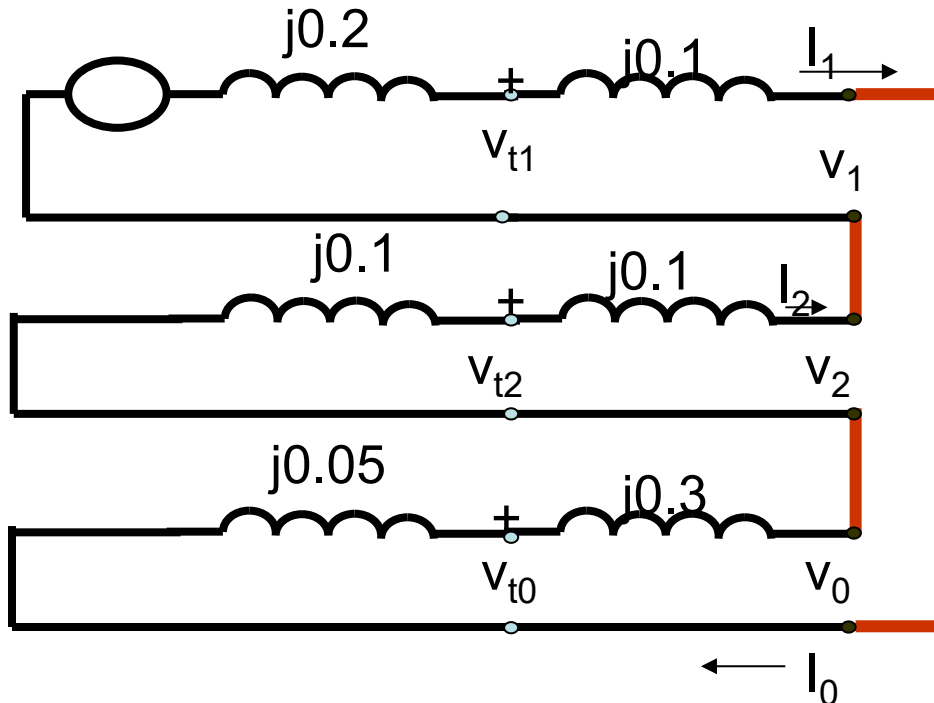
$$I_{\text{base}} = \frac{150,000}{13.8\sqrt{3}} = 6276 \quad \text{A}$$

$$I_a = 3.528 \times 6276 = 22,140 \quad \text{A}$$

$$I_b = 0 \quad \text{A}$$

$$I_c = 0 \quad \text{A}$$

Voltage at the terminals of the generator (symmetrical components)



$$\begin{aligned} V_{t0} &= -j0.05 I_0 \\ &= -j0.05 \times -j1.176 \\ &= -0.0588 \quad pu \end{aligned}$$



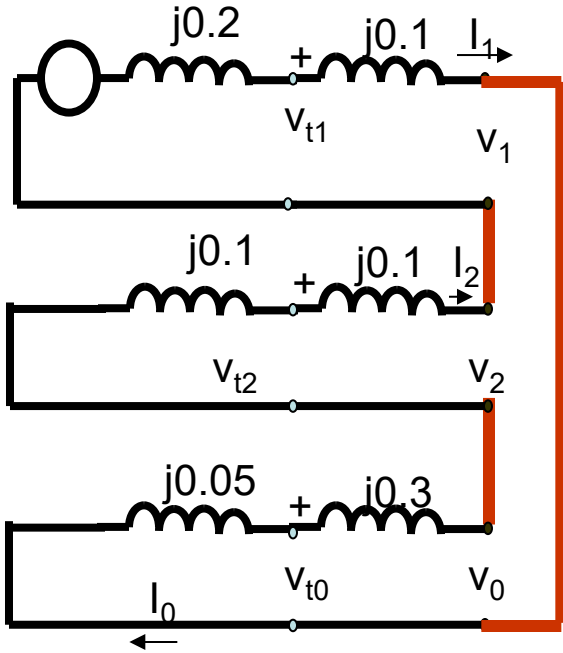
Voltage at the terminals of the generator

$$\begin{aligned}V_{t1} &= E_a - j0.20 \times I_1 \\ &= 1.0 \angle 0^\circ - (j0.20)(-j1.176) \\ &= 0.7648 \quad pu\end{aligned}$$

$$\begin{aligned}V_{t2} &= -j0.10 \times I_2 \\ &= -(j0.10)(-j1.176) \\ &= -0.1176 \quad pu\end{aligned}$$

$$\begin{aligned}V_{t0} &= -j0.05 I_0 \\ &= -j0.05 \times -j1.176 \\ &= -0.0588 \quad pu\end{aligned}$$

Phase voltage (line to ground) at the terminals

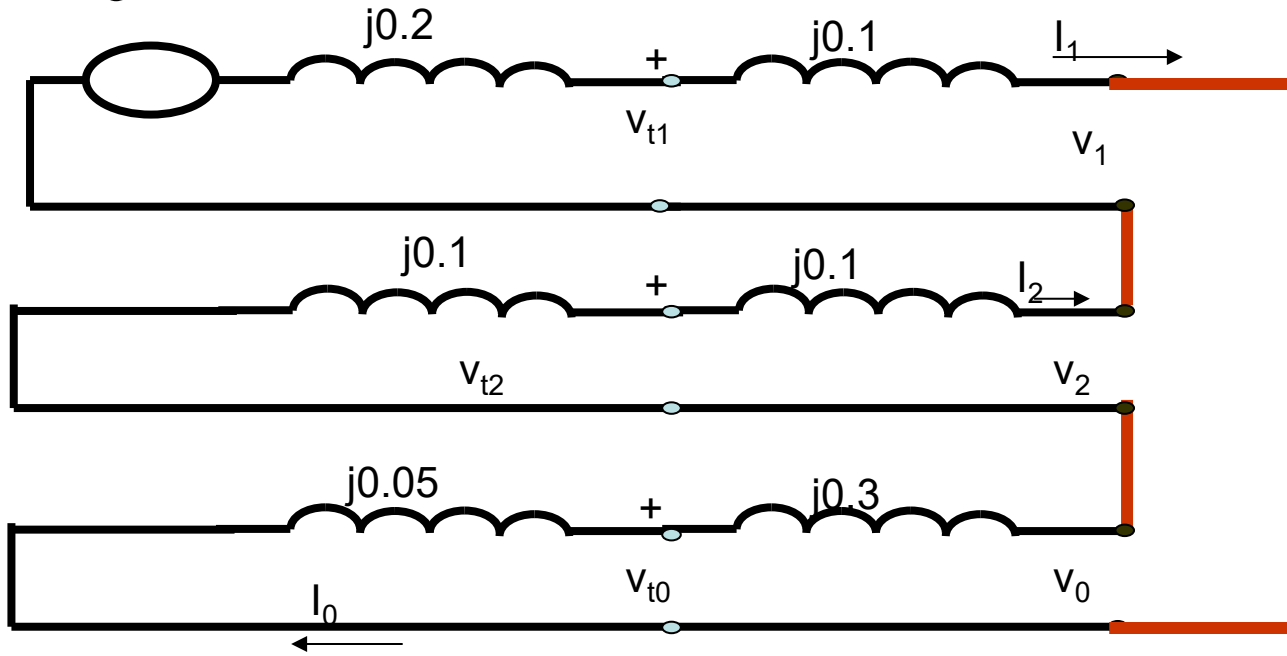


$$\begin{bmatrix} V_{ta} \\ V_{tb} \\ V_{tc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{t0} \\ V_{t1} \\ V_{t2} \end{bmatrix}$$

$$\begin{bmatrix} V_{ta} \\ V_{tb} \\ V_{tc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.0588 \\ 0.7648 \\ -0.1176 \end{bmatrix}$$

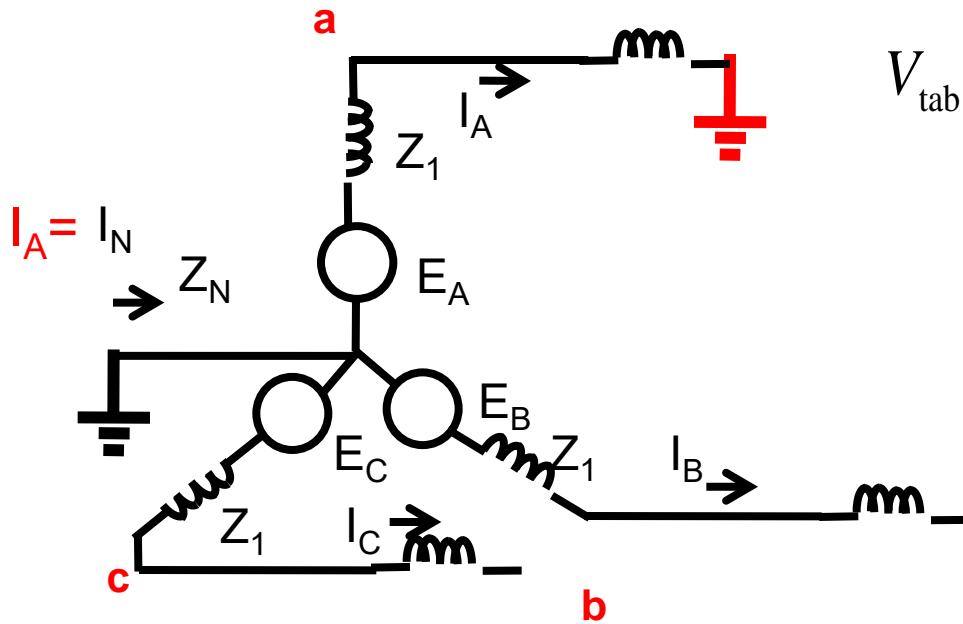
$$= \begin{bmatrix} 0.588 \angle 0^\circ \\ 0.855 \angle -117^\circ \\ 0.855 \angle 117^\circ \end{bmatrix} pu$$

Phase voltage at the terminals



$$\begin{bmatrix} V_{ta} \\ V_{tb} \\ V_{tc} \end{bmatrix} = \begin{bmatrix} 0.588 \angle 0^\circ \\ 0.855 \angle -117^\circ \\ 0.855 \angle 117^\circ \end{bmatrix} \left( \frac{13.8}{\sqrt{3}} \text{ kV} \right) \\
 = \begin{bmatrix} 4.685 \angle 0^\circ \\ 6.812 \angle -117^\circ \\ 6.812 \angle 117^\circ \end{bmatrix} \text{ kV}$$

Since the generated voltage to neutral  $E_A$  was taken as 1 p.u, the line to line voltages are expressed in p.u, of the base voltage to neutral. Expressed in volts, the post fault line voltages are



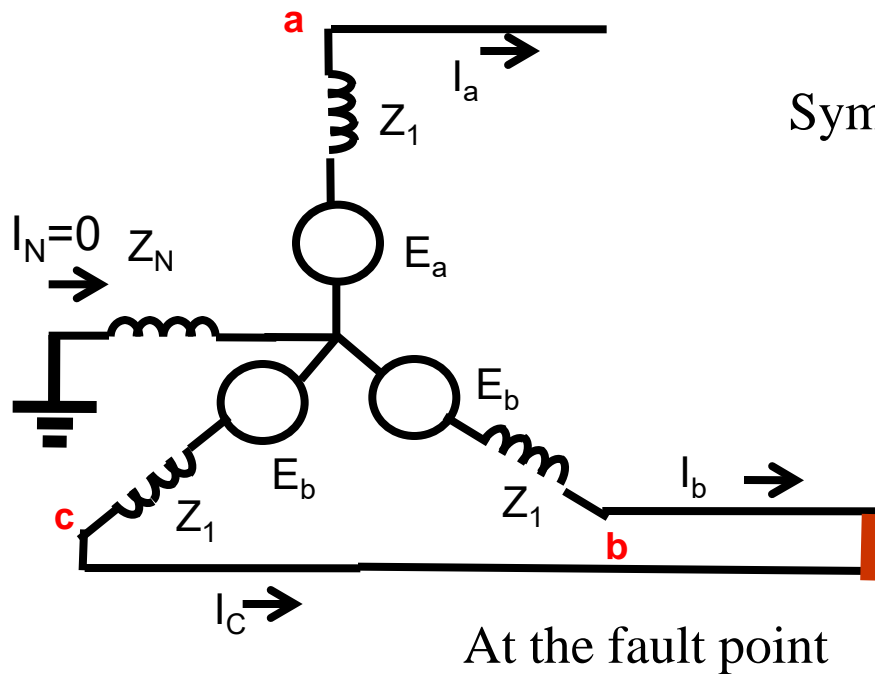
$$\begin{aligned}
 V_{tab} &= V_{ta} - V_{tb} \\
 &= 4.685 + j0 - (-3.0926 - j6.0695) \\
 &= 9.8656 \angle 37.97 \text{ kV}
 \end{aligned}$$

$$\begin{aligned}
 V_{tbc} &= V_{tb} - V_{tc} \\
 &= (-3.0926 - j6.0695) - (-3.0926 + j6.0695) \\
 &= -j12.1391 \\
 &= 12.1391 \angle -90 \text{ kV}
 \end{aligned}$$

## Line voltage at the terminals

$$\begin{aligned}
 V_{tca} &= V_{tc} - V_{ta} \\
 &= (-3.0926 + j6.0695) - 4.685 \\
 &= -7.7776 + j6.0695 \\
 &= 9.8656 \angle 142.03 \text{ kV}
 \end{aligned}$$

## Line-to-line fault on an unloaded generator



$$I_a = 0 \quad V_b = V_c \quad I_b = -I_c$$

Symmetrical components

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_1 = V_2$$

$$\text{As, } I_b = -I_c \quad \bar{I}_a = 0$$

The symmetrical components of current

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix} \leftarrow$$

$$\text{so, } I_0 = 0$$

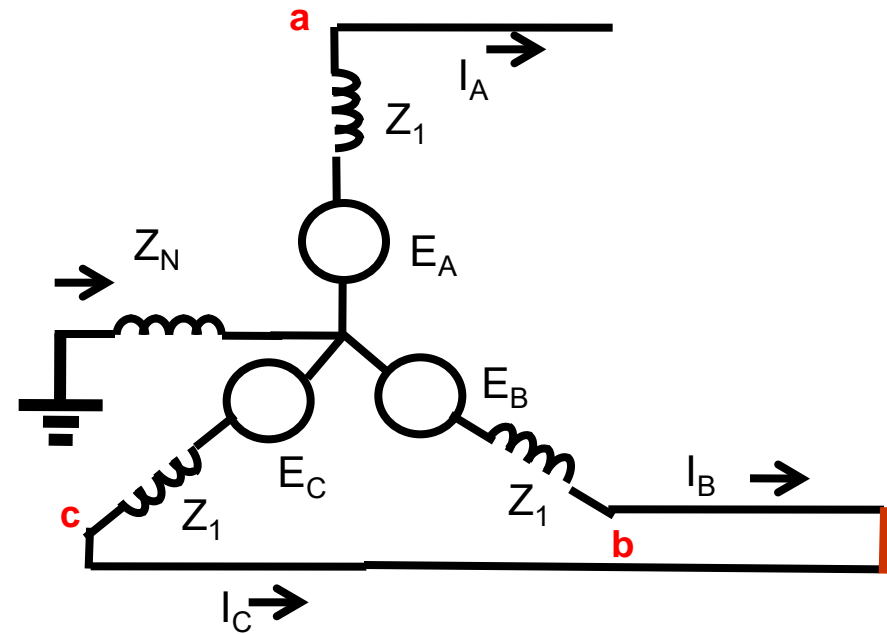
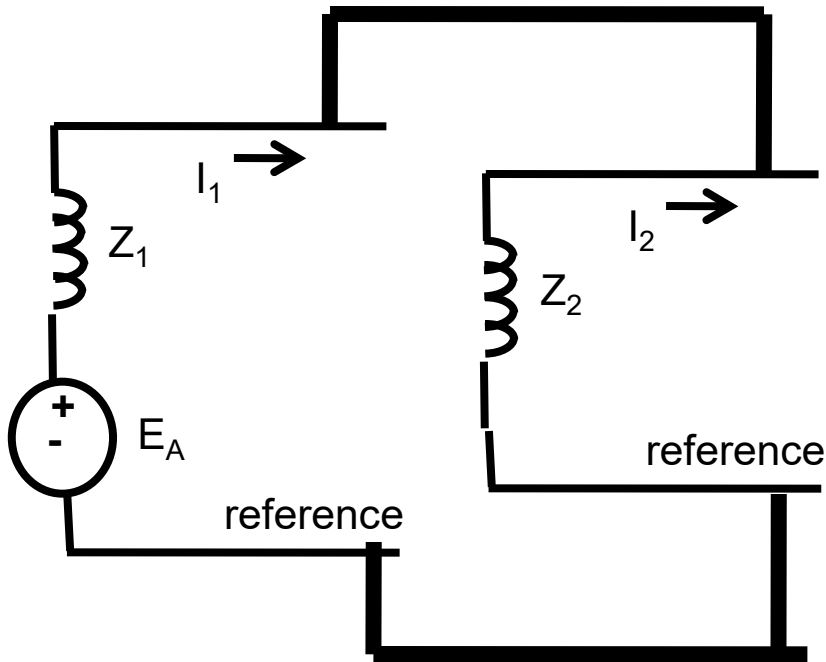
The zero sequence network is unconnected:  $I_2 = -I_1$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ I_1 \\ -I_1 \end{bmatrix}$$

$$V_0 = 0 \quad V_1 = E_a - I_1 Z_1 \quad V_2 = I_1 Z_2$$

But,  $V_1 = V_2$     $I_1 Z_2 = E_a - I_1 Z_1$     $\rightarrow$     $I_1 = \frac{E_a}{Z_1 + Z_2}$

The positive and negative sequence networks are connected in parallel:



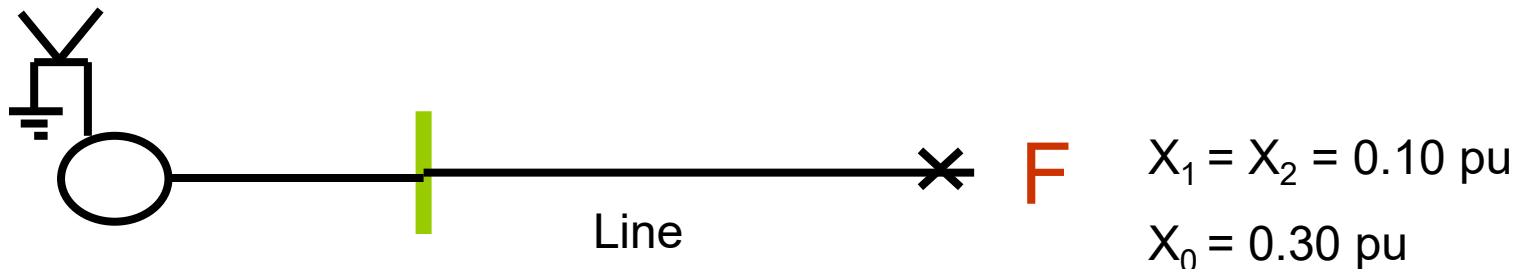
$$I_1 = \frac{E_a}{Z_1 + Z_2}$$

## Example

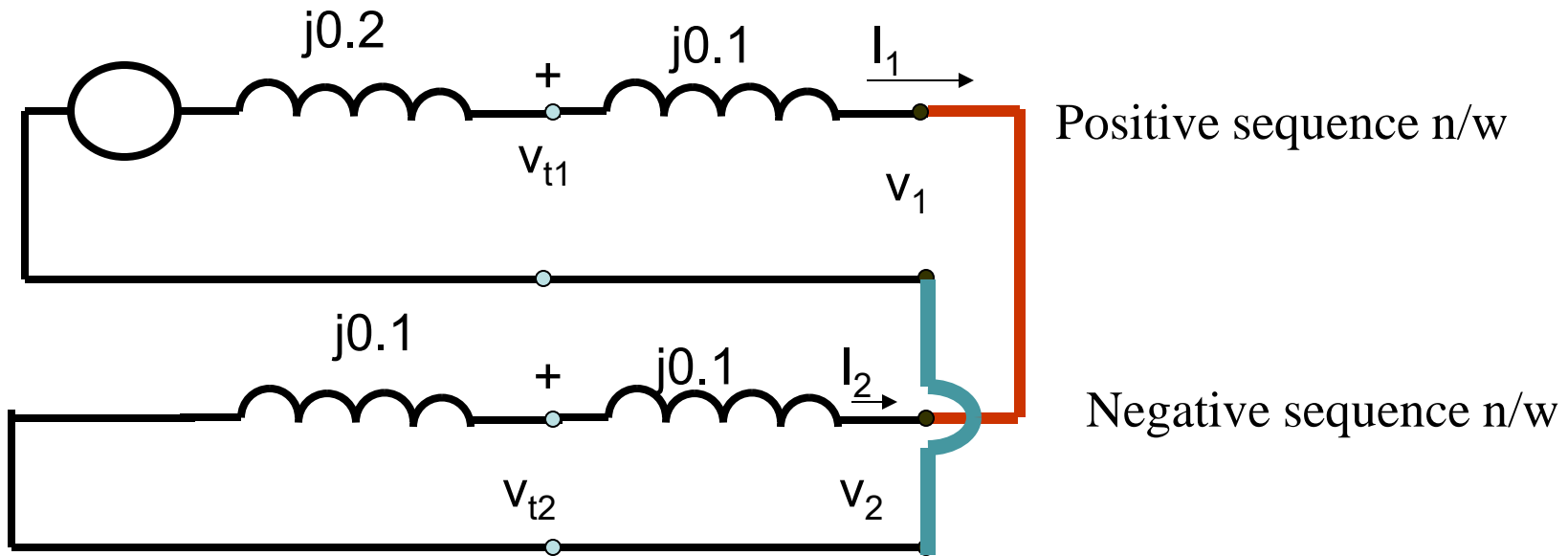
- A 150 MVA, 13.8 kV, Y connected synchronous generator has a synchronous reactance, negative sequence reactance and zero sequence reactance 20%, 10%, and 5% respectively. The neutral of the generator is solidly grounded.
- Generator is connected to short transmission line with positive and negative sequence reactance of 10%, and zero sequence impedance of 30%.
- The generator was unloaded when a line-to-line fault occurs on phase b and c at the end of the line. Find the voltages and fault currents at the fault point and at the generator terminals.

## Solution

150 MVA, 13.8 Kv,  $X_s=0.20$  pu,  $X_2 = 0.10$  pu,  $X_0= 0.05$  pu







$$E_a = 1.0 \angle 0^\circ$$

$$I_1 = \frac{1 \angle 0^\circ}{(Z_1 + Z_2)} = \frac{1 \angle 0^\circ}{j(0.3 + 0.2)}$$

$$= \frac{1 \angle 0^\circ}{j(0.5)} = 2 \angle -90^\circ = -j2.0 \text{ pu}$$

$$I_0 = 0 \text{ pu}$$

$$I_2 = -I_1 = 2 \angle 90^\circ = j2.0 \text{ pu}$$

Phase currents

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2 \\ j2 \end{bmatrix}$$
$$= \begin{bmatrix} 0 \\ -3.46 \\ 3.46 \end{bmatrix} pu = \begin{bmatrix} 0 \\ -21.74 \\ 21.74 \end{bmatrix} kA$$

Phase terminal voltages

$$\mathbf{V}_{t0} = 0$$

$$\begin{aligned} \mathbf{V}_{t1} &= \mathbf{E}_a - j0.20 \times \mathbf{I}_1 \\ &= 1.0 \angle 0^\circ - (j0.20)(-j2.0) \\ &= 0.6 \angle 0^\circ \quad pu \end{aligned}$$

Phase terminal voltages

$$\begin{aligned}V_{t2} &= -j0.10 \times I_2 \\ &= -(j0.10)(j2.0) \\ &= 0.2 \angle 0^\circ \text{ pu}\end{aligned}$$

$$\begin{aligned}\begin{bmatrix} V_{ta} \\ V_{tb} \\ V_{tc} \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.0 \\ 0.6 \\ 0.2 \end{bmatrix} \\ &= \begin{bmatrix} 0.8 \angle 0^\circ \\ 0.53 \angle -139^\circ \\ 0.53 \angle 139^\circ \end{bmatrix} \text{ pu} = \begin{bmatrix} 6.37 \angle 0^\circ \\ 4.22 \angle -139^\circ \\ 4.22 \angle 139^\circ \end{bmatrix} \text{ kV}\end{aligned}$$

## Double line-to-ground fault of an unloaded generator

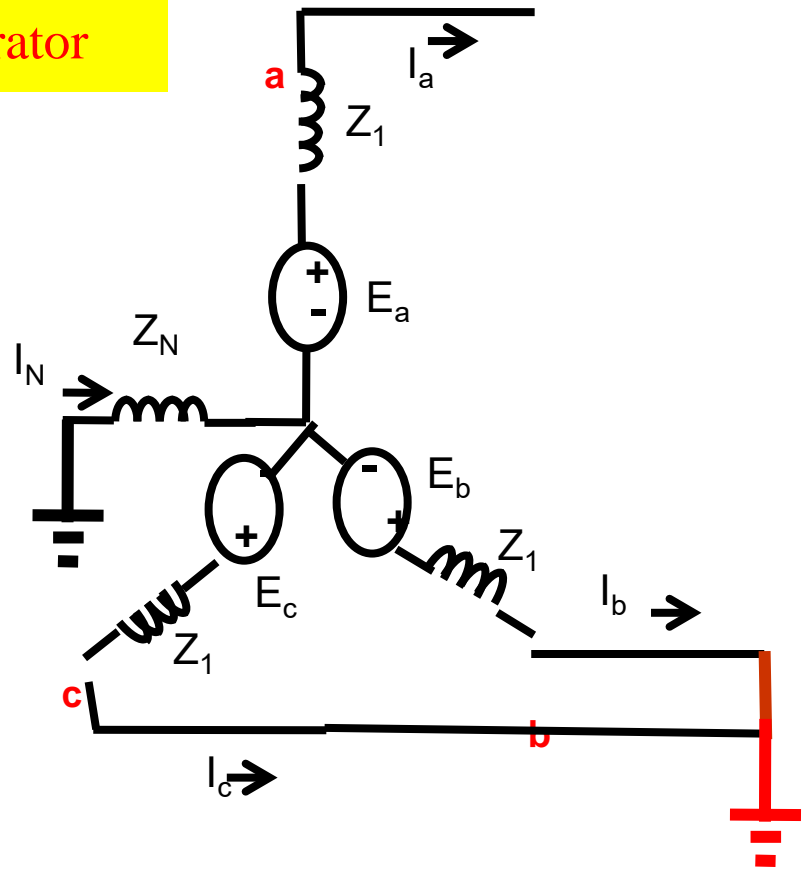
At the fault location:  $\mathbf{I}_a = \mathbf{0}$

$$\mathbf{V}_b = \mathbf{0} \quad \mathbf{V}_c = \mathbf{0},$$

Symmetrical components becomes:

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \mathbf{V}_{a0} &= \mathbf{V}_{a1} \\ &= \mathbf{V}_{a2} \\ &= \mathbf{V}_a / 3 \end{aligned}$$



$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$V_0 = V_1 = V_2 = \frac{V_a}{3}$$

$$\begin{bmatrix} E_a - I_1 Z_1 \\ E_a - I_1 Z_1 \\ E_a - I_1 Z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$



$$Z = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

$$\mathbf{Z}^{-1} = \begin{bmatrix} \mathbf{Z}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{\mathbf{Z}_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{1}{\mathbf{Z}_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{1}{\mathbf{Z}_2} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{E}_a - \mathbf{I}_1 \mathbf{Z}_1 \\ \mathbf{E}_a - \mathbf{I}_1 \mathbf{Z}_1 \\ \mathbf{E}_a - \mathbf{I}_1 \mathbf{Z}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{E}_a \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_0 \\ \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{Z} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{E}_a - \mathbf{I}_1 \mathbf{Z}_1 \\ \mathbf{E}_a - \mathbf{I}_1 \mathbf{Z}_1 \\ \mathbf{E}_a - \mathbf{I}_1 \mathbf{Z}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{Z} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{E}_a \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{Z} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{Z} \end{bmatrix} \begin{bmatrix} \mathbf{I}_0 \\ \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$[Z]^{-1} \begin{bmatrix} E_a - I_1 Z_1 \\ E_a - I_1 Z_1 \\ E_a - I_1 Z_1 \end{bmatrix} = [Z]^{-1} \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \quad Z^{-1} = \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix} \begin{bmatrix} E_a - I_1 Z_1 \\ E_a - I_1 Z_1 \\ E_a - I_1 Z_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_0} & 0 & 0 \\ 0 & \frac{1}{Z_1} & 0 \\ 0 & 0 & \frac{1}{Z_2} \end{bmatrix} \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$



Solving the equation gives:

$$\frac{E_a - I_1 Z_1}{Z_0} = -I_0$$

$$\frac{E_a - I_1 Z_1}{Z_2} = -I_2$$

$$\frac{\mathbf{E}_a - \mathbf{I}_1 \mathbf{Z}_1}{\mathbf{Z}_1} = \frac{\mathbf{E}_a}{\mathbf{Z}_1} - \mathbf{I}_1 \quad \mathbf{I}_a = \mathbf{I}_0 + \mathbf{I}_1 + \mathbf{I}_2 = 0$$

$$-\frac{\mathbf{E}_a}{\mathbf{Z}_0} + \mathbf{I}_1 \frac{\mathbf{Z}_1}{\mathbf{Z}_0} + \mathbf{I}_1 - \frac{\mathbf{E}_a}{\mathbf{Z}_2} + \mathbf{I}_1 \frac{\mathbf{Z}_1}{\mathbf{Z}_2} = 0$$

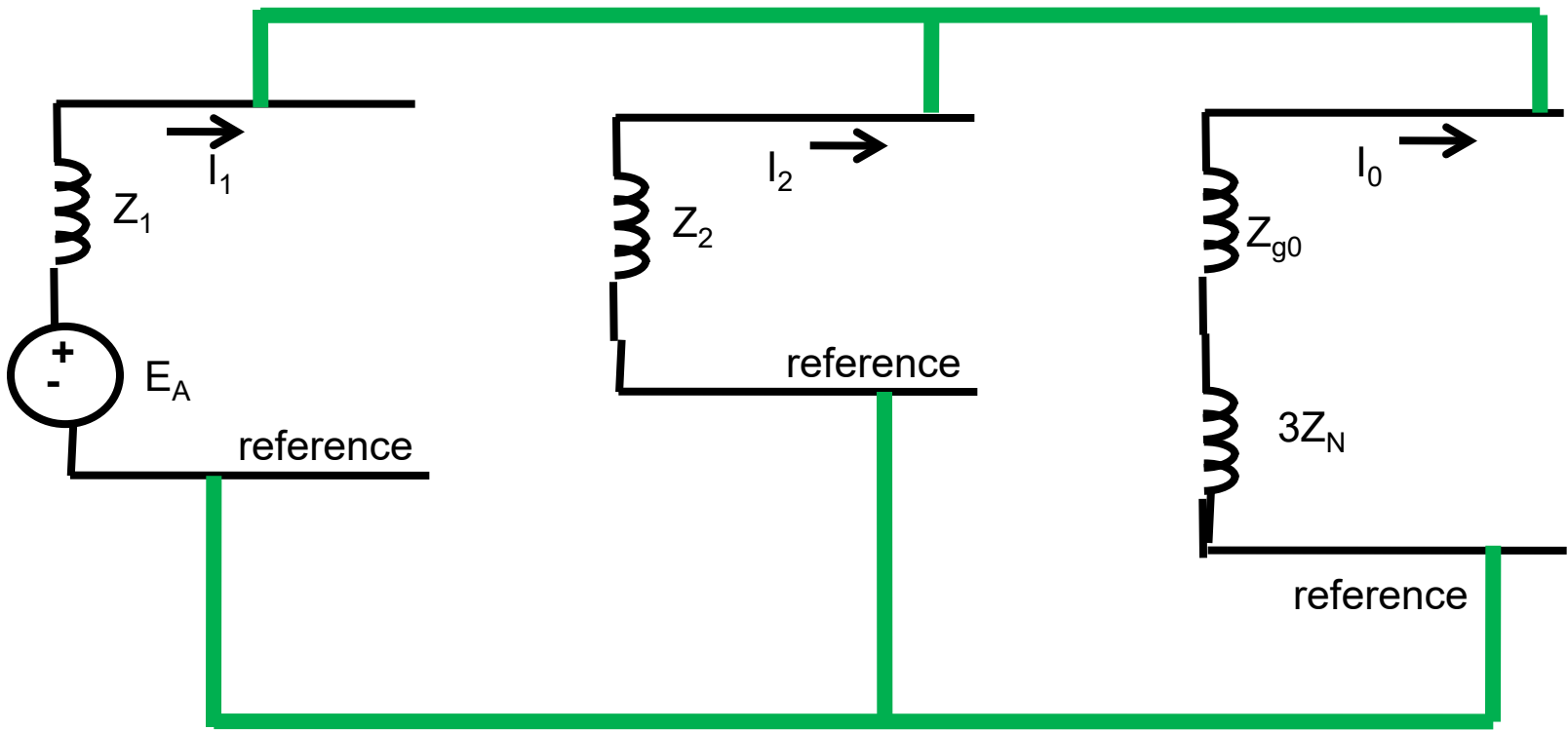
$$\mathbf{I}_1 \left( 1 + \frac{\mathbf{Z}_1}{\mathbf{Z}_0} + \frac{\mathbf{Z}_1}{\mathbf{Z}_2} \right) = \frac{\mathbf{E}_a (\mathbf{Z}_2 + \mathbf{Z}_0)}{\mathbf{Z}_2 \mathbf{Z}_0}$$

$$\mathbf{I}_1 = \frac{\mathbf{E}_a (\mathbf{Z}_2 + \mathbf{Z}_0)}{\mathbf{Z}_2 \mathbf{Z}_1 + \mathbf{Z}_2 \mathbf{Z}_0 + \mathbf{Z}_1 \mathbf{Z}_0}$$

$$\mathbf{I}_1 = \frac{\mathbf{E}_a}{\mathbf{Z}_1 + \frac{\mathbf{Z}_2 \mathbf{Z}_0}{\mathbf{Z}_2 + \mathbf{Z}_0}}$$



Networks connection becomes:



$$I_1 = \frac{E_a}{Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0}}$$

## Example

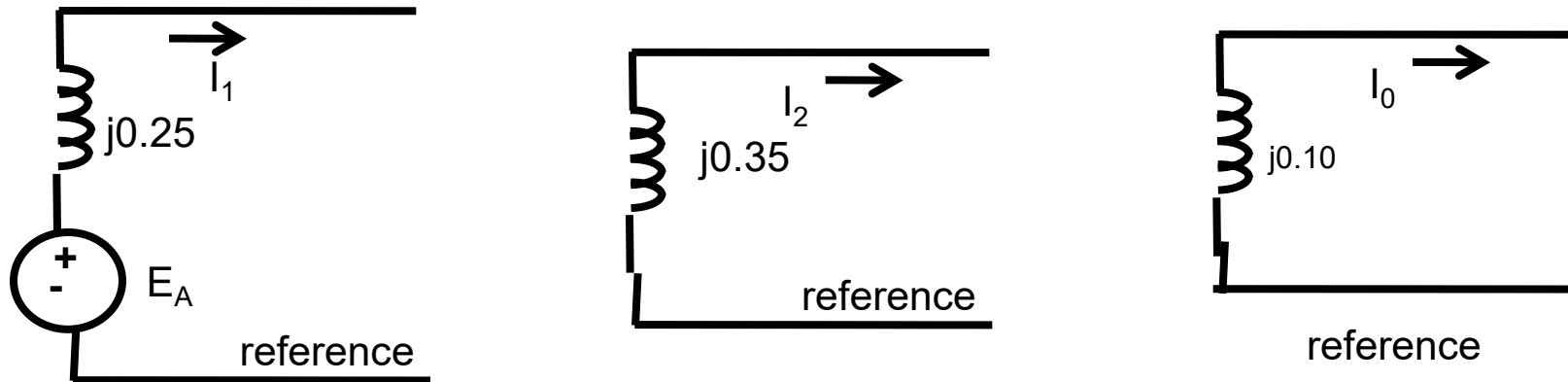
A 20 MVA, 13.8 kV, undamped salient pole synchronous generator, has the following data:  $X_d''=0.25$  pu,  $X_2 =0.35$  pu,  $X_0=0.10$  pu

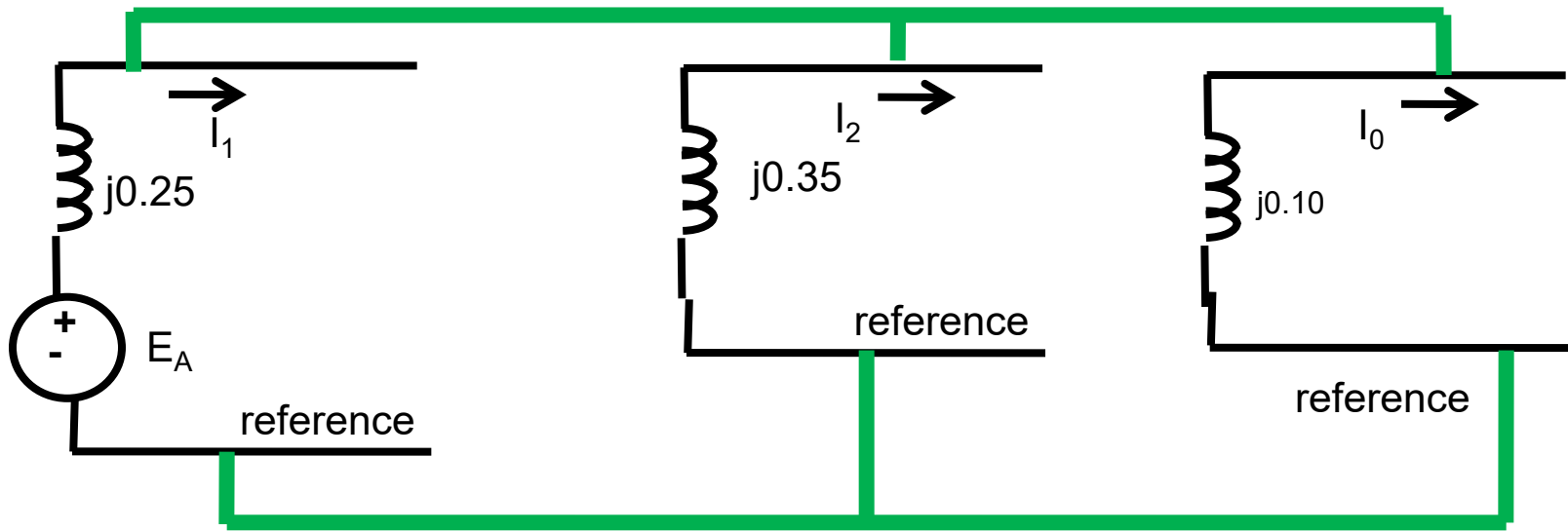
Neutral is solidly grounded.

Calculate subtransient fault current and line voltages when a double-line-to fault occurs at its terminal.

## Solution

Sequence networks for the generator:

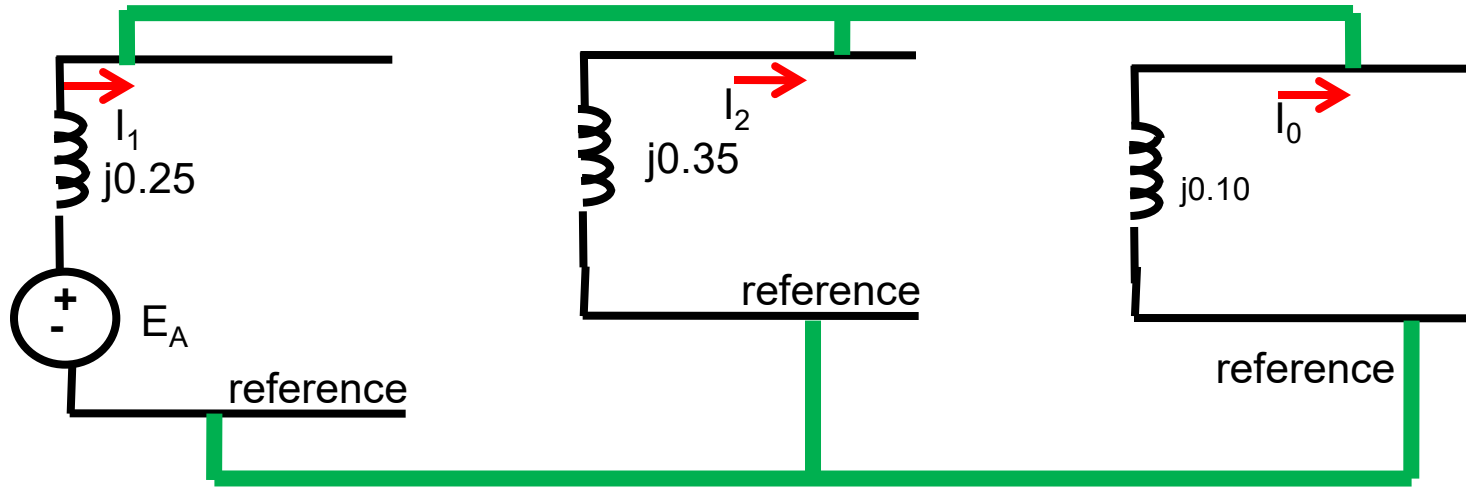




$$I_1 = \frac{E_a}{Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0}} = \frac{1.0 + j0}{j0.25 + (j0.35 \times j0.10)/(j0.35 + j0.10)}$$

$$= \frac{1.0}{j0.3278} = -j3.0506 \text{ pu}$$

$$V_0 = V_1 = V_2 = E_a - I_1 Z_1 = 1 - (-j3.05)(j0.25) = 0.237 \text{ pu}$$



$$I_1 = -j3.0506 \text{ pu}$$

$$I_2 = -\frac{V_2}{Z_2} = -\frac{0.237}{j0.35} = j0.68 \text{ pu}$$

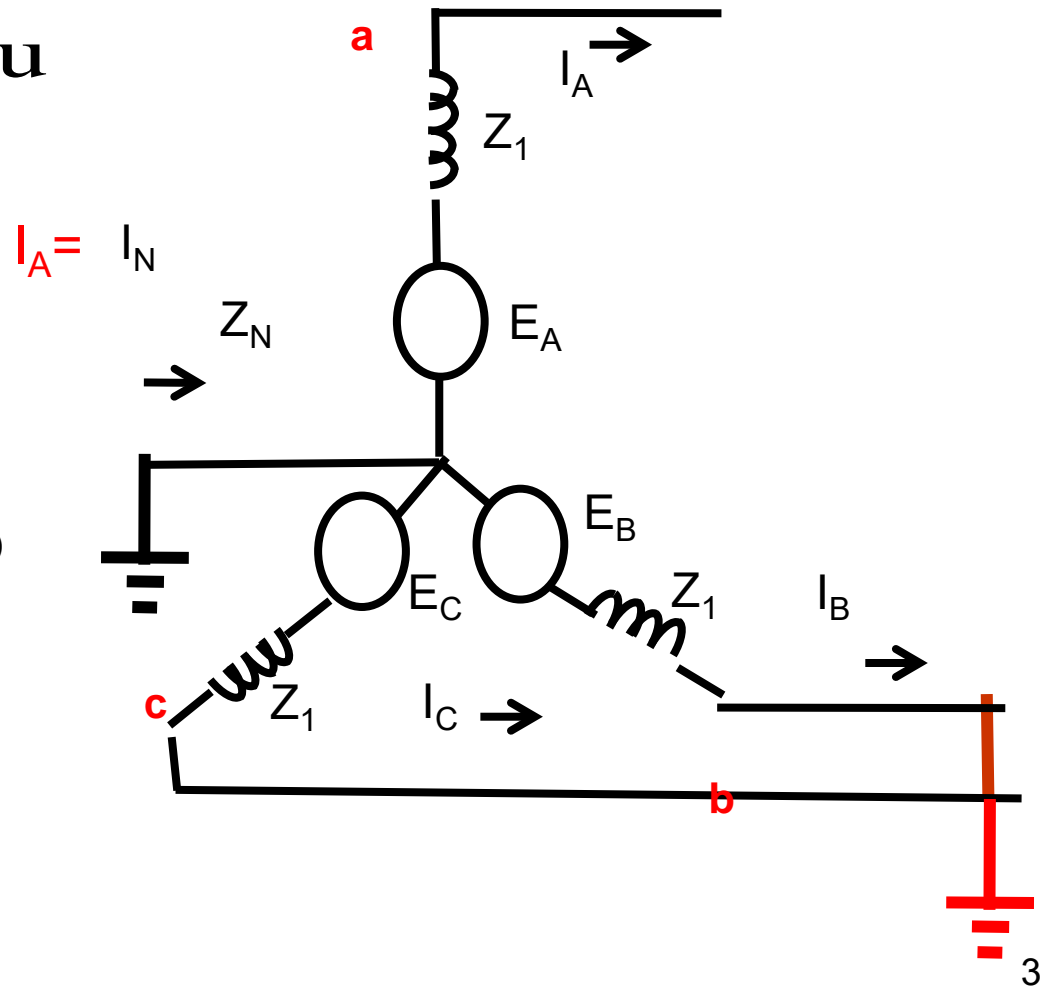
$$I_0 = -\frac{V_2}{Z_0} = -\frac{0.237}{j0.10} = j2.37 \text{ pu}$$

$$I_a = I_0 + I_1 + I_2$$

$$= -j3.05 + j0.68 + j2.37 = 0$$

$$\begin{aligned}
\mathbf{I}_b &= \mathbf{a}^2 \mathbf{I}_1 + \mathbf{a} \mathbf{I}_2 + \mathbf{I}_0 \\
&= (-0.5 - j0.866) \mathbf{I}_1 + (-0.5 + j0.866) \mathbf{I}_2 + \mathbf{I}_0 \\
&= -3.230 + j3.555 \\
&= 4.80 \angle 132.3^\circ \text{ pu}
\end{aligned}$$

$$\begin{aligned}
\mathbf{I}_c &= \mathbf{a} \mathbf{I}_1 + \mathbf{a}^2 \mathbf{I}_2 + \mathbf{I}_0 \\
&= (-0.5 + j0.866) \bar{\mathbf{I}}_1 + \\
&\quad (-0.5 - j0.866) \bar{\mathbf{I}}_2 + \bar{\mathbf{I}}_0 \\
&= 3.230 + j3.555 \\
&= 4.80 \angle 47.7^\circ \text{ pu}
\end{aligned}$$



$$\begin{aligned}
\mathbf{I}_N &= \mathbf{I}_b + \mathbf{I}_c \\
&= -3.23 + j3.555 + 3.236 + j3.555 \\
&= j7.11 \text{ pu}
\end{aligned}$$

$$\begin{aligned}
\mathbf{V}_a &= \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_0 \\
&= 3\overline{\mathbf{V}}_1 = 3 \times 0.237 = j0.711 \text{ pu}
\end{aligned}$$

$$\mathbf{V}_b = \mathbf{V}_c = \mathbf{0}$$

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b = 0.711 \text{ pu}$$

$$\mathbf{V}_{bc} = \mathbf{0}$$

$$\mathbf{V}_{ca} = \mathbf{V}_c - \mathbf{V}_a = -0.711 \text{ pu}$$

In amperes,

$$\mathbf{I}_a = \mathbf{0}$$

$$I_b = 837 \times 4.80 \angle 132.3^\circ = 4017 \angle 132.3^\circ$$

$$I_c = 837 \times 4.80 \angle 47.7^\circ = 4017 \angle 47.7^\circ$$

$$I_N = 837 \times 7.11 \angle 90^\circ = 5951 \angle 90^\circ \text{ A}$$

$$V_{ab} = 0.711 \times \frac{13.8}{\sqrt{3}} = 5.66 \angle 0^\circ \text{ kV}$$

$$V_{bc} = 0$$

$$V_{ca} = -0.711 \times \frac{13.8}{\sqrt{3}} = 5.66 \angle 180^\circ \text{ kV}$$

## Unsymmetrical Fault in a Power System: the assumptions

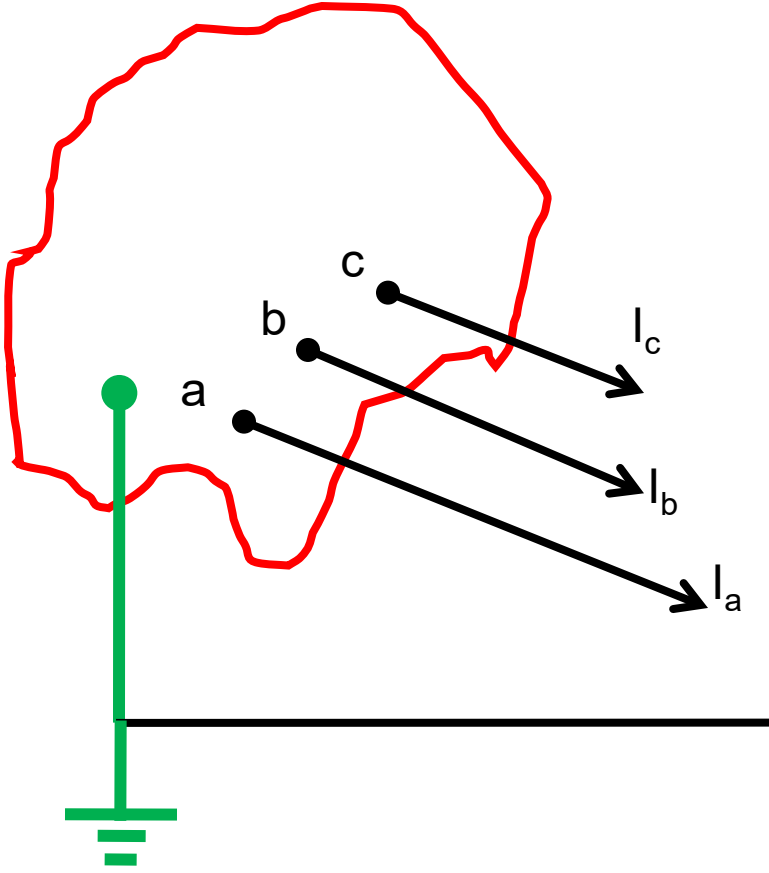
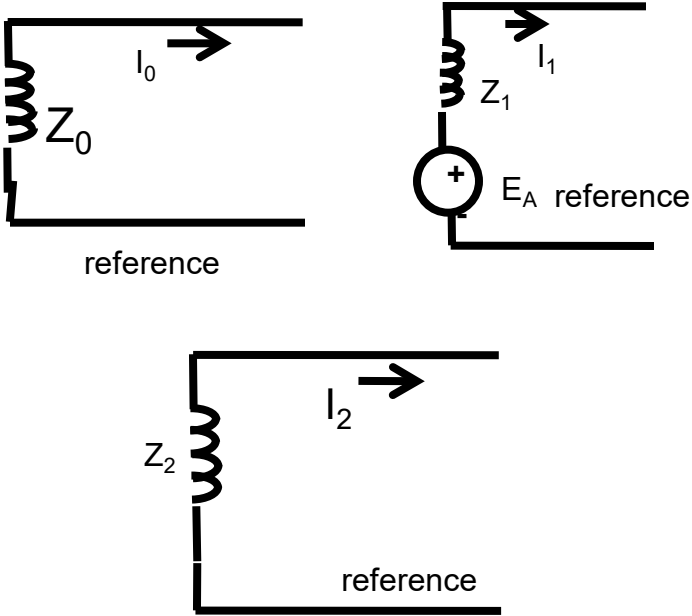
- Power system operates under balanced steady-state conditions before fault.
- Pre fault load current is neglected. Pre fault voltage at each bus in the positive-sequence network equals  $V_0$ .
- transformer winding resistance and shunt admittances are neglected.
- Transmission-line series resistance and shunt admittances are neglected.
- synchronous machine armature resistance, saliency and saturation are neglected.
- All non rotating impedance loads are neglected.
- Induction motors are either neglected or represented as synchronous machines.



# General three-phase bus:

Terminals abc, denoted the fault terminals, are brought out in order to make external connections that represent faults.

Sequence networks:



## Thevenin equivalents as viewed from faults terminals

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

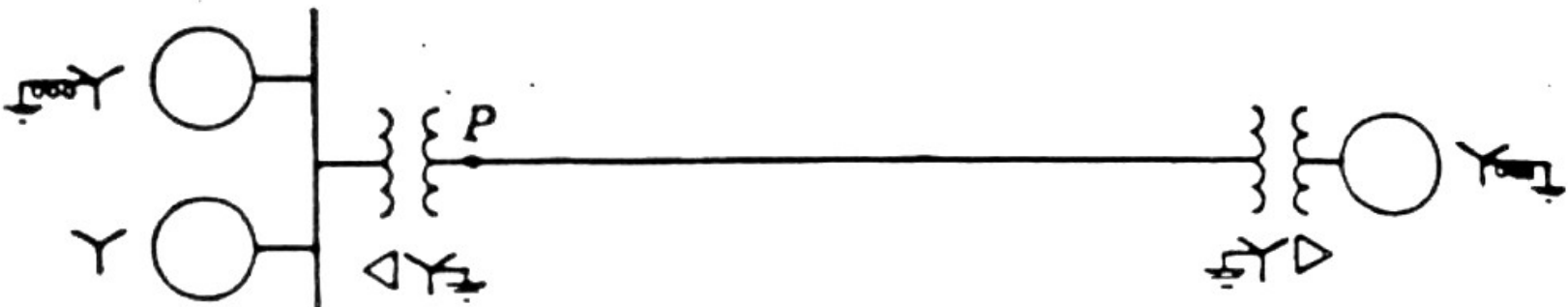
$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

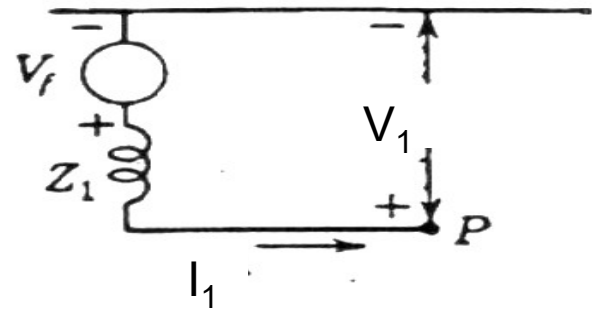
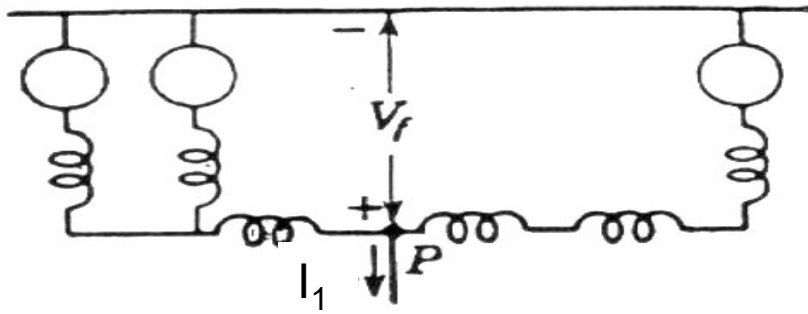
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

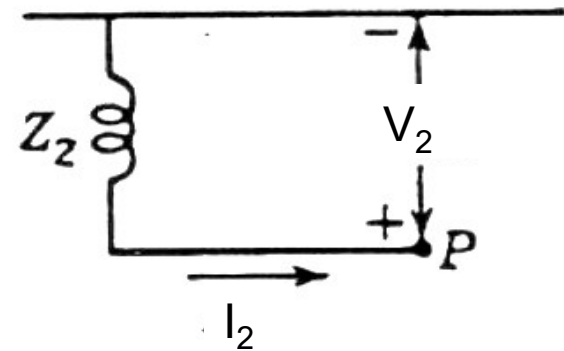
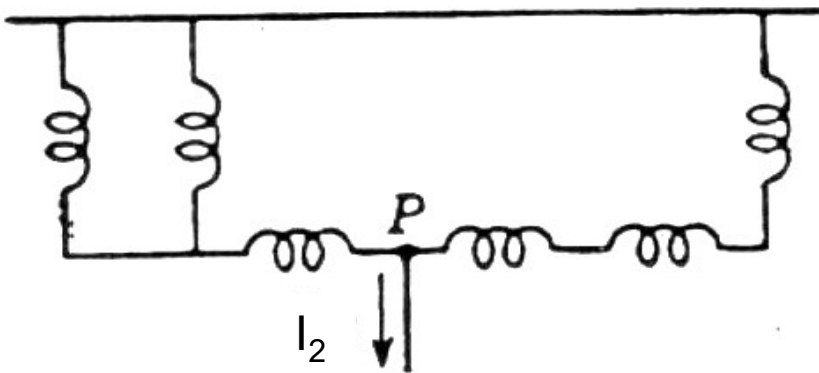
Three phase power system with fault at point **P**



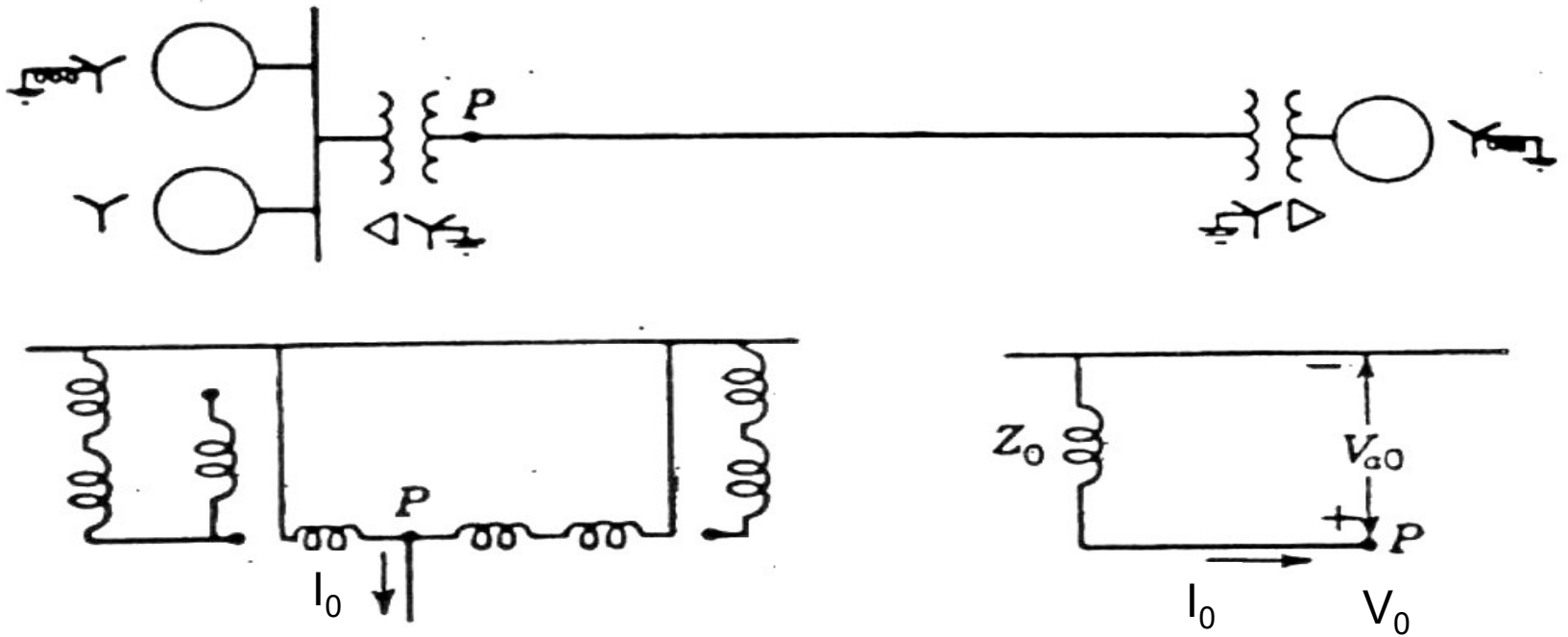
## Single-line diagram of a balanced power system



## Positive sequence network and thevenin equivalent



## Negative sequence network and thevenin equivalent



Zero sequence network and thevenin equivalent

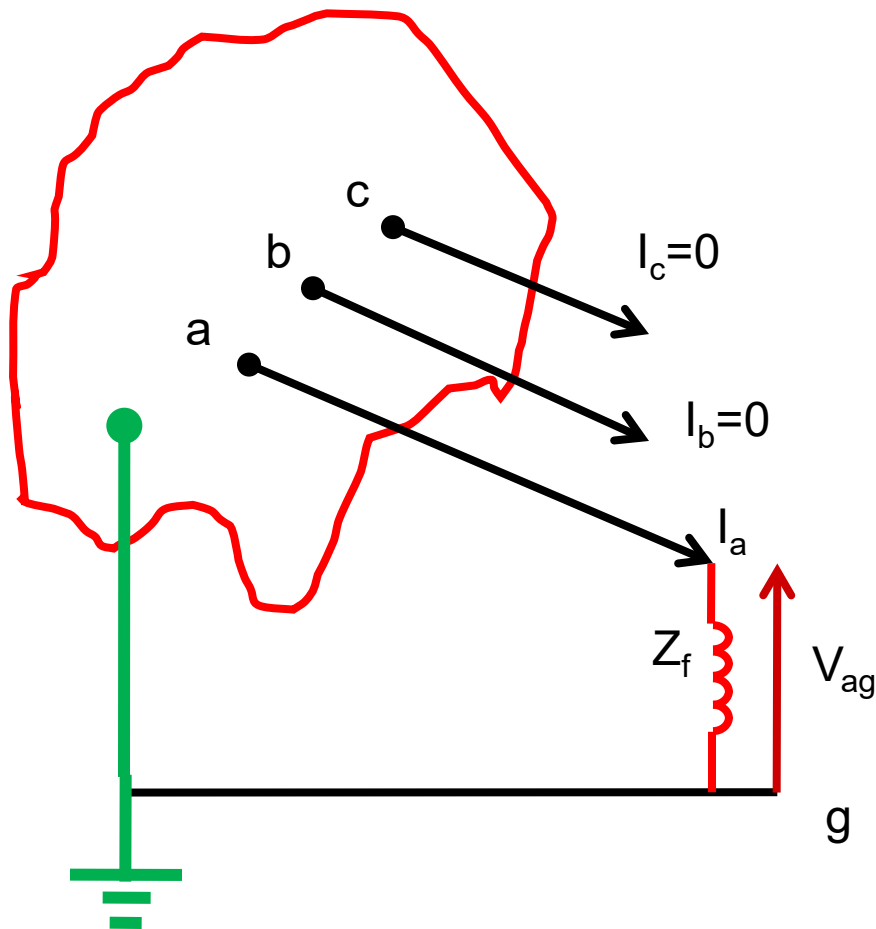
Sequence Voltages for phase a are:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

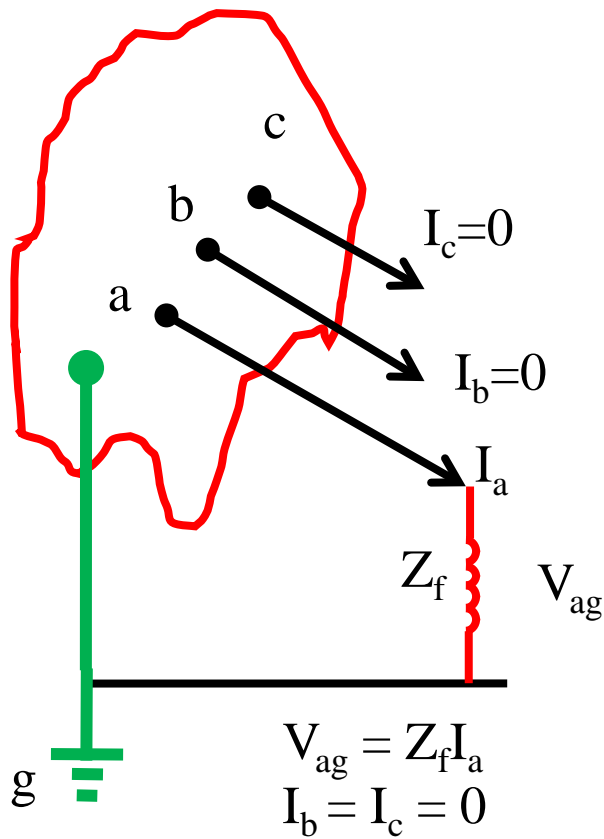
## Single line-to-ground Fault:



Fault conditions in phase domain:

$$I_b = I_c = 0$$

$$V_{ag} = Z_f I_a$$



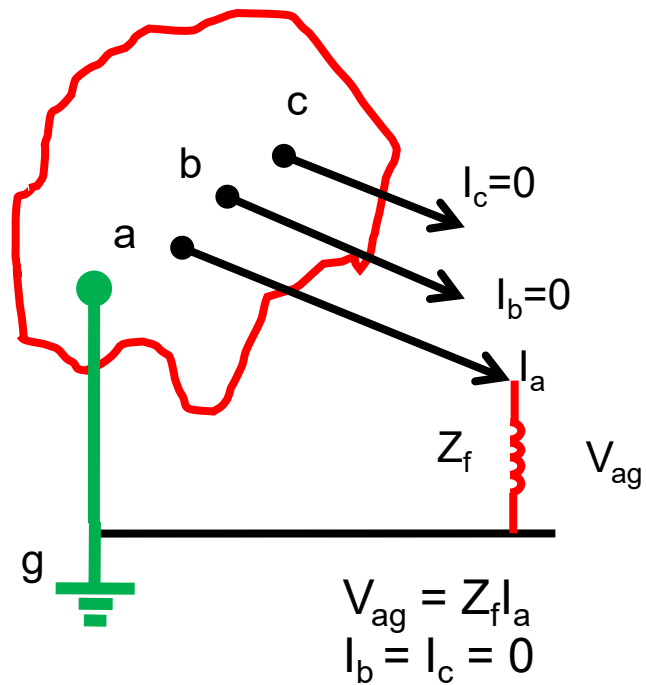
$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} I_a \\ I_a \\ I_a \end{bmatrix}$$

$$I_0 = I_1 = I_2$$

$$\underbrace{(V_0 + V_1 + V_2)}_{V_{ag}} = Z_f \underbrace{(I_0 + I_1 + I_2)}_{I_a}$$

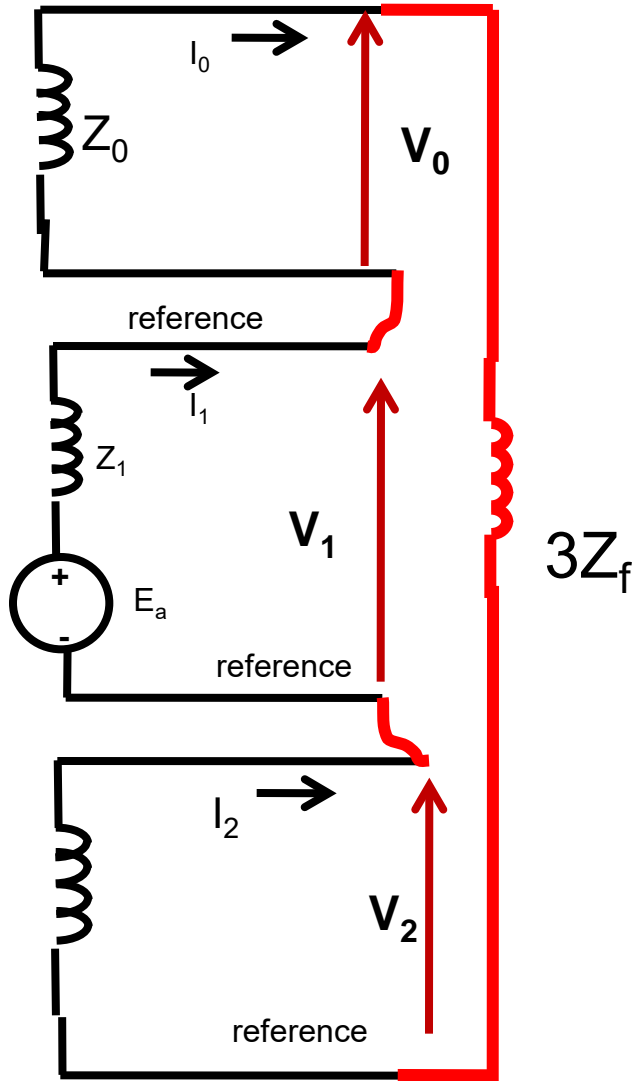
$$(V_0 + V_1 + V_2) = 3Z_f (I_1)$$

The equations can be satisfied by interconnecting the sequence network in series at the fault terminals through the impedance  $3Z_f$



Prove this your self

# Interconnected sequence network



$$\bar{I}_1 = \bar{I}_2 = \bar{I}_0$$

$$(V_0 + V_1 + V_2) = 3Z_f (I_1)$$

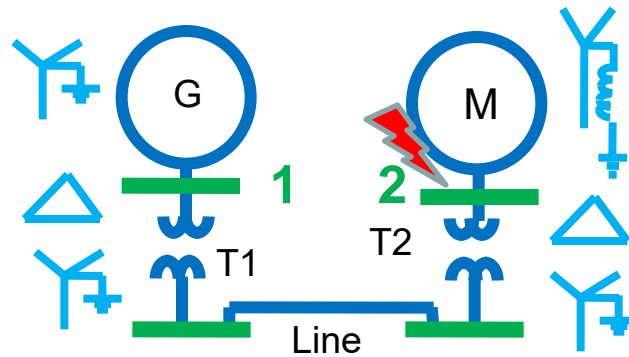
$$\bar{I}_1 = \frac{E_a}{Z_0 + Z_1 + Z_2 + (3Z_f)}$$

$$\bar{I}_a = \frac{3E_a}{Z_0 + Z_1 + Z_2 + (3Z_f)}$$



## Example

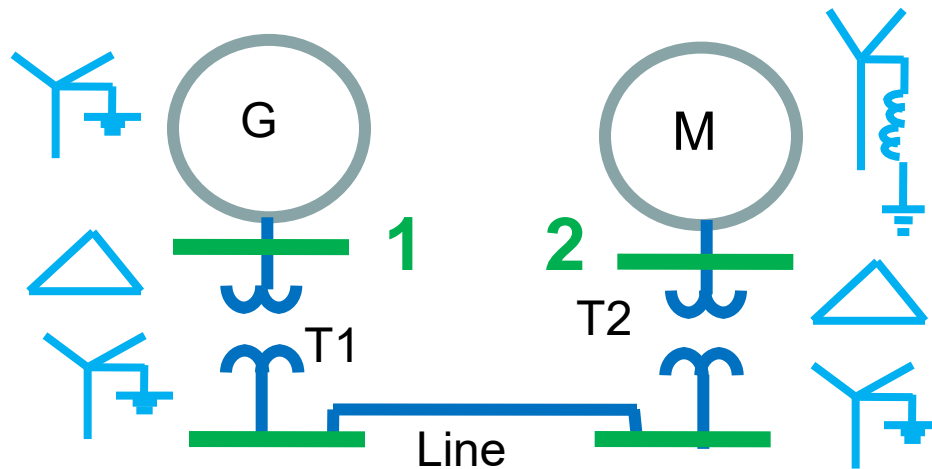
- Calculate the subtransient fault current in per-unit and in kA for a **bolted single line-to-ground short circuit** from phase a to ground at bus 2.
- Calculate the per-unit line-to-ground voltages at bus 2



	MVA	kV	X1	X''	X2	X0	Xn
G	100	13.8		0.15	0.17	0.05	
T1	100	13.8/138	0.10		0.10	0.10	
T2	100	138/13.8	0.10		0.10	0.10	
M	100	13.8		0.20	0.21	0.10	0.05
Line			20Ω		20Ω	60Ω	

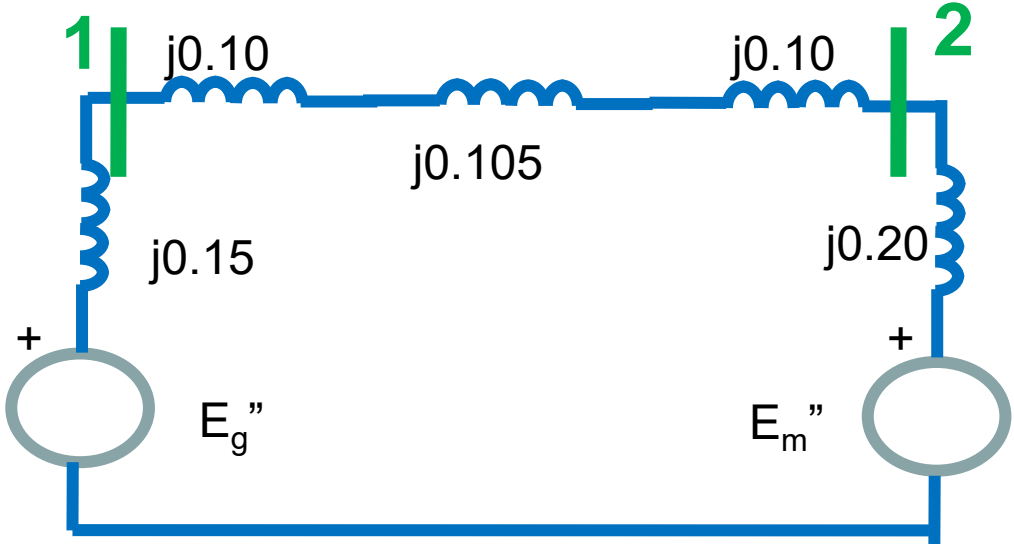
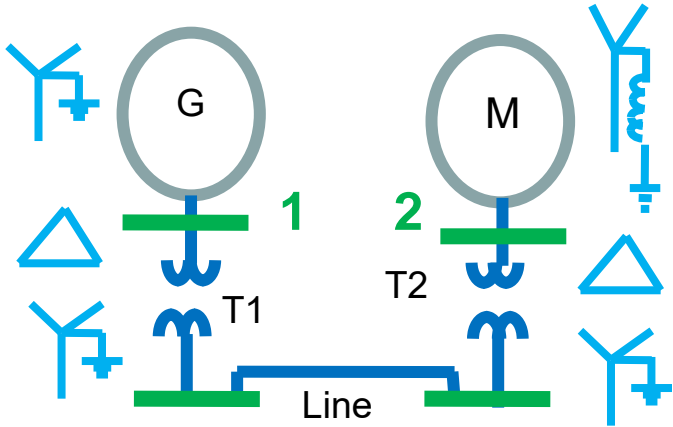
## Solution

Draw **positive, negative and zero** sequence networks of the given system

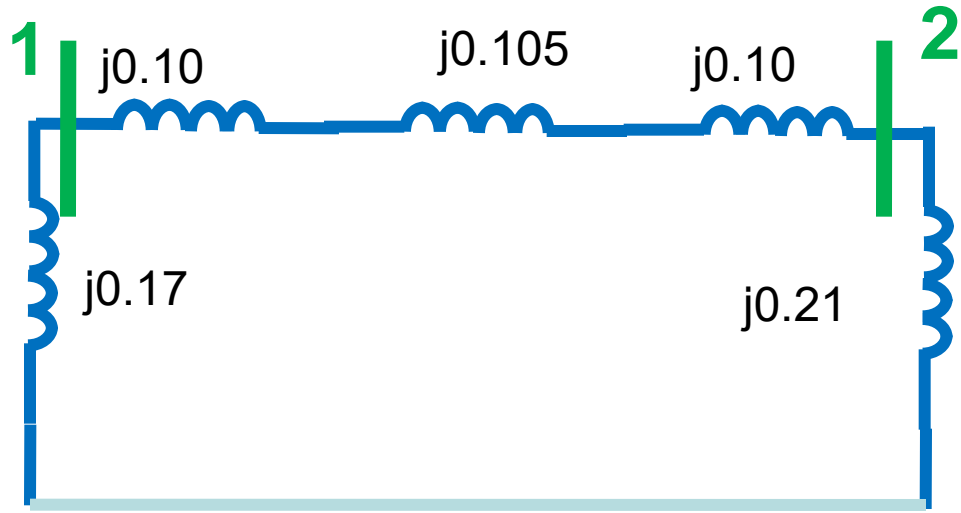
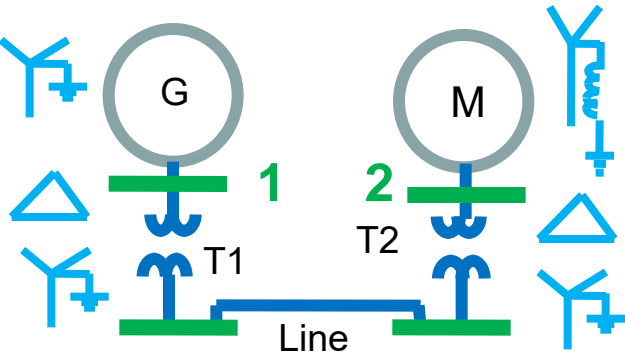


	MVA	kV	X1	X''	X2	X0	Xn
G	100	13.8		0.15	0.17	0.05	
T1	100	13.8/138	0.10		0.10	0.10	
T2	100	138/13.8	0.10		0.10	0.10	
M	100	13.8		0.20	0.21	0.10	0.05
Line			20Ω		20Ω	60Ω	

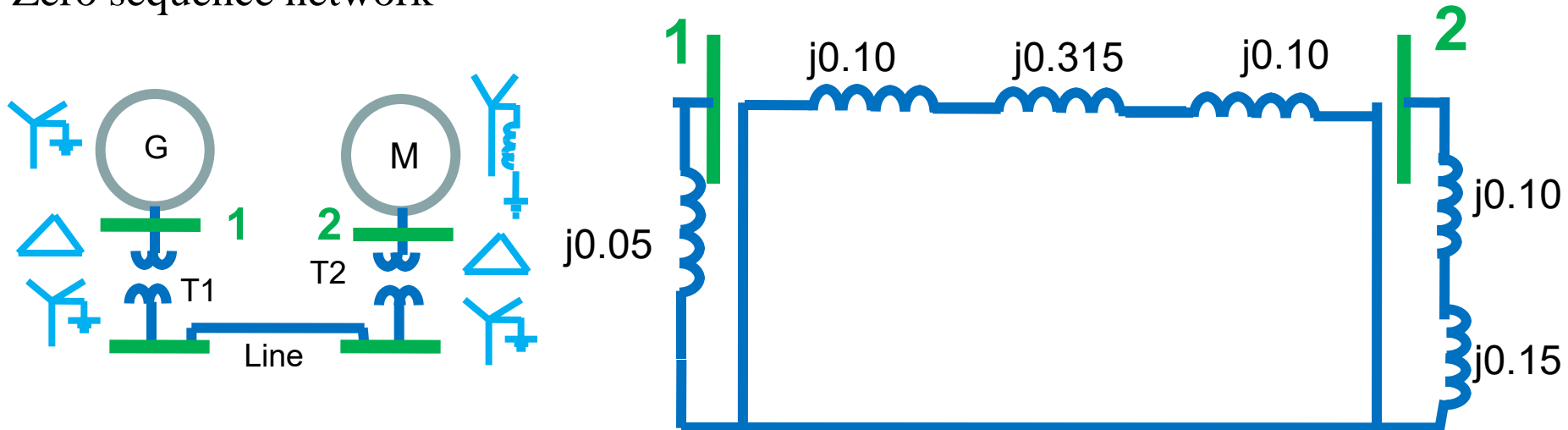
# Positive sequence network



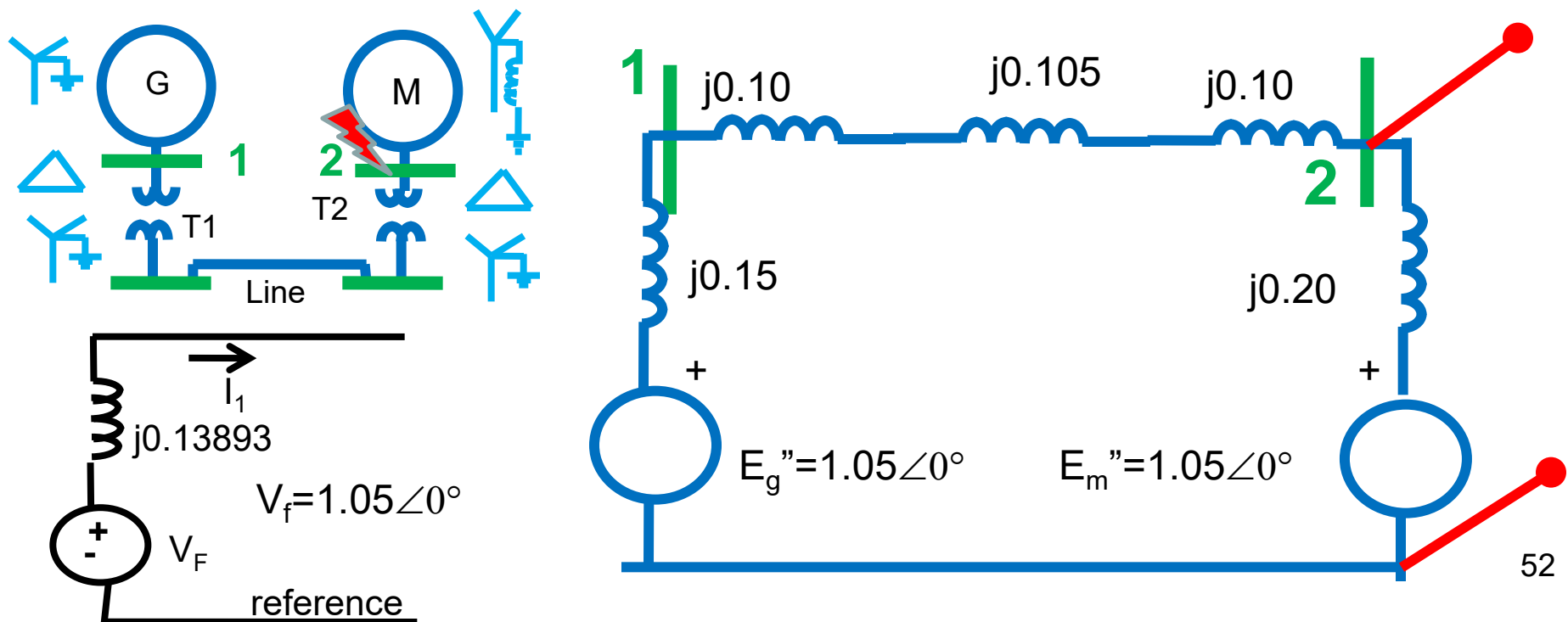
# Negative sequence network



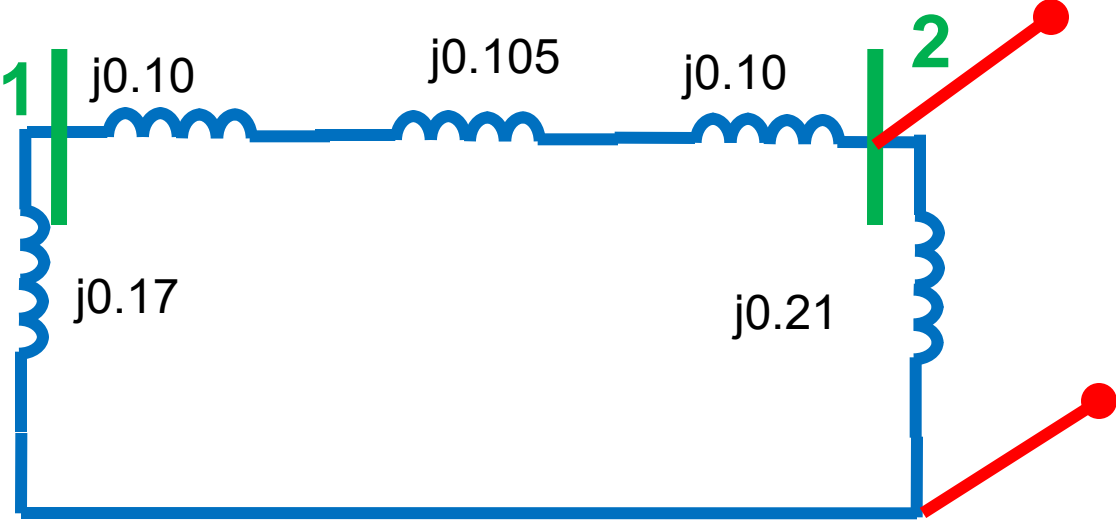
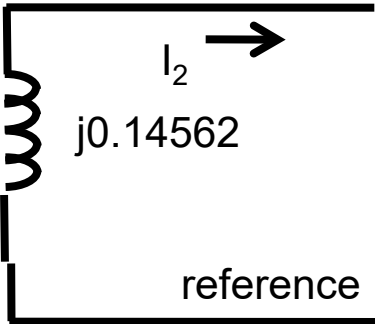
## Zero sequence network



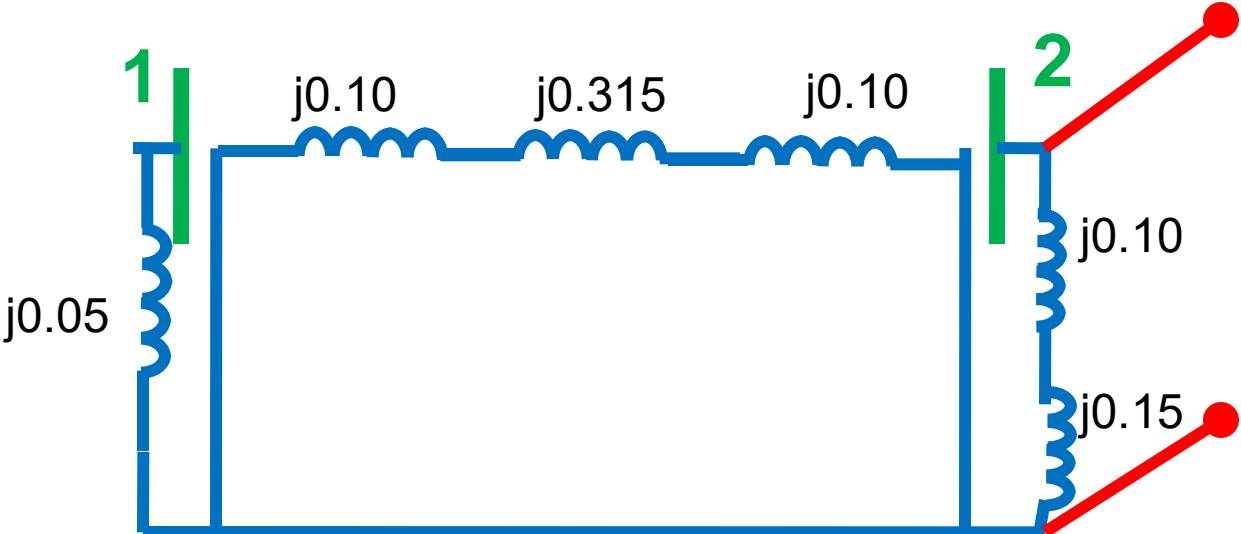
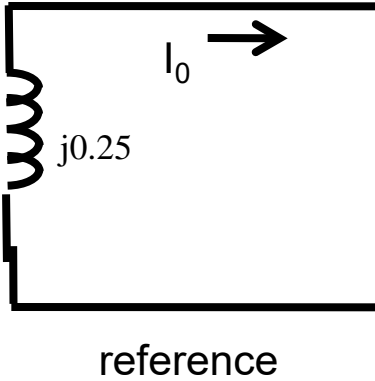
## Positive sequence network for fault at bus 2



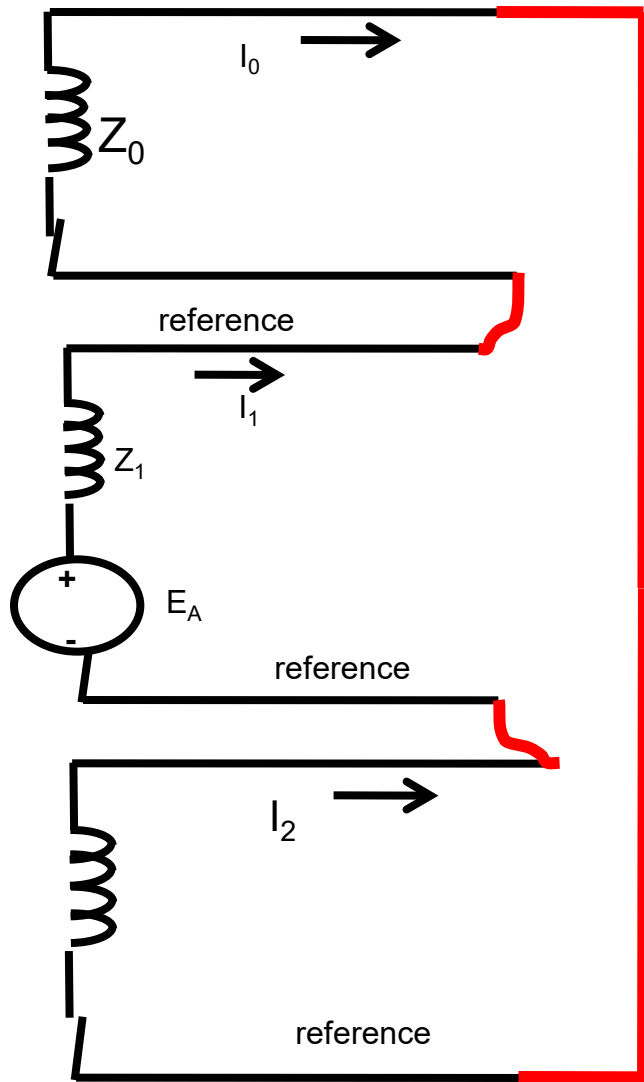
# Negative sequence network



# Zero sequence network



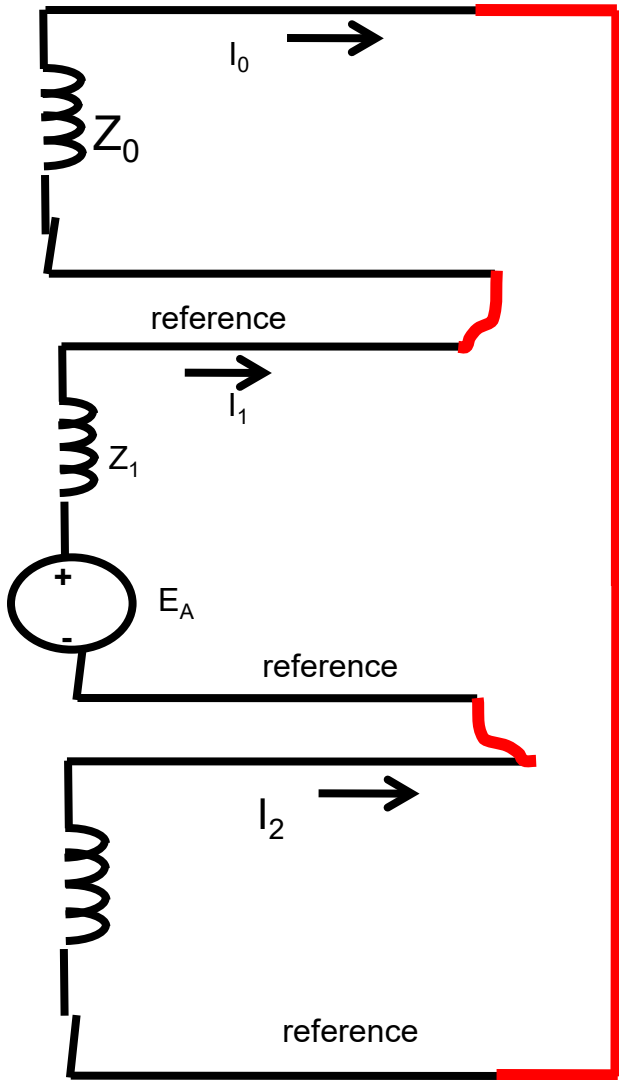
# Interconnected sequence network



$$\bar{I}_1 = \bar{I}_2 = \bar{I}_0$$

$$\begin{aligned} \bar{I}_1 &= \frac{E_f}{Z_0 + Z_1 + Z_2} \\ &= \frac{1.05 \angle 0}{j(0.25 + 0.13893 + 0.14562)} \\ &= \frac{1.05}{j0.53455} = -j1.96427 \text{ pu} \end{aligned}$$

# Interconnected sequence network



$$\begin{aligned}\bar{I}_a &= 3(-j1.96427) \\ &= -j5.8928 \text{ pu}\end{aligned}$$

$$\begin{aligned}I_{base} &= \frac{100}{\sqrt{3} \times 13.8} \\ &= 4.1837 \text{ kA}\end{aligned}$$

$$\begin{aligned}\bar{I}_a &= -j5.8928 \times 4.1837 \\ &= 24.65 \text{ kA}\end{aligned}$$

The sequence components of the voltages at the fault are

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.05 \angle 0 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.25 & 0 & 0 \\ 0 & j0.13893 & 0 \\ 0 & 0 & j0.14562 \end{bmatrix} \begin{bmatrix} -j1.96427 \\ -j1.96427 \\ -j1.96427 \end{bmatrix}$$

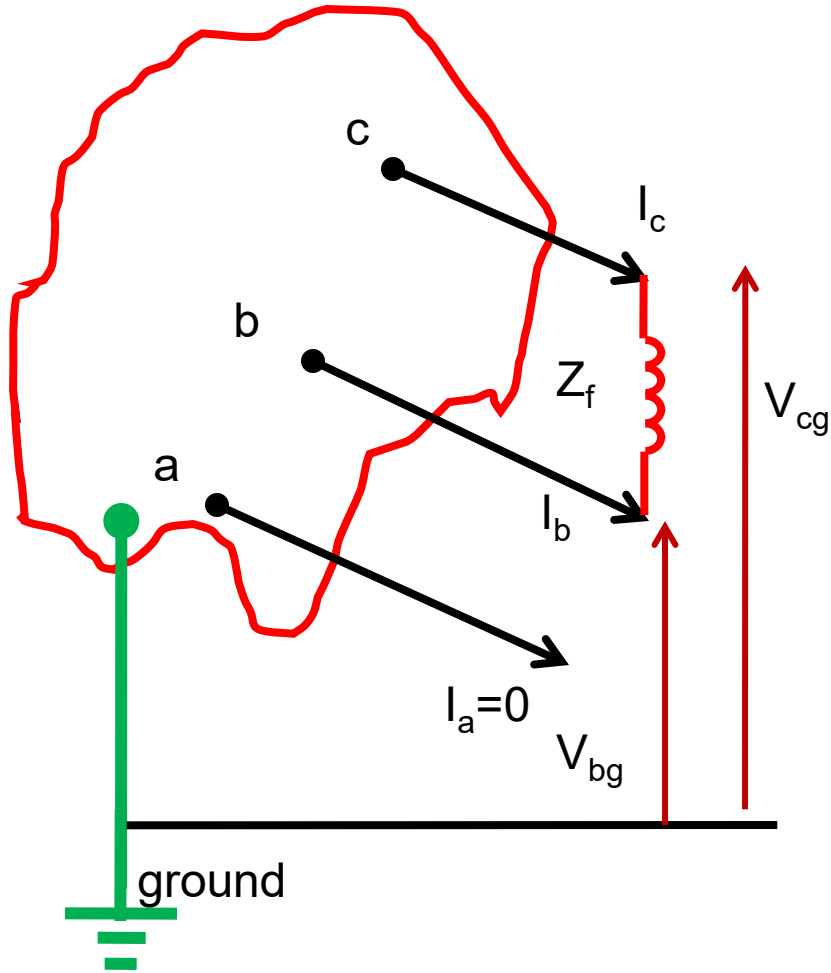
$$= \begin{bmatrix} -0.49107 \\ 0.77710 \\ -0.28604 \end{bmatrix} pu$$

The line-to-ground voltage at faulted bus 2 are

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.49107 \\ 0.77710 \\ -0.28604 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.179 \angle 231.3 \\ 1.179 \angle 128.7 \end{bmatrix} pu$$



## Line-to-line Fault:



Fault conditions in phase domain:

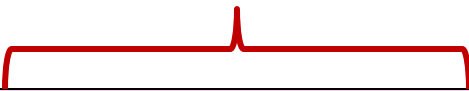
$$I_b = -I_c$$

$$I_a = 0$$

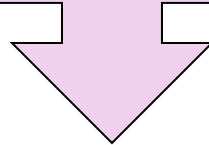
$$V_{bg} - V_{cg} = Z_f I_b$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{3}(a - a^2)I_b \\ \frac{1}{3}(a^2 - a)I_b \end{bmatrix} pu$$

$$V_{bg} - V_{cg} = Z_f I_b$$

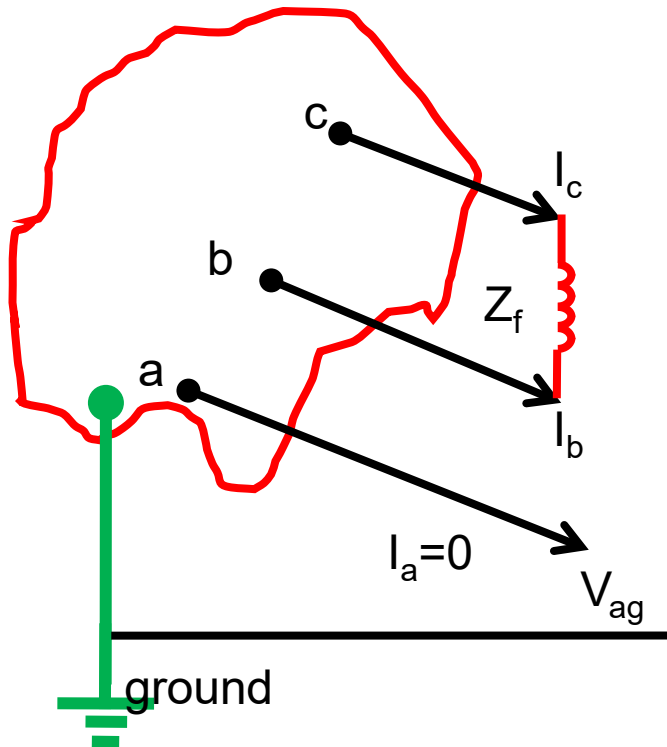
 $V_{bg}$ 
 $V_{cg}$ 
 $I_b$ 


$$(V_0 + a^2 V_1 + a V_2) - (V_0 + a V_1 + a^2 V_2) = Z_F (I_0 + a^2 I_1 + a I_2)$$



$I_0 = 0$

$I_2 = -I_1$

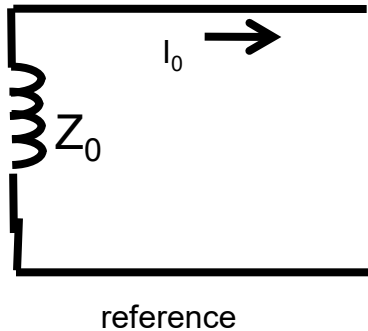


Simplifies to

$$(a^2 - a)V_1 - (a^2 - a)V_2 = Z_F (a^2 - a)I_1$$

$$V_1 - V_2 = Z_F I_1$$

## Fault conditions in sequence domain

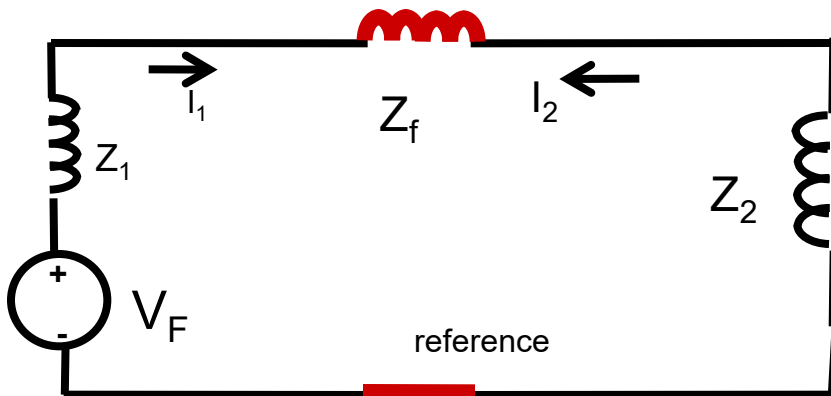


$$I_0 = 0$$

$$I_2 = -I_1$$

$$V_1 - V_2 = Z_F I_1$$

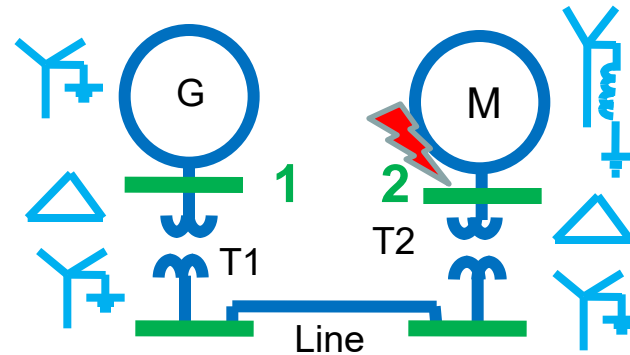
These equations are satisfied by connecting the positive sequence and negative sequence networks in parallel at the fault terminals through the fault impedance  $Z_f$



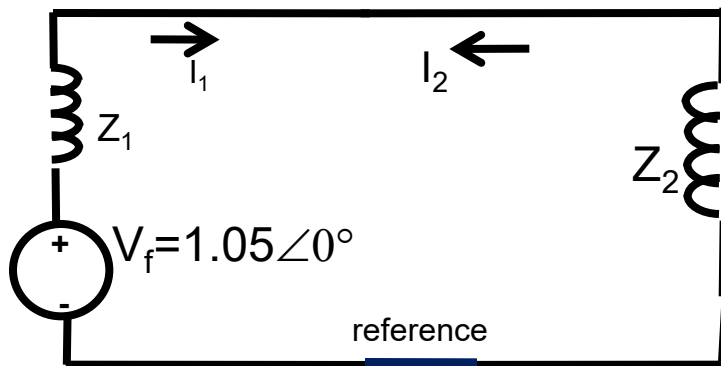
$$I_1 = -I_2 = \frac{V_F}{(Z_1 + Z_2 + Z_F)}$$

## Example

- Calculate the subtransient fault current in per-unit and kA for a bolted line-to-line fault from phase b to c at bus 2



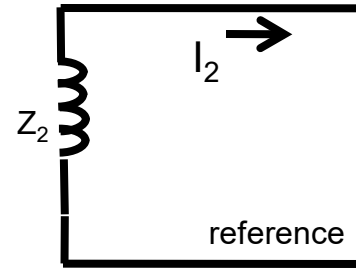
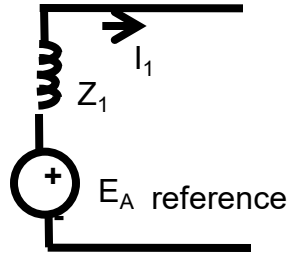
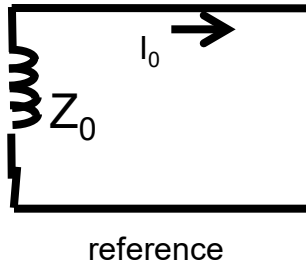
Fault conditions in sequence domain



$$\begin{aligned}
 I_1 &= -I_2 = \frac{V_F}{(Z_1 + Z_2 + Z_F)} \\
 &= \frac{1.05 \angle 0}{j(0.13893 + 0.14562)} \\
 &= 3.690 \angle -90
 \end{aligned}$$

$$\begin{aligned}
 I_0 &= 0 & I_b'' &= 6.391 \angle 180 \text{ pu} & I_c'' &= 6.391 \angle 0 \text{ pu} \\
 I_a'' &= 0 & &= 26.74 \text{ kA} & &= 26.74 \text{ kA}
 \end{aligned}$$

## Sequence networks:



Thevenin equivalents as viewed from faults terminals

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

## Revision

1. Write equation expressing phase and sequence voltages

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

2. Write equation expressing sequence and phase voltages

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

## Revision

1. Write equation expressing phase and sequence currents

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

2. Write equation expressing sequence and phase currents

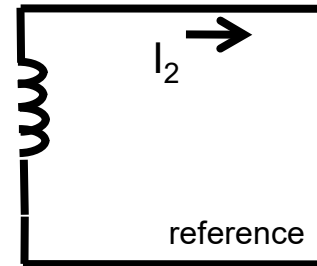
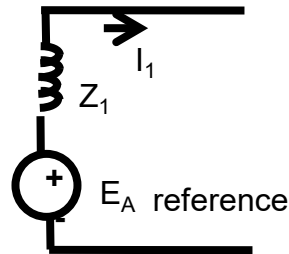
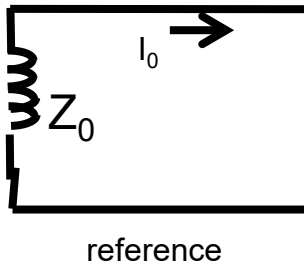
$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

## Revision

1. Write equation expressing sequence networks

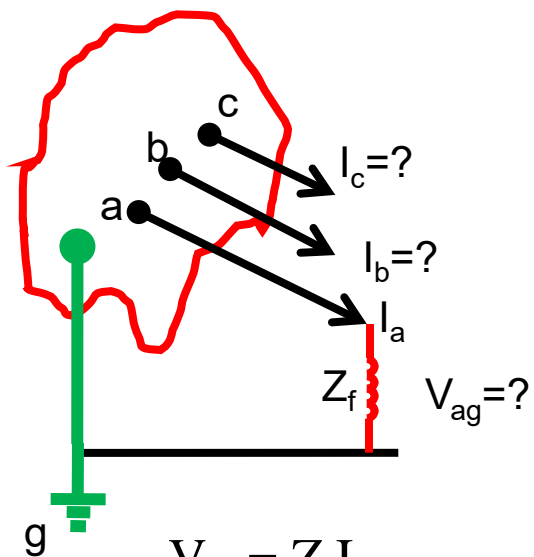
$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

2. Draw sequence networks





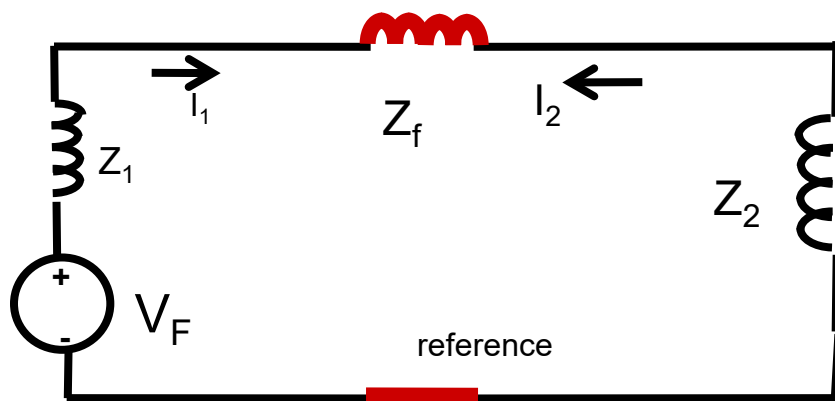
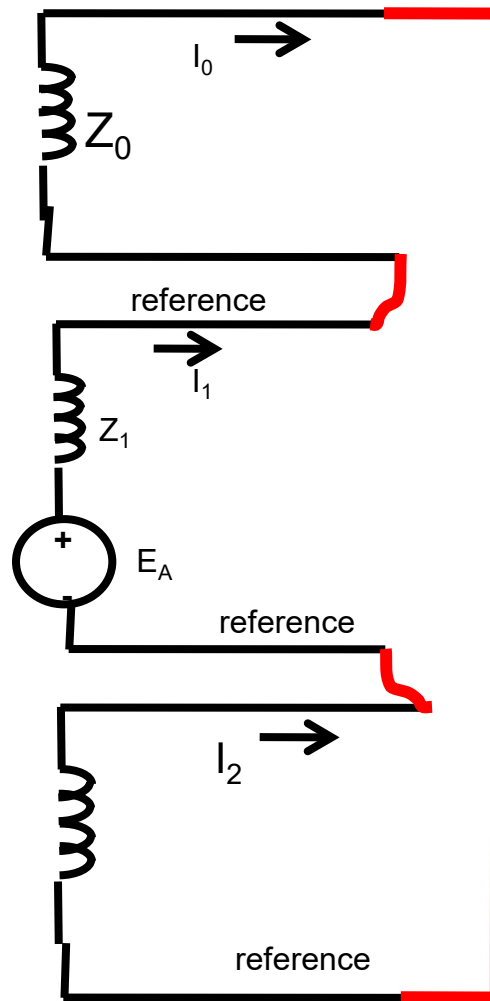
# Revision



$$V_{ag} = Z_f I_a$$

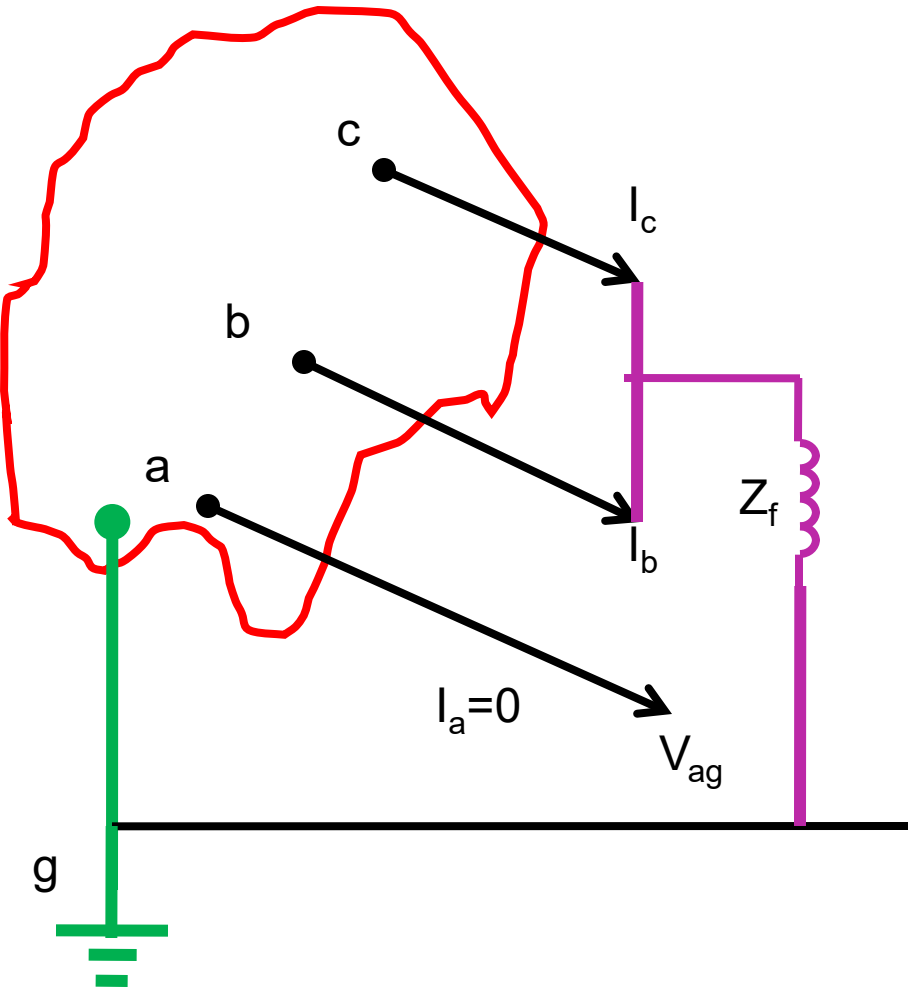
$$I_b = I_c = 0$$

# Interconnected sequence network



# Double Line-to-Ground Fault:

Fault conditions in phase domain:



$$I_a = 0$$

$$V_{bg} = V_{cg} = Z_f(I_b + I_c)$$

$$V_{bg} = V_{cg} \quad ??$$

$$(V_0 + a^2V_1 + aV_2) = (V_0 + aV_1 + a^2V_2)$$

$$(-a^2 + a)V_2 = (-a^2 + a)V_1$$

$$V_2 = V_1$$

$$V_{bg} = Z_F (I_b + I_c)$$

$$(V_0 + a^2 V_1 + a V_2) = Z_F (I_0 + a^2 I_1 + a I_2 + I_0 + a I_1 + a^2 I_2)$$

$$(V_0 - V_1) = Z_F (2I_0 - I_1 - I_2) \quad ?$$

$$(V_0 - V_1) = 3Z_F I_0 \quad ?$$

$$I_a = 0$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_f \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$$\begin{aligned} I_a &= 0 \\ (I_0 + I_1 + I_2) &= 0 \\ I_0 &= -(I_1 + I_2) \\ (a^2 + a) &= -1 \end{aligned}$$

$$\underbrace{V_{bg}} = \underbrace{V_{cg}} = \underbrace{Z_f(I_b + I_c)}$$

$$(-Z_0 I_0) - (V_f - I_1 Z_1) = 3Z_F I_0$$

$$(a^2 + a) = -1$$

$$I_0 = -\frac{V_f - Z_1 I_1}{Z_0 + 3Z_f} \quad ?$$

$$(V_0 - V_1) = 3Z_F I_0$$

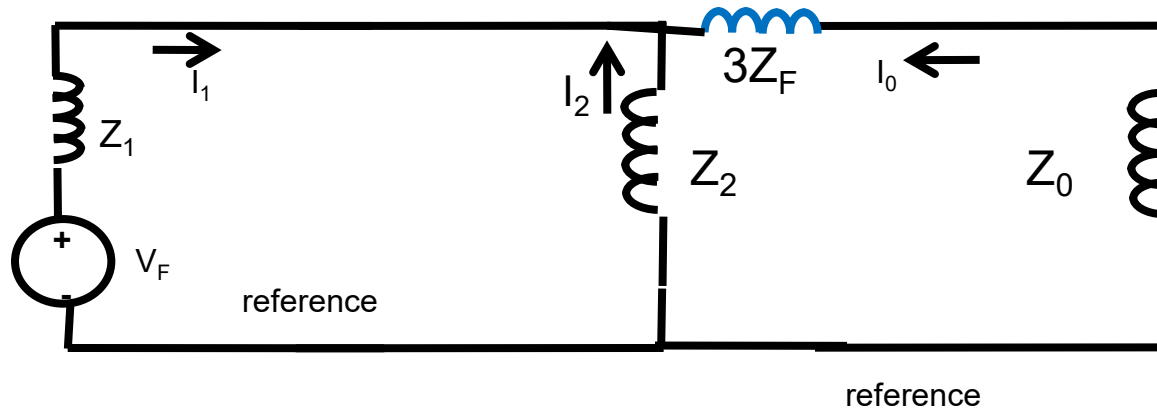
$$V_2 = V_1$$

$$(-I_2 Z_2) = V_f - Z_1 I_1$$

$$(I_0 + I_1 + I_2) = 0$$

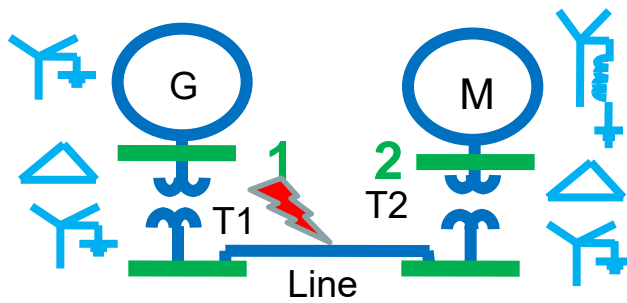
$$I_2 = -\frac{V_f - Z_1 I_1}{Z_2} \quad ?$$





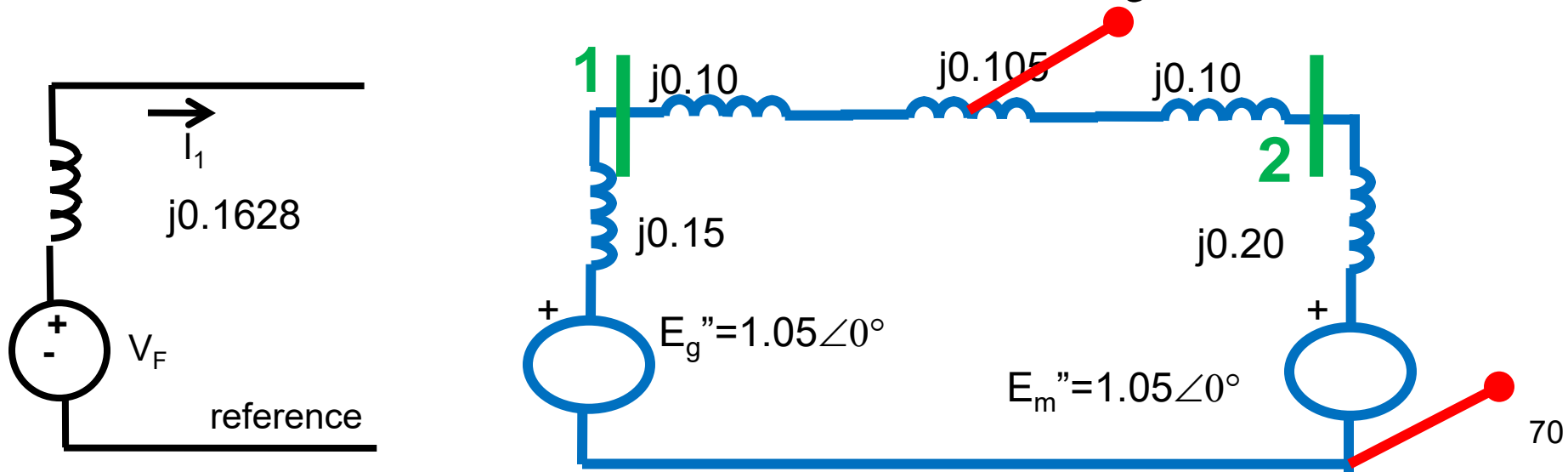
$$I_1 = \frac{V_F}{Z_1 + [Z_2 \parallel (Z_0 + 3Z_F)]} = \frac{V_F}{Z_1 + \left[ \frac{Z_2(Z_0 + 3Z_F)}{Z_2 + Z_0 + 3Z_F} \right]} \quad ?$$

$$I_2 = (-I_1) \frac{(Z_0 + 3Z_F)}{Z_2 + Z_0 + 3Z_F} \quad I_0 = (-I_1) \frac{Z_2}{Z_2 + Z_0 + 3Z_F} \quad ?$$

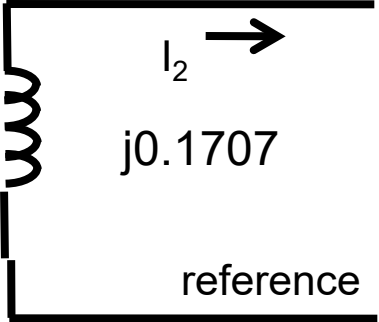
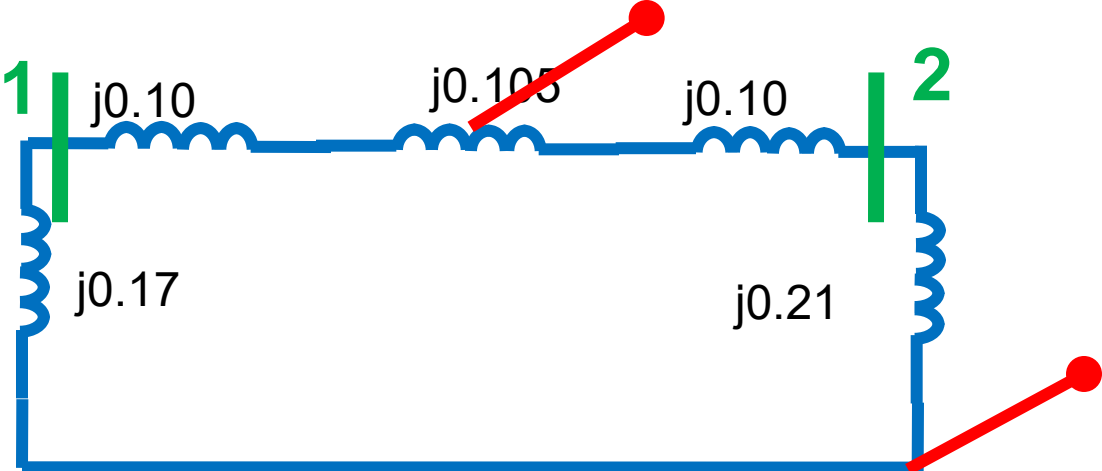


## Exercise

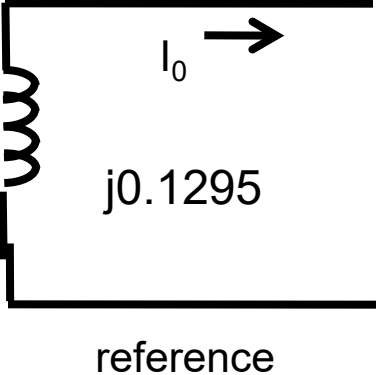
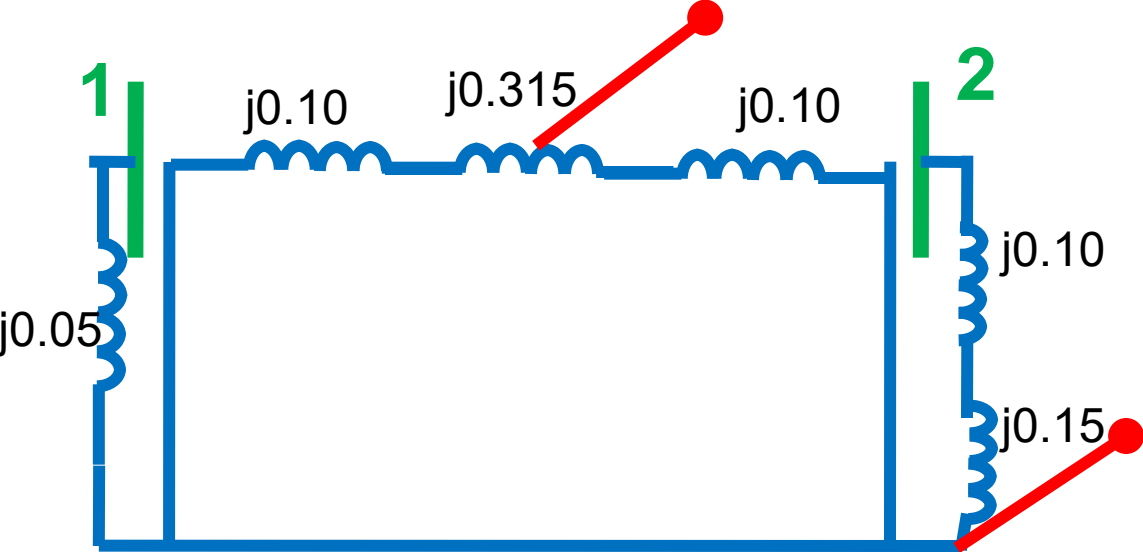
- Calculate the subtransient fault current in each phase, in per-unit and kA, for a bolted double line-to-ground fault from phase b to c in the middle of transmission line
- Neutral fault current
- Contribution to the fault current from the motor and the generator



# Negative sequence network



# Zero sequence network



## Example

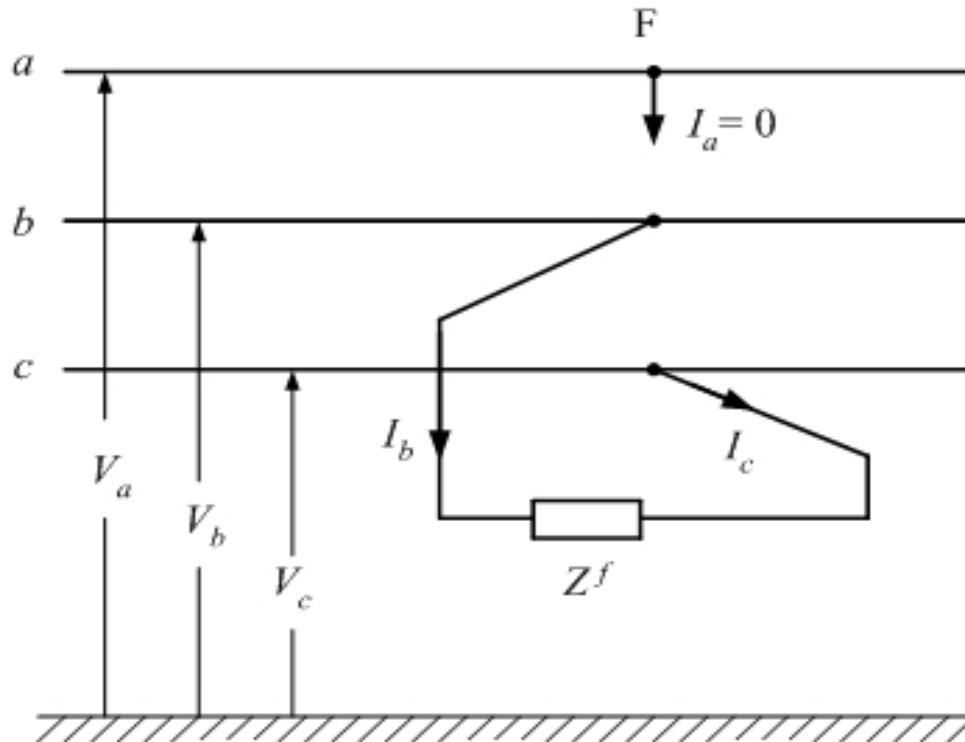
Q3. Explain how methods of symmetrical components is used to analyze unbalanced power systems. **(5 marks)**

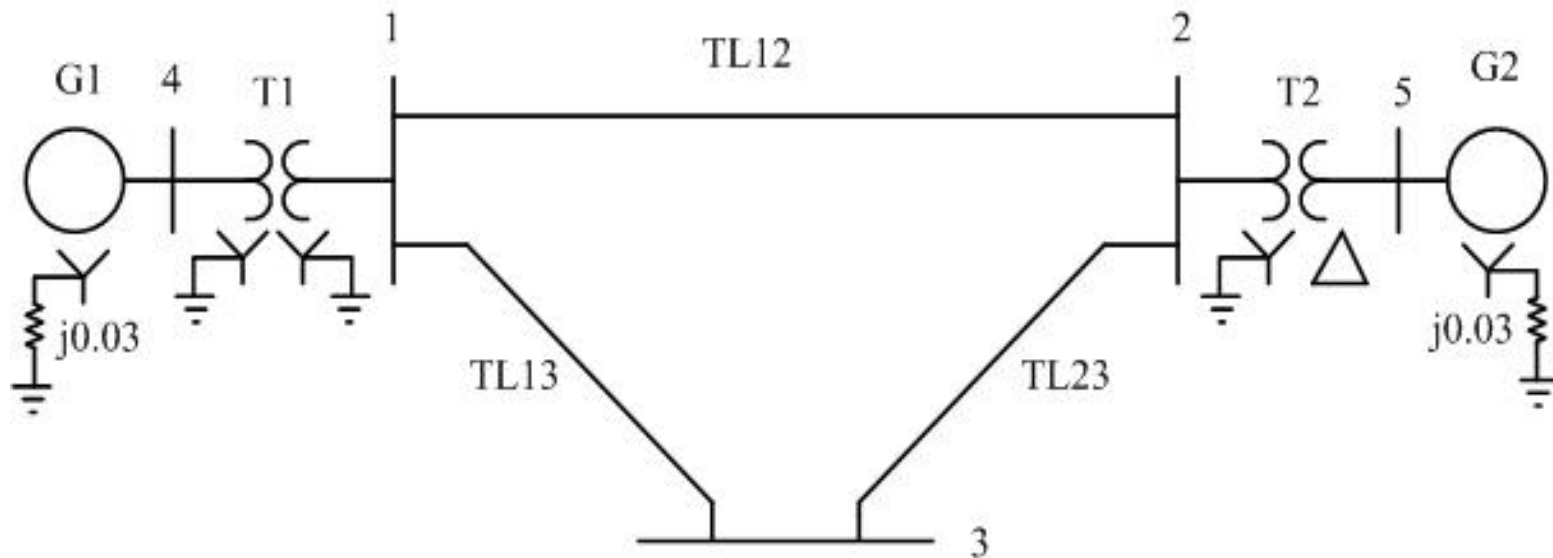
What are symmetrical components? Why are they used? **(5 marks)**

Q4. Show that the fault currents in a single line-to-ground fault of an unloaded generator can be calculated by connecting the three sequence networks in series so that  $I_{A1} = I_{A2} = I_{A0}$ . Where  $I_{A1}$ ,  $I_{A2}$ , and  $I_{A0}$  are the positive, negative and zero sequence currents respectively. **(6 marks)**



Figure Q3(a) shows a line to line fault at F in a power system on phases b and c through a fault impedance  $Z^f$ . Assume that the system is operating at no load before the occurrence of the fault. Show that the fault current can be calculated by connecting the positive sequence network and negative sequence network in parallel through a series impedance  $Z^f$ . [10 Marks]





### Synchronous generators

<b>G1</b>	<b>100 MVA</b>	<b>25 kV</b>	<b><math>X_1 = X_2 = 0.2</math></b>	<b><math>X_0 = 0.05</math></b>
<b>G2</b>	<b>100 MVA</b>	<b>13.8 kV</b>	<b><math>X_1 = X_2 = 0.2</math></b>	<b><math>X_0 = 0.05</math></b>

### Transformers

<b>T1</b>	<b>100 MVA</b>	<b>25/230 kV</b>	<b><math>X_1 = X_2 = X_0 = 0.05</math></b>	
<b>T2</b>	<b>100 MVA</b>	<b>13.8/230 kV</b>	<b><math>X_1 = X_2 = X_0 = 0.05</math></b>	

<b>TL12</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>
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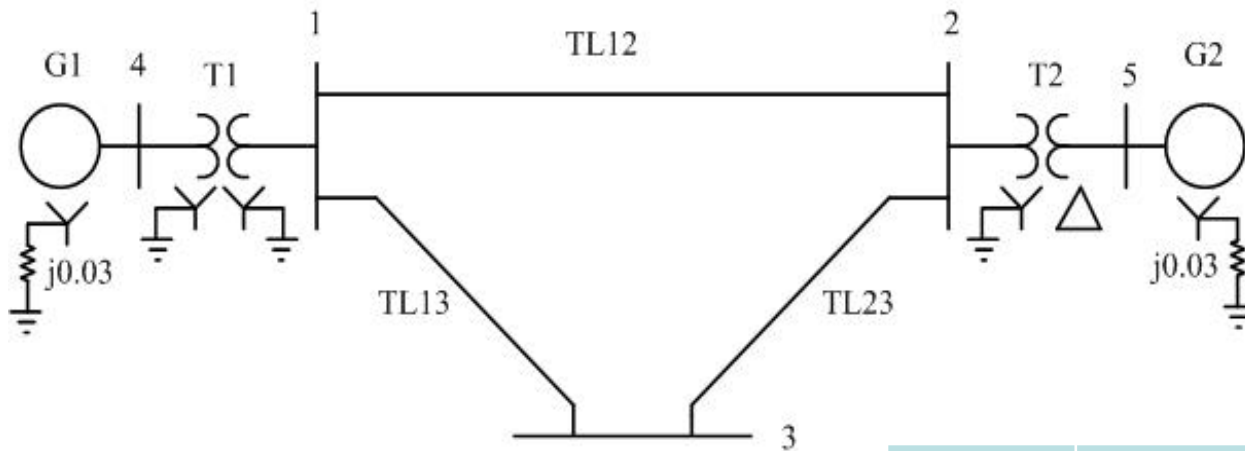
<b>TL13</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>
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<b>TL23</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>
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**All the reactances are based on 100 MVA and 230 kV at the transmission lines.**

**Compute fault current and voltages for double line to ground fault at the midpoint of transmission line TL12.**

**[20 marks]**

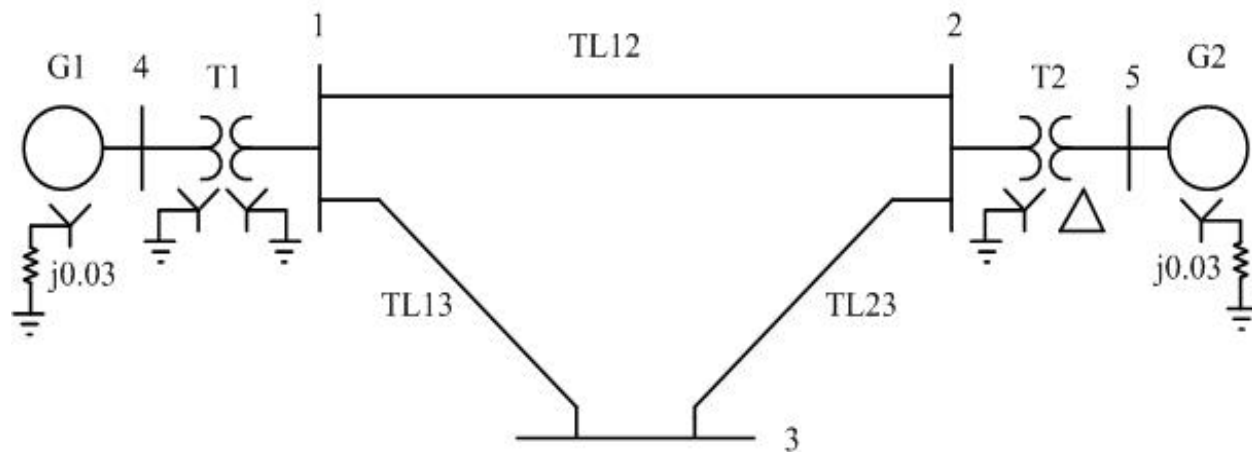


Synchronous generators			
<b>G1</b>	<b>100 MVA</b>	<b>25 kV</b>	<b><math>X_1 = X_2 = 0.2</math> <math>X_0 = 0.05</math></b>
<b>G2</b>	<b>100 MVA</b>	<b>13.8 kV</b>	<b><math>X_1 = X_2 = 0.2</math> <math>X_0 = 0.05</math></b>
Transformers			
<b>T1</b>	<b>100 MVA</b>	<b>25/230 kV</b>	<b><math>X_1 = X_2 = X_0 = 0.05</math></b>
<b>T2</b>	<b>100 MVA</b>	<b>13.8/230 kV</b>	<b><math>X_1 = X_2 = X_0 = 0.05</math></b>

<b>TL12</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>
<b>TL13</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>
<b>TL23</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>

**All the reactances are based on 100 MVA and 230 kV at the transmission lines.**

**Draw positive, negative and zero sequence network**



### Synchronous generators

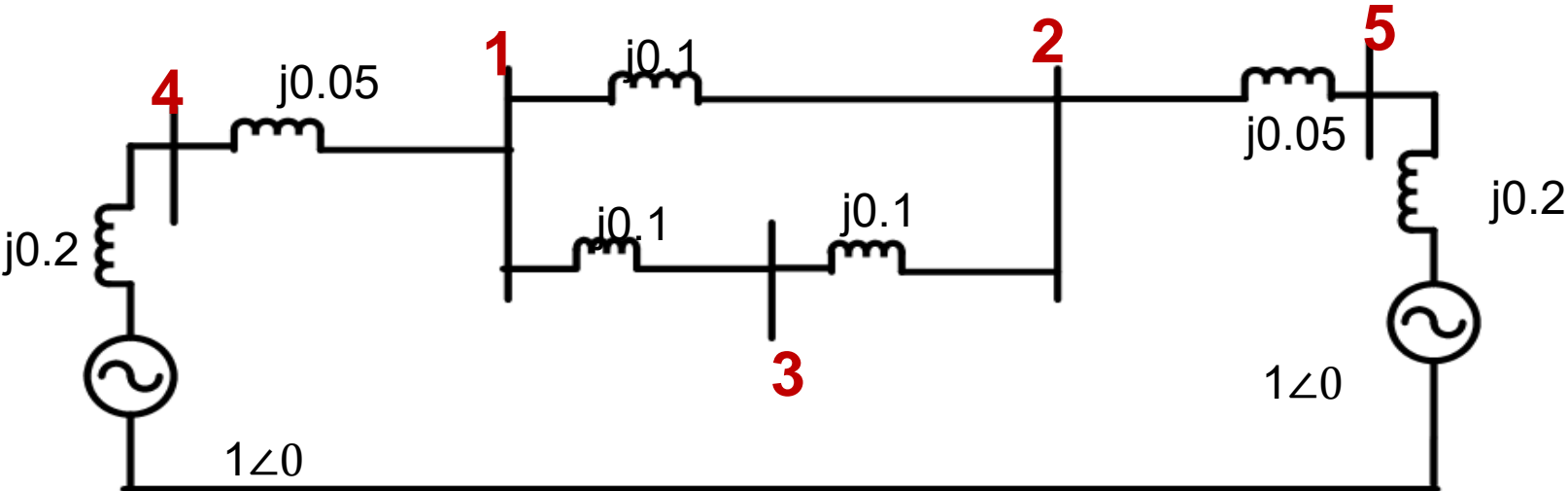
<b>G1</b>	<b>100 MVA</b>	<b>25 kV</b>	<b><math>X_1 = X_2 = 0.2</math></b>	<b><math>X_0 = 0.05</math></b>
<b>G2</b>	<b>100 MVA</b>	<b>13.8 kV</b>	<b><math>X_1 = X_2 = 0.2</math></b>	<b><math>X_0 = 0.05</math></b>
<b>Transformers</b>				
<b>T1</b>	<b>100 MVA</b>	<b>25/230 kV</b>	<b><math>X_1 = X_2 = X_0 = 0.05</math></b>	
<b>T2</b>	<b>100 MVA</b>	<b>13.8/230 kV</b>	<b><math>X_1 = X_2 = X_0 = 0.05</math></b>	

<b>TL12</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>
<b>TL13</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>
<b>TL23</b>	<b><math>X_1 = X_2 = 0.1</math></b>	<b><math>X_0 = 0.3</math></b>

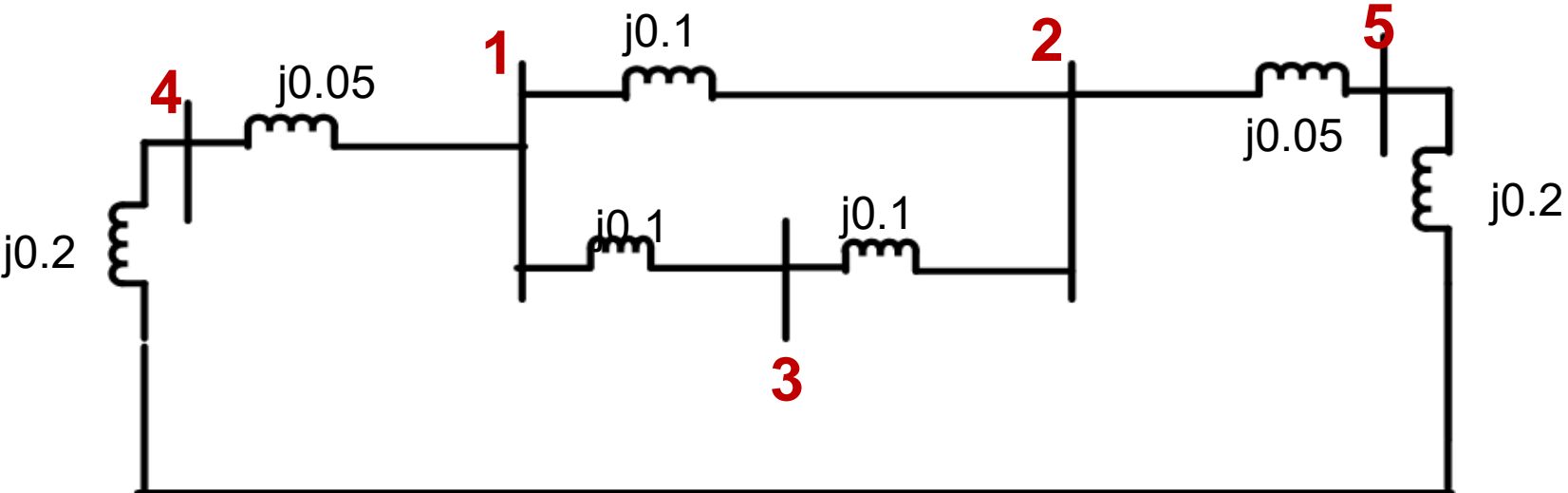
**All the reactances are based on 100 MVA and 230 kV at the transmission lines.**

**Draw positive, negative and zero sequence network**

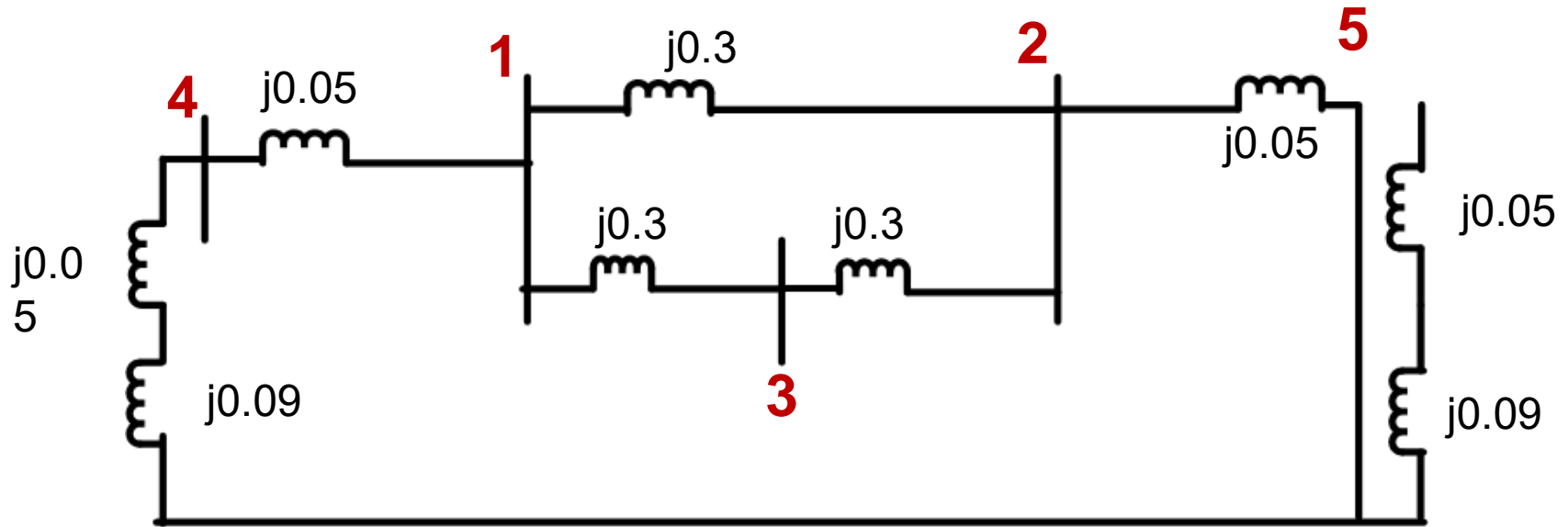
# Positive sequence network



# Negative sequence network



## Zero sequence network



- Reduce the networks to their Thevenin equivalents looking at **bus 1**
- **Neglect phase shifts, compute:**
- **Fault currents for a double line to earth fault at bus 1**

