

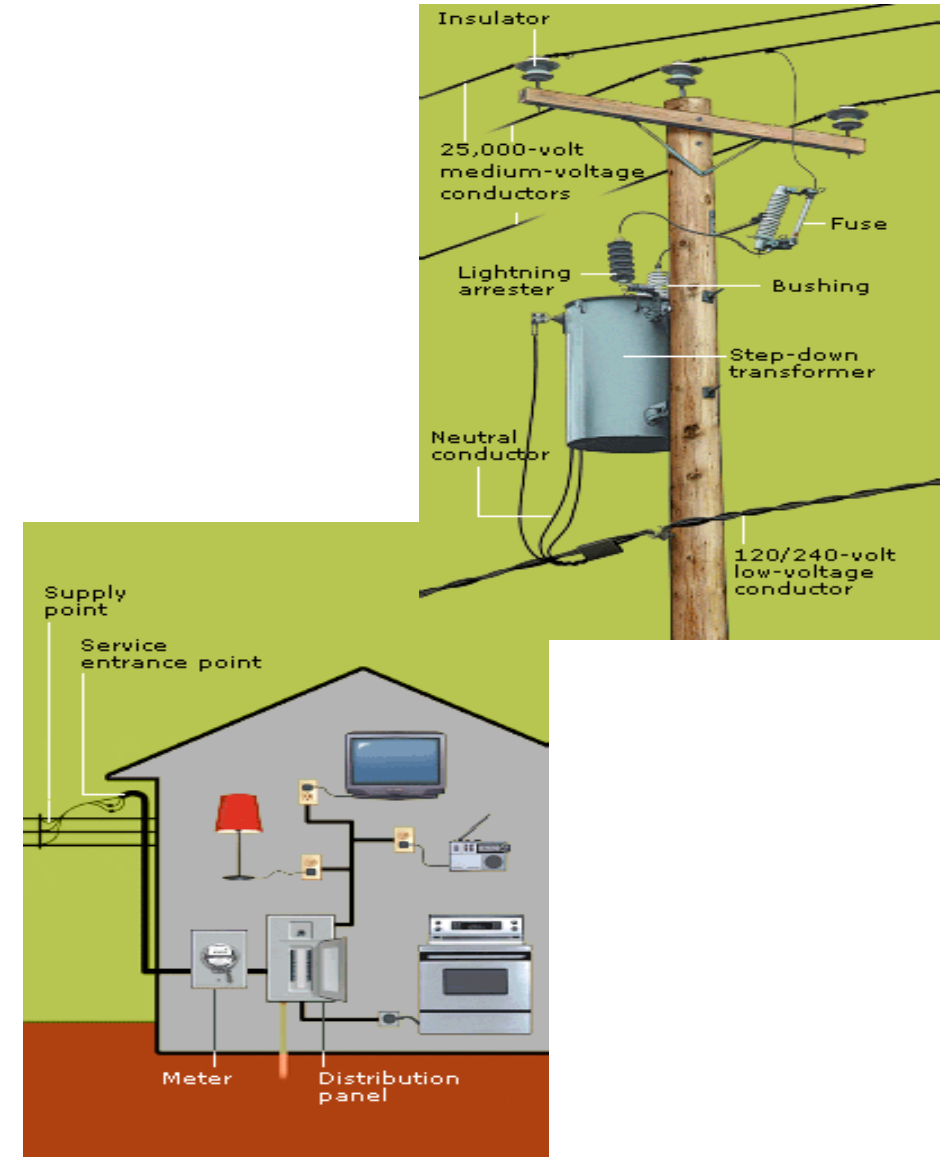
# CHAPTER FIVE

TOPIC:

**SECONDARY DISTRIBUTION SYSTEM**

# Introduction

- The part of the electric utility system that is between the **primary system** and **the consumer's property** is called the **secondary system**.
- Secondary distribution systems include:
  - step-down distribution transformers,
  - secondary circuits (secondary mains),
  - consumer services (or SDs), and
  - meters.
- Generally, the secondary distribution systems are designed in single phase for areas of residential customers and in three phase for areas of industrial or commercial customers with high-load densities.

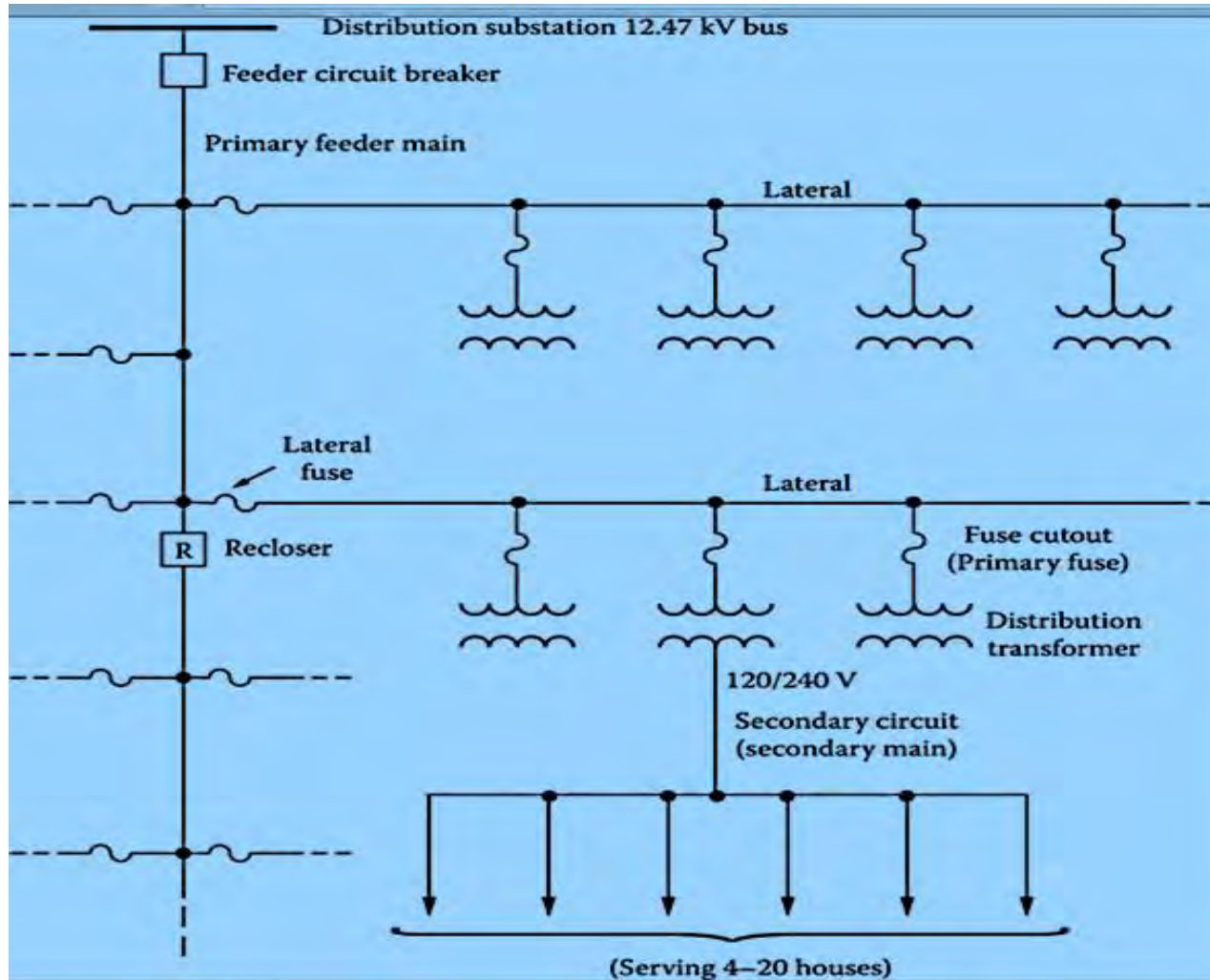


# Types of the secondary distribution systems

- The types of the secondary distribution systems include the following:
- 1. **The separate-service system for each consumer with separate distribution transformer and secondary connection :**
  - 1 transformer for 1 consumer
  - It is rarely used.
- 2. **The radial system with a common secondary main, which is supplied by one distribution transformer and feeding a group of consumers:**
  - 1 transformer for a group of consumers:
  - It is commonly used .
- 3. **The secondary-bank system with a common secondary main that is supplied by several distribution transformers, which are all fed by the same primary feeder :**
  - many transformer for a group of consumers:
  - fed by the same primary feeder
- 4. **The secondary-network system with a common grid-type main:**
  - supplied by a large number of the distribution transformers,
  - fed by the many primary feeder

# Types of the secondary distribution systems

- One-line diagram of a simple radial secondary system

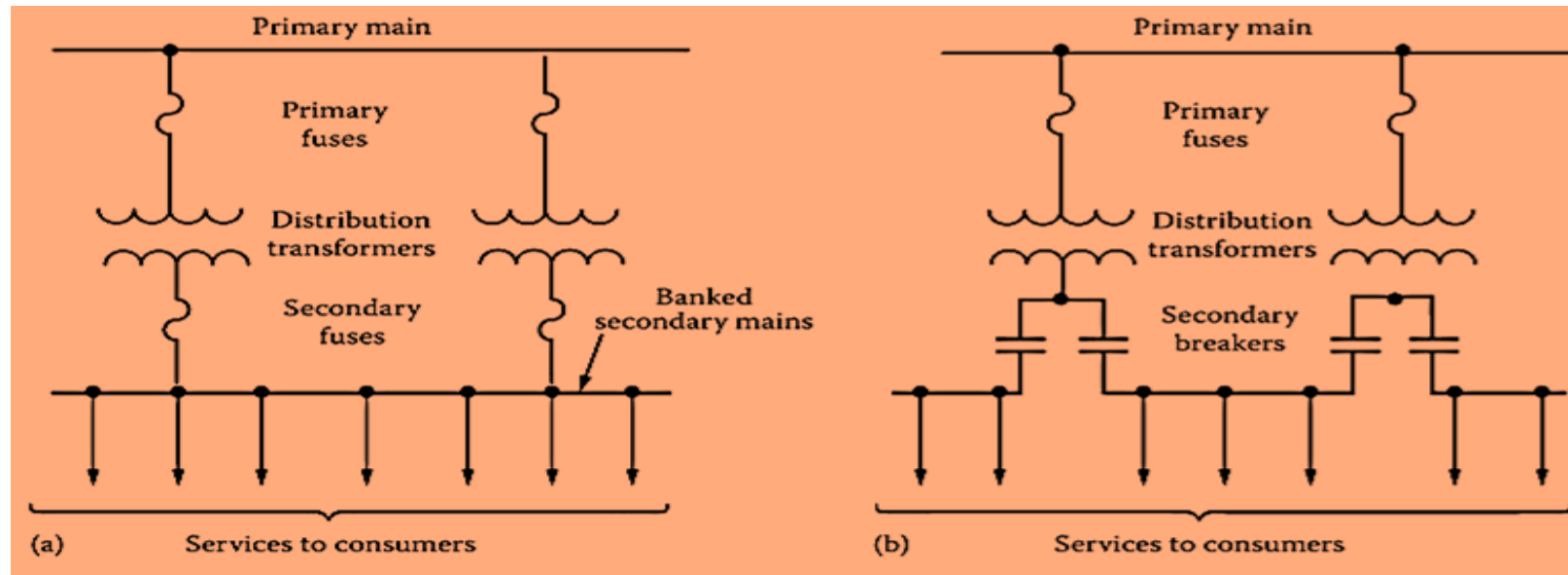


## Note:

Most of the secondary systems for serving residential, rural, and light-commercial areas are radial designed.

# Types of the secondary distribution systems

- One-line diagram of a secondary-bank system with a common secondary main



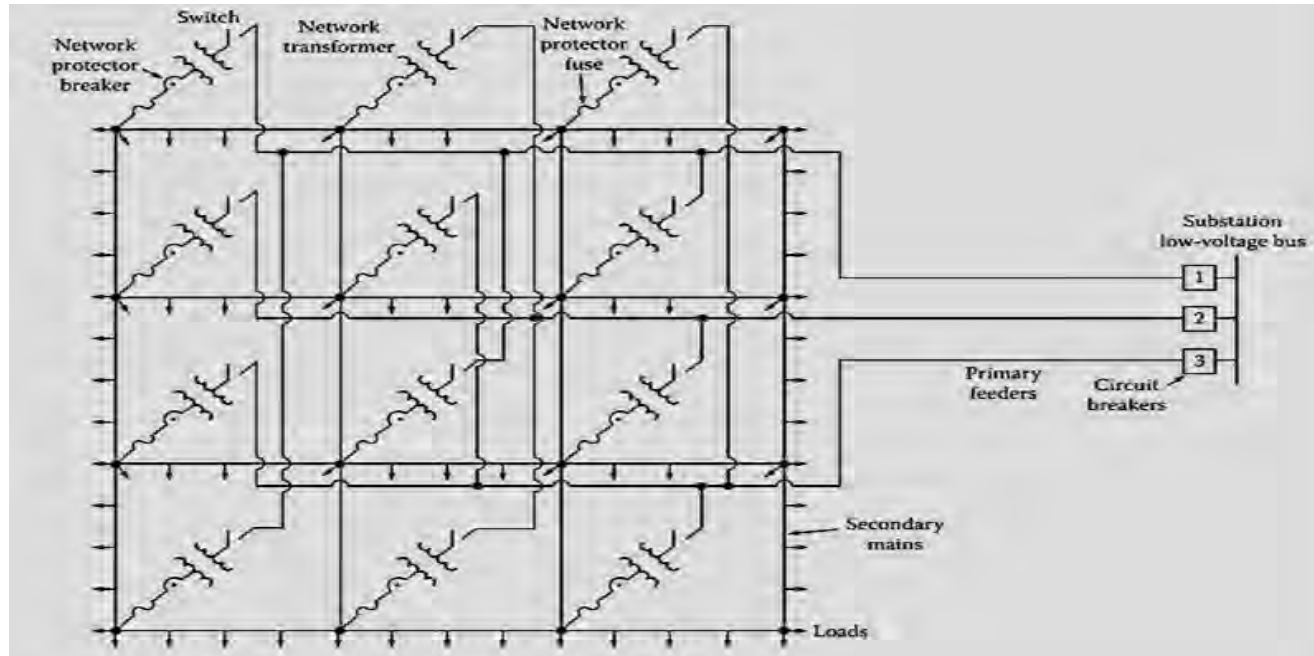
## Note:

This system have Improved voltage regulation, Reduced voltage dip, Improved reliability.

This system has difficulty in fuse coordination, and circulating current may exist.

# Types of the secondary distribution systems

- One-line diagram of a secondary-network system with a common grid-type main



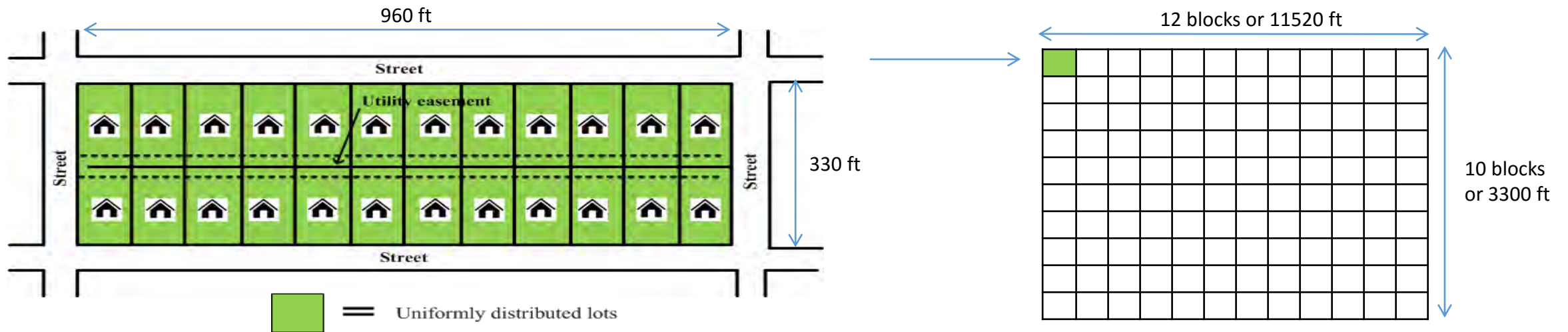
## Note:

This system is used in the areas of high-load density. Underground system is preferred to avoid overhead congestion

# Economic Design of Secondaries

## Patterns and Some of the Variables

- Economic design of a secondary distribution refers to minimizing the total annual cost (TAC) of owning and operating the secondary portion of a three-wire single phase distribution network.
- Can be applied either to OH or URD construction.
- The design must satisfy voltage-drop and voltage-dip performance
- It is hoped that a design for satisfactory VD performance will agree at least reasonably well with the design for minimum TAC
- Consider the following residential area, where secondary circuit is to be designed

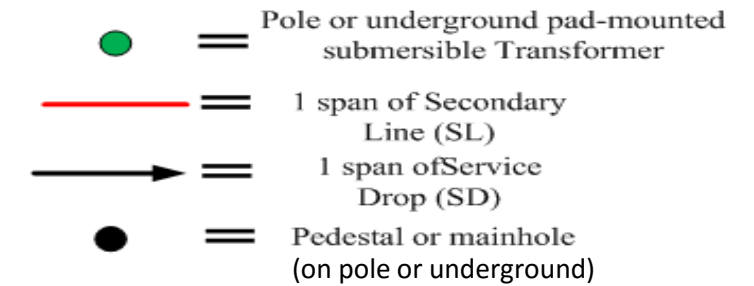
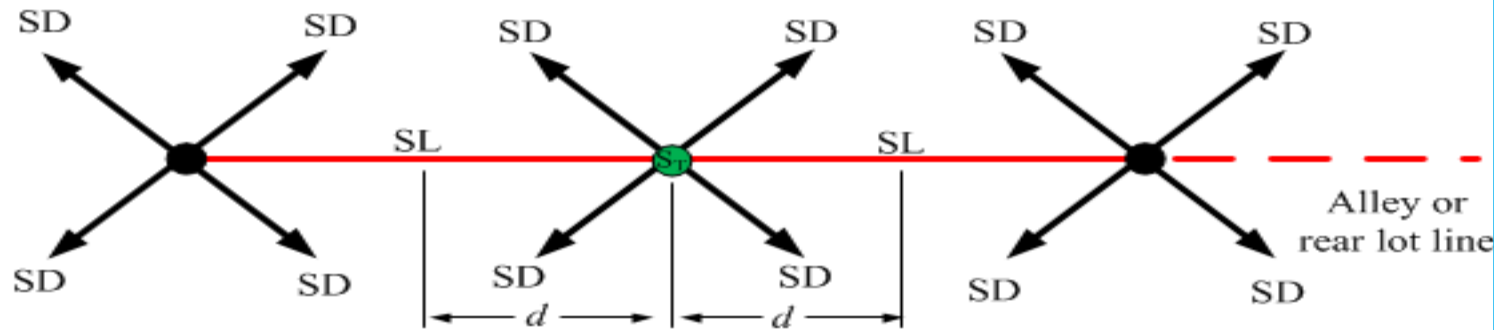


**Residential area(one block) lot layout and utility easement arrangement.**

# Economic Design of Secondaries

## Patterns and Some of the Variables

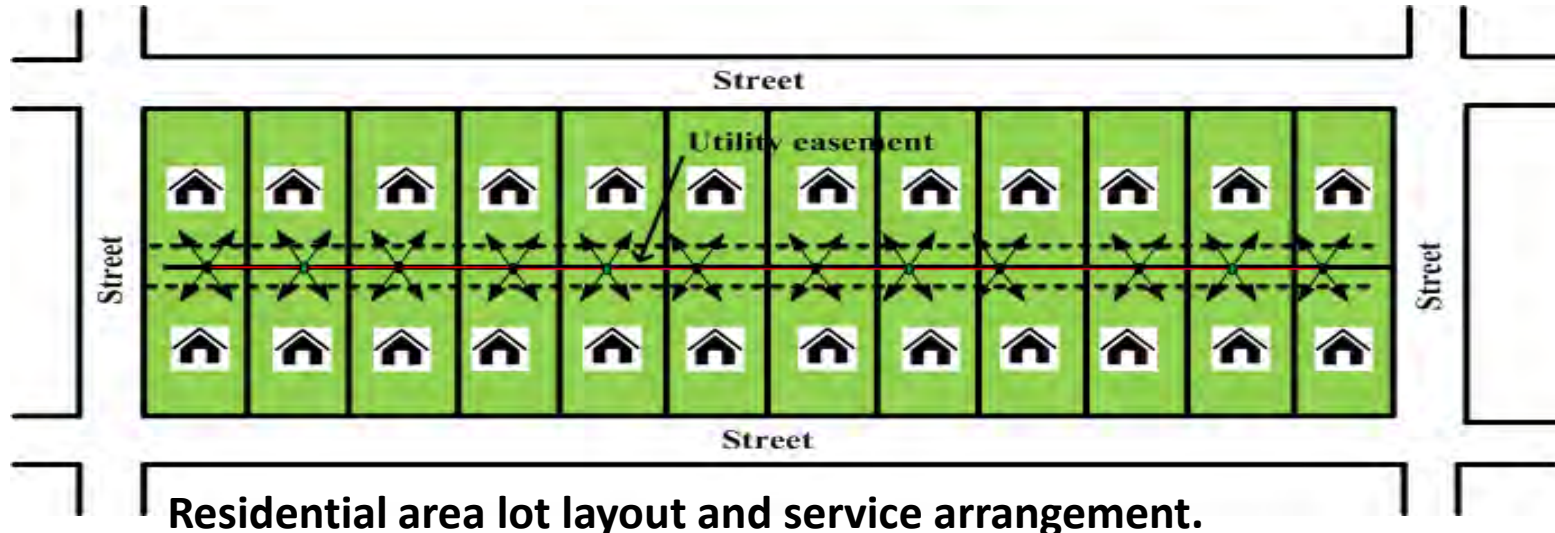
- Possible pattern of secondary circuit for such residential area, having 1 span of SL each way from the distribution transformer are shown below. The system is assumed to be built in a straight line along an alley or along rear lot lines. The lots are assumed to be of uniform width  $d$  so that each span of SL is of length  $2d$ . If SLs are not used, then there is a distribution transformer on every pole and OH construction, and every transformer supplies four SDs.



Number of consumers  $12=SD$ , length of  $SD = 70ft$   
 Number of secondary lines  $2$ , length of  $SL = 2d$   
 Number of transformer  $=1$

### Assumptions:

- lots have uniform width  $d$
- $SL = 2d$  (each span of SL)
- Transformer on every pole
- 1 Transformer serve 4 SD
- 2 wires/SD





# Economic Design of Secondaries

## Patterns and Some of the Variables

- The number of spans of SLs each way from a transformer is an important variable.
- No SL is used in **high-load density** areas. In light-load density areas, **three or more spans** of SL each way from the transformer may be used.
- OH system, the transformer with its arrester and fuse cutout is pole mounted.
- SL and SD may be of either open-wire or triplex cable construction.
- URD, the transformer is grade mounted on a concrete slab and completely enclosed in a grounded metal housing or it is submersibly installed in a hole lined with concrete.
- SL and SDs are triplex or twin concentric neutral direct-burial cable laid in narrow trenches

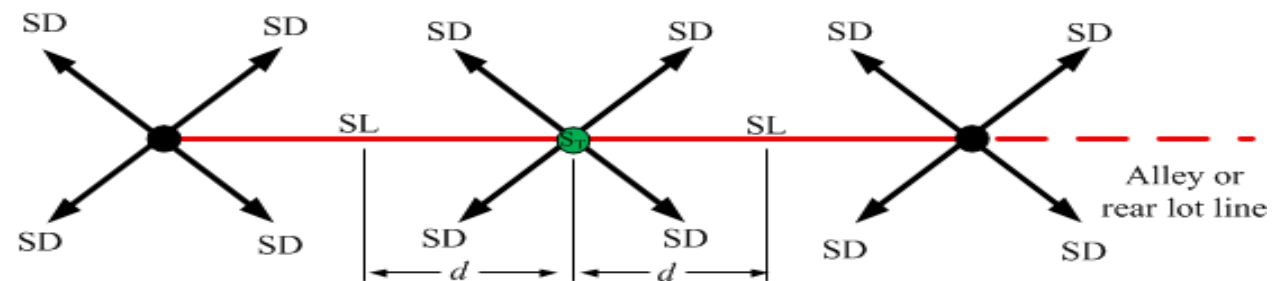
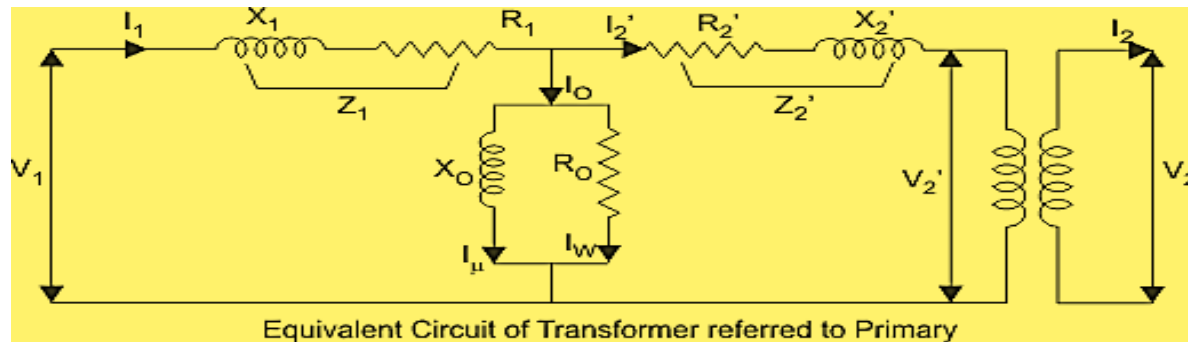


Illustration of a typical pattern

# Economic Design of Secondaries

## Patterns and Some of the Variables

- The distribution transformers have the parameters defined in the following:
  - $S_T$  is the transformer capacity, continuously rated kVA.
  - $I_{exc}$  is the per unit exciting current (based on  $S_T$ ).
  - $P_{T,Fe}$  is the transformer core loss at rated voltage and rated frequency, kW.
  - $P_{T,Cu}$  is the transformer copper loss at rated kVA load, kW.



# Economic Design of Secondaries

## Patterns and Some of the Variables

- The SL has the parameters defined in the following:
  - $A_{SL}$  is the conductor area, kcmil.
  - $\rho$  is the conductor resistivity,  $(\Omega \cdot \text{cmil})/\text{ft.} = 20.5$  at  $65^\circ\text{C}$  for aluminum cable.
  
- The SD has the parameters defined in the following:
  - $A_{SD}$  is the conductor area, kcmil.
  - $\rho$  is the conductor resistivity,  $(\Omega \cdot \text{cmil})/\text{ft.} = 20.5$  at  $65^\circ\text{C}$  for aluminum cable.

$r_s$ Resistance ( $\Omega/\text{Conductor}/\text{mi}$ )								$X_s$ Inductive Reactance ( $\Omega/\text{Conductor}/\text{mi}$ ) at 1 ft Spacing			$X'_s$ Shunt Capacitive Reactance ( $\text{M}\Omega \cdot \text{mi}/\text{Conductor}$ ) at 1 ft Spacing		
25°C (77°F)				50°C (122°F)									
DC	25 Cycles	50 Cycles	60 Cycles	DC	25 Cycles	50 Cycles	60 Cycles	25 Cycles	50 Cycles	60 Cycles	25 Cycles	50 Cycles	60 Cycles
0.0585	0.0594	0.0620	0.0634	0.0640	0.0648	0.0672	0.0685	0.1666	0.333	0.400	0.216	0.1081	0.0901
0.0650	0.0658	0.0682	0.0695	0.0711	0.0718	0.0740	0.0752	0.1693	0.339	0.406	0.220	0.1100	0.0916
0.0731	0.0739	0.0760	0.0772	0.0800	0.0808	0.0826	0.0837	0.1722	0.344	0.413	0.224	0.1121	0.0934

# Economic Design of Secondaries

## Patterns and Some of the Variables: Further Assumptions

- 1. All secondaries and services are single phase three wire and nominally 120/240 V.
- 2. Perfectly balanced loading obtains in all three-wire circuits.
- 3. The system is energized 100% of the time, that is, 8760 h/year.
- 4. The annual loss factor is estimated by using  $F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2$
- 5. The annual peak-load kilovolt-ampere loading in any element of the pattern, that is, SD, section of SL, or transformer is estimated by using the maximum diversified demand.
- 6. Current flows are estimated in kilovolt-amperes and nominal operating voltage, usually 240 V.
- 7. All loads have the same (and constant) power factor.

# Economic Design of Secondaries

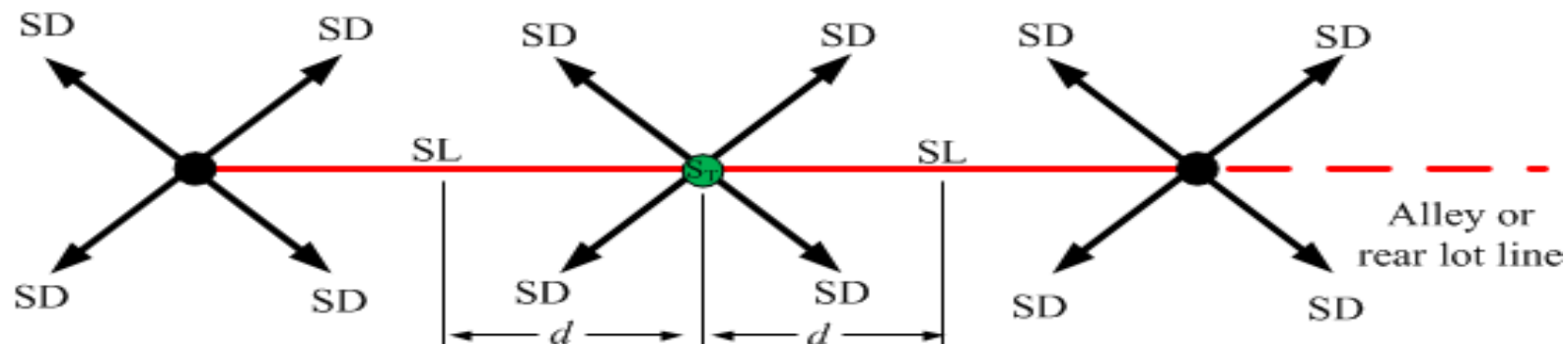
## Total Annual Cost

- The TAC of owning and operating one pattern of the secondary system is a summation of investment (fixed) costs (ICs) and operating (variable) costs (OCs).

- transformer related terms ( $IC_T, OC_{exc}, OC_{T,Fe}, OC_{T,Cu}$ )
- secondary line related terms ( $IC_{SL}, OC_{SL,Cu}$ )
- service drop related terms ( $IC_{SD}, OC_{SD,Cu}$ )
- poles related ( $IC_{PH}$ )

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

- The summations are to be taken for the one standard pattern being considered, like fig. below, but modified appropriately for the number of spans of SL being considered.
- The TAC may be divided by the number of customers per pattern so that the TAC can be allocated on a per customer basis.



# Economic Design of Secondaries

## Assembly of Cost Data

- The following cost data are sufficient for illustrative purposes but not necessarily of the accuracy required for engineering design in commercial practice.
- The cost data given may be quite inaccurate because of recent, severe inflation.
- The data are intended to represent an OH system using three-conductor triplex aluminum cable for both SLs and SDs.
- The important aspect of the following procedures is the finding of equations for all costs so that analytical methods can be employed to minimize the TAC:

# Economic Design of Secondaries

## Assembly of Cost Data

- $IC_T$  is the annual installed cost of the distribution transformer + associated protective equipment.

$$IC_T = (250 + 7.26 \times S_T) \times i \quad \$/\text{transformer}$$

Where

$S_T$  is the transformer-rated kVA

$i$  is the pu fixed charge rate on investment

- transformer related terms ( $IC_T, OC_{exc}, OC_{T,Fe}, OC_{T,Cu}$ )
- secondary line related terms ( $IC_{SL}, OC_{SL,Cu}$ )
- service drop related terms ( $IC_{SD}, OC_{SD,Cu}$ )
- poles related ( $IC_{PH}$ )

- $IC_{SL}$  is the annual installed cost of triplex aluminum SL cable

$$IC_{SL} = (60 + 4.5 \times A_{SL}) \times i \quad \$/1000 \text{ ft}$$

Where

$A_{SL}$  is the conductor area, kcmil

$i$  is the pu fixed charge rate on investment

**Note:**

that this cost is 1000 ft of cable, that is, 3000 ft of conductor.

- $IC_{SD}$  is the annual installed cost of triplex aluminum SD cable

$$IC_{SD} = (60 + 4.5 \times A_{SD}) \times i \quad \$/1000 \text{ ft}$$

Where

$A_{SD}$  is the conductor area, kcmil

$i$  is the pu fixed charge rate on investment

**Note:**

Both equations are alike because the same triplex aluminum cable, is assumed to be used for both SL and SD construction

# Economic Design of Secondaries

- transformer related terms( $IC_T, OC_{exc}, OC_{T,Fe}, OC_{T,Cu}$ )
- secondary line related terms( $IC_{SL}, OC_{SL,Cu}$ )
- service drop related terms( $IC_{SD}, OC_{SD,Cu}$ )
- poles related( $IC_{PH}$ )

## Assembly of Cost Data

- $IC_{PH}$  is the annual installed cost of pole and hardware on it but excluding transformer and transformer protective equipment

$$IC_{PH} = (\$160) \times i \quad \$/\text{pole}$$

Where

$i$  is the pu fixed charge rate on investment

### Note:

In case of URD design, the cost item  $IC_{PH}$  would designate the annual installed cost of a secondary pedestal or manhole.

- $OC_{exc}$  is the annual operating cost of transformer exciting current

$$OC_{exc} = (I_{exc} \times S_T \times IC_{cap}) \times i \quad \$/\text{transformer}$$

Where

$IC_{cap}$  is the total installed cost of primary-voltage shunt capacitors = \$5.00/kvar

$I_{exc}$  is the an average value of transformer exciting current based on  $S_T$  kVA rating = 0.015 pu

- $OC_{T,Fe}$  is the annual operating cost of transformer due to core (iron) losses

$$OC_{T,Fe} = (IC_{sys} \times i + 8760 \times EC_{off}) \times P_{T,Fe} \quad \$/\text{transformer}$$

Where

$IC_{sys}$  is the average investment cost of power system upstream, that is, toward generator, from distribution transformers = \$350/kVA

$EC_{off}$  is the incremental cost of electric energy (off-peak) = \$0.008/kWh

$P_{T,Fe}$  is the annual transformer core loss, kW =  $0.004 \times S_T$  15 kVA  $\leq S_T \leq$  100 kVA



# Economic Design of Secondaries

## Assembly of Cost Data

- transformer related terms( $IC_T, OC_{exc}, OC_{T,Fe}, OC_{T,Cu}$ )
- secondary line related terms( $IC_{SL}, OC_{SL,Cu}$ )
- service drop related terms( $IC_{SD}, OC_{SD,Cu}$ )
- poles related( $IC_{PH}$ )

- $OC_{T,Cu}$  is the annual operating cost of transformer due to copper losses

$$OC_{T,Cu} = (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times \left( \frac{S_{max}}{S_T} \right)^2 \times P_{T,Cu} \quad \$/\text{transformer}$$

Where

$EC_{on}$  is the incremental cost of electric energy (on-peak) = \$0.010/kWh

$S_{max}$  is the annual maximum kVA demand on transformer (from table = (no. of consumers/tx) X (kVA/consumer))

$P_{T,Cu}$  is the transformer copper loss, kW at rated kVA load=  $0.073 + 0.00905 \times S_T$

$F_{LS}$  is the annual loss factor, (from eq.)  $F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2$

- $OC_{SL,Cu}$  is the annual operating cost of copper loss in a unit length of SL

$$OC_{SL,Cu} = (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times P_{SL,Cu}$$

Where

$P_{SL,Cu}$  is the power loss in a unit of SL at time of annual peak load due to copper losses, kW

$P_{SL,Cu}$  is an  $I^2R$  loss, and it must be related to conductor area  $A_{SL}$  with  $R = \rho L/A_{SL}$

**Note:**

different sections of SLs may have different values of current and, therefore, different  $P_{SL,Cu}$ .

# Economic Design of Secondaries

## Assembly of Cost Data

- transformer related terms( $IC_T, OC_{exc}, OC_{T,Fe}, OC_{T,Cu}$ )
- secondary line related terms( $IC_{SL}, OC_{SL,Cu}$ )
- service drop related terms( $IC_{SD}, OC_{SD,Cu}$ )
- poles related( $IC_{pH}$ )

- $OC_{SD,Cu}$  is the annual operating cost of copper loss in a unit length of SD

$$OC_{SD,Cu} = (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times P_{SD,Cu} \quad \$/SD$$

Where

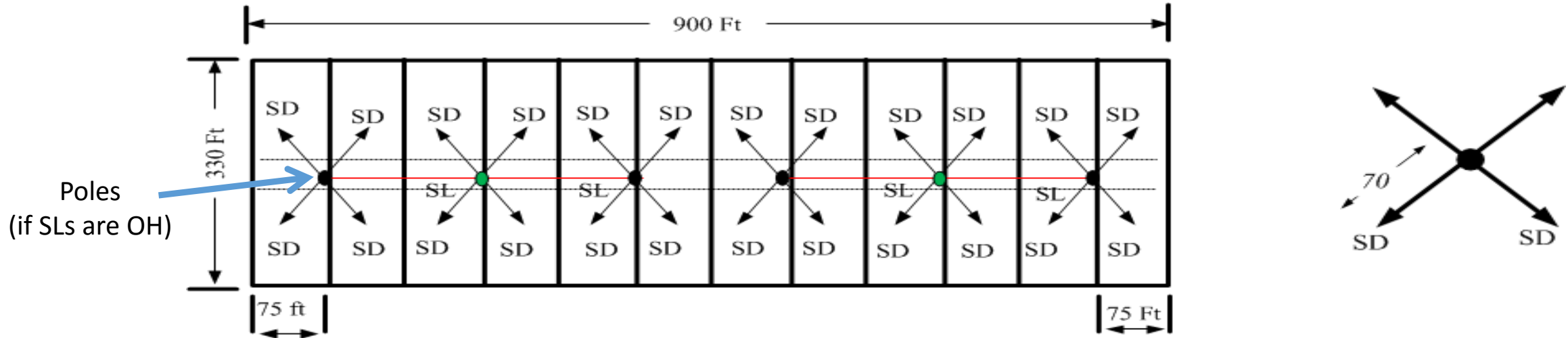
$P_{SD,Cu}$  is the power loss in a unit of SD at time of annual peak load due to copper losses, kW

$P_{SD,Cu}$  is an  $I^2R$  loss, and it must be related to conductor area ASD with  $R = \rho L/ASD$

## Example

# Economic Design of Secondaries

- This example deal with cost of a single-phase OH secondary distribution system in a residential area.
- For the layouts and the service arrangement shown in the figure below calculate the TAC as a function of ST, ASD and ASL
- Equal lot width and uniform load spacing are assumed. (All SDs = 70 ft long)



**Residential area lot layout and service arrangement.**

The calculations for one block of the residential area for the case of 12 services per transformer (2 transformer per block)

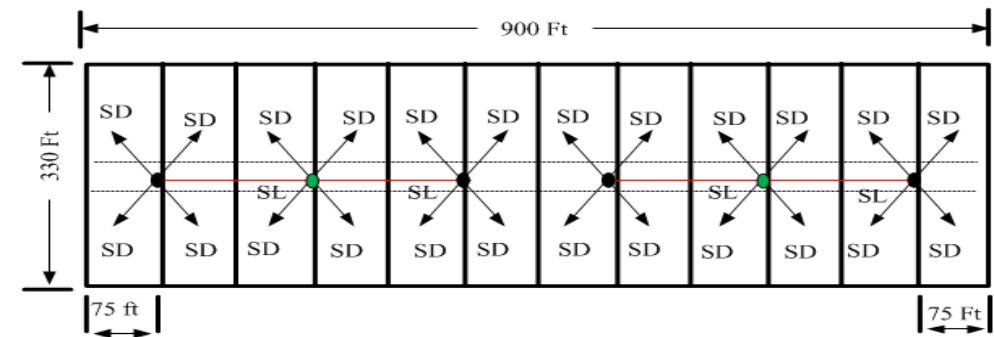
- One consumer per SD
- Four consumers per section of SL
- Twelve consumers per transformer

No. of customers being diversified	30-Min. Annual Max. Demands, kVA/Customer		
	Class 1	Class 2	Class 3
1	18	10	2.5
2	14.4	7.6	1.8
4	12	6	1.5
12	10	4.4	1.2
100	8.4	3.6	1.1

# Economic Design of Secondaries

## Example

- For the layouts and the service arrangement shown in the figure below calculate the TAC eq. applicable to one block of these residential areas for the case of 12 services per transformer (2 transformer per block). Use the following data and assumptions:
- 1. All secondaries and services are single phase three wire, nominally 120/240 V.
- 2. Assume perfectly balanced loading in all single-phase three-wire circuits.
- 3. Assume that the system is energized 100% of the time, that is, 8760 h/year.
- 4. Assume the annual load factor to be  $F_{LD} = 0.35$ .
- 5. Assume the annual loss factor to be  $F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2$
- 6. Assume that there are 12 services per transformers.
- 7. Assume nominal operating voltage of 240 V when computing currents.
- 8. Assume a 90% power factor for all loads.
- 9. Assume a fixed charge (capitalization) rate of 0.15.
- Assume  $\rho$  is 20.5 ( $\Omega \cdot \text{cmil}$ )/ft at 65°C for aluminum cable
- 10. Use 30 min annual maximum demands for customer class 2



# Economic Design of Secondaries

## Solution

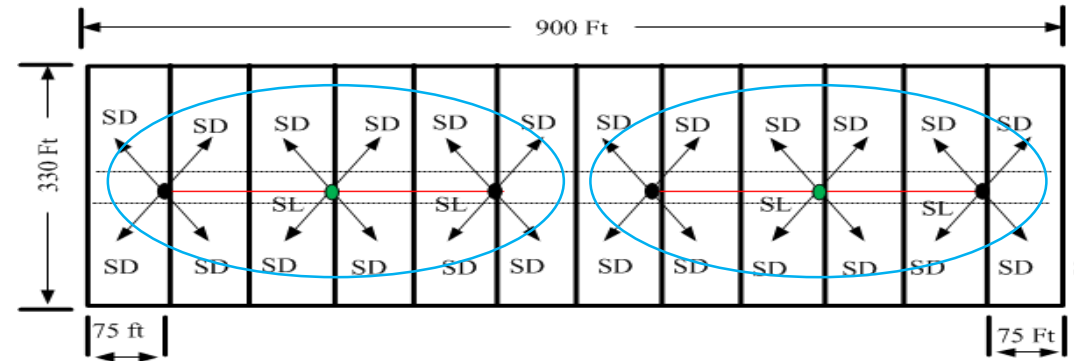
- TAC is given by

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

- Calculate each cost independently

### All investment (fixed) costs (ICs)

- Given : fixed charge (capitalization) rate of 0.15
- Known:
- 12 services per transformer
- 2 transformers per block



- The annual installed cost of the two distribution transformer and associated protective equipment:

$$IC_T = (250 + 7.26 \times S_T) \times i \quad \$/\text{transformer}$$

$$IC_T = 2(250 + 7.26 \times S_T) \times 0.15$$

$$= 75 + 2.175 S_T \quad \$/\text{block}$$

# Economic Design of Secondaries

**Solution**

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

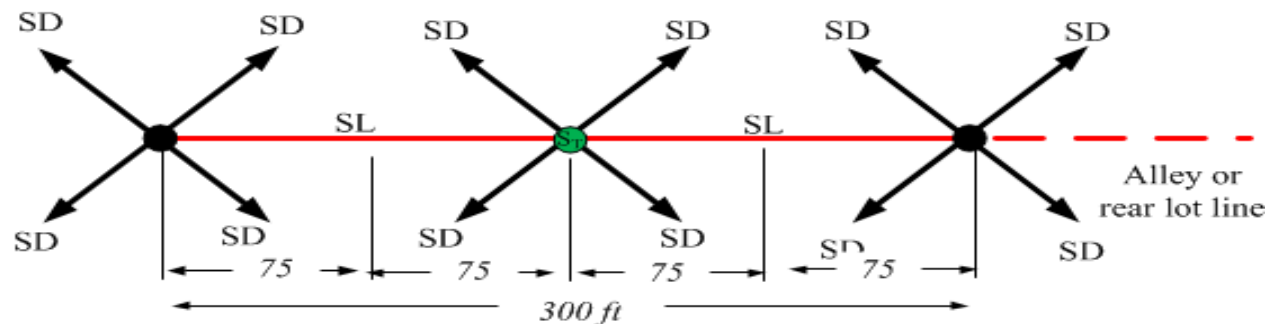
- The annual installed cost of the triplex aluminum SL cable used for 300 ft/transformer (since there is 150 ft SL at each side of each transformer) in the SLs is:

$$IC_{SL} = (60 + 4.5 \times A_{SL}) \times i \quad \$/1000 \text{ ft}$$

$$IC_{SL} = 2(60 + 4.5 \times A_{SL}) \times 0.15 \times \frac{300 \text{ ft/transformer}}{1000 \text{ ft}}$$

$$IC_{SL} = 5.4 + 0.405 A_{SL} \quad \$/\text{block}$$

1 span of SL each way from the transformer



$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

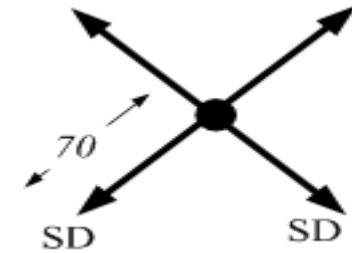
**Solution**

- The annual installed cost of triplex aluminum 24 SDs per block (each SD is 70 ft long)

$$IC_{SD} = (60 + 4.5 \times A_{SD}) \times i \quad \$/1000 \text{ ft}$$

$$IC_{SD} = 2(60 + 4.5 \times A_{SD}) \times 0.15 \times \frac{12 \times 70 \text{ft/SD}}{1000 \text{ ft}}$$

$$IC_{SD} = 15.12 + 1.134 A_{SD} \$/\text{block}$$



- The annual installed cost of poles per block (six poles per block)

$$IC_{PH} = (\$160) \times i \quad \$/\text{pole}$$

$$IC_{PH} = (\$160) \times 6 \times 0.15 = 144 / \text{block}$$



$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

➤ All operating (variable) costs (OCs)

➤  $OC_{exc}$  is the annual operating cost of transformer exciting current per block

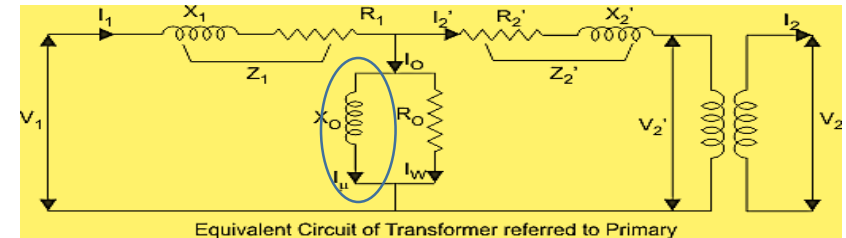
$$OC_{exc} = (I_{exc} \times S_T \times IC_{cap}) \times i \quad \$/\text{transformer}$$

Where

$IC_{cap}$  is the total installed cost of primary-voltage shunt capacitors = \$5.00/kvar

$I_{exc}$  is the an average value of transformer exciting current based on  $S_T$  kVA rating = 0.015 pu

$$OC_{exc} = 2(0.015 \times S_T \times 5) \times 0.15 = 0.0225 S_T \quad \$/\text{block}$$



➤ The annual operating cost of transformer due to core (iron) losses

$$OC_{T,Fe} = (IC_{sys} \times i + 8760 \times EC_{off}) \times P_{T,Fe} \quad \$/\text{transformer}$$

$$OC_{T,Fe} = 2(350 \times 0.15 + 8760 \times 0.008) \times 0.004 \times S_T$$

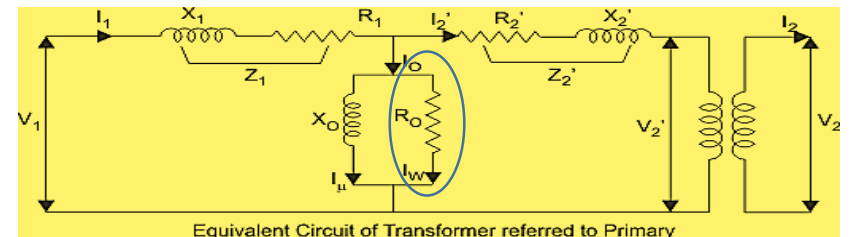
$$= 0.98 S_T \quad \$/\text{block}$$

Where

$IC_{sys}$  is the average investment cost of power system upstream, that is, toward generator, from distribution transformers = \$350/kVA

$EC_{off}$  is the incremental cost of electric energy (off-peak) = \$0.008/kWh

$P_{T,Fe}$  is the annual transformer core loss, kW =  $0.004 \times S_T$  15 kVA ≤  $S_T$  ≤ 100 kVA





# Solution

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

➤ The annual operating cost of transformer due to copper losses

$$OC_{T,Cu} = (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times \left( \frac{S_{max}}{S_T} \right)^2 \times P_{T,Cu} \quad \$/\text{transformer}$$

Where

$IC_{sys}$  is the average investment cost of power system upstream, that is, toward generator, from distribution transformers = \$350/kVA

$EC_{on}$  is the incremental cost of electric energy (on-peak) = \$0.010/kWh

$S_{max}$  is the annual maximum kVA demand on transformer

$P_{T,Cu}$  is the transformer copper loss, kW at rated kVA load =  $0.073 + 0.00905 \times S_T$

$F_{LS}$  is the annual loss factor

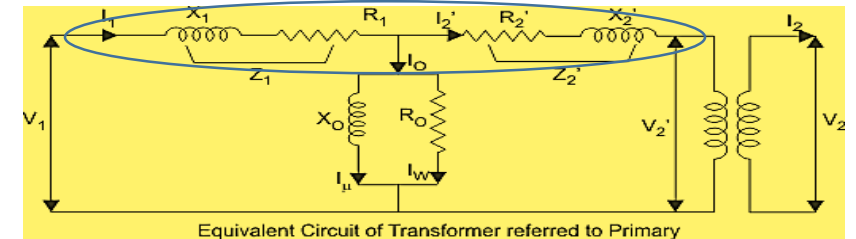
$F_{LD}$  is the annual load factor = 0.35

12 customers/ transformer; from the table maximum diversified demand = 4.4 kVA

$$F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2$$

$$F_{LS} = 0.3 \times 0.35 + 0.7 \times 0.35^2 = 0.1904$$

$$S_{max} = 12 \text{ customers / transformer} \times 4.4 \text{ kVA / customer}$$



No. of customers being diversified	30-Min. Annual Max. Demands, kVA/Customer		
	Class 1	Class 2	Class 3
1	18	10	2.5
2	14.4	7.6	1.8
4	12	6	1.5
12	10	4.4	1.2
100	8.4	3.6	1.1

$$OC_{T,Cu} = 2(350 \times 0.15 + 8760 \times 0.010 \times 0.1904) \times \left( \frac{12 \times 4.4}{S_T} \right)^2 \times (0.073 + 0.00905 \times S_T)$$

$$OC_{T,Cu} = \frac{28170}{S_T^2} + \frac{3492}{S_T} \quad \$/\text{block}$$

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

## Solution

➤ the annual operating cost of copper loss in a unit length of SL (4 SLs)

$$OC_{SL,Cu} = (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times P_{SL,Cu} \text{ \$/transformer}$$

Where

$EC_{on}$  is the incremental cost of electric energy (on-peak) = \$0.010/kWh

$IC_{sys}$  is the average investment cost of power system upstream, that is, toward generator, from distribution transformers = \$350/kVA

$F_{LS}$  is the annual loss factor

No. of customers being diversified	30-Min. Annual Max. Demands, kVA/Customer		
	Class 1	Class 2	Class 3
1	18	10	2.5
2	14.4	7.6	1.8
4	12	6	1.5
12	10	4.4	1.2
100	8.4	3.6	1.1

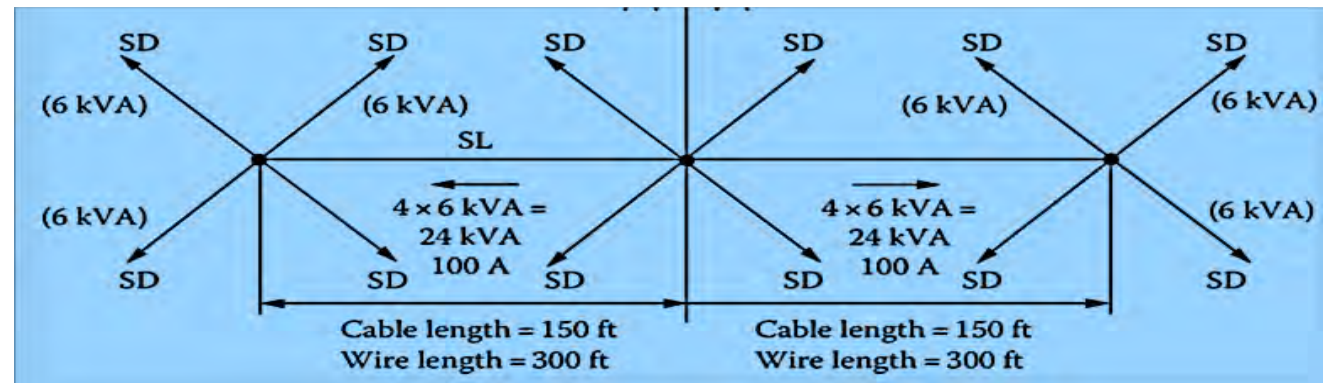
$$F_{LS} = 0.3 \times 0.35 + 0.7 \times 0.35^2 = 0.1904$$

$P_{SL,Cu}$  is the copper losses in two SLs at time of annual peak load, kW/transformer which is given by

$$P_{SL,Cu} = I^2 R$$

$$R = \frac{\rho l}{A_{SL}} = \frac{20.5(\Omega \text{cmil}) / \text{ft} \times 300 \text{ ft wire} \times 2}{1000 \times A_{SL}}$$

$$= \frac{12.3}{A_{SL}} \Omega \cdot \text{kcmil/transformer}$$



each span of SL is of length 300 ft

# Economic Design of Secondaries

**Solution**

➤ Therefore

$$P_{SL,Cu} = \left( \frac{24kVA}{240V} \right)^2 \times \frac{12.3}{A_{SL}} \times \frac{1}{1000}$$

$$= \frac{123}{A_{SL}} kW \quad kW/\text{transformer}$$

No. of customers being diversified	30-Min. Annual Max. Demands, kVA/Customer		
	Class 1	Class 2	Class 3
1	18	10	2.5
2	14.4	7.6	1.8
4	12	6	1.5
12	10	4.4	1.2
100	8.4	3.6	1.1

4 customers/ SL ; from the table maximum diversified demand = 6 kVA

Now

➤ the annual operating cost of copper loss in a unit length of SL

$$OC_{SL,Cu} = (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times P_{SL,Cu} \quad \$/\text{transformer}$$

$$OC_{SL,Cu} = 2(350 \times 0.15 + 8760 \times 0.01 \times 0.1904) \times \frac{123}{A_{SL}} = \frac{17018}{A_{SL}} \quad \$/\text{block}$$

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

**Solution**

➤ the annual operating cost of copper loss in a unit length of SD (24 SDs)

$$OC_{SD,Cu} = (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times P_{SD,Cu} \quad \$/\text{transformer}$$

Where

$EC_{on}$  is the incremental cost of electric energy (on-peak) = \$0.010/kWh

$IC_{sys}$  is the average investment cost of power system upstream, that is, toward generator, from distribution transformers = \$350/kVA

$F_{LD}$  is the annual load factor=0.35

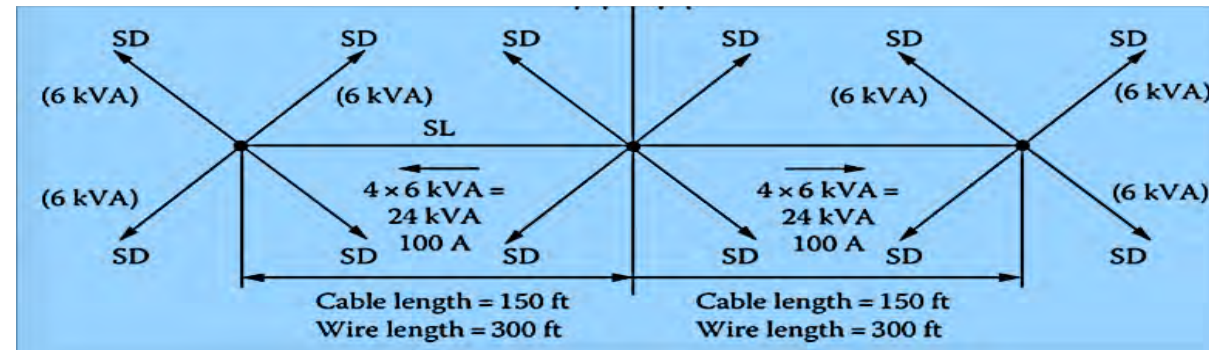
$$F_{LS} = 0.3 \times 0.35 + 0.7 \times 0.35^2 = 0.1904$$

$P_{SD,Cu}$  is the copper losses in the 24 SDs at the time of annual peak load, kW given by

$$P_{SD,Cu} = I^2 R$$

$$R = \frac{\rho l}{A_{SD}} = \frac{20.5(\Omega\text{cmil} / \text{ft}) \times 70 \text{ ft} \times 24 \text{ SD} / \text{block} \times 2 \text{ wires} / \text{SD}}{1000 \times A_{SD}}$$

$$= \frac{68.88}{A_{SD}} \Omega.\text{kcmil}/\text{block}$$



# Economic Design of Secondaries

**Solution**

➤ Therefore

$$P_{SD,Cu} = \left( \frac{10kVA}{240V} \right)^2 \times \frac{68.88}{A_{SD}} \times \frac{1}{1000}$$

$$= \frac{119.58}{A_{SD}} kW / block$$

No. of customers being diversified	30-Min. Annual Max. Demands, kVA/Customer		
	Class 1	Class 2	Class 3
1	18	10	2.5
2	14.4	7.6	1.8
4	12	6	1.5
12	10	4.4	1.2
100	8.4	3.6	1.1

one SD per one Class 2 customer, from the table maximum diversified demand = 10 kVA

Now

$$OC_{SD,Cu} = (350 \times 0.15 + 8760 \times 0.01 \times 0.1904) \times \frac{119.58}{A_{SD}} = \frac{8273}{A_{SD}} \text{ \$/block}$$

➤ Finally substituting all the equation in to TAC formula, we have

$$TAC = (75 + 2.178S_T) + (5.4 + 0.405A_{SL}) + (15.12 + 1.134A_{SD}) +$$

$$(144 + 0.0225S_T) + (0.98S_T) + \left( \frac{28170}{S_T^2} + \frac{3492}{S_T} \right) + \frac{17108}{A_{SL}} + \frac{8273}{A_{SD}}$$

# Economic Design of Secondaries

## Solution

- Simplifying TAC becomes

$$TAC = 239.52 + 3.1805S_T + \frac{3492}{S_T} + \frac{28170}{S_T^2} + 0.405A_{SL} + \frac{17108}{A_{SL}} + 1.134A_{SD} + \frac{8273}{A_{SD}}$$

- Note that TAC have the general form as

$$TAC = A + BS_T + \frac{C}{S_T} + \frac{D}{S_T^2} + EA_{SL} + \frac{F}{A_{SL}} + GA_{SD} + \frac{H}{A_{SD}}$$

# Minimization of the TAC

## Example

- TAC general equation shown below can be used to find the minimum TAC by taking three partial derivatives, and setting each derivative to zero:

$$TAC = A + BS_T + \frac{C}{S_T} + \frac{D}{S_T^2} + EA_{SL} + \frac{F}{A_{SL}} + GA_{SD} + \frac{H}{A_{SD}}$$

- That is

$$\begin{aligned} \frac{\delta(TAC)}{\delta S_T} &= 0 \\ &= B - \frac{C}{S_T^2} - \frac{2D}{S_T^3} = 0 \end{aligned}$$

$$\begin{aligned} \frac{\delta(TAC)}{\delta A_{SL}} &= 0 \\ &= E - \frac{F}{A_{SL}^2} = 0 \end{aligned}$$

$$\begin{aligned} \frac{\delta(TAC)}{\delta A_{SD}} &= 0 \\ &= G - \frac{H}{A_{SD}^2} = 0 \end{aligned}$$

### Note:

The solution obtained for  $S_T$ ,  $A_{SL}$  and  $A_{SD}$  are continuous variable. TAC for the standard commercial sizes of equipment nearest to the results should be obtained.

# Minimization of the TAC

## Example

- If the TAC found as a function of  $S_T$ ,  $A_{SL}$ ,  $A_{SD}$  is

$$TAC = 239.52 + 3.1805S_T + \frac{3492}{S_T} + \frac{28170}{S_T^2} + 0.405A_{SL} + \frac{17108}{A_{SL}} + 1.134A_{SD} + \frac{8273}{A_{SD}}$$

### Find

- a. The most economical SD size ( $A_{SD}$ ) and the nearest larger standard AWG wire size
- b. The most economical SL size ( $A_{SL}$ ) and the nearest larger standard AWG wire size
- c. The most economical distribution transformer size ( $S_T$ ) and the nearest larger standard transformer size
- d. The TAC per block for the theoretically most economical sizes of equipment
- e. The TAC per block for the nearest larger standard commercial sizes of equipment



# Minimization of the TAC

## Example

➤ If the TAC found as a function of  $S_T$ ,  $A_{SL}$ ,  $A_{SD}$  is

$$TAC = 239.52 + 3.1805S_T + \frac{3492}{S_T} + \frac{28170}{S_T^2} + 0.405A_{SL} + \frac{17108}{A_{SL}} + 1.134A_{SD} + \frac{8273}{A_{SD}}$$

## Solution

➤ a. The most economical SD size ( $A_{SD}$ ) and the nearest larger standard AWG wire size

$$\frac{\delta(TAC)}{\delta A_{SD}} = 0$$

$$= G - \frac{H}{A_{SD}^2} = 0$$

$$1.134 - \frac{8273}{A_{SD}^2} = 0$$

$$A_{SD} = 85.4 \text{ kcmil}$$

Nearest larger size = **106.6 kcmil** AWG (1/0)

Size of Conductor Circular Mils	AWG or B & S	Number of Strands	Diameter of Individual Strands (in.)	Outside Diameter (in.)	Breaking Strength (lb)	Weight (lb/mi)	Approx. Current Carrying Capacity* (amps)	Geometric Mean Radius at 60 Cycles (ft)
1,000,000	—	37	0.1644	1.151	43,830	16,300	1300	0.0368
900,000	—	37	0.1560	1.092	39,610	14,670	1220	0.0349
800,000	—	37	0.1470	1.029	35,120	13,040	1130	0.0329
750,000	—	37	0.1424	0.997	33,400	12,230	1090	0.0319
700,000	—	37	0.1375	0.963	31,170	11,410	1040	0.0306
500,000	—	37	0.1273	0.891	27,020	9,781	940	0.0285
500,000	—	37	0.1162	0.814	22,610	8,161	840	0.0260
500,000	—	19	0.1622	0.811	21,590	8,161	840	0.0256
450,000	—	19	0.1539	0.770	19,750	7,336	780	0.0243
400,000	—	19	0.1451	0.726	17,560	6,521	730	0.0229
350,000	—	19	0.1357	0.679	16,890	5,706	670	0.0214
350,000	—	12	0.1708	0.710	16,140	5,706	670	0.0225
300,000	—	19	0.1257	0.629	13,510	4,891	610	0.01987
300,000	—	12	0.1581	0.657	13,170	4,891	610	0.0208
250,000	—	19	0.1147	0.574	11,360	4,076	540	0.01813
250,000	—	12	0.1443	0.600	11,130	4,076	540	0.01902
211,600	4/0	19	0.1055	0.528	9,617	3,450	480	0.01668
211,600	4/0	12	0.1328	0.552	9,483	3,450	490	0.01750
211,600	4/0	7	0.1739	0.522	9,154	3,450	480	0.01579
167,800	3/0	12	0.1183	0.492	7,556	2,736	420	0.01569
167,800	3/0	7	0.1548	0.464	7,366	2,736	420	0.01404
133,100	2/0	7	0.1379	0.414	5,926	2,170	360	0.01252
106,600	1/0	7	0.1228	0.368	4,752	1,720	310	0.01113
83,690	1	7	0.1093	0.328	3,804	1,364	270	0.00992

# Minimization of the TAC

## Example

➤ If the TAC found as a function of  $S_T$ ,  $A_{SL}$ ,  $A_{SD}$  is

$$TAC = 239.52 + 3.1805S_T + \frac{3492}{S_T} + \frac{28170}{S_T^2} + 0.405A_{SL} + \frac{17108}{A_{SL}} + 1.134A_{SD} + \frac{8273}{A_{SD}}$$

## Solution

➤ b. The most economical SL size ( $A_{SL}$ ) and the nearest larger standard AWG wire size

$$\frac{\delta(TAC)}{\delta A_{SL}} = 0$$

$$= E - \frac{F}{A_{SL}^2} = 0$$

$$0.405 - \frac{17018}{A_{SL}^2} = 0$$

$$A_{SL} = 204.99 \text{ kcmil}$$

Nearest larger size = **211.6 kcmil** AWG (4/0)

Size of Conductor Circular Mils	AWG or B & S	Number of Strands	Diameter of Individual Strands (in.)	Outside Diameter (in.)	Breaking Strength (lb)	Weight (lb/mi)	Approx. Current Carrying Capacity* (amps)	Geometric Mean Radius at 60 Cycles (ft)
1,000,000	—	37	0.1644	1.151	43,830	16,300	1300	0.0368
900,000	—	37	0.1560	1.092	39,610	14,670	1220	0.0349
800,000	—	37	0.1470	1.029	35,120	13,040	1130	0.0329
750,000	—	37	0.1424	0.997	33,400	12,230	1090	0.0319
700,000	—	37	0.1375	0.963	31,170	11,410	1040	0.0306
500,000	—	37	0.1273	0.891	27,020	9,781	940	0.0285
500,000	—	37	0.1162	0.814	22,610	8,161	840	0.0260
500,000	—	19	0.1622	0.811	21,590	8,161	840	0.0256
450,000	—	19	0.1539	0.770	19,750	7,336	780	0.0243
400,000	—	19	0.1451	0.726	17,560	6,521	730	0.0229
350,000	—	19	0.1357	0.679	16,890	5,706	670	0.0214
350,000	—	12	0.1708	0.710	16,140	5,706	670	0.0225
300,000	—	19	0.1257	0.629	13,510	4,891	610	0.01987
300,000	—	12	0.1581	0.657	13,170	4,891	610	0.0208
250,000	—	19	0.1147	0.574	11,360	4,076	540	0.01813
250,000	—	12	0.1443	0.600	11,130	4,076	540	0.01902
211,600	4/0	19	0.1055	0.528	9,617	3,450	480	0.01668
211,600	4/0	12	0.1328	0.552	9,483	3,450	490	0.01750
211,600	4/0	7	0.1739	0.522	9,154	3,450	480	0.01579
167,800	3/0	12	0.1183	0.492	7,556	2,736	420	0.01569
107,800	2/0	7	0.1548	0.404	7,568	2,736	420	0.01404
133,100	2/0	7	0.1379	0.414	5,926	2,170	360	0.01252
106,600	1/0	7	0.1228	0.368	4,752	1,720	310	0.01113
83,690	1	7	0.1093	0.328	3,804	1,364	270	0.00992

# Minimization of the TAC

## Example

- If the TAC found as a function of  $S_T$ ,  $A_{SL}$ ,  $A_{SD}$  is

$$TAC = 239.52 + 3.1805S_T + \frac{3492}{S_T} + \frac{28170}{S_T^2} + 0.405A_{SL} + \frac{17108}{A_{SL}} + 1.134A_{SD} + \frac{8273}{A_{SD}}$$

## Solution

- b. The most economical Transformer size ( $S_T$ ) and the nearest larger standard size

$$\begin{aligned} \frac{\delta(TAC)}{\delta S_T} &= 0 \\ &= B - \frac{C}{S_T^2} - \frac{2D}{S_T^3} = 0 \end{aligned}$$

$$\begin{aligned} &= 3.1085 - \frac{3492}{S_T^2} - \frac{2 \times 28170}{S_T^3} = 0 \\ &= 39kVA \end{aligned}$$

Nearest larger size= 50 kVA

TABLE 3.1

Standard Transformer Kilovoltamperes and Voltages

Kilovoltamperes		High Voltages		Low Voltages	
Single Phase	Three Phase	Single Phase	Three Phase	Single Phase	Three Phase
5	30	2,400/4,160 Y	2,400	120/240	208 Y/120
10	45	4,800/8,320 Y	4,160 Y/2,400	240/480	240
15	75	4,800Y/8,320 YX	4,160 Y	2400	480
25	112½	7,200/12,470 Y	4,800	2520	480 Y/277
37½	150	12,470 Gnd Y/7,200	8,320 Y/4,800	4800	240 x 480
50	225	7,620/13,200 Y	8,320 Y	5040	2,400
75	300	13,200 Gnd Y/7,620	7,200	6900	4,160 Y/2,400
100	500	12,000	12,000	7200	4,800
167		13,200/22,860 Gnd Y	12,470 Y/7,200	7560	12,470 Y/7,200
250		13,200	12,470 Y	7980	13,200 Y/7,620
333		13,800 Gnd Y/7,970	13,200 Y/7,620		
500		13,800/23,900 Gnd Y	13,200 Y		
		13,800	13,200		
		14,400/24,940 Gnd Y	13,800		
		16,340	22,900		
		19,920/34,500 Gnd Y	34,400		
		22,900	43,800		
		34,400	67,000		
		43,800			
		67,000			

# Minimization of the TAC

## Example

- If the TAC found as a function of  $S_T$ ,  $A_{SL}$ ,  $A_{SD}$  is

$$TAC = 239.52 + 3.1805S_T + \frac{3492}{S_T} + \frac{28170}{S_T^2} + 0.405A_{SL} + \frac{17108}{A_{SL}} + 1.134A_{SD} + \frac{8273}{A_{SD}}$$

### Solution

- d. The TAC per block for the theoretically most economical sizes of equipment

$$TAC = 239.52 + 3.1805 \times 39 + \frac{3492}{39} + \frac{28170}{39^2} + 0.405 \times 209.99 + \frac{17108}{209.99} + 1.134 \times 85.41 + \frac{8273}{85.41}$$

**= \$838**

- e. The TAC per block for the nearest larger standard commercial sizes of equipment

$$TAC = 239.52 + 3.1805 \times 50 + \frac{3492}{50} + \frac{28170}{50^2} + 0.405 \times 211.6 + \frac{17108}{211.6} + 1.134 \times 106.5 + \frac{8273}{106.5}$$

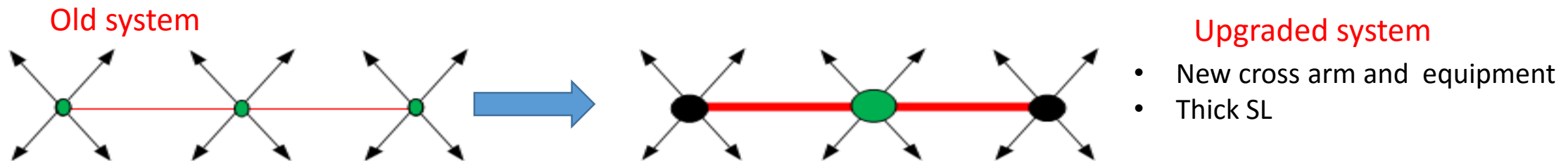
**= \$844**

# Secondary System Upgrading Costs

- In general, it costs more to upgrade given equipment to a higher capacity than to build to that capacity in the first place.
- Upgrading an existing SL entails removing the old conductor and installing new. Usually, new hardware is required, and sometimes poles and cross arms must be replaced.
- Therefore, usually, the cost of this conversion greatly exceeds the cost of building to the higher-capacity design in the first place.
- Because of this, T&D engineers have an incentive to look at long-term needs carefully and to install extra capacity for future growth.

# Secondary System Upgrading Costs

- It has been estimated that a 12.47 kV OH, three-phase feeder with 336 kcmil costs \$120,000/mile. It has been also estimated that to build the feeder with 600 kcmil conductor instead and a 15 MVA capacity would cost about \$150,000/mile. Upgrading the existing 9 MVA capacity line later to 15 MVA capacity entails removing the old conductor and installing new. The cost of upgrade is \$200,000/mile. Determine the following:
  - a. The cost of building the 9 MVA capacity line in dollars per kVA-mile
  - b. The cost of building the 15 MVA capacity line in dollars per kVA-mile
  - c. The cost of the upgrade in dollars per kVA-mile



# Secondary System Upgrading Costs

## Solution

- **Given:** 336 kcmil costs \$120,000/mile : System KVA Rating = 9MVA  
600 kcmil costs \$150,000/mile: System KVA Rating = 15MVA  
cost of upgrade from 9 MVA to 15 MVA is \$200,000/mile
- a. The cost of building the 9 MVA capacity line in dollars per kVA-mile

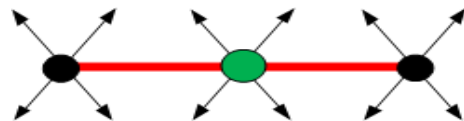
$$\mathit{Cost}_{9\text{ MVA line}} = \frac{\$120000}{9000\text{ kVA}} = 13.33\text{ \$/kVA mile}$$

- b. The cost of building the 15 MVA capacity line in dollars per kVA-mile

$$\mathit{Cost}_{15\text{ MVA line}} = \frac{\$150000}{15000\text{ kVA}} = 10\text{ \$/kVA mile}$$

- c. The cost of the upgrade in dollars per kVA-mile

$$\mathit{Cost}_{9\text{ to }15\text{ MVA line}} = \frac{\$200000}{(15000 - 9000)\text{ kVA}} = 33.33\text{ \$/kVA mile}$$

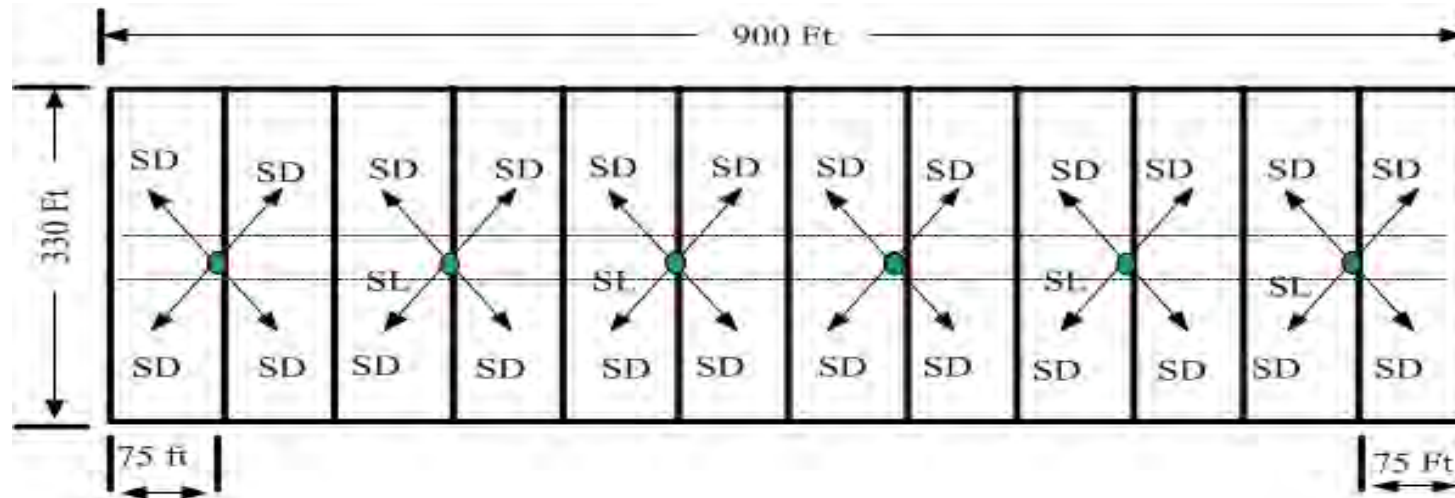


Note that the upgrade option is very costly

# Economic Design of Secondaries

## Example

- For the layouts and the service arrangement shown in the figure below calculate the TAC as a function of  $ST$ ,  $ASD$  and  $ASL$



**Residential area lot layout and service arrangement.**



# Economic Design of Secondaries

## Example

### ➤ System data

System parameter	Value or description
System phase	single
System voltage	120/240
Reliability	100% (8760 h/y)
Annual $F_{LD}$	0.35
Annual $F_{LS}$	$F_{LS} = 0.3F_{LD} + 0.7F_{LD}^2$
Consumer/transformer	4
Power factor	0.9
Fixed charge rate	0.15
$\rho$ for aluminium @ 65°	20.5 $\Omega$ .cmil/ft
Consumer class	2
Consumer/SD	1

No. of customers being diversified	30-Min. Annual Max. Demands, kVA/Customer		
	Class 1	Class 2	Class 3
1	18	10	2.5
2	14.4	7.6	1.8
4	12	6	1.5
12	10	4.4	1.2
100	8.4	3.6	1.1

# Economic Design of Secondaries

## Solution

- TAC is given by

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

- All investment (fixed) costs (ICs)

Annual installed cost of the six distribution transformer and associated protective equipment

$$\begin{aligned} IC_T &= (250 + 7.26 \times S_T) \times i \text{ \$/transformer} \\ &= 6(250 + 7.26 \times S_T) \times 0.15 \\ &= \$(225 + 6.534 S_T) \text{ \$/block} \end{aligned}$$

Annual cost of pole and hardware for the six poles per block

$$\begin{aligned} IC_{PH} &= (\$160) \times i \text{ \$/pole} \\ (\$160) \times 6 \times 0.15 &= \$(144)/\text{block} \end{aligned}$$

Annual installed cost of triplex aluminum 24 SDs per block (each SD is 70 ft long)

$$\begin{aligned} IC_{SD} &= (60 + 4.5 \times A_{SD}) \times i \text{ \$/1000 ft} \\ &= 6(60 + 4.5 \times A_{SD}) \times 0.15 \times \frac{4 \times 70 \text{ t/SD}}{1000 \text{ ft}} \\ &= \$(15.12 + 1.134 A_{SD}) \text{ \$/block} \end{aligned}$$

# Economic Design of Secondaries

## Solution

- TAC is given by

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

- All operating (variable) costs (OCs)

Annual OC of transformer exciting current per block

$$\begin{aligned} OC_{exc} &= (I_{exc} \times S_T \times IC_{cap}) \times i \text{ \$/transformer} \\ &= 6(0.015 \times S_T \times 5) \times 0.15 \\ &= 0.0675 S_T \text{ \$/block} \end{aligned}$$

Annual OC of core (iron) losses of the six transformer exciting current per block

$$\begin{aligned} OC_{T,Fe} &= (IC_{sys} \times i + 8760 \times EC_{off}) \times P_{T,Fe} \text{ \$/transformer} \\ &= 6(350 \times 0.15 + 8760 \times 0.008) \times 0.004 \times S_T \\ &= 2.94 S_T \text{ \$/block} \end{aligned}$$

# Economic Design of Secondaries

## Solution

- TAC is given by

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

- All operating (variable) costs (OCs)

$$F_{LS} = 0.3 \times 0.35 + 0.7 \times 0.35^2 = 0.1904$$

4 customers/ transformer; from the table maximum diversified demand = 6 kVA

$$S_{\max} = 4 \text{ customers / transformer} \times 6 \text{ kVA / customer} = 24 \text{ kVA / transformer}$$

Annual OC of transformer due to copper losses

$$\begin{aligned} OC_{T,Cu} &= (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times \left( \frac{S_{\max}}{S_T} \right)^2 \times P_{T,Cu} \quad \$/\text{transformer} \\ &= 6(350 \times 0.15 + 8760 \times 0.010 \times 0.1904) \times \left( \frac{24}{S_T} \right)^2 \times (0.073 + 0.00905 \times S_T) \\ &= \frac{84510}{S_T^2} + \frac{10476}{S_T} \quad \$/\text{block} \end{aligned}$$

# Economic Design of Secondaries

## Solution

- TAC is given by

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

- All operating (variable) costs (OCs)

one SD per one Class 2 customer, from the table  
maximum diversified demand = 10 kVA

$$R = \frac{\rho l}{A_{SD}} = \frac{20.5(\Omega \text{cmil} / \text{ft}) \times 70 \text{ ft} \times 24 \text{ SD} / \text{block} \times 2 \text{ wires} / \text{SD}}{1000 \times A_{SD}}$$

$$= \frac{68.88}{A_{SD}}$$

$$P_{SD,Cu} = \left( \frac{10 \text{ kVA}}{240 \text{ V}} \right)^2 \times \frac{68.88}{A_{SD}} \times \frac{1}{1000} = \frac{119.58}{A_{SD}} \text{ kW/block}$$

Annual OC of copper losses in the 24 SDs

$$OC_{SD,Cu} = (IC_{sys} \times i + 8760 \times EC_{on} \times F_{LS}) \times P_{SD,Cu}$$

$$= (350 \times 0.15 + 8760 \times 0.01 \times 0.1904) \times \frac{119.58}{A_{SD}} = \frac{8273}{A_{SD}} \text{ \$/block}$$

# Economic Design of Secondaries

## Solution

➤ TAC is given by

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

$$TAC = 384.12 + 9.54S_T + \frac{10476}{S_T} + \frac{84510}{S_T^2} + 1.134A_{SD} + \frac{8273}{A_{SD}}$$

➤ a. The most economical SD size ( $A_{SD}$ ) and the nearest larger standard AWG wire size

$$\frac{\delta(TAC)}{\delta A_{SD}} = 0$$

$$= G - \frac{H}{A_{SD}^2} = 0$$

$$1.134 - \frac{8273}{A_{SD}^2} = 0$$

$$A_{SD} = 85.4 \text{ kcmil}$$

Nearest larger size = 106.6 kcmil AWG (1/0)

Size of Conductor Circular Mils	AWG or B & S	Number of Strands	Diameter of Individual Strands (in.)	Outside Diameter (in.)	Breaking Strength (lb)	Weight (lb/mi)	Approx. Current Carrying Capacity* (amps)	Geometric Mean Radius at 60 Cycles (ft)
1,000,000	—	37	0.1644	1.151	43,830	16,300	1300	0.0368
900,000	—	37	0.1560	1.092	39,610	14,670	1220	0.0349
800,000	—	37	0.1470	1.029	35,120	13,040	1130	0.0329
750,000	—	37	0.1424	0.997	33,400	12,230	1090	0.0319
700,000	—	37	0.1375	0.963	31,170	11,410	1040	0.0306
500,000	—	37	0.1273	0.891	27,020	9,781	940	0.0285
500,000	—	37	0.1162	0.814	22,610	8,161	840	0.0260
500,000	—	19	0.1622	0.811	21,590	8,161	840	0.0256
450,000	—	19	0.1539	0.770	19,750	7,336	780	0.0243
400,000	—	19	0.1451	0.726	17,560	6,521	730	0.0229
350,000	—	19	0.1357	0.679	16,890	5,706	670	0.0214
350,000	—	12	0.1708	0.710	16,140	5,706	670	0.0225
300,000	—	19	0.1257	0.629	13,510	4,891	610	0.01987
300,000	—	12	0.1581	0.657	13,170	4,891	610	0.0208
250,000	—	19	0.1147	0.574	11,360	4,076	540	0.01813
250,000	—	12	0.1443	0.600	11,130	4,076	540	0.01902
211,600	4/0	19	0.1055	0.528	9,617	3,450	480	0.01668
211,600	4/0	12	0.1328	0.552	9,483	3,450	490	0.01750
211,600	4/0	7	0.1739	0.522	9,154	3,450	480	0.01579
167,800	3/0	12	0.1183	0.492	7,556	2,736	420	0.01569
167,800	3/0	7	0.1548	0.464	7,366	2,736	420	0.01404
133,100	2/0	7	0.1379	0.414	5,926	2,170	360	0.01252
106,600	1/0	7	0.1228	0.368	4,752	1,720	310	0.01113
83,690	1	7	0.1093	0.328	3,804	1,364	270	0.00992

# Economic Design of Secondaries

**Solution**

➤ TAC is given by

$$TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PH} + \sum OC_{exc} + \sum OC_{T,Fe} + \sum OC_{T,Cu} + \sum OC_{SL,Cu} + \sum OC_{SD,Cu}$$

$$TAC = 384.12 + 9.54S_T + \frac{10476}{S_T} + \frac{84510}{S_T^2} + 1.134A_{SD} + \frac{8273}{A_{SD}}$$

➤ b. The most economical transformer size ( $S_T$ ) and the nearest larger standard size

$$\frac{\delta(TAC)}{\delta S_T} = 0$$

$$= B - \frac{C}{S_T^2} - \frac{2D}{S_T^3} = 0$$

$$= 9.54 - \frac{10476}{S_T^2} - \frac{2 \times 84510}{S_T^3} = 0$$

$$\approx 39 \text{ kVA}$$

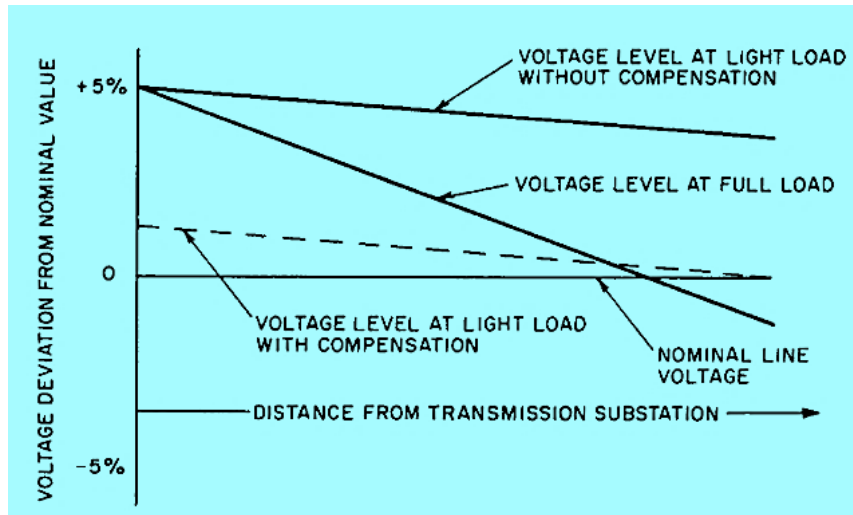
Nearest larger size = 50 kVA

TABLE 3.1  
Standard Transformer Kilovoltamperes and Voltages

Kilovoltamperes		High Voltages		Low Voltages	
Single Phase	Three Phase	Single Phase	Three Phase	Single Phase	Three Phase
5	30	2,400/4,160 Y	2,400	120/240	208 Y/120
10	45	4,800/8,320 Y	4,160 Y/2,400	240/480	240
15	75	4,800Y/8,320 YX	4,160 Y	2400	480
25	112½	7,200/12,470 Y	4,800	2520	480 Y/277
37½	150	12,470 Gnd Y/7,200	8,320 Y/4,800	4800	240 x 480
50	225	7,620/13,200 Y	8,320 Y	5040	2,400
75	300	13,200 Gnd Y/7,620	7,200	6900	4,160 Y/2,400
100	500	12,000	12,000	7200	4,800
167		13,200/22,860 Gnd Y	12,470 Y/7,200	7560	12,470 Y/7,200
250		13,200	12,470 Y	7980	13,200 Y/7,620
333		13,800 Gnd Y/7,970	13,200 Y/7,620		
500		13,800/23,900 Gnd Y	13,200 Y		
		13,800	13,200		
		14,400/24,940 Gnd Y	13,800		
		16,340	22,900		
		19,920/34,500 Gnd Y	34,400		
		22,900	43,800		
		34,400	67,000		
		43,800			
		67,000			

# Introduction

- To satisfy the operation of motors, lamps and other loads, substantially constant voltage is required.
- A wide variation of voltage may cause malfunctioning of consumer's appliances.
- When the load in the system increases, the voltage in the consumer terminals falls due to the increase in the voltage drop in
  - Generator synchronous impedance
  - Transmission lines
  - Transformer impedance
  - Feeders and secondary lines
- The voltage variation are undesirable and must be kept within the prescribed limits.





# System Voltage Terms:

- 1. **System voltage:** The root-mean-square phase-to-phase voltage of a portion of an ac electric system.
- 2. **Nominal system voltage:** The voltage by which a portion of the system is designated, and to which certain operating characteristics of the system are related.
- 3. **Maximum system voltage:** The highest system voltage that occurs under normal operating conditions, and the highest system voltage for which equipment and other components are designed for satisfactory continuous operation without derating of any kind.
- 4. **Service voltage:** The voltage at the point where the electric system of the supplier and the electric system of the user are connected.
- 5. **Utilization voltage:** The voltage at the line terminals of utilization equipment.

# Voltage Drops in Different Type of Circuits

## Single phase two-wire system

- For a single phase two-wire system shown in Figure, assuming that the resistance  $R_p = R_N$  and reactance  $X_p = X_N$ .

- Applying KVL at loop

$$-\mathbf{V}_S + \Delta V_p + \mathbf{V}_{load} + \Delta V_N = 0$$

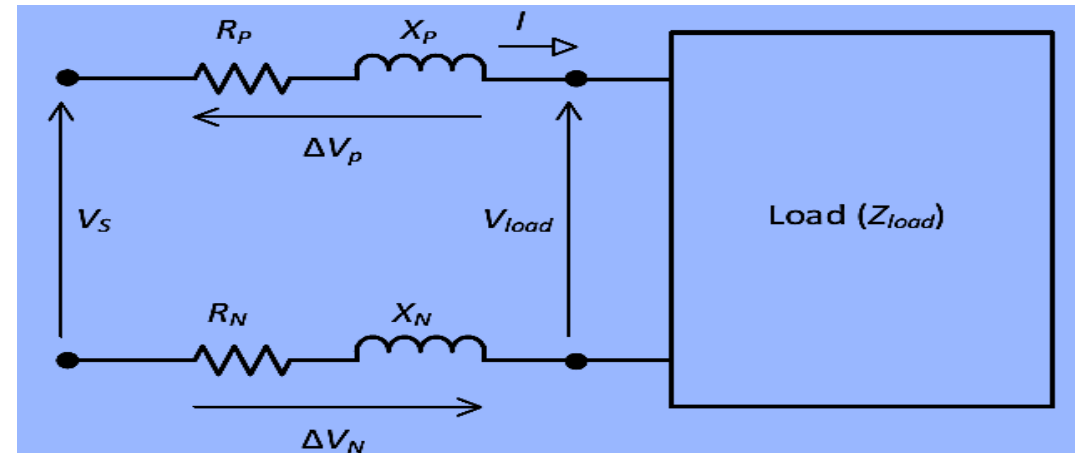
- If the neutral conductor impedance ( $\mathbf{Z}_N$ ) is the same with phase conductor

- impedance ( $\mathbf{Z}_p$ ),  $\Delta V_p = \Delta V_N$

$$-\mathbf{V}_S + \mathbf{V}_{load} + 2\Delta V_p = 0$$

$$\mathbf{V}_S - \mathbf{V}_{load} = 2\Delta V_p$$

$$\Delta V_{1\phi} = 2\Delta V_p = 2\Delta V$$

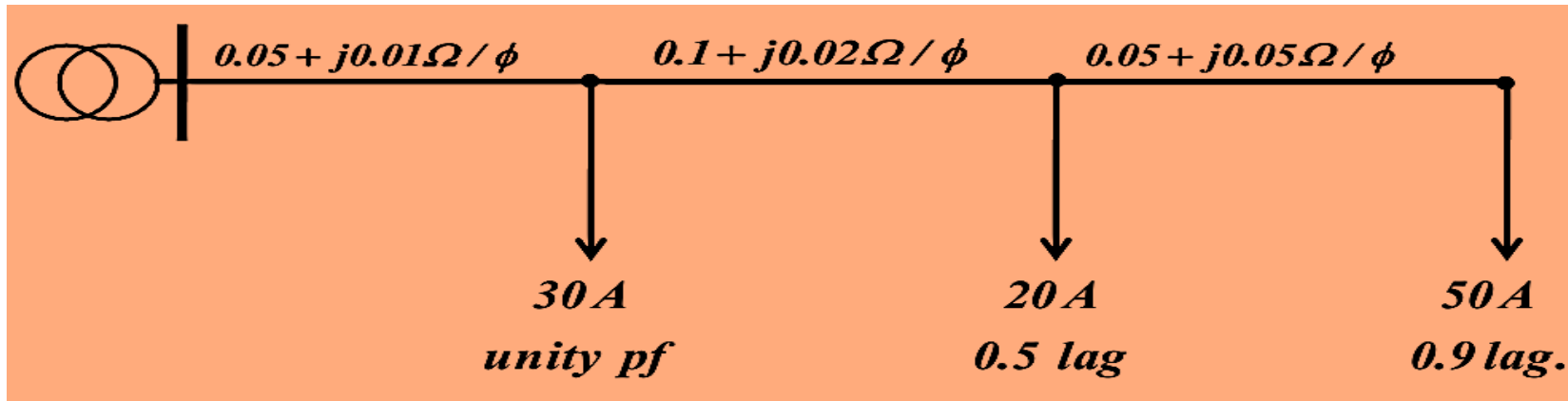




# Voltage Drops in Different Type of Circuits

## Example

- Consider the three-phase 4-wire 416 V secondary radial system with balanced per-phase loads as shown in the Figure. Determine: The total voltage drop in one phase .

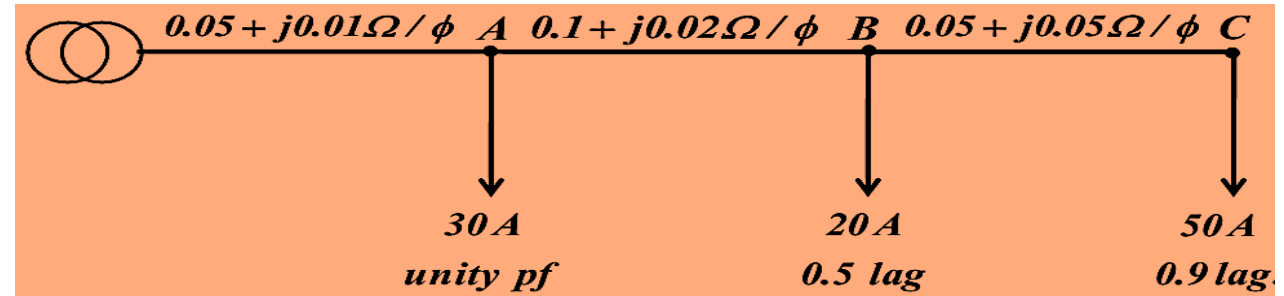


# Voltage Drops in Different Type of Circuits

## Example

- Consider the three-phase 4-wire 416 V secondary radial system with balanced per-phase loads as shown in the Figure. Determine: The total voltage drop in one phase .

## Approximate solution



$$VD = V_s - V_r = IR \cos \theta + IX \sin \theta$$

$$VD_{Load-A} = 30[0.05(1) + 0.01(0)] = 1.5V$$

$$VD_{Load-B} = 20[(0.05 + 0.1)(0.5) + (0.01 + 0.02)(0.866)] = 2.02V$$

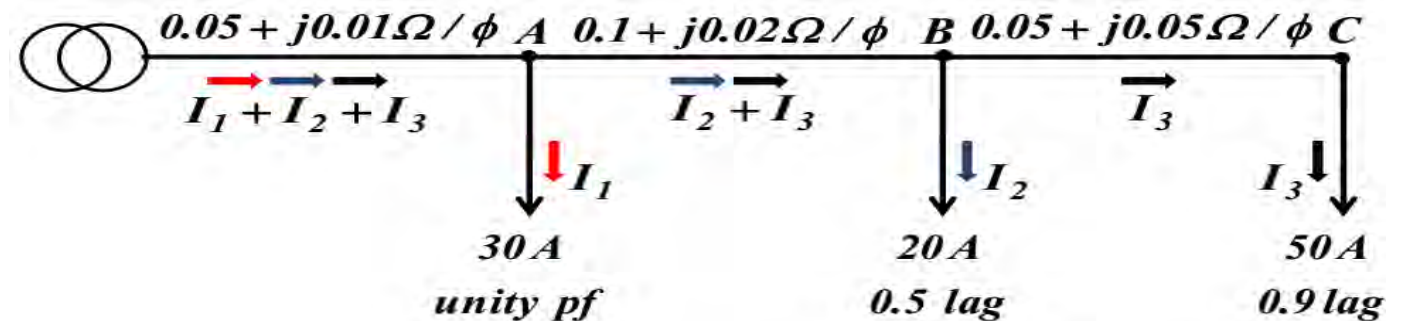
$$VD_{Load-C} = 50[(0.05 + 0.1 + 0.05)(0.9) + (0.01 + 0.02 + 0.05)(0.436)] = 10.744V$$

$$VD_T = VD_A + VD_B + VD_C = 14.264V$$

# Voltage Drops in Different Type of Circuits

## Example

- Consider the three-phase 4-wire 416 V secondary radial system with balanced per-phase loads as shown in the Figure. Determine: The total voltage drop in one phase .



## Actual solution

$$VD_{Node-A} = (I_1 + I_2 + I_3)Z_{SA} = 4.6412 - j1.1058$$

$$VD_{Node-B} = (I_2 + I_3)Z_{AB} = 6.2823 - j2.8115$$

$$VD_{Node-C} = (I_3)Z_{BC} = 3.3397 + j1.1603$$

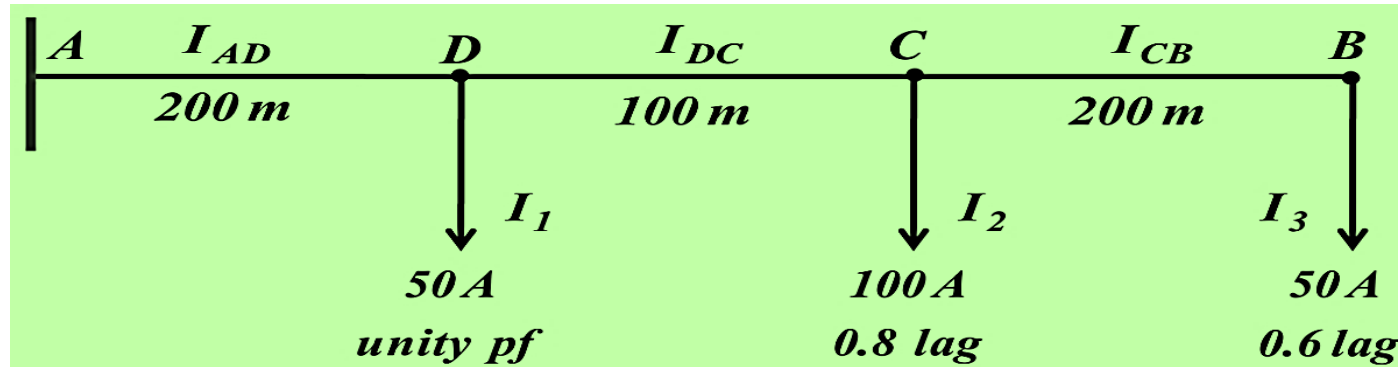
$$VD_T = VD_A + VD_B + VD_C = 14.2632 - j2.7570$$

$$|VD_T| = 14.5272$$

# Voltage Drops in Different Type of Circuits

## Example

- Find the total voltage drop for the following single phase feeder which has a total length of 500 m and total impedance of  $0.02+j0.04 \Omega$  and the voltage at point A is 250V



# Voltage Drops in Different Type of Circuits

## Solution

Current in section AD is the vector sum of the three load currents:

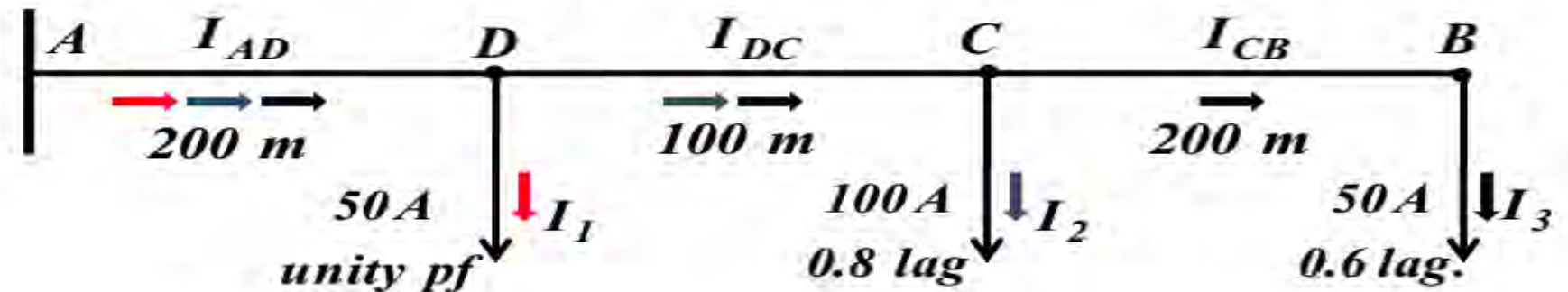
$$I_{AD} = 50 + 100(0.8 - j0.6) + 50(0.6 - j0.8) = 160 - j100 \text{ A}$$

The impedance of section AD is:

$$Z_{AD} = \frac{200}{500} \times (0.02 + j0.04) = 0.008 + j0.016 \Omega$$

The voltage drop of section AD is:

$$V_{AD} = (160 - j100) \times (0.008 + j0.016) = 2.88 + j1.76 \text{ V}$$



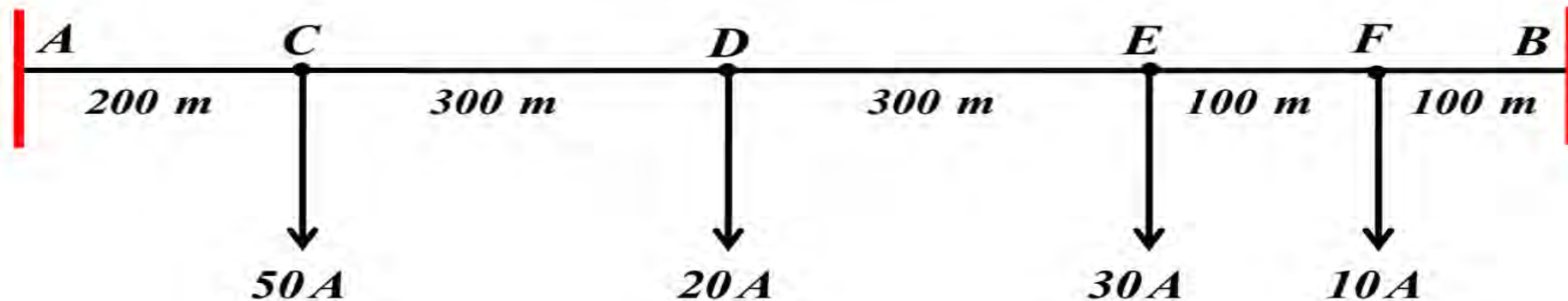
- The same procedure can be applied to sections DC and CB and the total voltage drop will be the summation of the voltage drop of all three sections



# Voltage Drops in Different Type of Circuits

## Example

- Assume the resistance of the following loop connected feeder is  $0.001\Omega/\text{m}$ , the voltage at point A is 250 V and at point B is 246.5 V, find the voltage drop between points C and D. Assume we have DC system.



# Voltage Drops in Different Type of Circuits

## Solution

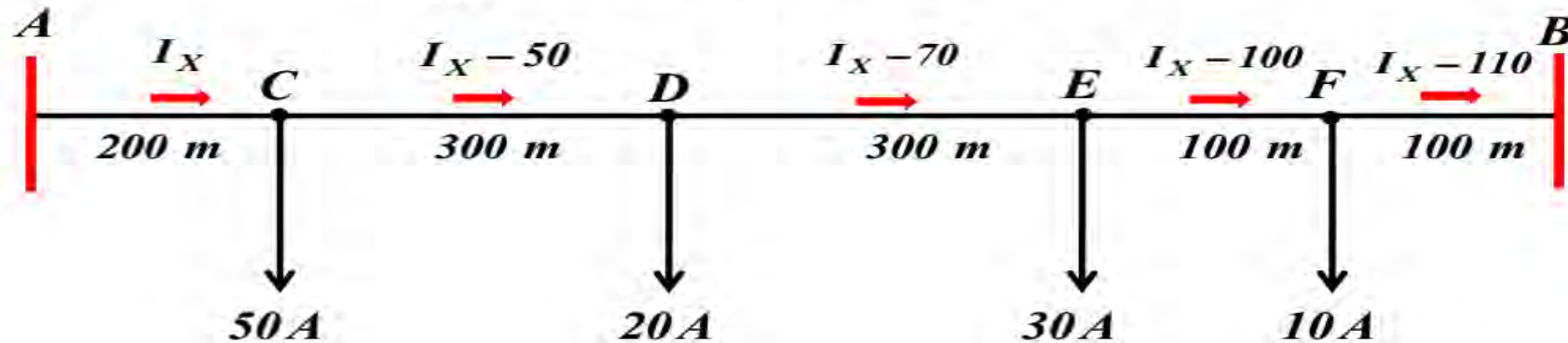
$$V_{AB} = I_x(0.001 \times 200) + (I_x - 50)(300 \times 0.001) + (I_x - 70)(300 \times 0.001) \\ + (I_x - 100)(100 \times 0.001) + (I_x - 110)(100 \times 0.001)$$

Solving the equation will result in:

$$I_x = 60.5 A$$

$$R_{CD} = 0.001 \times 300 = 0.3 \Omega$$

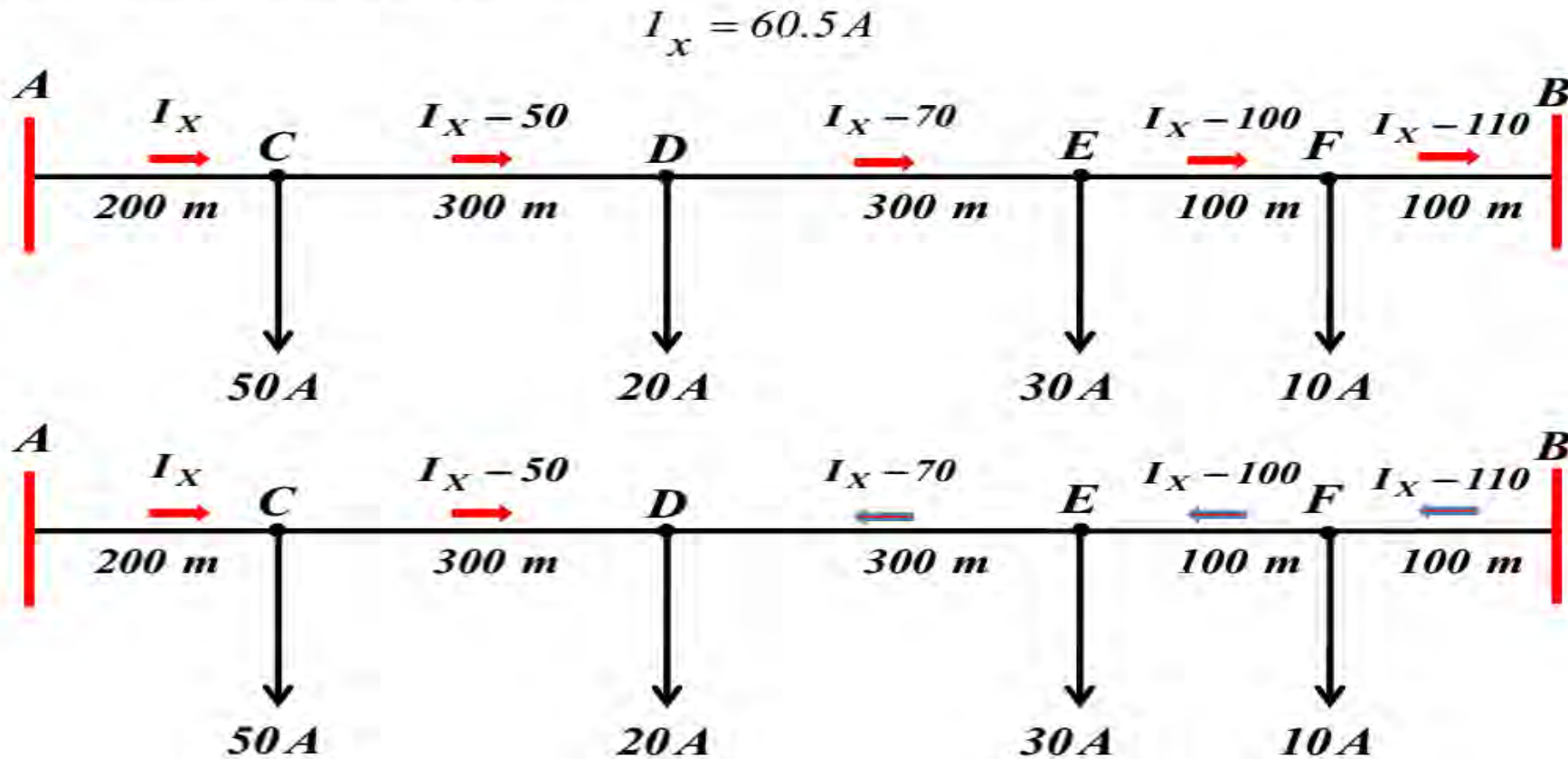
$$V_{CD} = (60.5 - 50) A \times 0.3 \Omega = 3.15 V$$



# Voltage Drops in Different Type of Circuits

## Solution

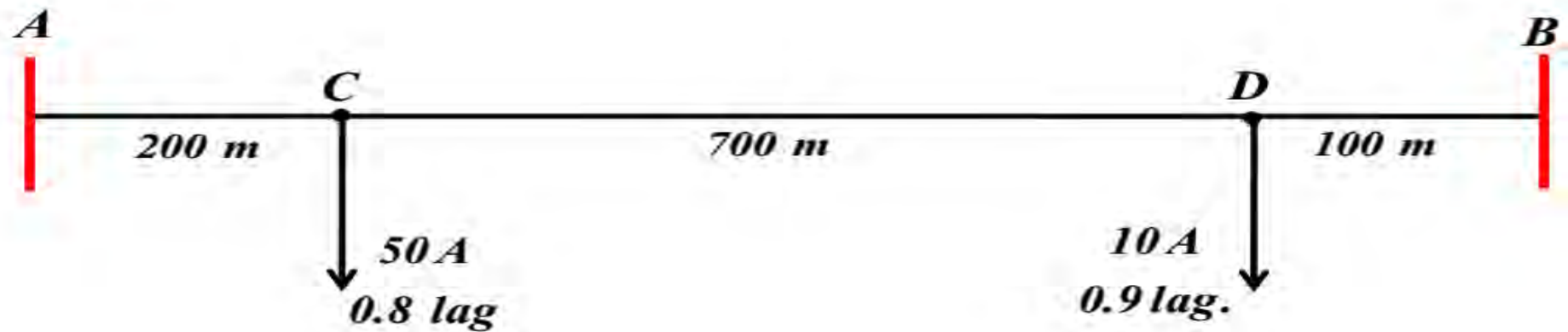
Current actual Direction:



# Voltage Drops in Different Type of Circuits

## Example

- Assume the impedance of the feeder is  $0.001+j0.005 \Omega/m$ , the voltage at point A is  $250 \angle 0^\circ \text{ V}$  and at point B is  $240 \angle -1.5^\circ \text{ V}$ , find the voltage drop between points C and D. Assume we have AC system now.



# Voltage Drops in Different Type of Circuits

## Solution

Assume the current in segment AC as:  $I_{AC} = I_X + jI_Y$

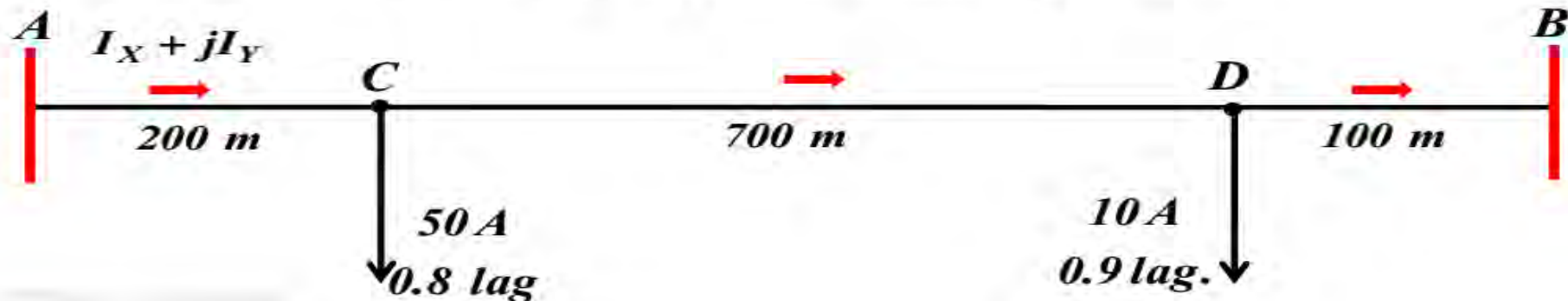
The current in segment CD is:  $I_{CD} = I_X + jI_Y - (39 - j30)$

And in segment DB is:  $I_{DB} = I_X + jI_Y - (39 - j30) - (9 - j4.35)$

$$VD_{AB} = I_{AC}Z_{AC} + I_{CD}Z_{CD} + I_{DB}Z_{DB} = 250 \angle 0 - 240 \angle 1.5$$

Substituting for the currents, the impedances and solving the equation, the current that flows in AC is:

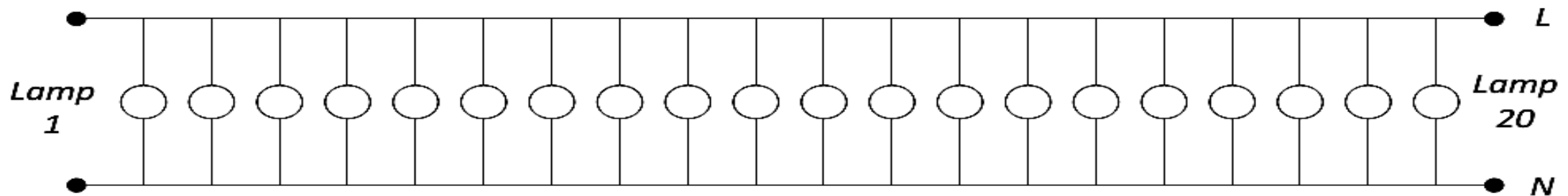
$$I_{AC} = I_X + jI_Y = 31.3 - j26.8 \text{ A}$$



# Voltage Drops in Different Type of Circuits

## Example

- For the circuit given in Figure, determine the voltage drop. Assume the following:
- i. Street lighting wire size is  $16 \text{ mm}^2$  where  $R = 2.33 \text{ } \Omega/\text{km}$ ,  $X = 0.095 \text{ } \Omega/\text{km}$ ,
- ii. Neutral cable size is  $120 \text{ mm}^2$  where  $R = 0.309 \text{ } \Omega/\text{km}$ ,  $X = 0.095 \text{ } \Omega/\text{km}$ ,
- iii. Each street lamp operates at  $150 \text{ W}$  with  $0.65$  lagging  $pf$ ,
- iv. Rated voltage of the lamp is  $240 \text{ V}$ ,
- v. All 20 street lights are evenly distributed within a distance of  $1 \text{ km}$  (distance between each pole is  $50 \text{ m}$ ).



# Voltage Drops in Different Type of Circuits

## Solution

$$\text{Apparent power, } S = \frac{P}{pf} = \frac{20 \times 150}{0.65} = \frac{3000}{0.65} = 4615.4 \text{ VA}$$

$$\text{Current, } I = \frac{S}{V_p} = \frac{4615.4}{240} = 19.23 \text{ A}$$

- assuming the total load is located at the end of the line (1km away)

$\Delta V$  along the street lighting wire is:

$$\begin{aligned}\Delta V &= I(R \cos \theta + X \sin \theta) \\ &= 19.23 [2.33 \times 0.65 + 0.095 \times \sin(\cos^{-1} 0.65)]\end{aligned}$$

$$\begin{aligned}\Delta V &= 19.23 [1.515 + 0.095 \times \sin(49.46^\circ)] \\ &= 19.23 (1.515 + 0.095 \times 0.7599) \\ &= 19.23 (1.587) \\ &= \mathbf{30.52 \text{ V}}\end{aligned}$$

# Voltage Drops in Different Type of Circuits

## Solution

$\Delta V$  along the neutral wire is:

$$\begin{aligned}\Delta V &= I(R \cos \theta + X \sin \theta) \\ &= 19.23 [0.309 \times 0.65 + 0.095 \times \sin(\cos^{-1} 0.65)] \\ &= 19.23 [0.2009 + 0.095 \times \sin(49.46^\circ)] \\ &= 19.23 (0.2009 + 0.095 \times 0.7599) \\ &= 19.23 (0.2731) \\ &= \mathbf{5.252 \text{ V}}\end{aligned}$$

➤ Hence, total voltage drop is:  $\Delta V = 30.52 + 5.252 = 35.77$

➤ Since the loads are evenly distributed,  $\Delta V = \frac{35.77}{2} = \mathbf{17.89 \text{ V}}$

➤ Therefore, for evenly distributed load the phase voltage at the end of load line is:

$$V_{load} = 240 \text{ V} - 17.89 \text{ V} = 222.1 \text{ V}$$

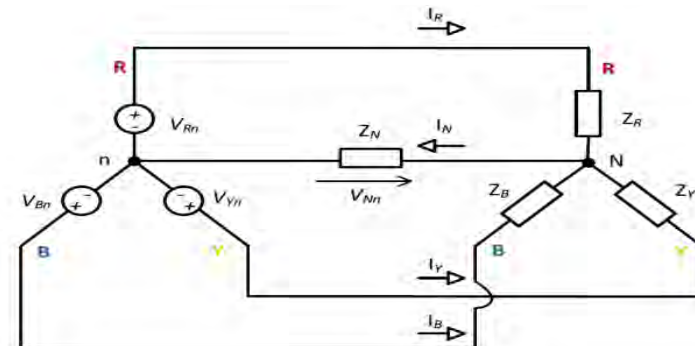


# Floating Neutral using Millman's Theorem

- Millman's theorem is used to determine the voltage at the ends of a circuit made up of only branches in parallel.
- Since nodes  $N$  and  $n$  are common to all three phase (R,Y,B), Millman's theorem is used to determine voltage drop across neutral impedance ( $V_{Nn}$ ).

$$V_{Nn} = \frac{(V_{Rn} Y_R + V_{Yn} Y_Y + V_{Bn} Y_B)}{(Y_R + Y_Y + Y_B + Y_N)}$$

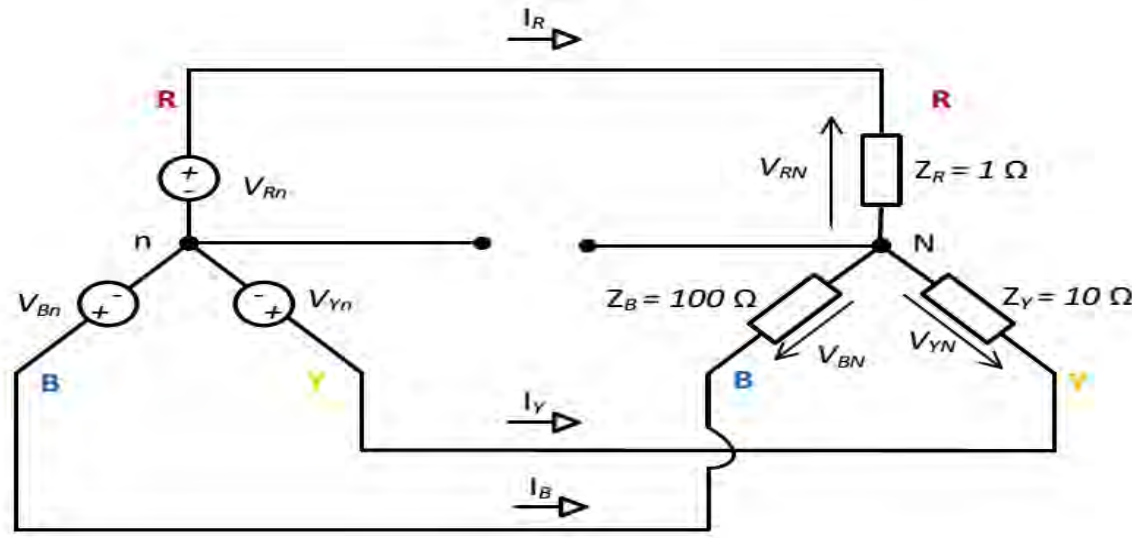
- By knowing  $V_{Nn}$ , voltage drop across the loads ( $V_{RN}$ ,  $V_{YN}$ ,  $V_{BN}$ ) can be obtained.
- Floating neutral may damage the connected loads or create hazardous touch voltage at equipment body.



# Floating Neutral using Millman's Theorem

## Example

- For the circuit given in Figure be a balanced RYB-sequence Y-connected source is connected to an unbalanced Y-load. Given  $V_{Rn} = 240\angle 0^\circ \text{ V}$ , determine voltage across the loads ( $V_{RN}$ ,  $V_{YN}$  and  $V_{BN}$ ) using Millman's theorem. Assume neutral line is open-circuit ( $Z_N = \infty \Omega$ ).

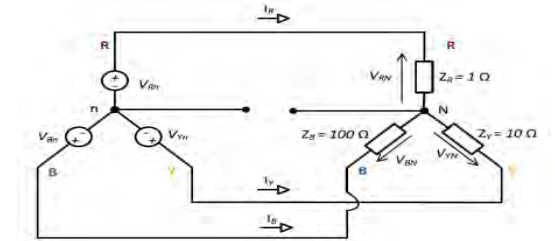


# Floating Neutral using Millman's Theorem

- Given
- $Z_R = 1 \Omega$ ,  $Z_Y = 10 \Omega$ ,  $Z_B = 100 \Omega$  and  $Z_N = \infty \Omega$ ;
- $V_{Rn} = 240\angle 0^\circ \text{ V}$ ,  $V_{Yn} = 240\angle 240^\circ \text{ V}$ ,  $V_{Bn} = 240\angle 120^\circ \text{ V}$
- Hence:
- $Y_R = 1/1 = 1 \text{ S}$ ,  $Y_Y = 1/10 = 0.1 \text{ S}$ ,  $Y_B = 1/100 = 0.01 \text{ S}$ ,  $Y_N = 1/\infty = 0 \text{ S}$ ,

$$\begin{aligned}
 V_{Nn} &= \frac{((240\angle 0^\circ)(1) + (240\angle 240^\circ)(0.1) + (240\angle 120^\circ)(0.01))}{(1 + 0.1 + 0.01 + 0)} \\
 &= \frac{(240\angle 0^\circ + (24\angle 240^\circ) + (2.4\angle 120^\circ))}{1.11} \\
 &= \frac{240 + 24 [\cos(240^\circ) + j \sin(240^\circ)] + 2.4 [\cos(120^\circ) + j \sin(120^\circ)]}{1.11} \\
 &= \frac{240 + 24 (-0.5 - j0.866) + 2.4 (-0.5 + j0.866)}{1.11} \\
 &= \frac{240 - 12 - j20.78 - 1.2 + j2.078}{1.11} \\
 &= \frac{226.8 - j18.71}{1.11} \\
 &= 204.3 - j16.85 \\
 &= \sqrt{204.3^2 + 16.85^2} \angle \tan^{-1} \left( -\frac{16.85}{204.3} \right) \\
 &= \sqrt{42022} \angle \tan^{-1}(-0.08248) \\
 &= \mathbf{204.9 \angle -4.715^\circ \text{ V}}
 \end{aligned}$$

## Solution

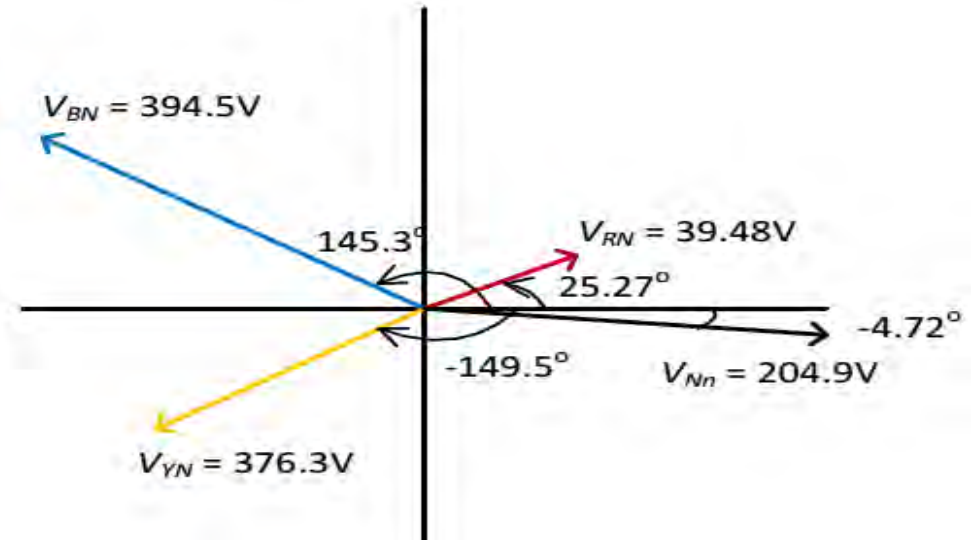


# Floating Neutral using Millman's Theorem

## Solution

$$\begin{aligned}\mathbf{V}_{RN} &= \mathbf{V}_{Rn} - \mathbf{V}_{Nn} \\ &= 240\angle 0^\circ - (204.9\angle -4.715^\circ) \\ &= 240 - (204.3 - j16.85) \\ &= 240 - 204.3 + j16.85 \\ &= \mathbf{35.7 + j16.85}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_{RN} &= \sqrt{35.7^2 + 16.85^2} \angle \tan^{-1}\left(\frac{16.85}{35.7}\right) \\ &= \sqrt{1558} \angle \tan^{-1}(0.4719) \\ &= \mathbf{39.48 \angle 25.27^\circ V}\end{aligned}$$



From Millman's theorem, phase voltages calculated are as follows:

$$\mathbf{V}_{RN} = 39.48 \angle 25.27^\circ V$$

$$\mathbf{V}_{YN} = 376.3 \angle -149.5^\circ V$$

$$\mathbf{V}_{BN} = 394.5 \angle 145.3^\circ V$$

### Note:

It can be seen that phases that have higher equivalent impedances ( $Z_Y$  and  $Z_B$ ) will experience overvoltage compared to phase with small equivalent impedance ( $Z_R$ ).

# Motor-Starting and Voltage Dips

- **When a motor is started, it typically draws a current 6-7 times its full load current for a short duration (commonly called the locked rotor current).**
- **This means that there can be large momentary voltage drops system-wide, from the power source (e.g. transformer or generator) through the intermediary buses, all the way to the motor terminals.**
- **A system-wide voltage drop can have a number of adverse effects:**
  - **Equipment with minimum voltage tolerances (e.g. electronics) may malfunction or behave aberrantly**
  - **Undervoltage protection may be tripped.**
  - **The motor itself may not start. Induction motors are typically designed to start with a terminal voltage  $>80\%$**



# Motor-Starting and Voltage Dips

- A starting voltage requirement and locked-rotor current should be stated as part of the motor specification.
- To determine the starting current for an induction motor, starting power of the motor must be determined using

$$S_{start} = (\text{rated horsepower})(\text{code letter factor})$$

- And the starting current can be found from

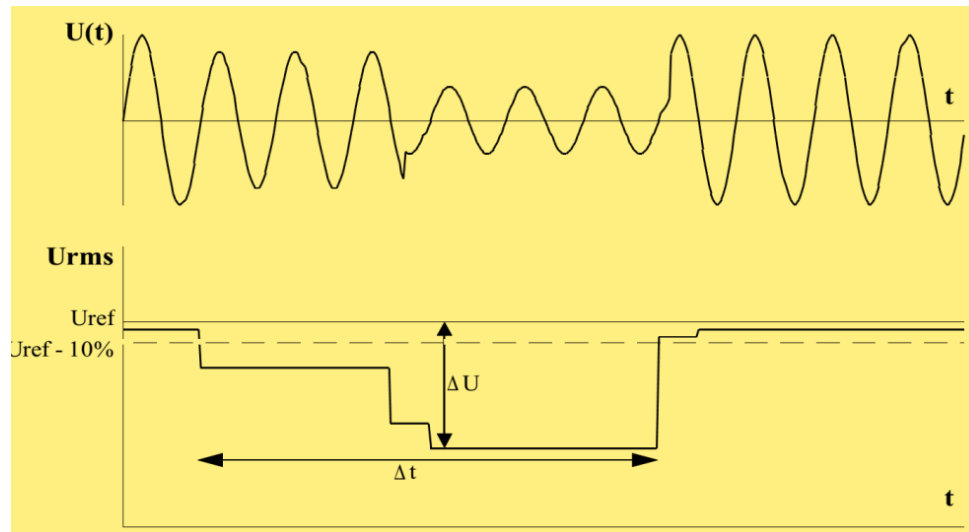
$$I_{L-start} = \frac{S_{start}}{\sqrt{3}V_L}$$

- The KVA required by a motor starting at full voltage is quite close to the "locked rotor KVA" requirement of the motor, which is easily determined from the actual motor's nameplate or from the manufacturer.

Code Letter	Kilovolt-Amperes per Horsepower with Locked Rotor
A	0-3.14
B	3.15-3.55
C	3.55-3.99
D	4.0-4.49
E	4.5-4.99
F	5.0-5.59
G	5.6-6.29
H	6.3-7.09
J	7.1-7.99
K	8.0-8.99
L	9.0-9.99
M	10.0-11.19
N	11.2-12.49
P	12.5-13.99
R	14.0-15.99
S	16.0-17.99
T	18.0-19.99
U	20.0-22.39
V	22.4-and up

# Motor-Starting and Voltage Dips

- A voltage dip is a short (from milliseconds up to seconds) decrease of more than 10 per cent of the supply voltage, but without the supply voltage disappearing completely.



- The Voltage dip due to motor starting is calculated as:

$$VDIP = |I_{start}|(R\cos\theta + X\sin\theta)$$

# Motor-Starting and Voltage Dips

## Example

- For the three-phase, 400V system shown in the Fig, determine the voltage drop percentage detected at the bus supplying load 1 (bus 1) during starting period of the Motor connected to bus 2.

- **System data**

Transformer:

11/0.4 kV, 250 kVA

$Z = 0.0101 + j 0.0143 \text{ pu}$

Line:

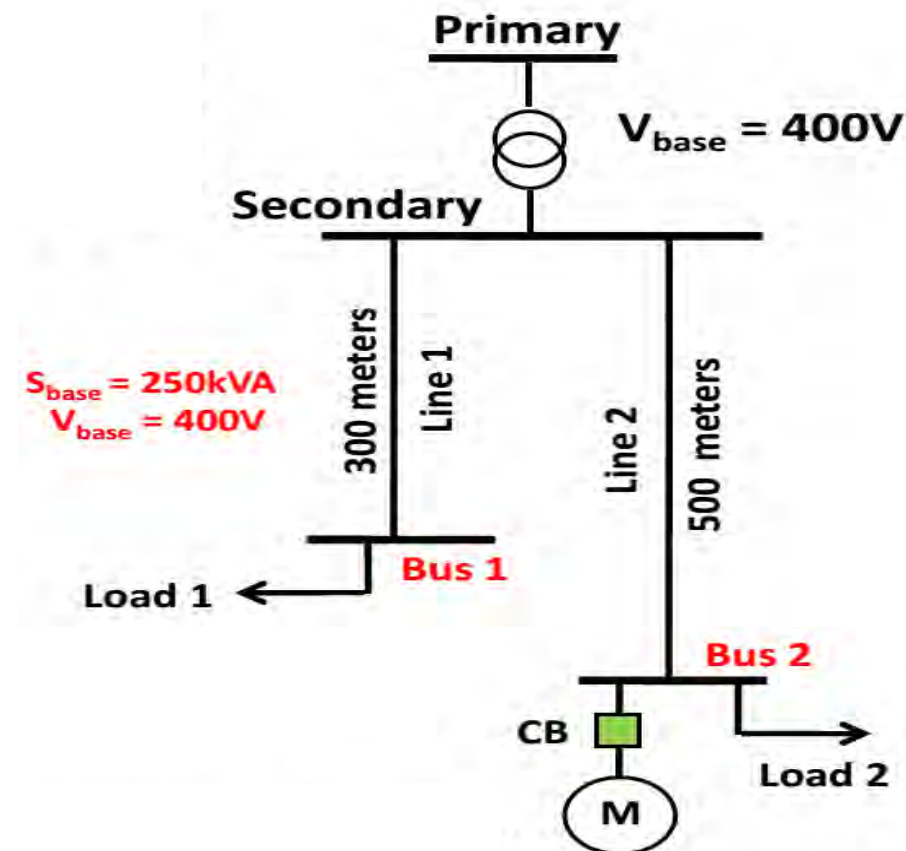
$Z_{\text{Line1}} = 0.025 + j 0.10 \text{ pu}$

$Z_{\text{Line2}} = 0.015 + j 0.06 \text{ pu}$

Load 1: 400V, 80 kVA, 0.9 pf lagging

Load 2: 400V, 50 kVA, 0.9 pf lagging

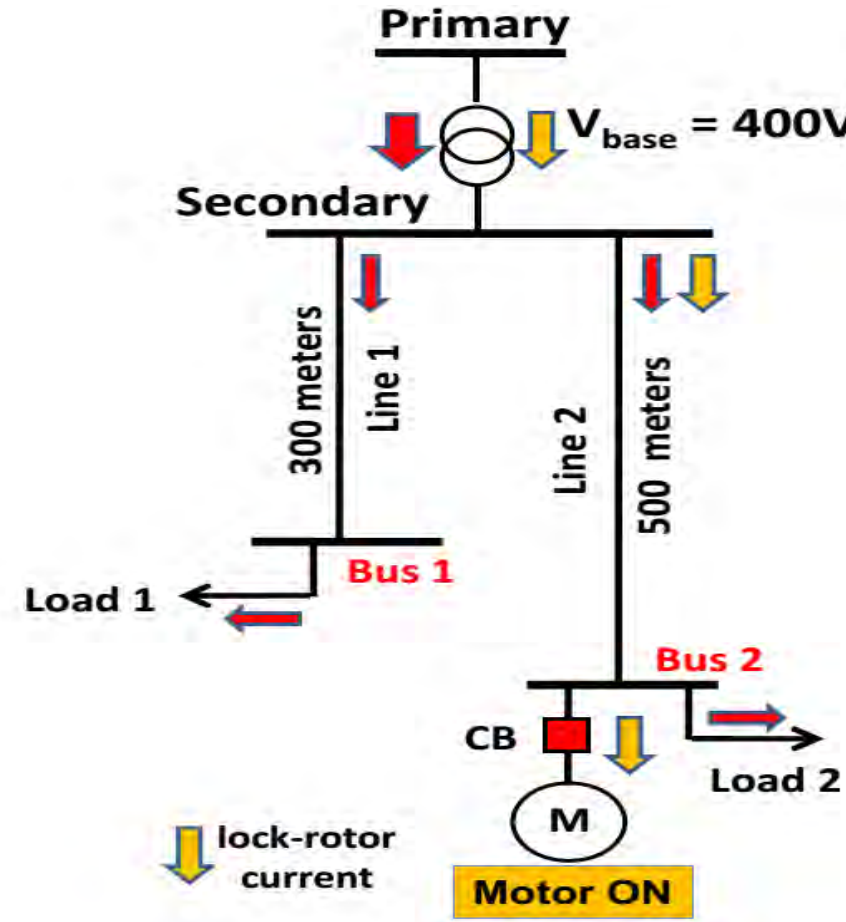
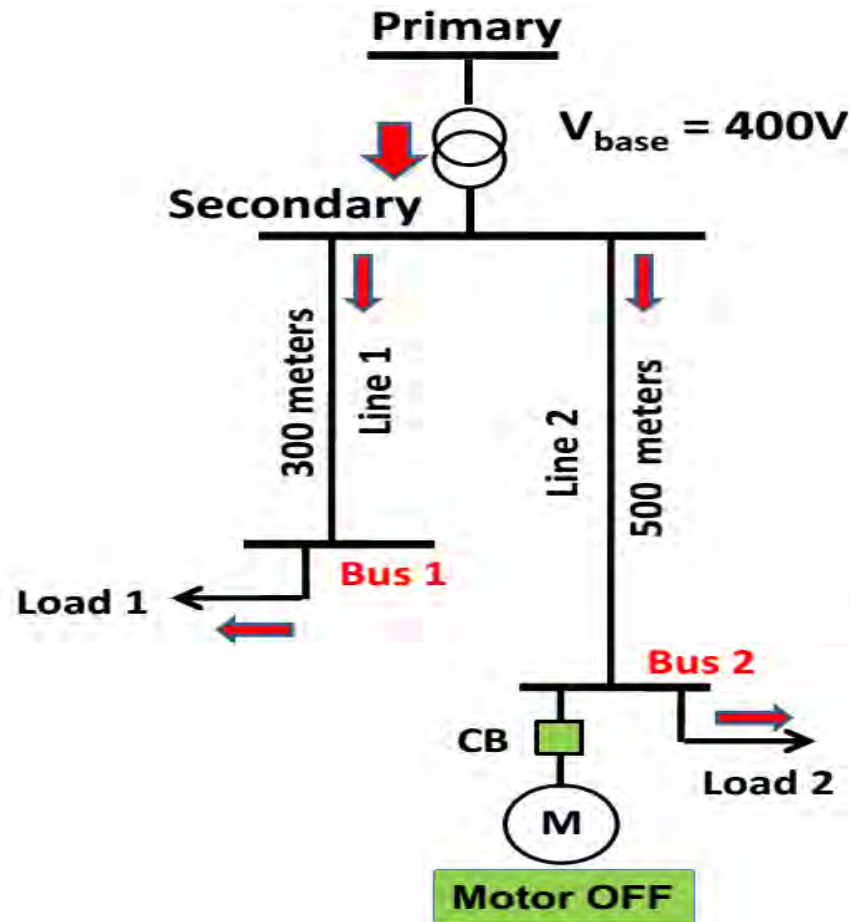
Motor: 400V, 20hp, 0.9 pf lagging and has a lock-rotor current of 5 times full load current & at 0.5 lag pf.





# Motor-Starting and Voltage Dips

**Solution**



# Motor-Starting and Voltage Dips

## Solution

$$VDIP_{bus1} = VD_{T,pu} + VDIP_{T,loc} + VD_{Line1,pu}$$

$$VDIP_{Trans} = VD_{T,pu} + VDIP_{T,loc}$$

(Transformer  
VD – NOC)
(Transformer  
VDIP – Motor Start)

$$I_T = I_{T,Load1} + I_{T,Load2}$$

### Transformer VD – NOC:

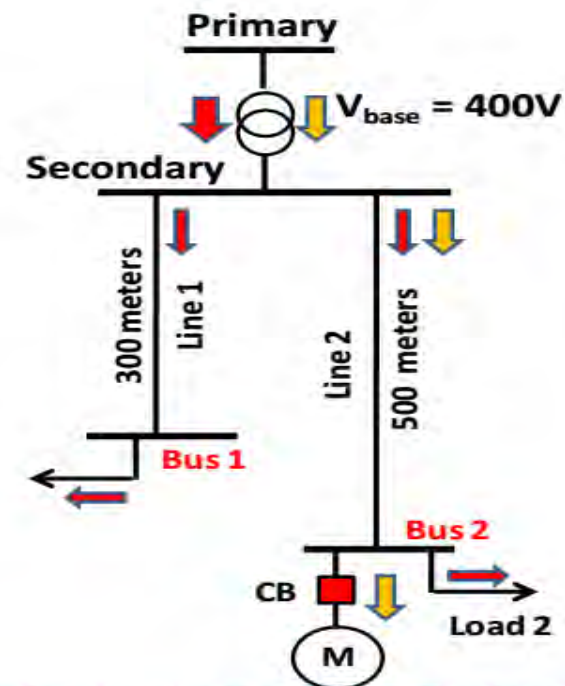
$$VD_T = I_T (R \cos\theta + X \sin\theta)$$

### Transformer VDIP – Motor Start:

$$VDIP_{T,loc} = |I_{loc}| (R \cos\theta + X \sin\theta)$$

### VD – Line 1 due to Load 1:

$$VD_{Line1,pu} = |I_{Load1}| (R_{line1} \cos\theta + X_{line1} \sin\theta)$$



### Transformer:

11/0.4 kV, 250 kVA

$Z = 0.0101 + j 0.0143 \text{ pu}$

$Z_{Line1} = 0.025 + j 0.10 \text{ pu}$

$Z_{Line2} = 0.015 + j 0.06 \text{ pu}$

Load 1: 400V, 80 kVA, 0.9 pf lagging

Load 2: 400V, 50 kVA, 0.9 pf lagging

# Motor-Starting and Voltage Dips

**Solution**

$$\begin{aligned} S_{\text{Base}} &= 250.0 \text{ kVA} \\ V_{\text{LL\_Base}} &= 400 \text{ V} \\ I_{\text{Base}} &= 360.8439 \text{ A} \end{aligned}$$

$$\begin{aligned} V_{\text{LL,source}} &= 400 \angle 30^\circ \\ V_{\text{LL,source}} &= 346.41 + j200.0 \end{aligned}$$

$$S_{\text{Load1}} = 80k \angle \cos^{-1}(pf_1) , S_{\text{Load1}} = 720004 + j34871$$

$$S_{\text{Load2}} = 50k \angle \cos^{-1}(pf_2) , S_{\text{Load2}} = 45000 + j21794$$

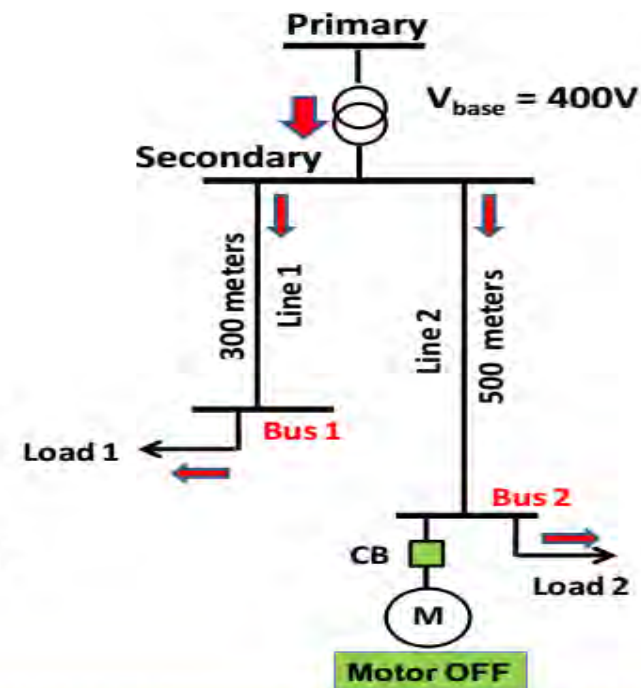
$$S_{\text{Load1,2}} = S_{\text{Load1}} + S_{\text{Load2}} , S_{\text{Load1,2}} = 117000 + j56667$$

$$I_{\text{Mag}} = \frac{S_{\text{Mag}}}{\sqrt{3} V_{\text{LL}}} , I_{\text{Mag pu}} = \frac{I_{\text{Mag}}}{I_{\text{Base}}} = \frac{S_{\text{Mag}}}{S_{\text{Base}}}$$

$$I_{\text{Mag1}} = 115.4701 \text{ A} , I_{\text{Mag1 pu}} = 0.3200 \text{ pu}$$

$$I_{\text{Mag2}} = 72.1688 \text{ A} , I_{\text{Mag2 pu}} = 0.2000 \text{ pu}$$

$$I_{\text{Mag1,2}} = 187.6389 \text{ A} , I_{\text{Mag1,2 pu}} = 0.5200 \text{ pu}$$



## Transformer:

11/0.4 kV, 250 kVA

$Z = 0.0101 + j 0.0143 \text{ pu}$

$Z_{\text{Line1}} = 0.025 + j 0.10 \text{ pu}$

$Z_{\text{Line2}} = 0.015 + j 0.06 \text{ pu}$

**Load 1:** 400V, 80 kVA, 0.9 pf lagging

**Load 2:** 400V, 50 kVA, 0.9 pf lagging

# Motor-Starting and Voltage Dips

**Solution**

$$VDIP_{bus1} = VD_{T,pu} + VDIP_{T,loc} + VD_{Line1,pu}$$

**Transformer VD – NOC:**

$$\downarrow I_T = I_{T,Load1} + I_{T,Load2}$$

$$S_T = S_{Load1} + S_{Load2}$$

$$VD_T = I_{MAG1,2} (R_T \cos\theta_{S_T} + X_T \sin\theta_{S_T})$$

$$VD_T = 0.520 (0.0101 \times 0.9 + 0.0143 \times 0.4359)$$

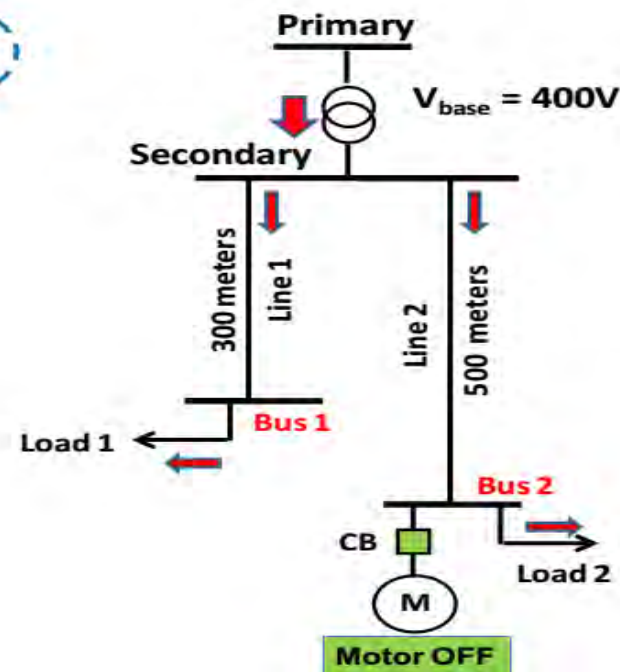
$$VD_T = 0.0080 \text{ pu}$$

**VD – Line 1 due to Load 1:**

$$VD_{Line1} = I_{Mag1} (R_{line1} \cos\theta_{Load1} + X_{line1} \sin\theta_{Load1})$$

$$VD_{Line1} = 0.3200 (0.025 \times 0.9 + 0.100 \times 0.4359)$$

$$VD_{Line1} = 0.0211 \text{ pu}$$



**Transformer:**

11/0.4 kV, 250 kVA

$Z = 0.0101 + j 0.0143 \text{ pu}$

$Z_{Line1} = 0.025 + j 0.10 \text{ pu}$

$Z_{Line2} = 0.015 + j 0.06 \text{ pu}$

**Load 1:** 400V, 80 kVA, 0.9 pf lagging

**Load 2:** 400V, 50 kVA, 0.9 pf lagging

# Motor-Starting and Voltage Dips

**Solution**

$$VDIP_{bus1} = VD_{T,pu} + VDIP_{T,loc} + VD_{Line1,pu}$$

$$I_{M, Mag} = \frac{P_{Motor}}{\sqrt{3} V_{LL} (pf_{motor})} = \frac{(20hp * 746) Watt}{\sqrt{3} 400 (0.9)}$$

$$I_{M, Mag} = 23.9280 A = 0.0663 pu$$

$$I_{loc\_M,pu} = 5 (23.9280) A = 5 (0.0663 pu)$$

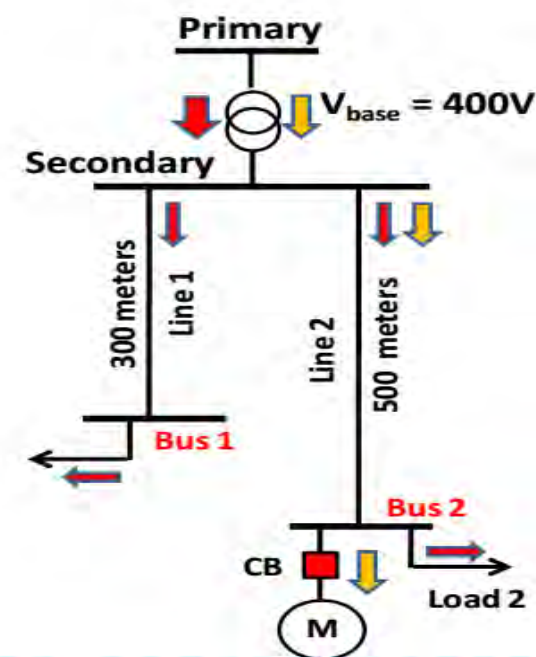
$$I_{loc\_M,pu} = 0.3316 pu$$

## Transformer VDIP – Motor Start:

$$VDIP_T = |I_{loc,M}| (R_T \cos\theta_{locM} + X_T \sin\theta_{locM})$$

$$VDIP_{T,loc} = 0.3316 (0.0101 (0.5) + 0.0143 (0.866))$$

$$VDIP_{T,loc} = 0.0058 pu$$



## Transformer:

11/0.4 kV, 250 kVA

$Z = 0.0101 + j 0.0143 pu$

$Z_{Line1} = 0.025 + j 0.10 pu$

$Z_{Line2} = 0.015 + j 0.06 pu$

**Load 1:** 400V, 80 kVA, 0.9 pf lagging

**Load 2:** 400V, 50 kVA, 0.9 pf lagging

# Motor-Starting and Voltage Dips

**Solution**

$$VDIP_{bus1} = VD_{T,pu} + VDIP_{T,loc} + VD_{Line1,pu}$$

$$VDIP_{Trans} = VD_{T,pu} + VDIP_{T,loc}$$

(Transformer VD - NOC)      (Transformer VDIP - Motor Start)

$$VD_T = 0.0080 \text{ pu}$$

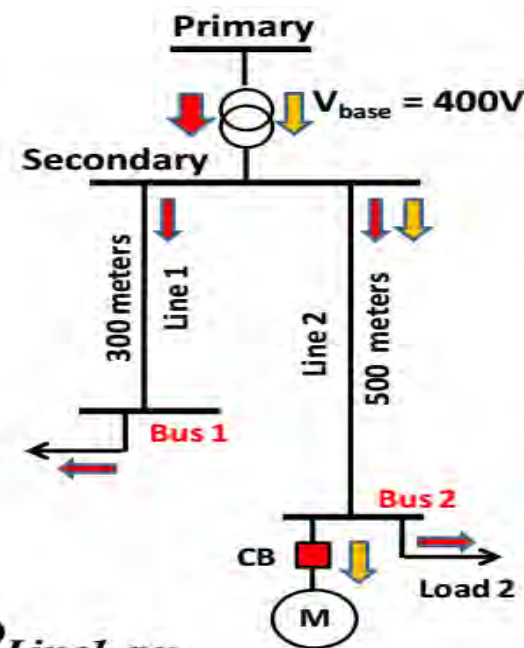
$$VD_{Line1} = 0.0211 \text{ pu}$$

$$VDIP_{T,loc} = 0.0058 \text{ pu}$$

$$VDIP_{bus1} = VD_{T,pu} + VDIP_{T,loc} + VD_{Line1,pu}$$

$$VDIP_{bus1} = 0.0080 + 0.0058 + 0.021$$

$$VDIP_{bus1} = 0.0348 = 3.48\% < 3.50\%$$



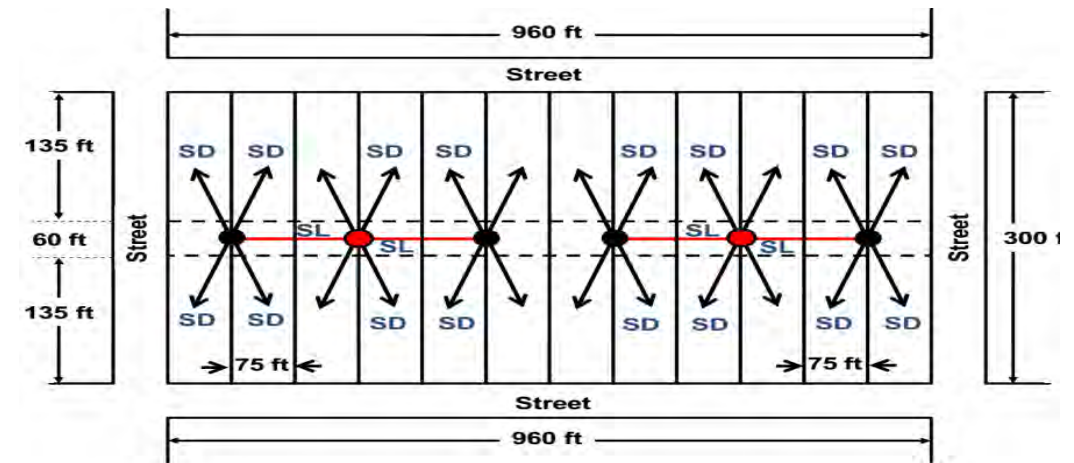
# Motor-Starting and Voltage Dips

## Example

- A three-phase URD distribution system in a residential area with the layout shown in the Figure below.
- A standard transformer size is selected as 75 kVA. See Table A for more information. The SL standard cable size is selected as # 4/0 AWG. The SD standard cable size is selected as # 1/0 AWG of 70 ft. See Table B for more information. The maximum limit of the VD = 3.5%.

Determine:

- The steady-state VD in pu at the most remote consumer meter for the annual system load assuming  $\text{pf} = 0.9$  lag.
- The VDIP in pu for motor starting at the most unfavorable location. The motor is three-phase 240-V, 50A locked rotor current, with a 50% power factor locked rotor.



# Motor-Starting and Voltage Dips

## Example

**Table A: Three-phase 7200-240 V distribution transformer data at 65 °C**

Rated kVA	Core Loss kW	Copper Loss kW	R pu	X pu	Excitation Current A
15	0.083	0.194	0.013	0.0094	0.014
25	0.115	0.309	0.0123	0.0138	0.015
37.5	0.17	0.4	0.0107	0.0126	0.014
50	0.178	0.537	0.0107	0.0139	0.014
75	0.28	0.755	0.0101	0.0143	0.014
100	0.335	0.975	0.0098	0.0145	0.014

**Table B: Twin concentric Al/Cu XLPE 600 V cable data**

Size	R( $\Omega$ /1000 ft) per conductor		X( $\Omega$ /1000 ft) per phase conductor	Direct-burial Ampacity A	$\tilde{K}$ Per unit voltage drop per $10^4$ A.ft	
	Phase conductor 90°C	Neutral Conductor 80°C			90 % PF	50 % PF
2 AWG	0.334	0.561	0.0299	180	0.02613	0.01608
1 AWG	0.265	0.419	0.305	205	0.02098	0.01324
1/0 AWG	0.21	0.337	0.0297	230	0.01683	0.01089
2/0 AWG	0.167	0.259	0.029	265	0.0136	0.00905
3/0 AWG	0.132	0.211	0.028	300	0.01092	0.00752
4/0 AWG	0.105	0.168	0.0275	340	0.00888	0.00636
250 kcmil	0.089	0.133	0.028	370	0.00769	0.00573
350 kcmil	0.063	0.085	0.027	445	0.00571	0.00458
500 kcmil	0.044	0.066	0.026	540	0.00424	0.00371

No. of customers being diversified	30-Min. Annual Max. Demands, kVA/Customer		
	Class 1	Class 2	Class 3
1	18	10	2.5
2	14.4	7.6	1.8
4	12	6	1.5
12	10	4.4	1.2
100	8.4	3.6	1.1

**NOTE:**  $\tilde{K}$  : per unit voltage drop per  $10^4$  A.ft



# Motor-Starting and Voltage Dips

## Solution

- (a) The steady-state VD in pu at the most remote consumer meter for the annual system load assuming pf = 0.9 lagging.

$$VD_{Total} = VD_{T,pu} + VD_{SL,pu} + VD_{SD,pu}$$

### The Transformer Load

No. of customers = 12

Therefore, the Transformer Load = 12\*4.4 = 52.8 kVA

$$I_T = \frac{S_T}{\sqrt{3} V} = \frac{52.8 \text{ kVA}}{\sqrt{3} 240 \text{ V}} = 127.0171 \text{ A}$$

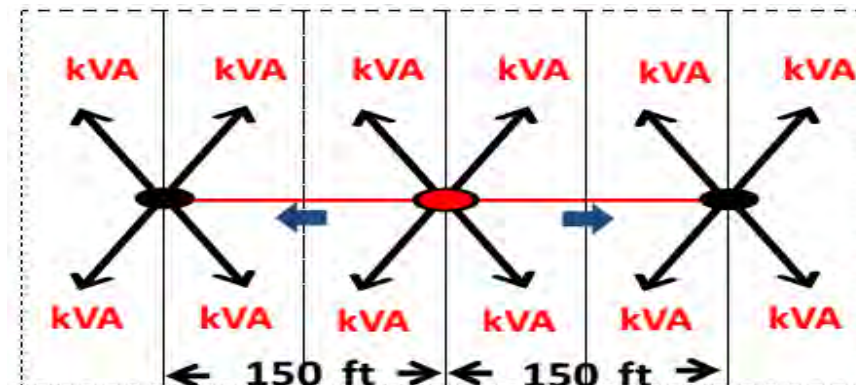
$$I_{T,pu} = \frac{127.0171}{\frac{S_B}{\sqrt{3} V_B}} = \frac{127.0171}{\frac{75 \text{ kVA}}{\sqrt{3} 240}} = 0.7040 \text{ pu}$$

### Transformer VD

$$VD_T = I_T ( R \cos\theta + X \sin\theta )$$

$$VD_T = 0.7040(0.0101 \times 0.9 + 0.0143 \times 0.4359)$$

$$VD_T = 0.0108 \text{ pu}$$



Rated kVA	Core Loss kW	Copper Loss kW	R pu	X pu	Excitation Current A
50	0.178	0.537	0.0107	0.0139	0.014
75	0.28	0.755	0.0101	0.0143	0.014



# Motor-Starting and Voltage Dips

## Solution

- (a) The steady-state VD in pu at the most remote consumer meter for the annual system load assuming pf = 0.9 lagging.

$$VD_{Total} = VD_{T,pu} + VD_{SL,pu} + VD_{SD,pu}$$

### The Secondary Line Load

No. of customers = 4

Therefore, the SL Load = 4\*6= 24 kVA

$$I_{SL,pu} = \frac{24.0 \text{ kVA}}{\frac{S_B}{\sqrt{3} V_B}} = \frac{24.0 \text{ kVA}}{\frac{75 \text{ kVA}}{\sqrt{3} 240}} = 0.320 \text{ pu}$$

$$I_{SL} = 57.7350 \text{ A}$$

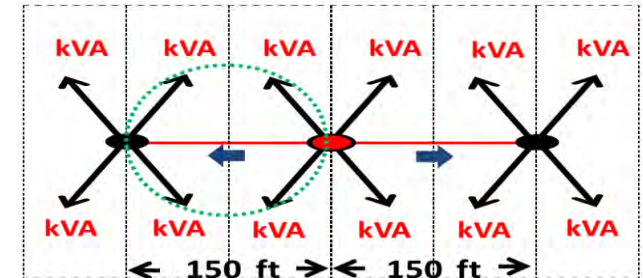
### Secondary line VD

$$I_{SL} = 57.7350 \text{ A}$$

$$VD_{SL,pu} = K \left[ \frac{I \times l}{10^4} \right]$$

$$VD_{SL,pu} = 0.0088 \left[ \frac{57.735 \times 150 \text{ ft}}{10^4} \right]$$

$$VD_{SL,pu} = 0.0077 \text{ pu}$$



### NOTE:

The pu VD is calculated using “K” factor and actual current (A) and conductor length (ft). No need for current pu values.

Size	R(Ω/1000 ft) per conductor		X(Ω/1000 ft) per phase conductor	Direct-burial Ampacity A	Per unit voltage drop per 10 <sup>4</sup> A .ft	
	Phase conductor 90°C	Neutral Conductor 80°C			90 % PF	50 % PF
2/0 AWG	0.167	0.259	0.029	265	0.0136	0.00905
3/0 AWG	0.132	0.211	0.028	300	0.01092	0.00752
4/0 AWG	0.105	0.168	0.0275	340	<u>0.00888</u>	0.00636
250 kcmil	0.089	0.133	0.028	370	0.00769	0.00573

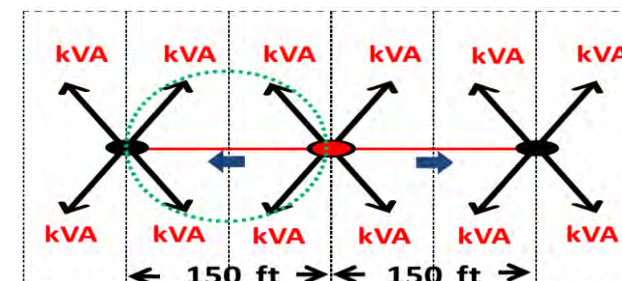


# Motor-Starting and Voltage Dips

## Solution

- (a) The steady-state VD in pu at the most remote consumer meter for the annual system load assuming pf = 0.9 lagging.

$$VD_{Total} = VD_{T,pu} + VD_{SL,pu} + VD_{SD,pu}$$



### The Service Drop Load

No. of customers = 1

Therefore, the SL Load = 1\*10= 10 kVA

### Transformer VD

$$I_{SD} = 24.0563 \text{ A}$$

$$VD_{SD,pu} = K \left[ \frac{I \times l}{10^4} \right]$$

$$VD_{SD,pu} = 0.01683 \left[ \frac{24.0563 \times 70 \text{ ft}}{10^4} \right]$$

$$VD_{SD,pu} = 0.0028 \text{ pu}$$

### The load Total Voltage Drop is

$$VD_T = 0.0108 \text{ pu} \quad VD_{SL,pu} = 0.0077 \text{ pu} \quad VD_{SD,pu} = 0.0028 \text{ pu}$$

$$VD_{Total} = VD_{T,pu} + VD_{SL,pu} + VD_{SD,pu}$$

$$VD_{Total} = 0.0213 \text{ pu} < 0.035 \text{ pu}$$

**Less than the given criteria (0.035 pu)**

### NOTE:

The pu VD is calculated using “K” factor and actual current (A) and conductor length (ft). No need for current pu values.

Size	R(Ω/1000 ft) per conductor		X(Ω/1000 ft) per phase conductor	Direct-burial Ampacity A	K Per unit voltage drop per 10 <sup>4</sup> A.ft	
	Phase conductor 90°C	Neutral Conductor 80°C			90 % PF	50 % PF
1/0 AWG	0.21	0.337	0.0297	230	0.01683	0.01089

# Motor-Starting and Voltage Dips

## Solution

- (a) The steady-state VD in pu at the most remote consumer meter for the annual system load assuming pf = 0.9 lagging.

$$VD_{Total} = VD_{T,pu} + VD_{SL,pu} + VD_{SD,pu}$$

### The Service Drop Load

No. of customers = 1

Therefore, the SL Load = 1\*10= 10 kVA

### Service drop VD

$$I_{SD} = 24.0563 \text{ A}$$

$$VD_{SD,pu} = K \left[ \frac{I \times l}{10^4} \right]$$

$$VD_{SD,pu} = 0.01683 \left[ \frac{24.0563 \times 70 \text{ ft}}{10^4} \right]$$

$$VD_{SD,pu} = 0.0028 \text{ pu}$$

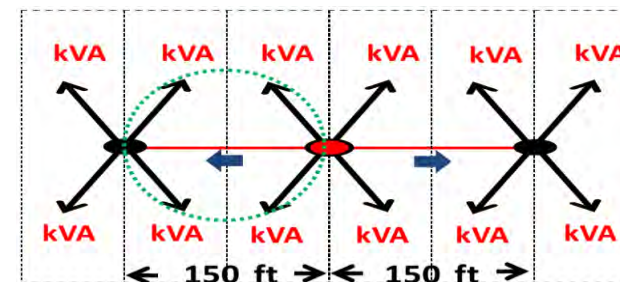
### The load Total Voltage Drop is

$$VD_T = 0.0108 \text{ pu} \quad VD_{SL,pu} = 0.0077 \text{ pu} \quad VD_{SD,pu} = 0.0028 \text{ pu}$$

$$VD_{Total} = VD_{T,pu} + VD_{SL,pu} + VD_{SD,pu}$$

$$VD_{Total} = 0.0213 \text{ pu} < 0.035 \text{ pu}$$

**Less than the given criteria (0.035 pu)**



### NOTE:

The pu VD is calculated using “K” factor and actual current (A) and conductor length (ft). No need for current pu values.

Size	R(Ω/1000 ft) per conductor		X(Ω/1000 ft) per phase conductor	Direct-burial Ampacity A	K Per unit voltage drop per 10 <sup>4</sup> A.ft	
	Phase conductor 90°C	Neutral Conductor 80°C			90 % PF	50 % PF
1/0 AWG	0.21	0.337	0.0297	230	0.01683	0.01089



# Motor-Starting and Voltage Dips

**Solution**

- (b) The VDIP in pu for motor starting at the most unfavorable location.

The motor is three-phase 240-V, 50A locked rotor current, with a 50% power factor locked rotor.

$$VDIP_T = VD_{T,pu} + VDIP_{T,loc} \qquad VD_T = 0.0108 \text{ pu}$$

$$VDIP_{T,loc} = |I_{loc}| (R \cos\theta + X \sin\theta)$$

$$VDIP_{T,loc} = \frac{50}{75kVA / (\sqrt{3} * 240)} (0.0101 (0.5) + 0.0143 (0.866))$$

$$VDIP_{T,loc} = 0.0048 \text{ pu}$$

$$I_{loc} = 0.2771 \text{ p5}$$

Rated kVA	Core Loss kW	Copper Loss kW	R pu	X pu	Excitation Current A
50	0.178	0.537	0.0107	0.0139	0.014
75	0.28	0.755	<u>0.0101</u>	<u>0.0143</u>	0.014

# Motor-Starting and Voltage Dips

**Solution**

- (b) The VDIP in pu for motor starting at the most unfavorable location.

The motor is three-phase 240-V, 50A locked rotor current, with a 50% power factor locked rotor.

$$VDIP_{SL} = VD_{SL,pu} + VDIP_{SL,loc} \qquad VD_{SL,pu} = 0.0077 \text{ pu}$$

$$VDIP_{SL,pu} = K \left[ \frac{(I_{MLoc}) \times l}{10^4} \right]$$

$$VDIP_{SL,pu} = 0.00636 \left[ \frac{50 \times 150 \text{ ft}}{10^4} \right]$$

$$VDIP_{SL,pu} = 0.0048 \text{ pu}$$

$$I_{SL} = 57.7350 \text{ A}$$

$$VD_{SL,pu} = K \left[ \frac{I \times l}{10^4} \right]$$

$$VD_{SL,pu} = 0.0088 \left[ \frac{57.735 \times 150 \text{ ft}}{10^4} \right]$$

$$VD_{SL,pu} = 0.0077 \text{ pu}$$

Size	R(Ω/1000 ft) per conductor		X(Ω/1000 ft) per phase conductor	Direct-burial Ampacity A	Per unit voltage drop per 10 <sup>4</sup> A .ft	
	Phase conductor 90°C	Neutral Conductor 80°C			90 % PF	50 % PF
2/0 AWG	0.167	0.259	0.029	265	0.0136	0.00905
3/0 AWG	0.132	0.211	0.028	300	0.01092	0.00752
4/0 AWG	0.105	0.168	0.0275	340	0.00888	0.00636
250 kcmil	0.089	0.133	0.028	370	0.00769	0.00573

# Motor-Starting and Voltage Dips

**Solution**

- (b) The VDIP in pu for motor starting at the most unfavorable location.

The motor is three-phase 240-V, 50A locked rotor current, with a 50% power factor locked rotor.

$$VDIP_{SD} = VD_{SD,pu} + VDIP_{SD,loc} \quad VD_{SD,pu} = 0.0028 \text{ pu}$$

$$VDIP_{SL,pu} = K \left[ \frac{(I_{MLoc}) \times l}{10^4} \right]$$

$$VDIP_{SD,pu} = 0.01089 \left[ \frac{50 \times 70 \text{ ft}}{10^4} \right]$$

$$VDIP_{SD,pu} = 0.00381 \text{ pu}$$

$$I_{SD} = 24.0563 \text{ A}$$

$$VD_{SD,pu} = K \left[ \frac{I \times l}{10^4} \right]$$

$$VD_{SD,pu} = 0.01683 \left[ \frac{24.0563 \times 70 \text{ ft}}{10^4} \right]$$

$$VD_{SD,pu} = 0.0028 \text{ pu}$$

**Table B: Twin concentric Al/Cu XLPE 600 V cable data**

Size	R(Ω/1000 ft) per conductor		X(Ω/1000 ft) per phase conductor	Direct-burial Ampacity A	$\tilde{K}$ Per unit voltage drop per 10 <sup>4</sup> A .ft	
	Phase conductor 90°C	Neutral Conductor 80°C			90 % PF	50 % PF
2 AWG	0.334	0.561	0.0299	180	0.02613	0.01608
1 AWG	0.265	0.419	0.305	205	0.02098	0.01324
1/0 AWG	0.21	0.337	0.0297	230	0.01683	0.01089

# Motor-Starting and Voltage Dips

## Solution

- (b) The VDIP in pu for motor starting at the most unfavorable location.

The Total Voltage Dip due to motor starting is:

$$VD_T = 0.0108 \text{ pu} \quad VDIP_{T,loc} = 0.0048 \text{ pu}$$

$$VD_{SL,pu} = 0.0077 \text{ pu} \quad VDIP_{SL,pu} = 0.0048 \text{ pu}$$

$$VD_{SD,pu} = 0.0028 \text{ pu} \quad VDIP_{SD,pu} = 0.00381 \text{ pu}$$

$$\begin{aligned} VDIP_{Total} &= VD_{T,pu} + VDIP_{T,loc} \\ &+ VD_{SL,pu} + VDIP_{SL,loc} \\ &+ VD_{SD,pu} + VDIP_{SD,loc} \end{aligned}$$

$$VDIP_{Total} = 0.0347 < 0.035$$

