SKEE 4443
POWER SYSTEM ANALYSIS

CHAPTER 1

Power System Representation and Per Unit System

Introduction

- It is clear that a detailed representation of each of the three phases in the system is cumbersome and can also obscure information about the system.
- A balanced three-phase system is solved as a single-phase circuit made of one line and the neutral return; thus a simpler representation.
- Further simplification by omitting the neutral.

Introduction

- The simplified diagram is called the *single-line diagram or one-line diagram*.

- The one-line diagram summarizes the relevant information about the system for the particular problem studied.

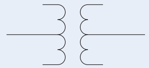
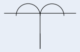
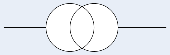
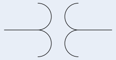
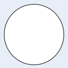
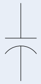
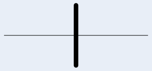
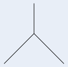
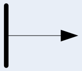
- For example, relays and circuit breakers are not important when dealing with a normal state problem. However, when fault conditions are considered, the location of relays and circuit breakers is important and is thus included in the single-line diagram.

Introduction

- The International Electrotechnical Commission (IEC), the American National Standards Institute (ANSI), and the Institute of Electrical and Electronics Engineers (IEEE) have published a set of standard symbols for electrical diagrams.

- The main component of a one-line (or single line) diagram are : Buses, Branches, Loads, Machines, 2 winding Transformers, Switched Shunts, Reactor and Capacitor Banks.

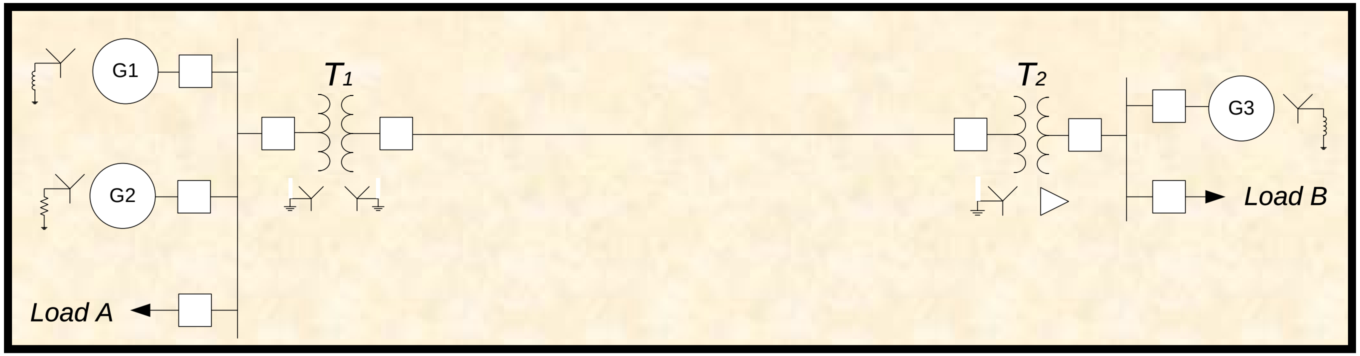
One Line Diagram - Symbols

	two-winding transformer		current transformer
	two-winding transformer		voltage transformer
	generator		capacitor
	bus		circuit breaker
	transmission line		circuit breaker
	delta connection		fuse
	wye connection		surge arrestor
	static load		disconnect

Symbols used in one line diagram (from ANSI and IEEE)

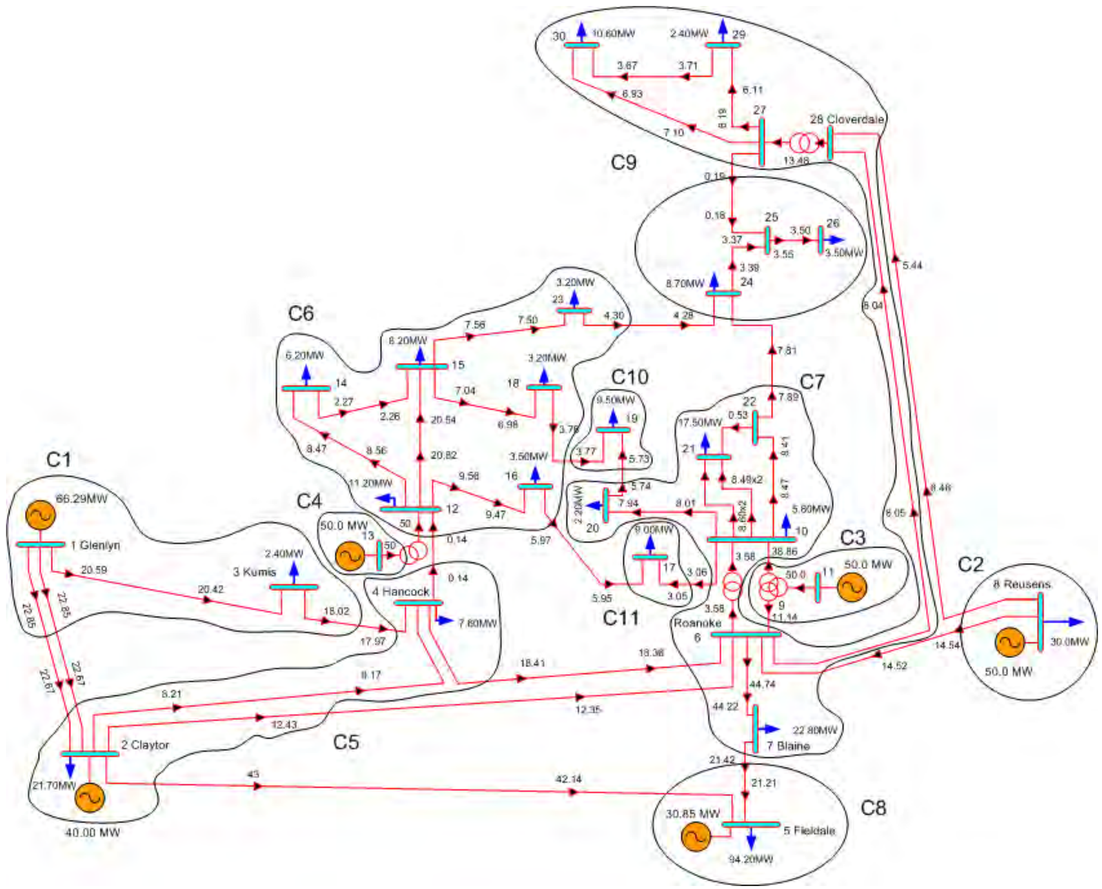
Single-line diagram

- It is important to know the location of points where a system is connected to ground in order to calculate the amount of current flow when an unsymmetrical fault involving ground occurs. The standard symbol to designate a three-phase Y with the neutral solidly grounded is shown in Figure below.



One-line diagram

- IEEE 30bus system

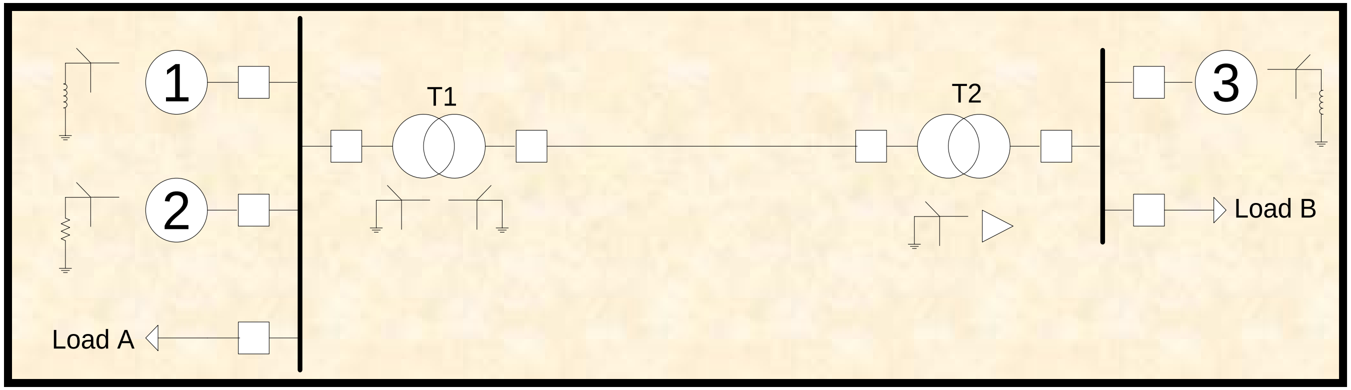


Impedance and Reactance Diagrams

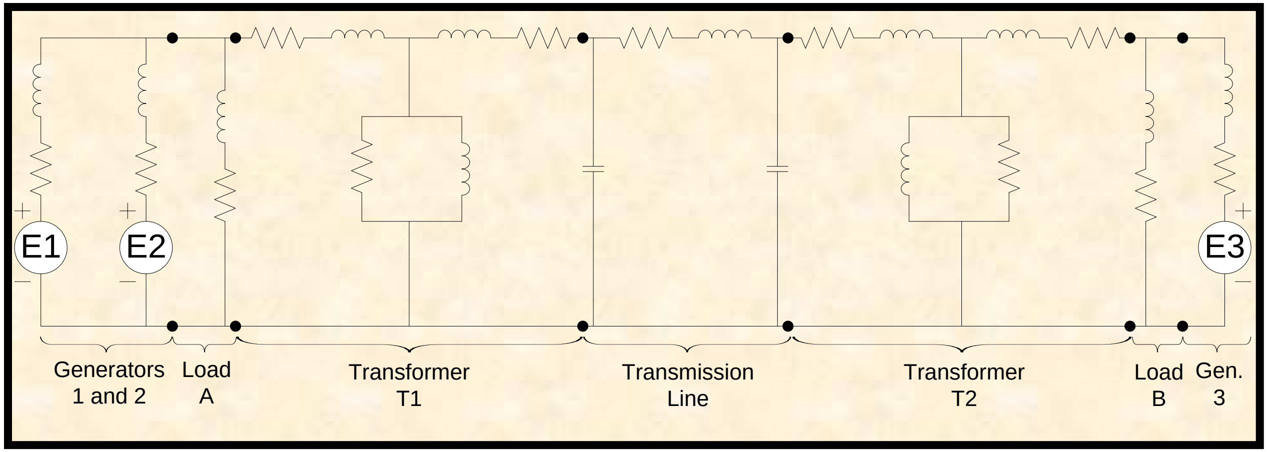
- The impedance ($Z = R + jX$) diagram is converted from one-line diagram showing the equivalent circuit of each component of the system.

- It is needed in order to calculate the performance of a system under load conditions (Load flow studies) or upon the occurrence of a short circuit (fault analysis studies).

Impedance and Reactance Diagrams



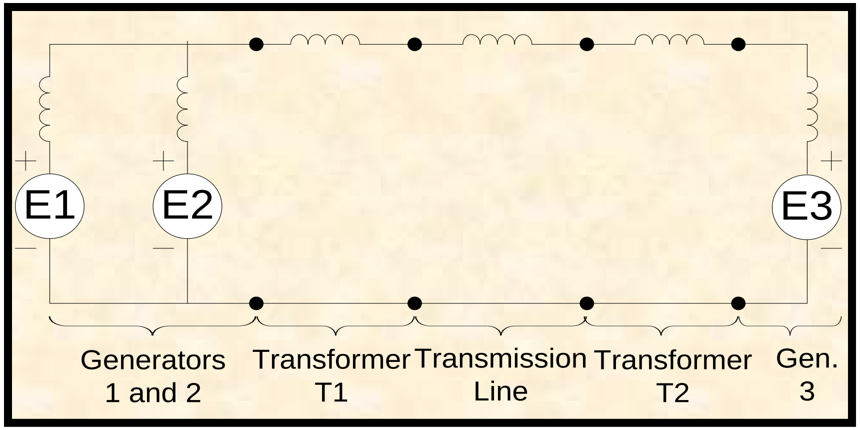
One-Line Diagram of an Electric Power System



Impedance Diagram Corresponding to the One-Line Diagram above

Impedance and Reactance Diagrams

- Reactance (jX) diagram is further simplified from impedance diagram by omitting
  - all static loads,
  - all resistances,
  - the magnetizing current of each transformer, and
  - the capacitance of the transmission line.



Reactance Diagram Corresponding to the One-Line Diagram of Example

Per Unit System

- In power systems large amounts of power being transmitted in the range of kilowatts to megawatts, at different voltage levels.

- As a result, in analysis, it is useful to scale, or normalize quantities with large physical values and this is commonly called per unit system in power system analysis.

Per Unit System

- The per unit system is widely used in the power system industry to express values of voltages, currents, powers, and impedances of various power equipment.
- It is mainly used for transformers and AC machines
- Per unit system used extensively along with one-line diagram to further simplify the process.

Per Unit Calculations

- All base values are only magnitude. They are not associated with any angle.
- The per unit values, however, are phasors.
- The phase angles of the currents and voltages and the power factor of the circuit are not affected by the conversion to per unit values.
- In general, the per unit value is the ratio of the actual value and the base value of the same quantity.

$$\boxed{\text{per unit value} = \frac{\text{actual value}}{\text{base value}}}$$

- The per unit system values can also be expressed as per cent values.

Base value

- Specify the base values of current and voltage, base impedance, kilovoltamperes can be determined
- Quantities and base value selected

    voltage, base value in *kilovolts, kV*

    current, base value in *ampere, A*

Base values

Generally the following two base values are chosen :

The base power = nominal power of the equipment

The base voltage = nominal voltage of the equipment

The base current and The base impedance are determined by the natural laws of electrical circuits

Base values

- Usually, the nominal apparent power (S) and nominal voltage (V) are taken as the base values for power (S_{base}) and voltage (V_{base}).

- The base values for the current (I_{base}) and impedance (Z_{base}) can be calculated based on the first two base values.

Per-unit System for 1- ϕ Circuits

$$P_{B,1\phi} = Q_{B,1\phi} = S_{B,1\phi}$$

$$I_B = \frac{S_{B,1\phi}}{V_{B,LN}}$$

$$Z_B = R_B = X_B = \frac{V_{B,LN}}{I_B} = \frac{V_{B,LN}^2}{S_{B,1\phi}}$$

Base value

- For single phase system

$$\text{Base impedance}, \Omega = \frac{(\text{base voltage}, KV_{LN})^2}{MVA_{1\phi}}$$

$$\text{Base power}, kW_{1\phi} = \text{base } kVA_{1\phi}$$

$$\text{Base power}, MW_{1\phi} = \text{base } MVA_{1\phi}$$

Per-Unit System

Per-unit system:

$$V_{p.u.} = \frac{V_{actual}}{V_B} \qquad I_{p.u.} = \frac{I_{actual}}{I_B}$$

$$S_{p.u.} = \frac{S_{actual}}{S_B} \qquad Z_{p.u.} = \frac{Z_{actual}}{Z_B}$$

$$Z\% = Z_{p.u.} \times 100\%$$

Percent of base Z

Per-unit System for 1- ϕ Circuits

For actual quantity that has complex number,
Per unit can be expressed as:

$$Z_{pu}\angle\theta = \frac{Z\angle\theta}{Z_B}$$

Per unit can be expressed in rectangular form:

$$Z_{pu} = R_{pu} + jX_{pu}$$

$$S_{pu} = R_{pu} + jQ_{pu}$$

Power:

$$S_{pu} = \frac{S}{S_b} = V_{pu} I^*_{pu}$$

$$P_{pu} = \frac{P}{S_b} = V_{pu} I_{pu} \cos\theta$$

$$Q_{pu} = \frac{Q}{S_b} = V_{pu} I_{pu} \sin\theta$$

exercise

- A generator has an impedance of 2.65 ohms. What is its impedance in per-unit, using bases 500MVA and 22kV

Per Unit system for 3- ϕ Circuits

- Have the same per unit values for line to line and line to neutral quantities.
- Make everything look like a single phase circuit.
- Balanced three phase circuits can be solved in per unit on a per phase basis after converting delta load impedance to equivalent Y impedance.
- Base value can be selected on a per phase basis or on a three phase basis.

Per Unit system for 3- ϕ Circuits (Voltage)

- In a three phase system, we have:

$$\boxed{V_{LL}^{pu} = V_{LN}^{pu}}$$

- Consider Y connected: $\boxed{V_{LL} = \sqrt{3}V_{LN}}$

- So:

$$\boxed{\begin{aligned} &V_{LL}^{pu} = V_{LN}^{pu} \quad \Leftrightarrow \quad \frac{V_{LL}}{V_{B,LL}} = \frac{V_{LN}}{V_{B,LN}} \\ &\therefore \quad \Rightarrow \quad V_{B,LL} = \sqrt{3}V_{B,LN} \end{aligned}}$$

Per Unit system for 3- ϕ Circuits (Power)

- In a three phase system, we have:

$$\boxed{S_{3\phi}^{pu} = S_{1\phi}^{pu}}$$

- We know:

$$\boxed{S_{3\phi} = 3S_{1\phi}}$$

- So:

$$S_{3\phi}^{pu} = S_{1\phi}^{pu} \quad \Leftrightarrow \quad \frac{S_{3\phi}}{S_{B,3\phi}} = \frac{S_{1\phi}}{S_{B,1\phi}}$$

$$\therefore \quad \Rightarrow \quad S_{B,3\phi} = 3S_{B,1\phi}$$

Per Unit system for 3- ϕ Circuits (Current)

- In a three phase system, we have:

$$\boxed{S_{3\phi} = 3V_{LN} I_{LN}^{*} = \sqrt{3} V_{LL} I_{LL}}$$

- We know:

- So:

$$\boxed{S^{pu} = V^{pu} I^{pu}}$$

$$S^{pu} = V^{pu} I^{pu} \quad \Leftrightarrow \quad \frac{S}{S_{B,3\phi}} = \frac{V_{LL}}{V_{B,LL}} \frac{I_{LL}}{I_B}$$

$$\therefore \quad \Rightarrow \quad I_B = \frac{S_{B,3\phi}}{\sqrt{3} V_{B,LL}}$$

Per Unit system for 3- ϕ Circuits (Impedance)

$$V^{pu} = Z^{pu} I^{pu} \quad \Leftrightarrow \quad \frac{V_{LN}}{V_{B,LN}} = \frac{Z_{LN}}{Z_B}\frac{I_{LN}}{I_B}$$

$$\therefore \quad \Rightarrow \quad Z_B = \frac{V_{B,LN}}{I_B} = \frac{\dfrac{V_{B,LL}}{\sqrt{3}}}{\dfrac{S_{B,3\phi}}{\sqrt{3}V_{B,LL}}} = \frac{\left(V_{B,LL}\right)^2}{S_{B,3\phi}}$$

Example 1

- Given base kVA for 3 phase systems is 30 000kVA and voltage base line to line 120kV. Find:

$$S_{B,1\phi}$$

$$V_{pu}$$

$$V_{B,LN}$$

$$V_{LN}$$

- Given actual line to line voltage is 108kV.

Example 1

Base $kVA_{3\Phi}$ = 30,000 kVA

and Base kV_{LL} = 120 kVA

therefore Base kVA $_{1\Phi}$ = 30,000 / 3 = 10,000 kVA

and Base kV $_{LN}$ = 120 / √3 = 69.2 kVA

$$\text{per unit value of any quantity} = \frac{\text{actual value of the quantity}}{\text{based value}}$$

For actual line-to-line voltage 108 kV, the line-to-neutral voltage, V_{LN} is 108/ √3 = 62.3

Per unit value - example

$$\text{per unit value of any quantity} = \frac{\text{actual value of the quantity}}{\text{based value}}$$

and

Per-unit voltage = 108/120 (3ϕ) OR
= 62.3/69.2 (1ϕ)
= 0.9

For three-phase power of 18,000 kW,

Per-unit power = 18,000/30,000 (3ϕ) OR
= 6,000/10,000(1ϕ)
= 0.6

Change of Base

- The impedance of individual generators & transformer, are generally in terms of *percent/per unit* based on their own ratings.
- Impedance of transmission line in *ohmic value*
- When pieces of equipment with various different ratings are connected to a system, it is necessary to convert their impedances to a per unit value expressed on the same base.

Change of Base

- In other word, since all impedances in any one part of the a system must be expressed on the same impedance base when making computations, it is necessary to have a means of converting per-unit impedances from one based to another.

$$Z_{actual} = Z_{new}^{pu} Z_{B,new} = Z_{old}^{pu} Z_{B,old}$$

$$\Rightarrow Z_{new}^{pu} = Z_{old}^{pu} \frac{Z_{B,old}}{Z_{B,new}}$$

$$\text{But we always have : } Z_B = \frac{V_B^2}{S_B}$$

$$Z_{new}^{pu} = Z_{old}^{pu} \frac{S_{B,new}}{S_{B,old}} \left(\frac{V_{B,old}}{V_{B,new}} \right)^2$$

Change of Base

- If $V_{B,new} = V_{B,old}$
- So

$$\boxed{Z_{new}^{pu} = Z_{old}^{pu} \frac{S_{B,new}}{S_{B,old}}}$$

Example 2

The reactance of a generator designated X" is given as 0.25 per unit based on the generator's nameplate rating of 18 kV, 500 MVA. The base for calculations is 20kV, 100 MVA. Find X" on the new base

$$Z_{pu}^{new} = Z_{pu}^{old}\left(\frac{S_B^{new}}{S_B^{old}}\right)\left(\frac{V_B^{old}}{V_B^{new}}\right)^2$$

$$\mathrm{Z_{pu}^{old}} = 0.25 \qquad \mathrm{V_B^{old}} = 18\ \mathrm{kV} \qquad \mathrm{S_B^{old}} = 500\ \mathrm{MVA}$$

$$\mathrm{V_B^{new}} = 20\ \mathrm{kV} \qquad \mathrm{S_B^{new}} = 100\ \mathrm{MVA}$$

Example 2

$$Z_{pu}^{old} = 0.25 \qquad V_B^{old} = 18\text{ kV} \qquad S_B^{old} = 500\text{ MVA}$$

$$V_B^{new} = 20\text{ kV} \qquad S_B^{new} = 100\text{ MVA}$$

$$Z_{pu}^{new} = X'' = 0.25\left(\frac{100}{500}\right)\left(\frac{18}{20}\right)^2 = 0.0405\text{ per unit}$$

Exercise 2

- Generator rated at 10MVA, 20kV
- $X_S = 0.9pu$ on the basis of the generator rating
- Given $S_{B,new} = 100MVA$ and $V_{B,new} = 20kV$
- Find: X_S^{new}

Solution:

Exercise 3

- Transformer rated at 10MVA, 33/11kV
- *Z = 10% and R = 1%*
- Given $S_{B,new}$=200MVA and $V_{B,new}$ =22kV (HV side transformer)
- Find: i. Z_{base}(HV and LV sides)

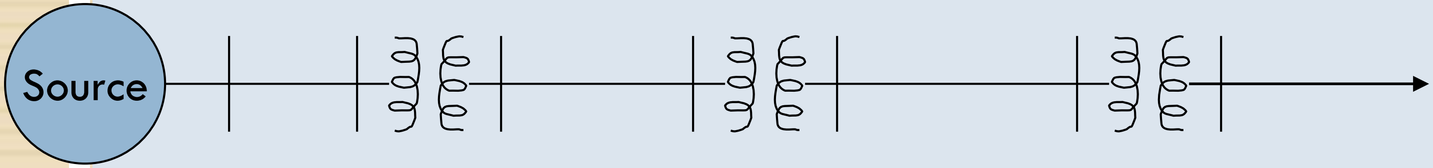
  ii. actual Z and R referred to primary and secondary

  iii. Transformer losses in kW, if 0.033 p.u (selected base) of current flow through R

Solution:

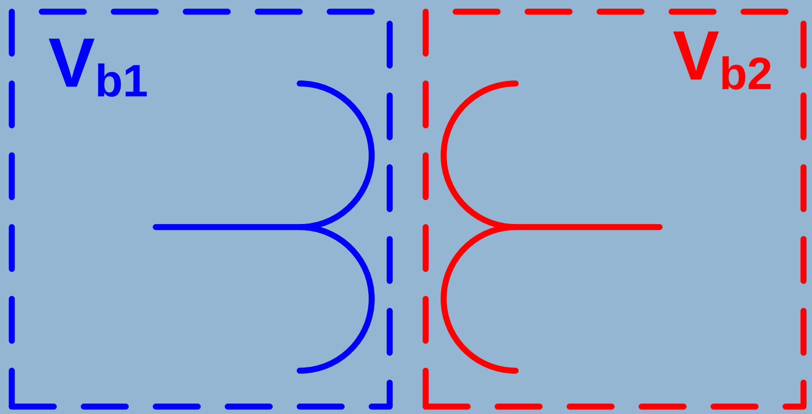
Procedure for Per Unit Analysis

1. Pick $|S_{\text{Base}}|$ for the system.
2. Pick V_{Base} according to line-to-line voltage.
3. Calculate Z_{Base} for different zones.
4. Express all quantities in p.u.
5. Draw impedance diagram and solve for p.u. quantities.
6. Convert back to actual quantities if needed.



How to solve problems containing multiple transformer?

Transformer Voltage Base

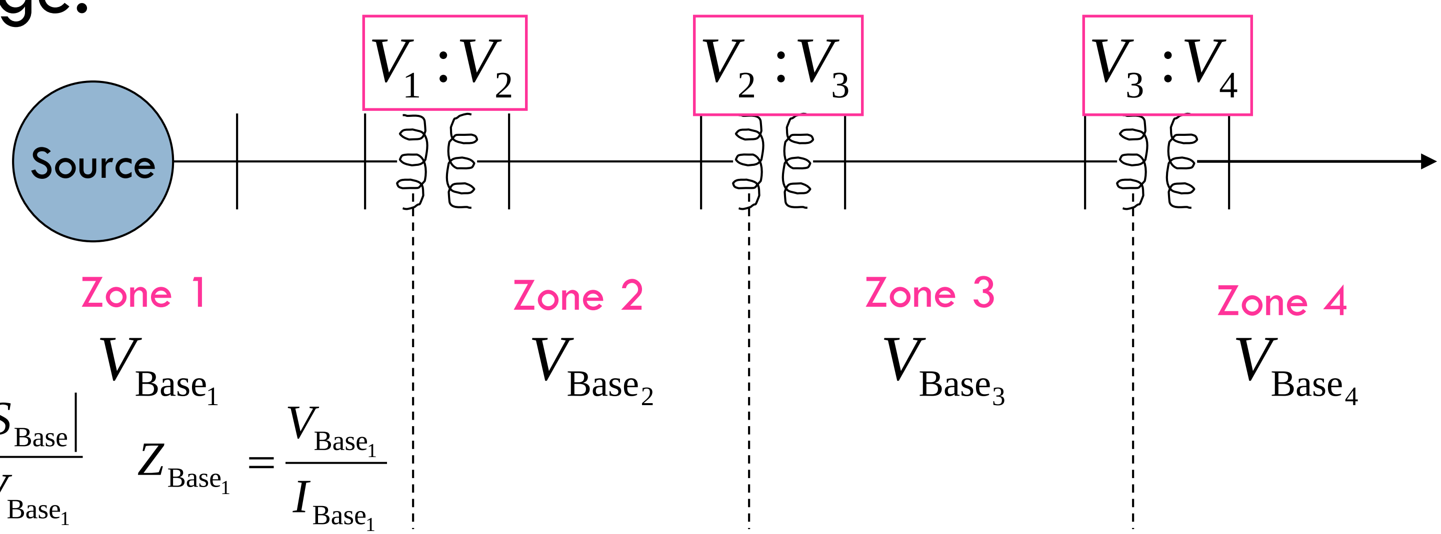


V_1/V_2

$$V_{b2} = \left(\frac{V_2}{V_1}\right) \bullet V_{b1}$$

How to Choose Base Values ?

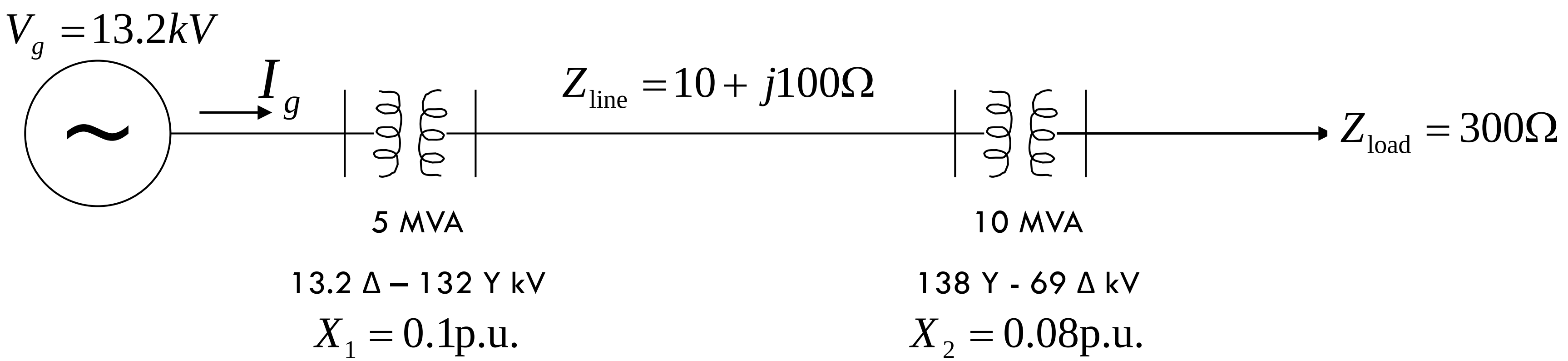
- Divide circuit into zones by transformers.
- Specify two base values out of I_B, V_B, Z_B, S_B ; for example, $|S_{\text{Base}}|$ and V_{Base}
- Specify voltage base in the ratio of zone line to line voltage.



$$I_{\text{Base}_1} = \frac{|S_{\text{Base}}|}{V_{\text{Base}_1}} \qquad Z_{\text{Base}_1} = \frac{V_{\text{Base}_1}}{I_{\text{Base}_1}}$$

Example 5

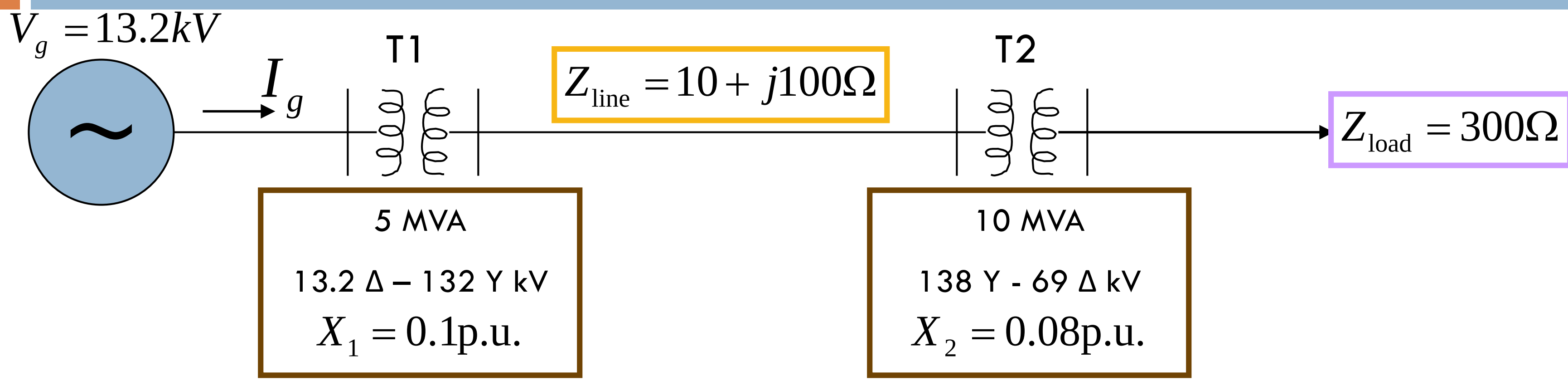
- Given a one line diagram,



Choosing a base apparent power of 10MVA and a base line voltage L1 of 69 kV ; find

I_g $\quad I_{\text{t-line}}$ $\quad I_{\text{load}}$ $\quad V_{\text{load}}$ $\quad P_{\text{load}}$

Step 1, 2, and 3: Base Values



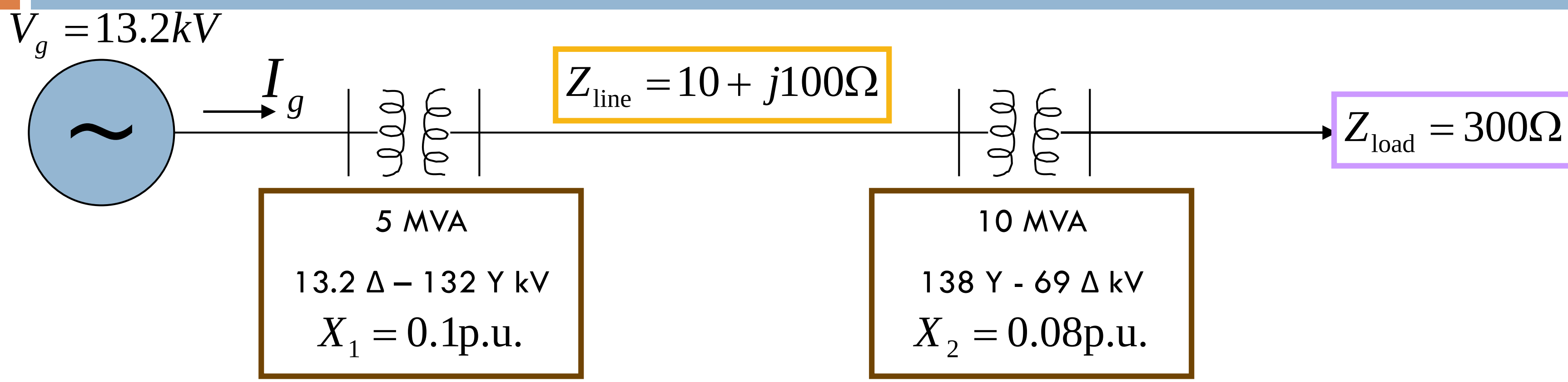
$S_{\text{B},new} = 10MVA$ $V_{\text{B,new}} = 69kV$

Zone 1	Zone 2	Zone 3
$V_{\text{B}_1} = 13.8kV$	$V_{\text{B}_2} = 138kV$	$V_{\text{B}_3} = 69kV$

$$V_{B_1} = \frac{13.2kV}{132kV} x138kV = 13.8kV \qquad V_{B_2} = \frac{138kV}{69kV} x69kV = 138kV$$

$\Rightarrow$ Calculate the base voltage in each zone

Step 1, 2, and 3: Base Values



$$S_{\mathrm{B},new} = 10MVA \qquad V_{\mathrm{B,new}} = 69kV$$

Zone 1

$$V_{\mathrm{B}_1} = 13.8kV$$

$$Z_{\mathrm{B}_1} = \frac{\left|V_{\mathrm{B}_1}\right|^2}{S_{\mathrm{B}}} = \frac{(13.8k)^2}{10M} = 19.04\Omega$$

$$I_{\mathrm{B}_1} = \frac{\left|S_{\mathrm{B}_1}^{3\Phi}\right|}{\sqrt{3}\left|V_{\mathrm{B}_1}\right|} = \frac{10M}{\sqrt{3}\cdot 13.8k} = 418.4$$

Zone 2

$$V_{\mathrm{B}_2} = 138kV$$

$$Z_{\mathrm{B}_2} = \frac{\left|V_{\mathrm{B}_2}\right|^2}{S_{\mathrm{B}}} = \frac{(138k)^2}{10M} = 1904\Omega$$

$$I_{\mathrm{B}_2} = \frac{\left|S_{\mathrm{B}_2}^{3\Phi}\right|}{\sqrt{3}\left|V_{\mathrm{B}_2}\right|} = \frac{10M}{\sqrt{3}\cdot 138k} = 41.84$$

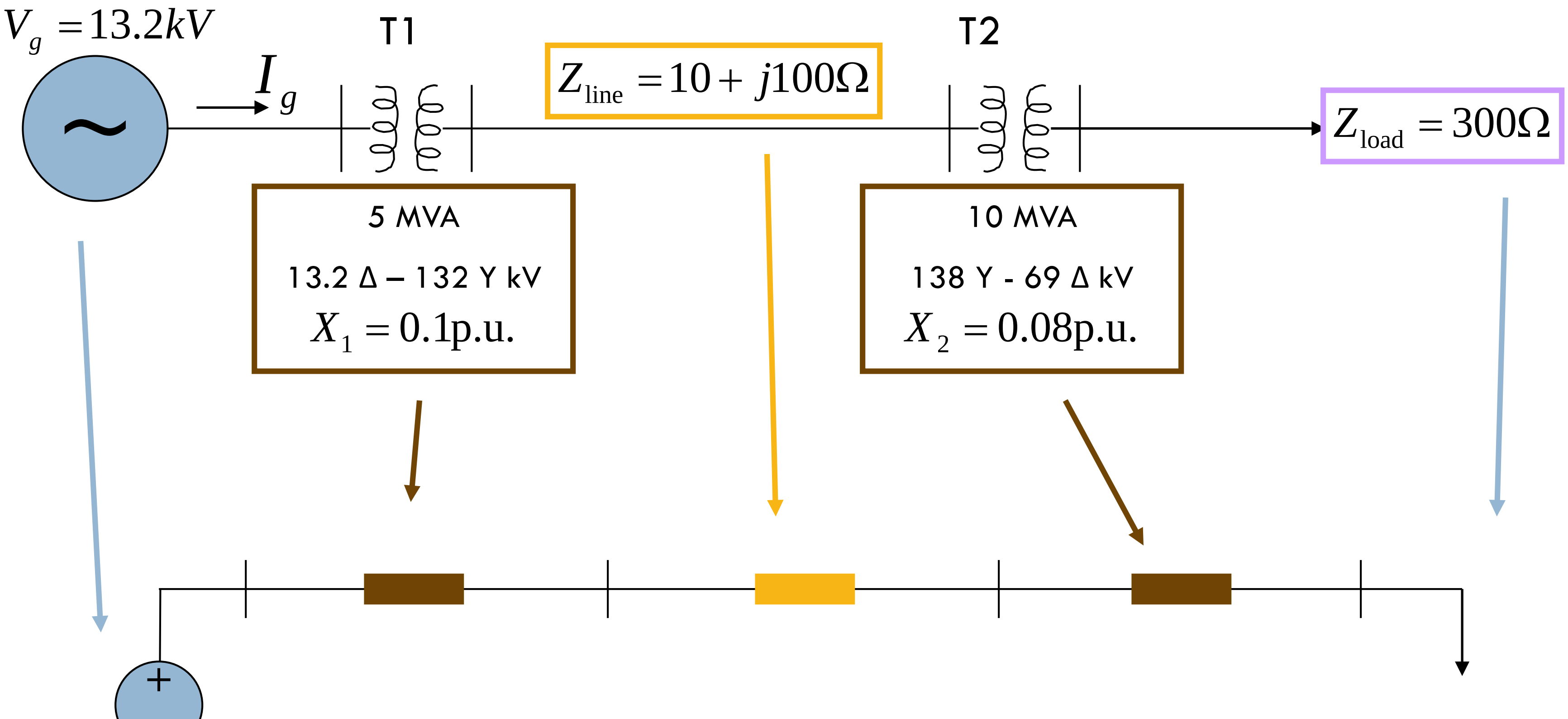
Zone 3

$$V_{\mathrm{B}_3} = 69kV$$

$$Z_{\mathrm{B}_3} = \frac{\left|V_{\mathrm{B}_3}\right|^2}{S_{\mathrm{B}}} = \frac{(69k)^2}{10M} = 476\Omega$$

$$I_{\mathrm{B}_3} = \frac{\left|S_{\mathrm{B}_3}^{3\Phi}\right|}{\sqrt{3}\left|V_{\mathrm{B}_3}\right|} = \frac{10M}{\sqrt{3}\cdot 69k} = 83.67$$

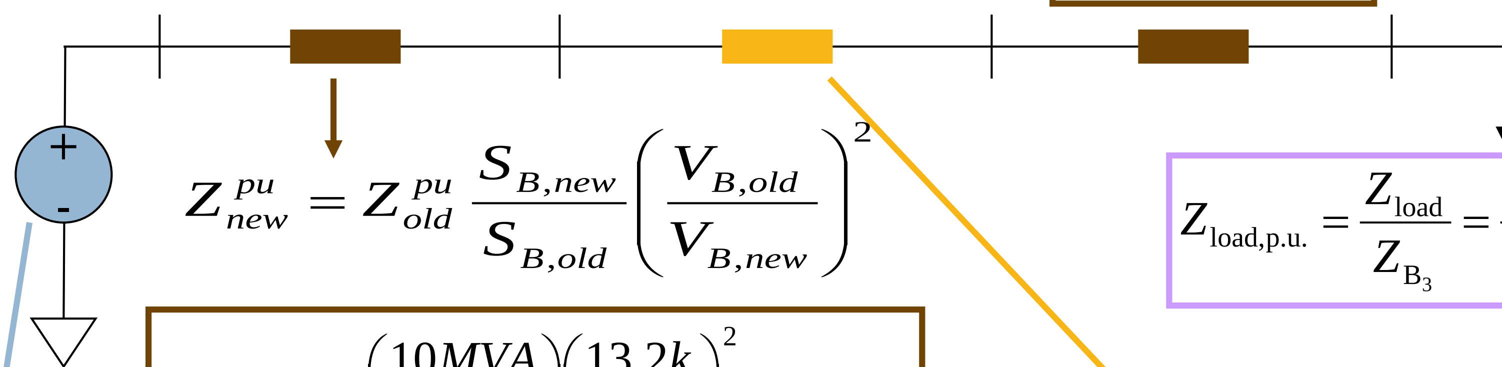
Step 4: All in Per Unit Quantities



- ⇒ Calculate V_g in per unit
- ⇒Calculate the new reactance for X_1 and X_2 in per unit
- ⇒Calculate the impedance in per unit
- ⇒ Draw the impedance diagram

Step 4: All in Per Unit Quantities

$$X_{2,pu} = 0.08\text{p.u.}$$



$$Z_{new}^{pu} = Z_{old}^{pu} \frac{S_{B,new}}{S_{B,old}} \left(\frac{V_{B,old}}{V_{B,new}} \right)^2$$

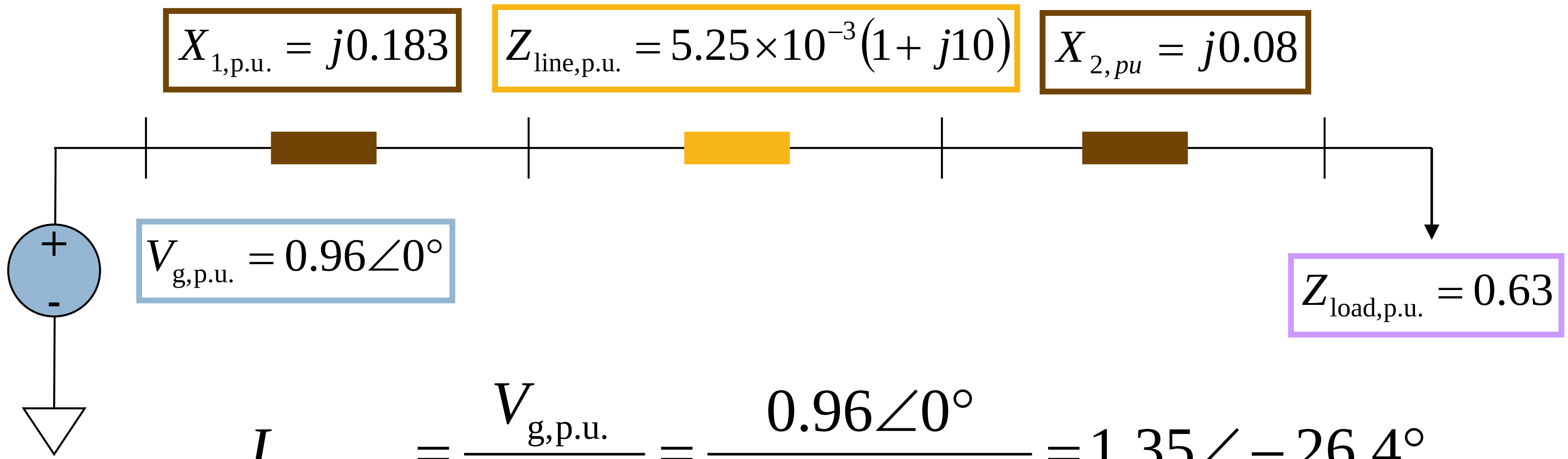
$$Z_{\text{load,p.u.}} = \frac{Z_{\text{load}}}{Z_{\text{B}_3}} = \frac{300\Omega}{476\Omega} = 0.63$$

$$X_{\text{1,p.u.}} = 0.1 \left(\frac{10MVA}{5MVA} \right) \left(\frac{13.2k}{13.8k} \right)^2 = 0.183$$

$$V_{\text{g,p.u.}} = \frac{V_{\text{g}}}{V_{\text{B}_1}} = \frac{13.2kV}{13.8kV} = 0.96\angle 0^\circ$$

$$Z_{\text{line,p.u.}} = \frac{Z_{\text{line}}}{Z_{\text{B}_2}} = \frac{10 + j100\Omega}{1904} = 5.25\times 10^{-3}(1 + j10)$$

Step 5: One Phase Diagram & Solve



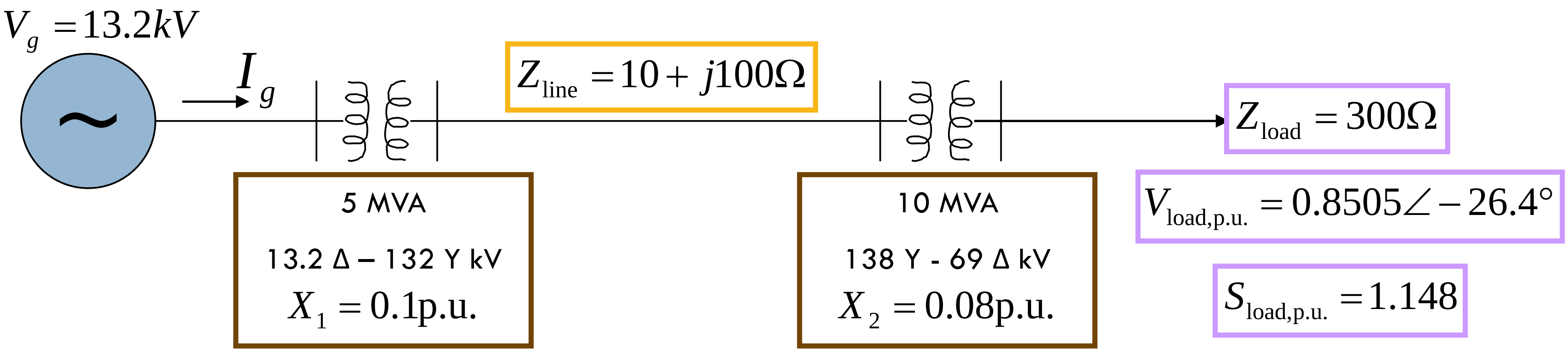
$$I_{\text{load,p.u.}} = \frac{V_{\text{g,p.u.}}}{Z_{\text{total,p.u.}}} = \frac{0.96\angle 0°}{0.709\angle 26.4°} = 1.35\angle -26.4°$$

$$V_{\text{load,p.u.}} = I_{\text{load,p.u.}} Z_{\text{load,p.u.}} = 0.8505\angle -26.4°$$

$$S_{\text{load,p.u.}} = V_{\text{load,p.u.}} I^{*}_{\text{load,p.u.}} = 1.148$$

$$I_{\text{g,p.u.}} = I_{\text{t-line,p.u.}} = I_{\text{load,p.u.}} = 1.35\angle -26.4°$$

Step 6: Convert back to actual quantities



$$I_{\text{g,p.u.}} = I_{\text{t-line,p.u.}} = I_{\text{load,p.u.}} = 1.35\angle -26.4°$$

Zone 1	Zone 2	Zone 3
$I_{\text{g}} = I_{\text{g,p.u.}} I_{\text{B}_1}$	$I_{\text{t-line}} = I_{\text{t-line,p.u.}} I_{\text{B}_2}$	$I_{\text{load}} = I_{\text{load,p.u.}} I_{\text{B}_3}$ $V_{\text{load}} = V_{\text{load,p.u.}} V_{\text{B}_3}$ $S_{\text{load}} = S_{\text{load,p.u.}} S_{\text{B}}$

Advantages of P.U. system

- Transformer equivalent circuit can be simplified by properly specifying base quantities.
  - Give a clear idea of relative magnitudes of various quantities such as voltage, current, power and impedance.
  - Avoid possibility of making serious calculation error when referring quantities from one side of transformer to the other.

Advantages of P.U. system

- Per-unit impedances of electrical equipment of similar type usually lie within a narrow numerical range when the equipment ratings are used as base values.
  - Manufacturers usually specify the impedances of machines and transformers in per-unit or percent in nameplate rating.

Advantages of P.U. system

- The circuit laws are valid in per unit systems, and the power and voltage equation are simplified since the factor $\sqrt{3}$ and 3 are eliminates in the per-unit systems.
- Ideal for the computerized analysis and simulation of complex power system problems.

Advantages

- **Why Use the Per Unit System Instead of the Standard SI Units?**
- Here are the main reasons for using the per unit system:
- When values are expressed in pu, the comparison of electrical quantities with their "normal" values is straightforward.
- For example, a transient voltage reaching a maximum of 1.42 pu indicates immediately that this voltage exceeds the nominal value by 42%.

Advantages

- The values of impedances expressed in pu stay fairly constant whatever the power and voltage ratings.
- For example, for all transformers in the 3 kVA to 300 kVA power range, the leakage reactance varies approximately between 0.01 pu and 0.03 pu, whereas the winding resistances vary between 0.01 pu and 0.005 pu, whatever the nominal voltage. For transformers in the 300 kVA to 300 MVA range, the leakage reactance varies approximately between 0.03 pu and 0.12 pu, whereas the winding resistances vary between 0.005 pu and 0.002 pu.

Advantages

- Similarly, for salient pole synchronous machines, the synchronous reactance Xd is generally between 0.60 and 1.50 pu, whereas the subtransient reactance X′d is generally between 0.20 and 0.50 pu.
- It means that if you do not know the parameters for a 10 kVA transformer, you are not making a major error by assuming an average value of 0.02 pu for leakage reactances and 0.0075 pu for winding resistances.

Advantages

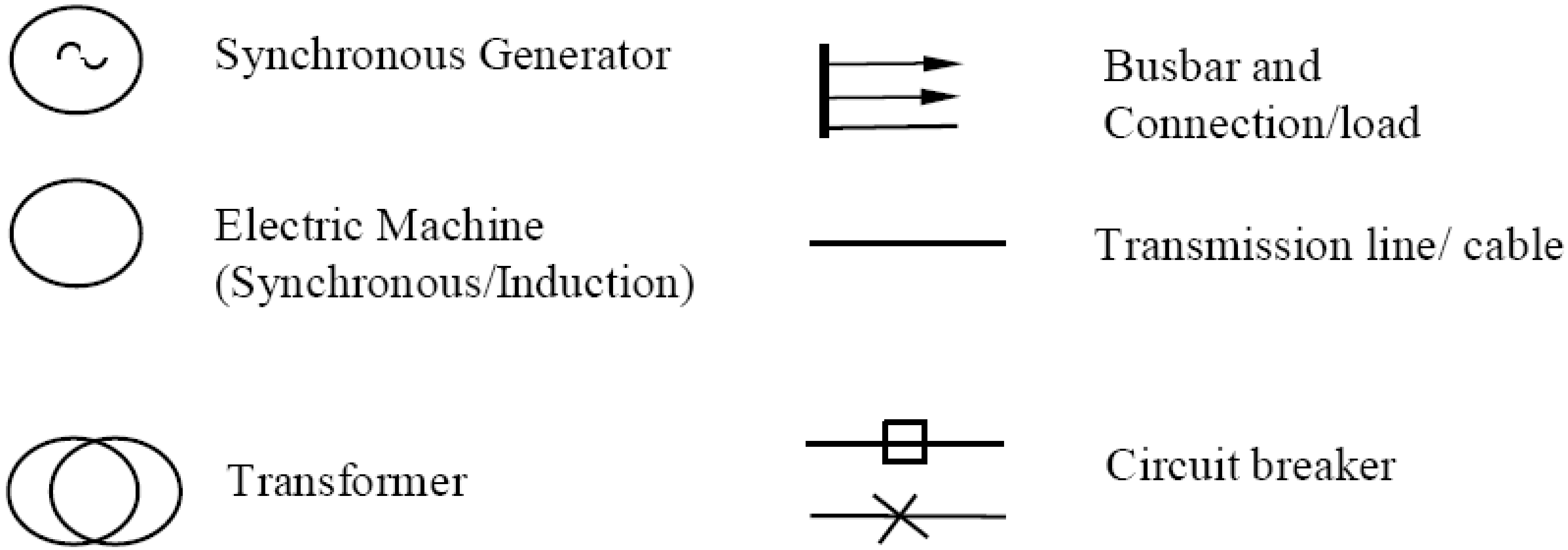
- The calculations using the per unit system are simplified. When all impedances in a multivoltage power system are expressed on a common power base and on the nominal voltages of the different subnetworks, the total impedance in pu seen at one bus is obtained by simply adding all impedances in pu, without taking into consideration the transformer ratios.

In a balanced 3-Φ system, the magnitude of voltages and currents in every phase, is the same and is displaced by 120°.

Therefore in a balanced condition, any 3-Φ system can be represented by one of the phases and the neutral connection.

To simplify further we can remove the neutral connection, thus leaving only the live connection as a single line.

This kind of diagram is called, the **single line diagram**.



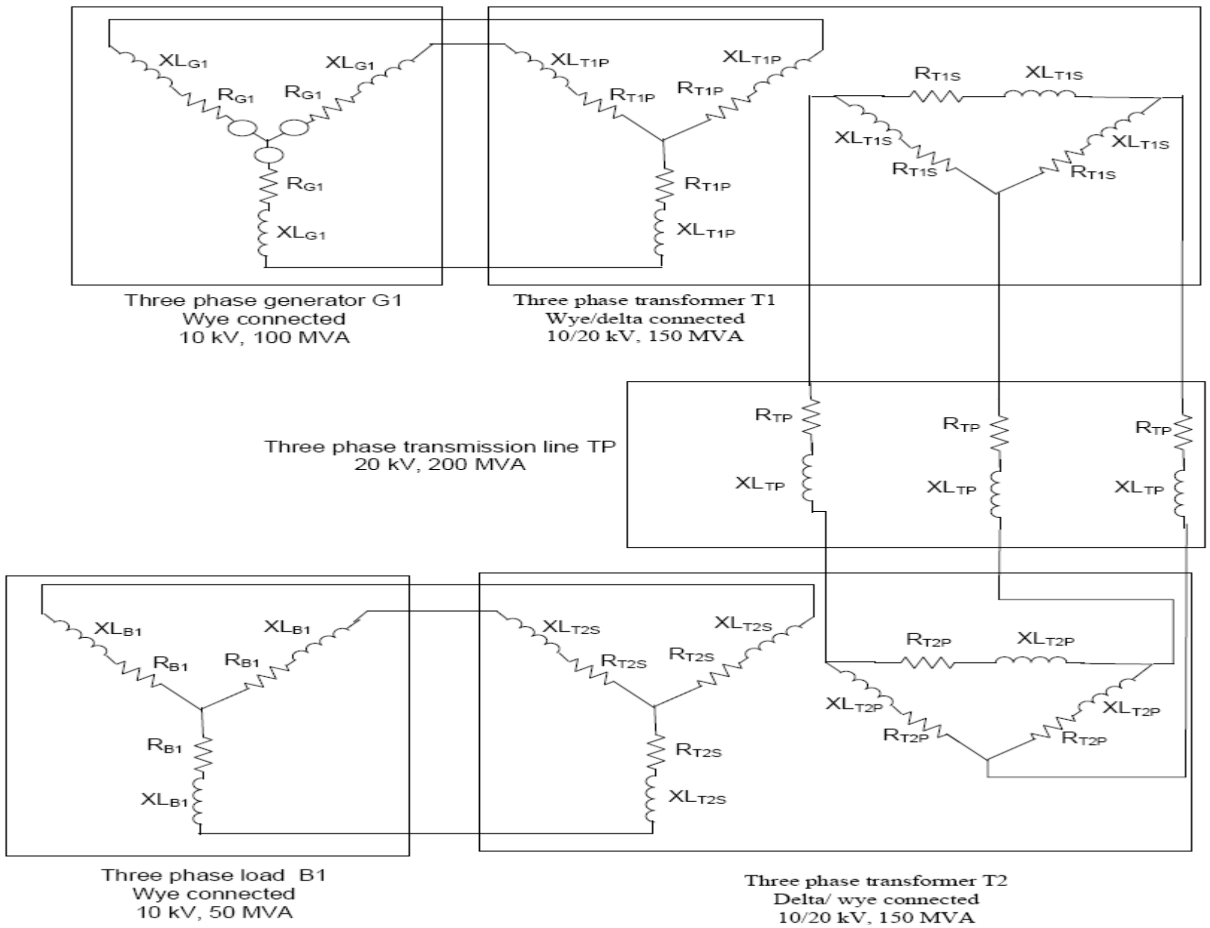
Symbol for common components in power system

The parameters supplied with each component in the one line diagram are:

i. rated voltage,
ii. rated power,
iii. phase impedance,
iv. type of connection (either Wye or Delta connection).

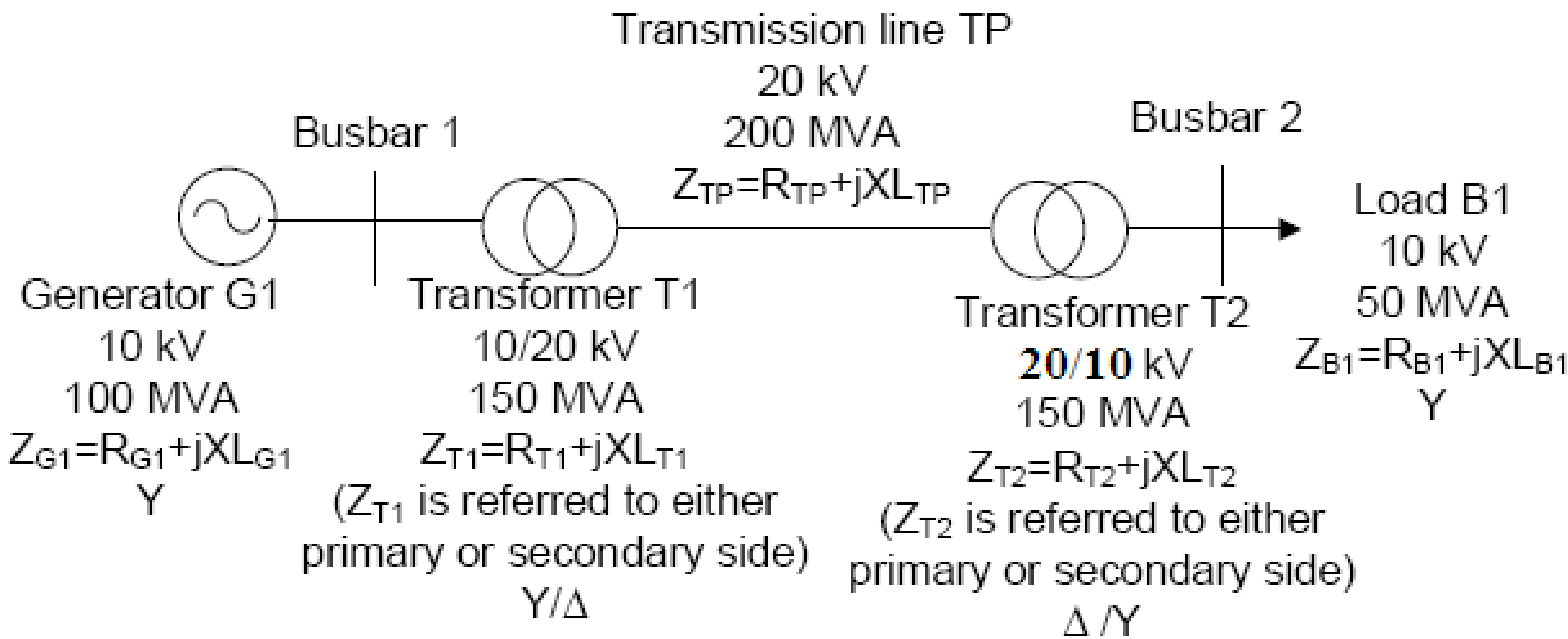
For 3-Φ system, – V_{line} , $P_{3\Phi}$, $Z_{1\Phi}$ (in pu or percentage)

Types of connection should be stated for the three phase **generators, motors, transformers, and loads**. Typically, the connection is assumed in **wye**.



(a)

Three phase system



(b)

Single line diagram

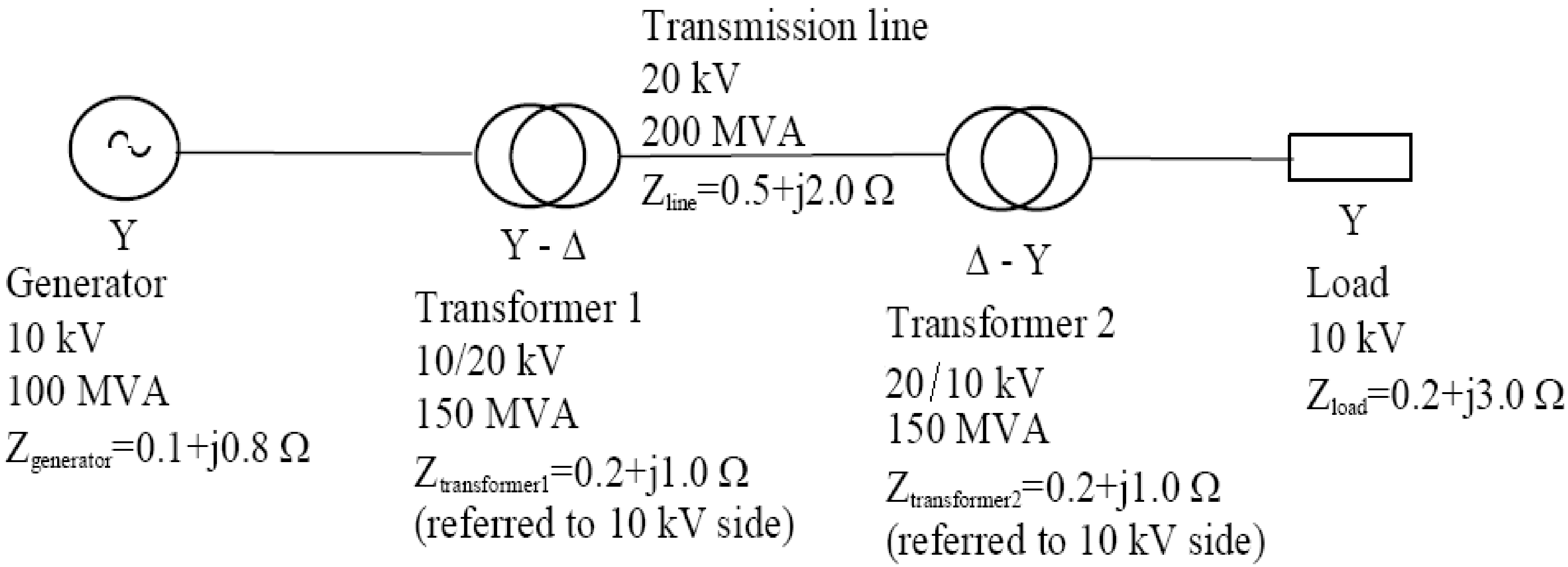
The one line diagram for a power system can be converted into per phase equivalent circuit which is used to analyse the performance of the system.

The equivalent circuit is known as ‘**Impedance Diagram**’.

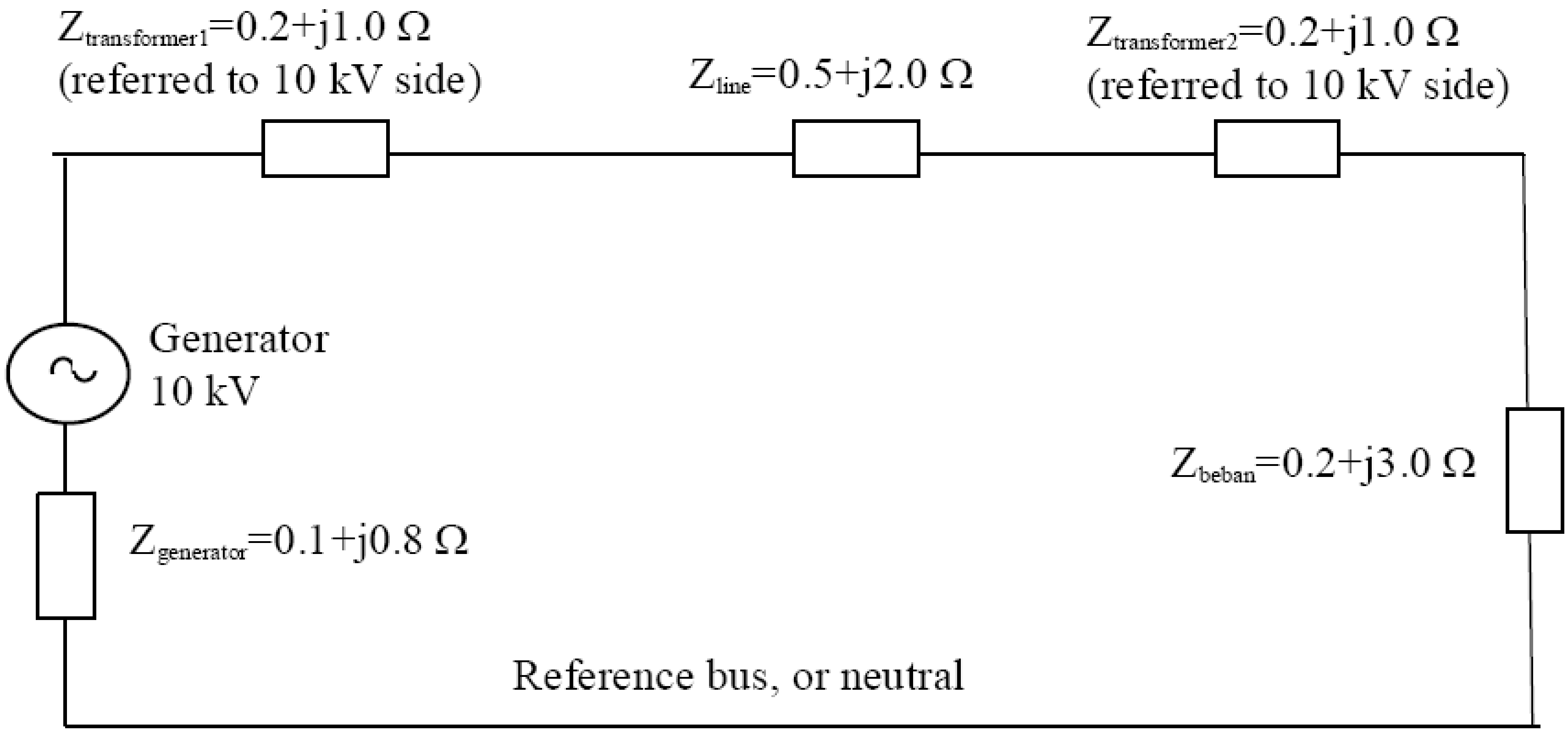
The impedance diagram is an equivalent single phase circuit with neutral line taken as the return path.

The analysis on the impedance diagram is similar to that on the alternating current circuit.

Example, of one line diagram has to be converted into the impedance diagram.



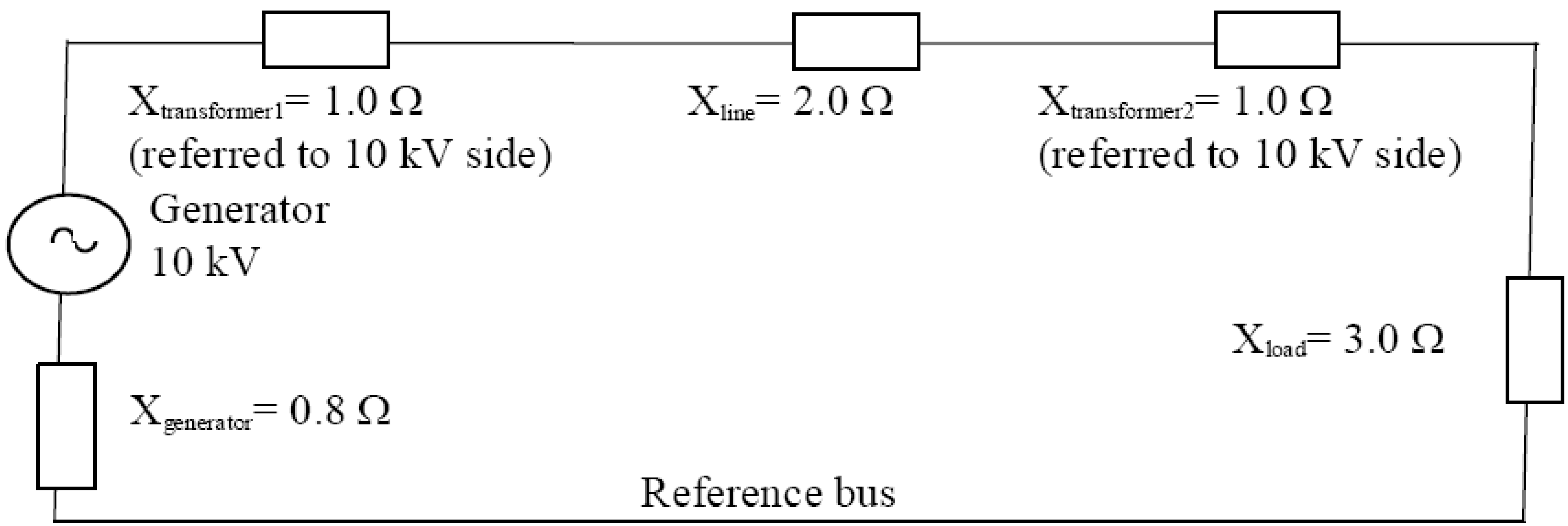
One line diagram



Impedance diagram

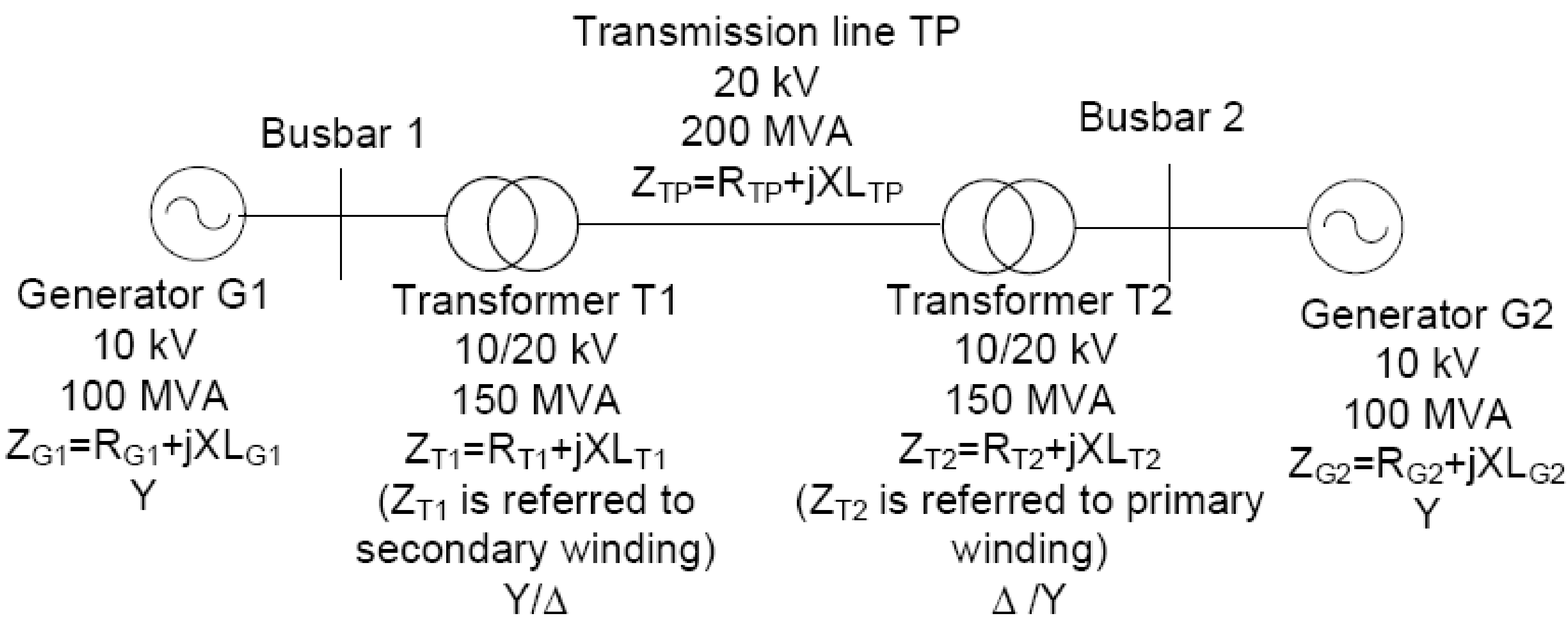
For most components of power system, the value of resistance is very small compared to that of inductive reactance.

Thus, by neglecting resistance in the impedance diagram we can construct **reactance diagram** representation for simpler analysis of the system.

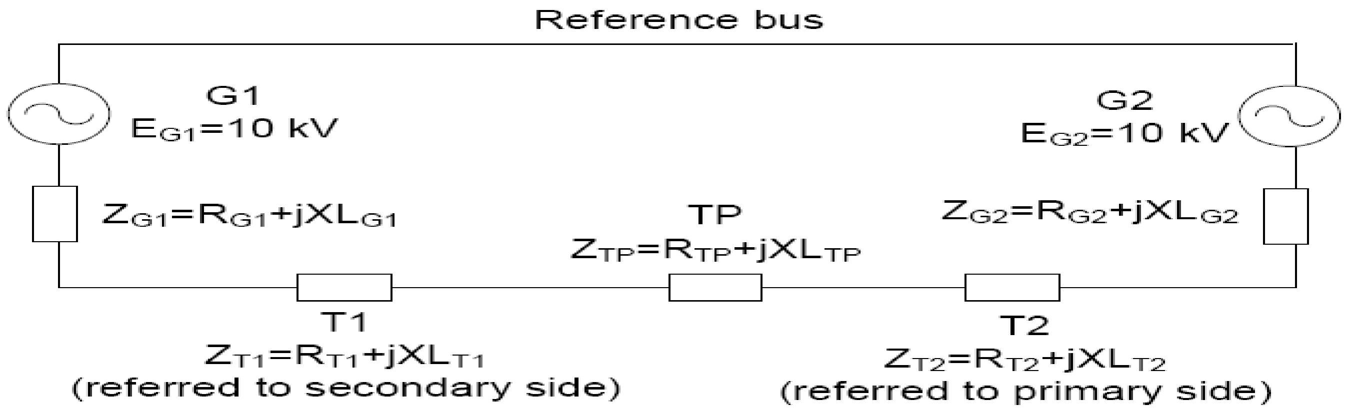


Reactance diagram

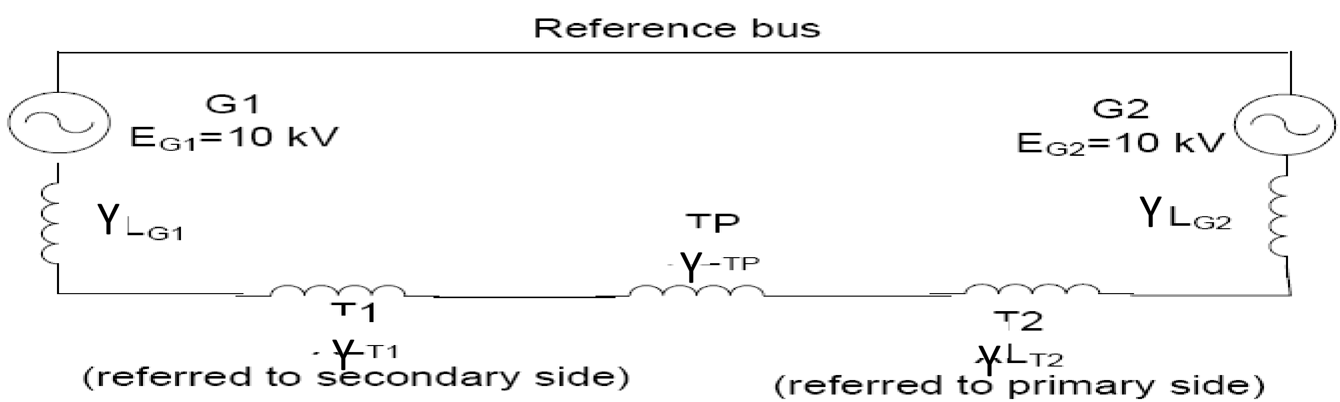
Q1. Obtain impedance and reactance diagram for power system in Figure shown below.



Question 1.



Impedance diagram for Question 1



Admittance diagram for Question 1

Per unit (p.u.) value for a quantity is defined as a ratio of the actual value over the base value.

$$\text{p.u. value} = \frac{\text{Actual value}}{\text{Base value}}$$

The p.u. value is commonly given either in fractional or percentage forms.

Four main parameters in power system analysis:

i. Voltage : Line voltage (V_L) or phase voltage (V_P)
ii. Power : 3Φ apparent power ($VA_{3\Phi}$) or $1\ \Phi$ ($VA_{1\Phi}$)
iii. Current : Line current (I_L) or phase current (I_P).
iv. Impedance : Phase impedance (Z_P).

By taking two parameters as the base parameters, the other base parameters can be obtained.

$V_{\text{P-base}}$ and $VA_{1\Phi\text{-base}}$,

$$I_{\text{P-base}} = \frac{VA_{1\phi-base}}{V_{\text{P-base}}}.$$

$$Z_{\text{P-base}} = \frac{(V_{\text{P-base}})^2}{VA_{1\phi-base}}.$$

Relationship between base current and base impedance.

Base parameters	1 phase system	3 phase system
Base current ($I_{P\text{-base}}$)	$\frac{VA_{1\phi-base}}{V_{P\text{-base}}}$	$\frac{VA_{3\phi-base}}{\sqrt{3}V_{L\text{-base}}}$
Base impedance ($Z_{P\text{-base}}$)	$\frac{(V_{P\text{-base}})^2}{VA_{1\phi-base}}$	$\frac{(V_{L\text{-base}})^2}{VA_{3\phi-base}}$

When all base parameters are obtained, any parameter in the system can be converted into p.u. using the above equations.

Exercises

2.5.1 Calculate base parameters for the following single phase system:

(a) Base impedance and base voltage are 10 Ω and 400 V, respectively. Calculate the values of base power and base current.

[16 kVA, 40 A]

(b) Base current and voltage are 3000 A and 300 kV, respectively. Calculate the values of base power and base impedance.

The following items are to be noted when there is transformer connected in the system:

i. The base parameter for power is applicable for both sides of the transformer since the transformer ratio does not affect the power through it.

ii. The base parameter for voltage changes according to the transformer ratio.

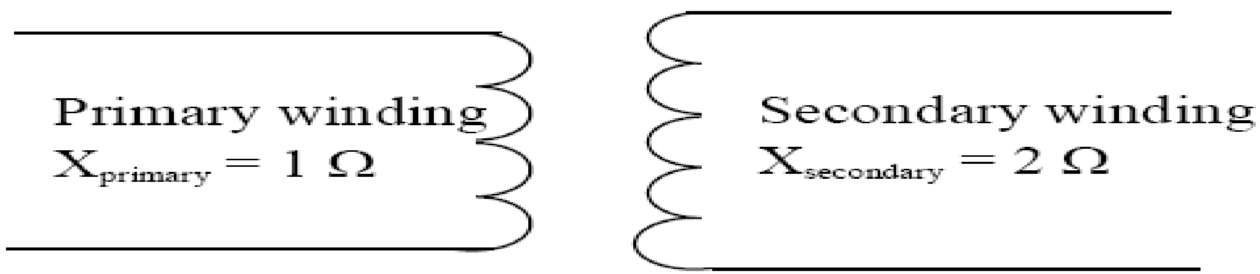
   eg : for a 10/20 kV transformer, if the base voltage at 10 kV is taken as 10 kV, then the base voltage at the 20 kV side should be 20 kV.

Transformer impedance in Ohm, referred to the primary winding is not equal to that referred to the secondary winding.

As a result, the analysis involving transformer is complicated since it has to be done by referring all parameters to either side of the transformer.

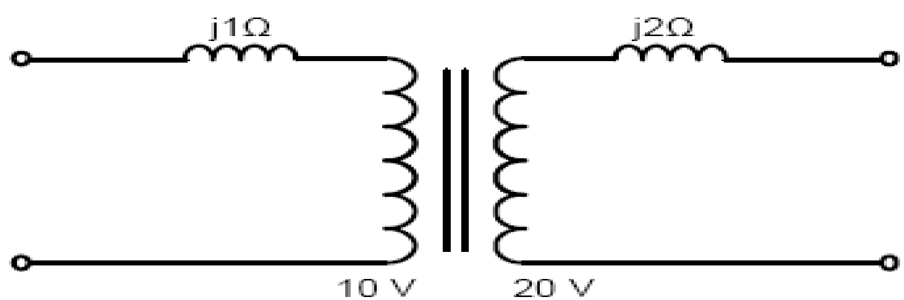
However, in per unit representation, the equivalent impedance values are the same at both sides of the transformer.

To simplify the analysis, assume that the resistance of the transformer can be neglected.



10/20 V 100 VA Transformer

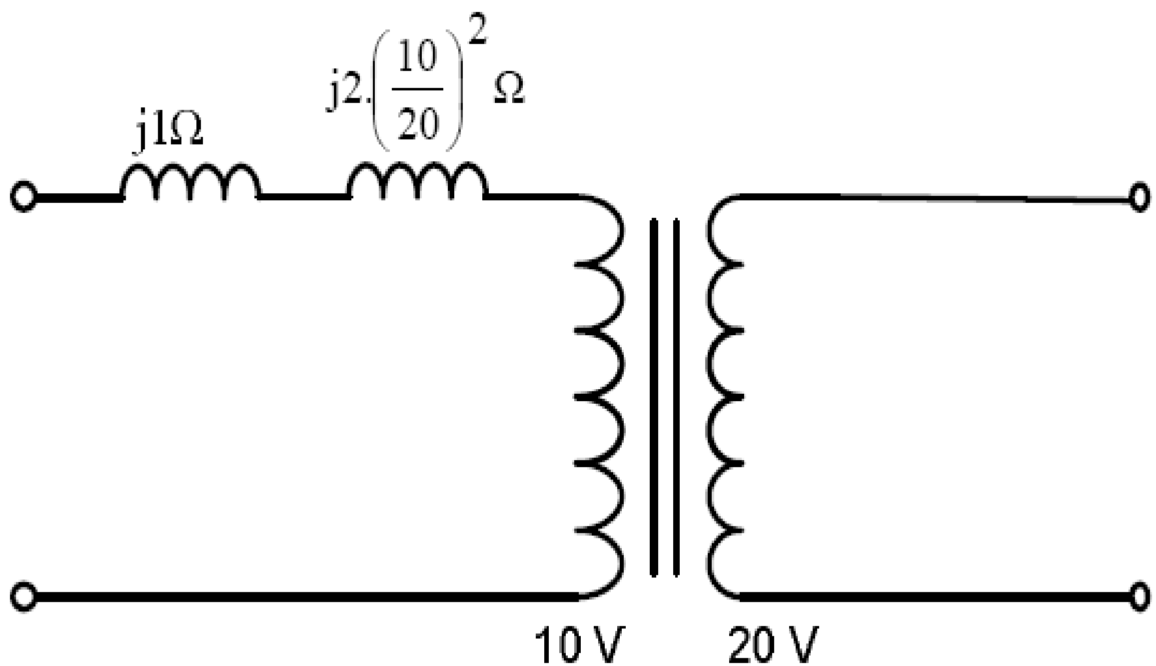
Figure 2.7



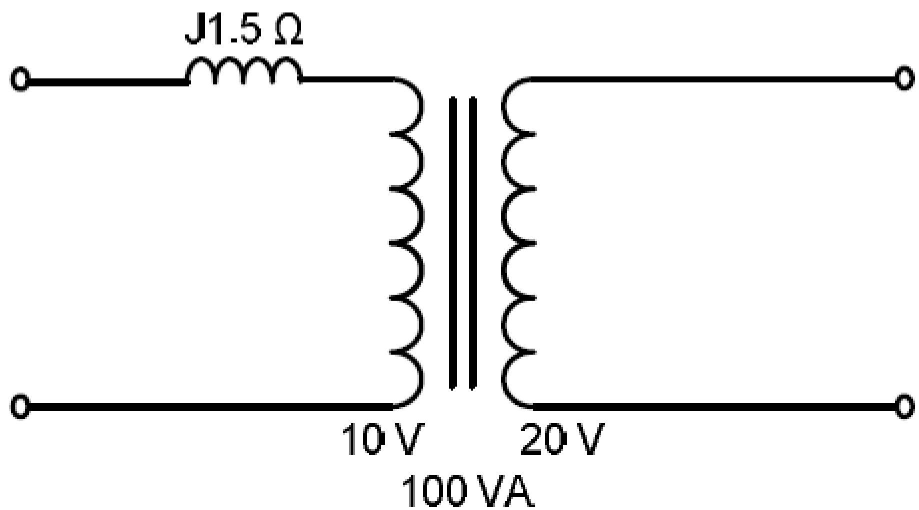
Rating 100 VA

The reactance referred to the primary side is

$$X_{\text{referred-primary}} = X_{\text{primary}} + X_{\text{secondary-referred-primary}} = 1 + \left(\frac{10}{20}\right)^2 2 = 1.5\ \Omega.$$

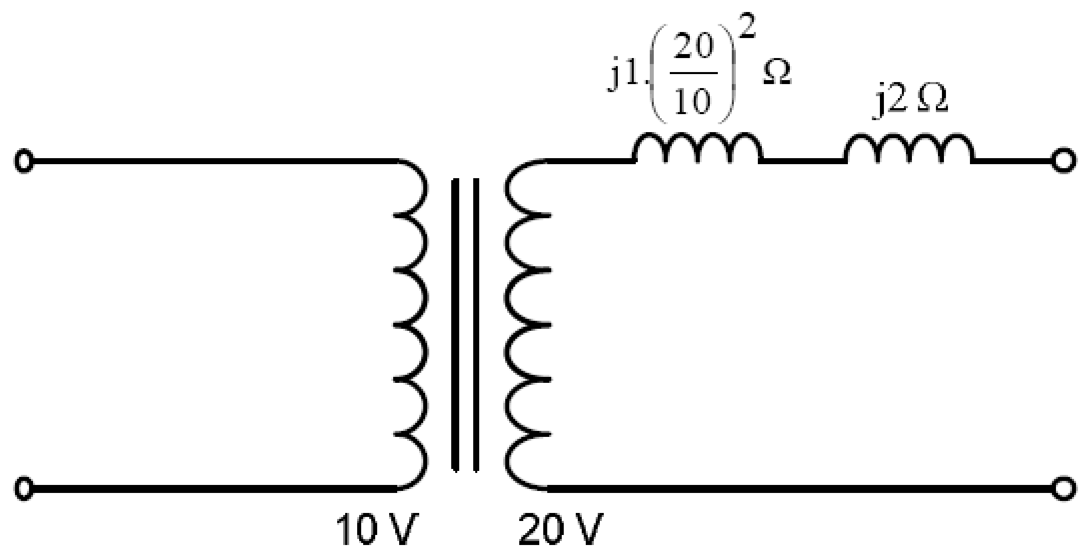


The final equivalent is circuit is as shown below

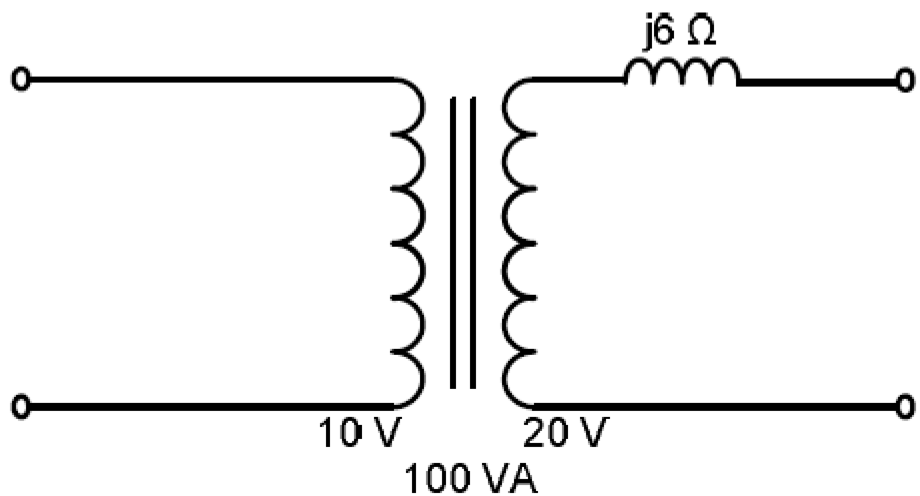


If the impedance is referred to the secondary :

$$X_{\text{referred-secondary}} = X_{\text{secondary}} + X_{\text{primary-referred-secondary}} = 2 + \left(\frac{20}{10}\right)^2 1 = 6\,\Omega.$$



The final equivalent is circuit is as shown below



with **base voltage of 10 V and 20 V** respectively at the primary and secondary windings and **base power of 100 VA,** thus the base reactance at the primary side is

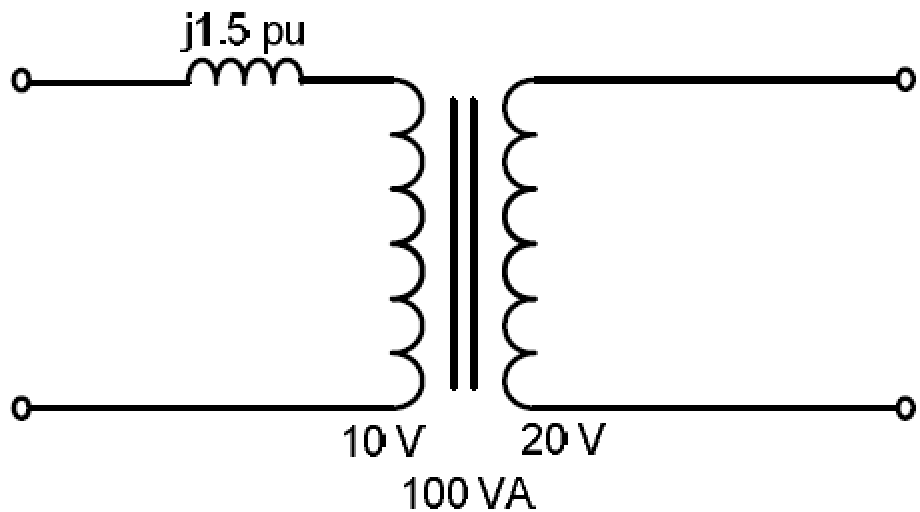
$$X_{\text{base-primary}} = \frac{10^2}{100} = 1\ \Omega$$

and base reactance at the secondary side is

$$X_{\text{base-secondary}} = \frac{20^2}{100} = 4\ \Omega$$

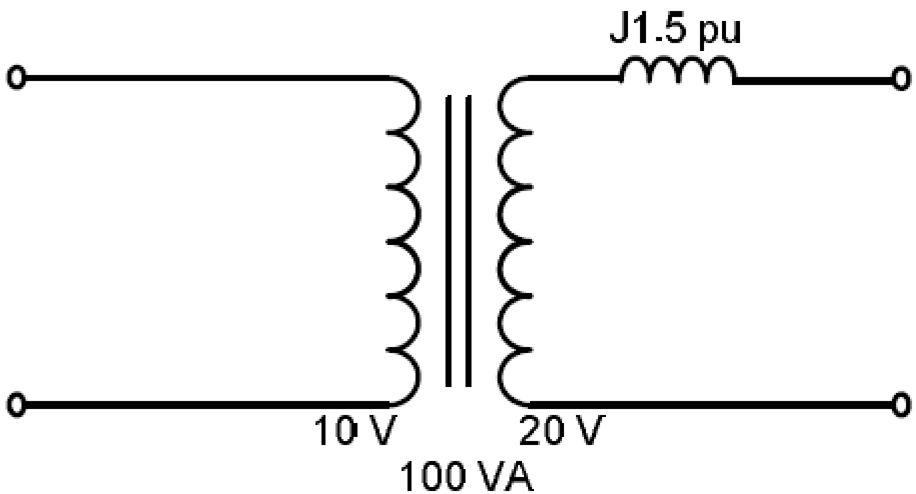
The reactance referred to the primary side in per unit is

$$X_{\text{referred-primary}} = \frac{1.5}{1} = 1.5 \text{ p.u.}$$

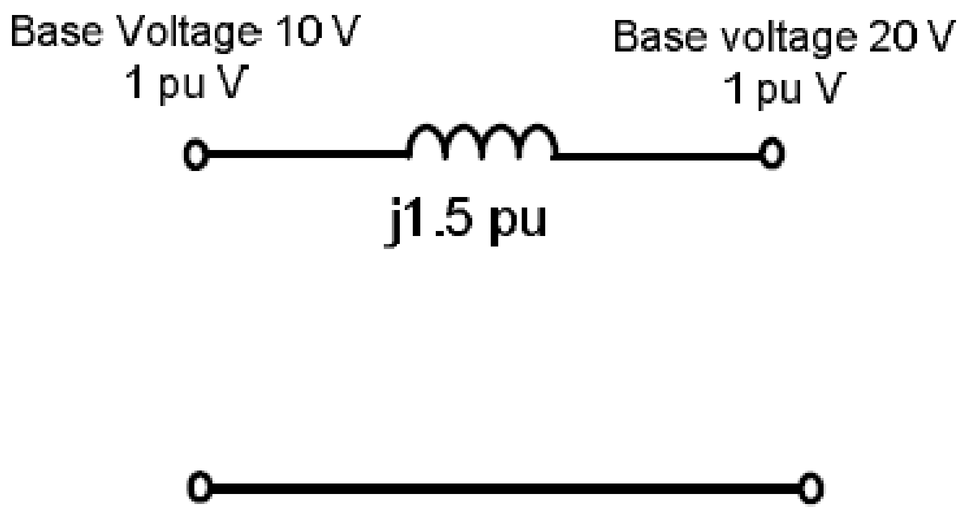


And the reactance referred to the secondary side is

$$X_{\text{referred-secondary}} = \frac{6}{4} = 1.5\ \text{p.u.}$$



Thus the equivalent circuit in pu is:



Example 5.1: *A single phase two-winding transformer is rated 25 kVA, 1100/440 volts, 50 Hz. The equivalent leakage impedance of the transformer referred to the low voltage side is* $0.06\angle 78^\circ$ *Ω. Using transformer rating as base values, determine the per-unit leakage impedance referred to low voltage winding and referred to high voltage winding.*

Solution: Let us assume high voltage side is primary and low voltage side is secondary windings.

Transformer rating = 25 kVA = 0.025 MVA

$$V_p = 1100 \text{ volt} = 1.1 \text{ kV};\ V_S = 440 \text{ volt} = 0.44 \text{ kV}$$

$$(\text{MVA})_B = 0.025,\ V_{pB} = 1.1 \text{ kV},\ V_{SB} = 0.44 \text{ kV}.$$

Base impedance on the 440 volt side of the transformer is

$$Z_{SB} = \frac{V_{SB}^2}{(\text{MVA})_B} = \frac{(0.44)^2}{(0.025)} = 7.744 \text{ ohm}$$

Per-unit leakage impedance referred to the low voltage side is

$$Z_S^{(pu)} = \frac{Z_{s,\,eq}}{Z_{SB}} = \frac{0.06\angle 78^\circ}{7.744} = 7.74 \times 10^{-3}\angle 78^\circ \text{ pu}.$$

If $Z_{p,\,eq}$ referred to primary winding (HV side),

$$Z_{p,\,eq} = a^2.Z_{s,\,eq} = \left(\frac{N_1}{N_2}\right)^2 . Z_{S,\,eq} = \left(\frac{1.1}{0.44}\right)^2 \times 0.06\underline{|78^\circ}$$

$$\therefore \qquad Z_{p,\,eq} = 0.375\underline{|\,78^\circ} \text{ ohm.}$$

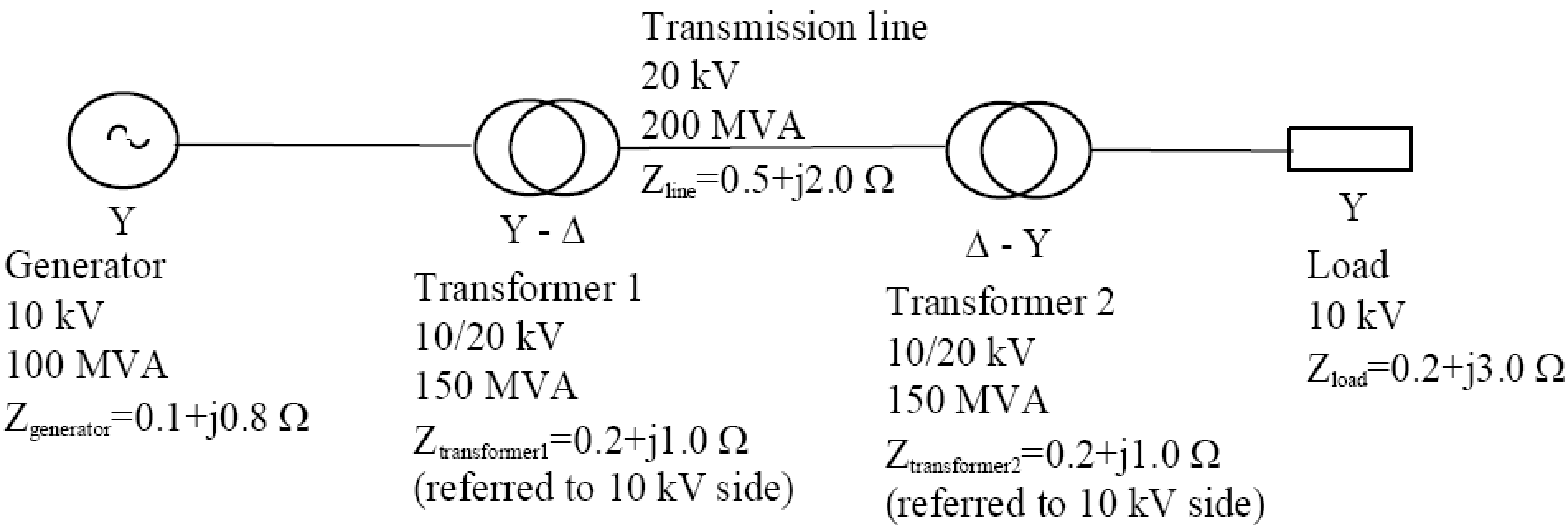
Base impedance on the 1.1 KV side is

$$Z_{pB} = \frac{V_{pB}^2}{(\text{MBA})_B} = \frac{(1.1)^2}{0.025} = 48.4\ \Omega$$

$$Z_p(pu) = \frac{Z_{p,\,eq}}{Z_{pB}} = \frac{0.375\underline{|\,78^\circ}}{48.4} = 7.74 \times 10^{-3}\underline{|\,78^\circ} \text{ pu}$$

Therefore, per-unit leakage impedance remains unchanged

Example : Find the per unit value for each component and draw the impedance diagram.



The base parameters taken for the system, both at the generator, are:

VL-base = 10 kV.
VA3Φ-base = 100 MVA.

The base impedance values at components in the system are:

Generator: $$Z_{\text{base-generator}} = \frac{(V_{\text{L-base}})^2}{VA_{3\phi-base}} = \frac{(10\ \text{kV})^2}{100\ \text{MVA}} = 1\ \Omega.$$

Transformer 1: $$Z_{\text{base-transformer1}} = \frac{(10\ \text{kV})^2}{100\ \text{MVA}} = 1\ \Omega.$$

Transmission line: $$Z_{\text{base-line}} = \frac{(20\text{ kV})^2}{100\text{ MVA}} = 4\ \Omega.$$

Transformer 2: $$Z_{\text{base-transformer2}} = \frac{(10\text{ kV})^2}{100\text{ MVA}} = 1\ \Omega.$$

Load: $$Z_{\text{base-load}} = \frac{(10\text{ kV})^2}{100\text{ MVA}} = 1\ \Omega.$$

Thus, the impedance in per unit for each component is:

Generator: $$Z_{pu\text{-}generator} = \frac{Z_{generator}}{Z_{base\text{-}generator}} = \frac{0.1 + j0.8}{1} = 0.1 + j0.8 \text{ p.u.}$$

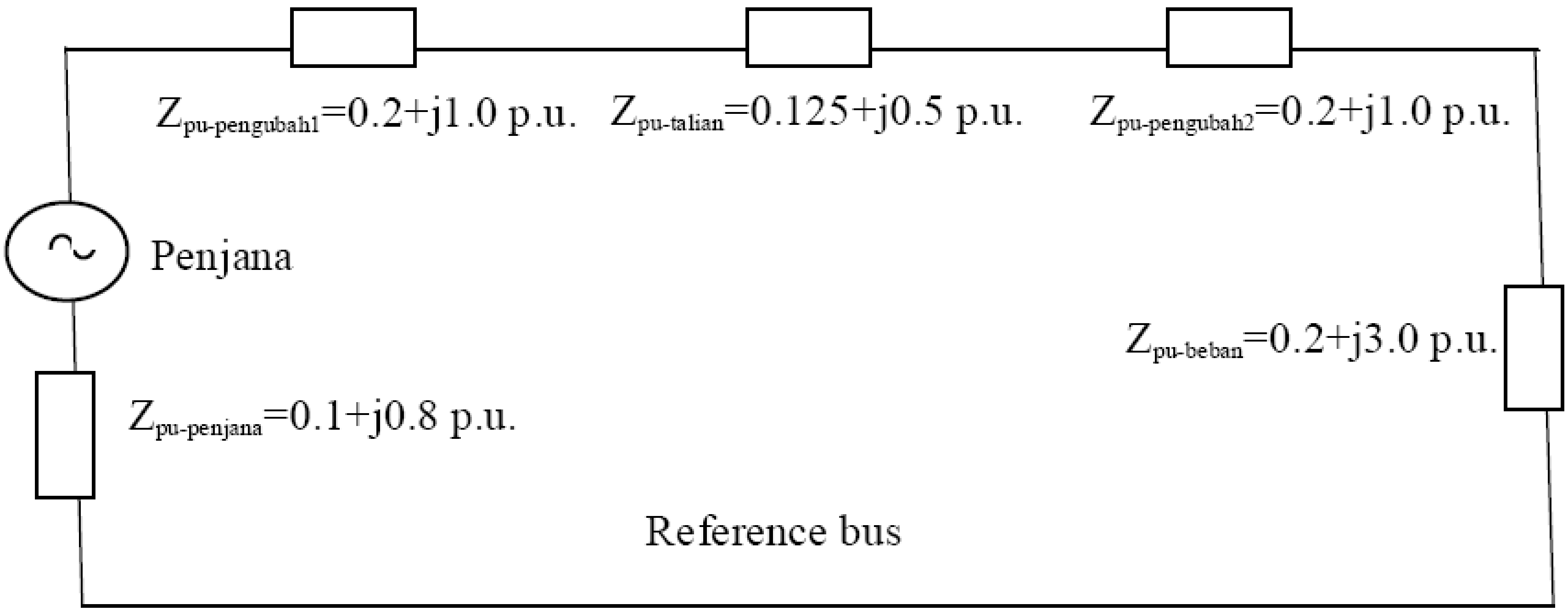
Transformer 1: $$Z_{pu\text{-}transformer1} = \frac{Z_{transformer1}}{Z_{base\text{-}transformer1}} = \frac{0.2 + j1.0}{1} = 0.2 + j1.0 \text{ p.u.}$$

Line: $$Z_{pu\text{-}line} = \frac{Z_{line}}{Z_{base\text{-}line}} = \frac{0.5 + j2.0}{4} = 0.125 + j0.5 \text{ p.u.}$$

Transformer 2: $$Z_{pu\text{-}transformer2} = \frac{Z_{transformer2}}{Z_{base\text{-}transformer2}} = \frac{0.2 + j1.0}{1} = 0.2 + j1.0 \text{ p.u.}$$

Load: $$Z_{pu\text{-}load} = \frac{Z_{load}}{Z_{base\text{-}load}} = \frac{0.2 + j3.0}{1} = 0.2 + j3.0 \text{ p.u.}$$

The impedance diagram :



Example :

Find the load current, actual current in every component and voltage drop in every component if the load voltage is 1pu.

$$I_{pu\text{-}load} = \frac{V_{pu\text{-}load}}{Z_{pu\text{-}load}} = \frac{1.0}{0.2 + j3.0} = 0.33\angle\text{-}86.2^{\circ}\ \text{p.u.}$$

The load current value in per unit is the same throughout the system.

The base current value at each component is

$$I_{\text{base-load}} = \frac{VA_{3\phi-base}}{\sqrt{3}V_{\text{L-base}}} = \frac{100\,\text{MVA}}{\sqrt{3}\,10\,\text{kV}} = 5773.5\text{ A.}$$

$$I_{\text{base-line}} = \frac{100\text{ MVA}}{\sqrt{3}\;20\text{ kV}} = 2886.8\text{ A.}$$

$$I_{\text{base-generator}} = \frac{100\text{ MVA}}{\sqrt{3}\;10\text{ kV}} = 5773.5\text{ A.}$$

The actual current value at each component is

$$I_{load} = I_{base\text{-}load} \text{ x } I_{pu\text{-}load} = 5773.5 \text{ x } 0.33\angle\text{-}86.2^{\circ} = 1905.3\ \angle\text{-}86.2^{\circ}\ \text{A.}$$

$$I_{line} = I_{base\text{-}line} \text{ x } I_{pu\text{-}load} = 2886.8 \text{ x } 0.33\angle\text{-}86.2^{\circ} = 952.6\ \angle\text{-}86.2^{\circ}\ \text{A.}$$

$$I_{generator} = I_{base\text{-}generator} \text{ x } I_{pu\text{-}load} = 5773.5 \text{ x } 0.33\angle\text{-}86.2^{\circ} = 1905.3\ \angle\text{-}86.2^{\circ}\ \text{A.}$$

The voltage drop at every component is:

Transformer 2: $= I_{pu\text{-}load}\, Z_{pu\text{-}transformer2} = (0.33\angle\text{-}86.2^{\circ})(0.2 + j1.0)$

$= 0.34\angle\text{-}7.5^{\circ}$ p.u.

Line: $= I_{pu\text{-}load}\, Z_{pu\text{-}line} = (0.33\angle\text{-}86.2^{\circ})(0.125 + j0.5)$

$= 0.17\angle\text{-}10.2^{\circ}$ p.u.

Transformer 1: $= I_{pu\text{-}load}\, Z_{pu\text{-}transformer1} = (0.33\angle\text{-}86.2^{\circ})(0.2 + j1.0)$

$= 0.34\angle\text{-}7.5^{\circ}$ p.u.

Generator: $= I_{pu\text{-}load}\, Z_{pu\text{-}generator} = (0.33\angle\text{-}86.2^{\circ})(0.1 + j0.8)$

$= 0.27\angle\text{-}3.3^{\circ}$ p.u.

The generated voltage is :

$$V_{pu\text{-}generator} = V_{pu\text{-}load} + KV_{pu\text{-}transformer2} + KV_{pu\text{-}line} + KV_{pu\text{-}transformer1} + Kv_{pu\text{-}generator}$$
$$= 1 + 0.34\angle\text{-}7.5^{\circ} + 0.17\angle\text{-}10.2^{\circ} + 0.34\angle\text{-}7.5^{\circ} + 0.27\angle\text{-}3.3^{\circ}$$
$$= 2.12 - j0.13 = 2.12\angle\text{-}3.5^{\circ}\ \text{p.u.}$$

The actual value of the generated voltage is

$$V_{generator} = V_{pu\text{-}generator} \times V_{base\text{-}generator} == (2.12\angle 30^{\circ} - 3.5^{\circ}) \times 10\ \text{kV}$$
$$= 21.2\angle 26.5^{\circ}\ \text{kV (line-to-line voltage).}$$

it can be observed that the generated voltage is far beyond the nominal voltage. This occurs due to the excessive voltage drop in the components of the system.

Question 2.

Repeat the previous by replacing the load with a new value of $50 + j\,100\ \Omega$.

Base impedances are similar to those given in the example:

Generator and 10 kV side of Transformer 1: $Z_{base\text{-}generator} = 1\ \Omega$.

Transmission line: $Z_{base\text{-}line} = 4\ \Omega$.

Transformer 2: $Z_{base\text{-}transformer2} = 1\ \Omega$.

Load: $Z_{base\text{-}load} = 1\ \Omega$.

The impedance value in per unit for each component, except load is similar to that from the example:

Generator: $Z_{pu\text{-}generator} = 0.1 + j0.8$ p.u.

Transformer 1: $Z_{pu\text{-}transformer1} = 0.2 + j1.0$ p.u.

Line: $Z_{pu\text{-}line} = 0.125 + j0.5$ p.u.

Transformer 2: $Z_{pu\text{-}transformer2} = 0.2 + j1.0$ p.u.

The new value for load impedance is

$$Z_{pu\text{-}load} = (50+j100) / 1 = 50 + j100 \text{ p.u.}$$

$$I_{pu\text{-}load} = 1/(50+j100) = 0.009\angle\text{-}63.4^\circ \text{ p.u.}$$

Base current is similar to that from the example:

$$I_{base\text{-}load} = 5773.5 \text{ A.}$$

$$I_{base\text{-}line} = 2886.8 \text{ A.}$$

$$I_{base\text{-}generator} = 5773.5 \text{ A.}$$

Thus, the current flowing into each component in Ampere is

$$I_{load} = I_{base\text{-}load} \text{ x } I_{pu\text{-}load} = 5773.5 \text{ x } 0.009\angle\text{-}63.4^\circ = 52.0 \angle\text{-}63.4^\circ \text{ A.}$$

$$I_{line} = I_{base\text{-}line} \text{ x } I_{pu\text{-}load} = 2886.8 \text{ x } 0.009\angle\text{-}63.4^\circ = 26.0 \angle\text{-}63.4^\circ \text{ A.}$$

$$I_{generator} = I_{base\text{-}generator} \text{ x } I_{pu\text{-}load} = 5773.5 \text{ x } 0.009\angle\text{-}63.4^\circ = 52.0 \angle\text{-}63.4^\circ \text{ A.}$$

Thevoltage drops in the components of the system are:

Transformer 2: $KV_{pu\text{-}transformer2} = (0.009\angle{-63.4^{\circ}})(0.2 + j1.0) = 9.2 \times 10^{-3}\angle 15.3^{\circ}$ p.u.

Line: $KV_{pu\text{-}line} = (0.009\angle{-63.4^{\circ}})(0.125 + j0.5) = 4.7 \times 10^{-3}\angle 12.6^{\circ}$ p.u.

Transformer 1: $KV_{pu\text{-}transformer1} = (0.009\angle{-63.4^{\circ}})(0.2 + j1.0) = 9.2 \times 10^{-3}\angle 15.3^{\circ}$ p.u.

Generator: $KV_{pu\text{-}generator} = (0.009\angle{-63.4^{\circ}})(0.1 + j0.8) = 7.3 \times 10^{-3}\angle 19.5^{\circ}$ p.u.

Generated voltage is

$$V_{pu\text{-}generator} = V_{pu\text{-}load} + KV_{pu\text{-}transformer2} + KV_{pu\text{-}line} + KV_{pu\text{-}transformer1} + KV_{pu\text{-}generator}$$
$$= 1 + 9.2 \times 10^{-3}\angle 15.3^{\circ} + 4.7 \times 10^{-3}\angle 12.6^{\circ} + 9.2 \times 10^{-3}\angle 15.3^{\circ} + 7.3 \times 10^{-3}\angle 19.5^{\circ}$$
$$= 1.029 + j0.008 = 1.03\angle 0.5^{\circ} \text{ p.u.}$$

In actual unit, the generated voltage is

$$V_{generator} = V_{pu\text{-}generator} \times V_{base\text{-}generator} == (1.03\angle 30^{\circ} + 0.5^{\circ}) \times 10 \text{ kV}$$
$$= 10.3\angle 30.5^{\circ} \text{ kV.}$$

List of advantages of per unit compared to normal unit :

i. Manufacturers of power system components conveniently provide impedance data for the components in per unit based on the voltage and power ratings of the components.

ii. The calculation for system parameters such as current and voltage is relatively simple and straightforward using per unit system in the impedance

iii. The impedance of a transformer in per unit is the same for both side.

iv. per unit are scaling mechanisms, useful to power system engineers because it can easily help indicate overvoltage, over current and overloading.

v. Simplify the relationship involves the multiplication of current and voltage.

In certain situation, conversion of impedance in per unit from a given base to another need to be done.

For instance, the impedance for an individual component is initially based on the ratings of the component.

However, if the base values used for the whole system differ from the component's ratings, then the per unit value of the impedance has to be corrected to the new system base.

A three phase generator has the following voltage and power ratings:

$$\text{Vbase} = 10 \text{ kV}$$
$$\text{VAbase} = 50 \text{ MVA}$$

Given the impedance of the generator

$$X_{pu} = 0.1 \text{ p.u.}$$

In Ohm, the impedance is

$$X_{Ohm} = X_{pu} \frac{(V_{L\text{-base}})^2}{VA_{3\phi-base}} = 0.1 \frac{(10\,\text{kV})^2}{50\,\text{MVA}} = 0.2\ \Omega$$

if the following new bases are used,

Vbase-new = 20 kV

VAbase-new = 10 MVA,

Then

$$Z_{\text{base-new}} = \frac{(V_{\text{L-base-new}})^2}{VA_{3\phi-base-new}} = \frac{(20\,\text{kV})^2}{10\,\text{MVA}} = 40\ \Omega$$

Thus, the impedance value in the new bases

$$X_{pu} = \frac{0.2}{40} = 0.005\ \text{p.u.},$$

Given the old base values for voltage and power and the impedance based on these values,

$$V_{base\text{-}old}$$
$$VA_{base\text{-}old}$$
$$Z_{pu\text{-}base\text{-}old}$$

The new base values

$$V_{base\text{-}new}$$
$$VA_{base\text{-}new}$$

Therefore, the impedance value in the new base is :

$$Z_{\text{pu-base-new}} = Z_{\text{pu-base-old}} \left[\frac{V_{\text{base-old}}}{V_{\text{base-new}}}\right]^2 \left[\frac{VA_{\text{base-new}}}{VA_{\text{base-old}}}\right]$$

Question 3

A generator has reactance of 0.25 p.u. based on 18 kV 500 MVA. The generator is connected to a power system that uses base values of 20 kV 100 MVA. Calculate

i. reactance of the generator in Ohm.
ii. the reactance in per unit based on 20 kV 100 MVA base values.

(i) With base values of 18 kV 500 MVA,

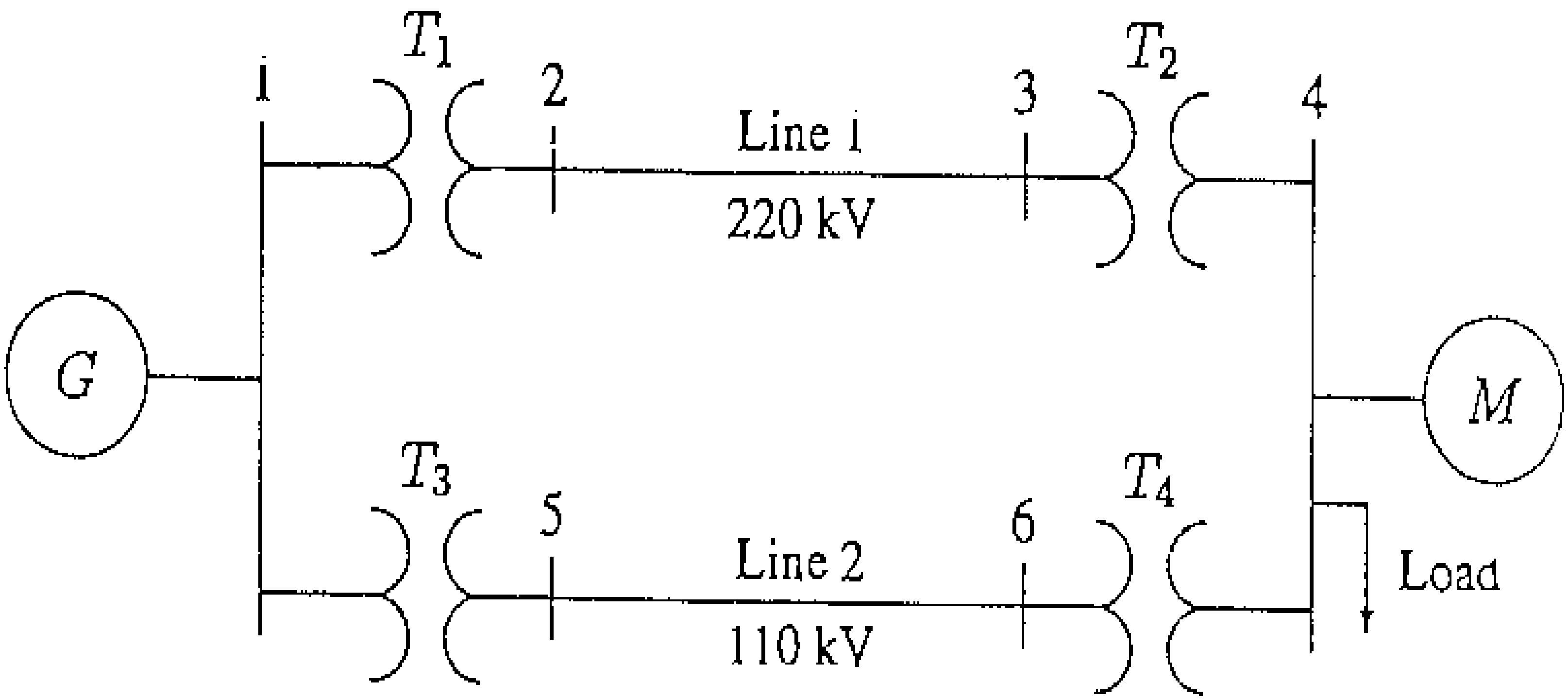
$$X_{base} = \frac{(18\text{ kV})^2}{500\text{ MVA}} = 0.65\ \Omega.$$

Thus,, $X_{generator} = 0.25 \text{ x } 0.65 = 0.16\ \Omega$.

(ii) By base conversion method,

$$X_{generator} = 0.25\left(\frac{18\text{ kV}}{20\text{ kV}}\right)^2 \frac{100\text{ MVA}}{500\text{ MVA}} = 0.04\text{ p.u.}$$

Example :



Data :

G:	90 MVA	22 kV	$X = 18\ \%$
T_1:	50 MVA	22/220 kV	$X = 10\ \%$
T_2:	40 MVA	220/11 kV	$X = 6.0\ \%$
T_3:	40 MVA	22/110 kV	$X = 6.4\ \%$
T_4:	40 MVA	110/11 kV	$X = 8.0\ \%$
M:	66.5 MVA	10.45 kV	$X = 18.5\ \%$

Find the new per-unit impedance for each component by taking 11 kV at the motor side and 100MVA as the new base.

The new per unit impedance

$$G \quad X = 0.18\left(\frac{100}{90}\right) = 0.20 \quad \text{pu}$$

$$T_1 \quad X = 0.10\left(\frac{100}{50}\right) = 0.20 \quad \text{pu}$$

$$T_2, \quad X = 0.06\left(\frac{100}{40}\right) = 0.15 \quad \text{pu}$$

$$T_3 \quad X = 0.064\left(\frac{100}{40}\right) = 0.16 \quad \text{pu}$$

$$T_4, \quad X = 0.08\left(\frac{100}{40}\right) = 0.2 \quad \text{pu}$$

$$M: \quad X = 0.185\left(\frac{100}{66.5}\right)\left(\frac{10.45}{11}\right)^2 = 0.25 \quad \text{pu}$$

Exercises

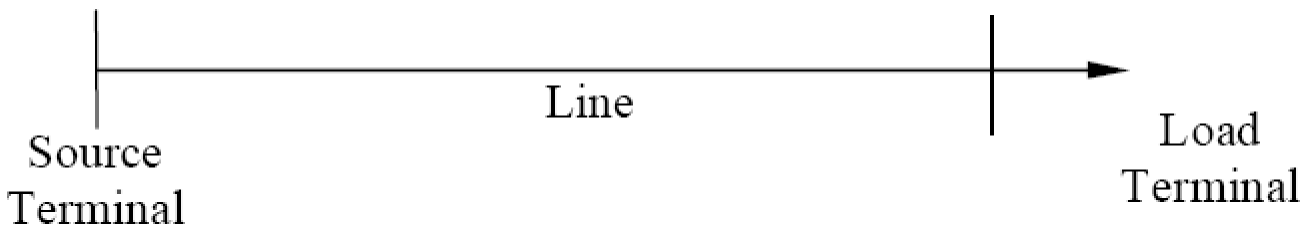
2.5.3 A three phase power system has two generator connected in parallel to a transformer and subsequently to a 230 kV transmission line. The ratings for the components are given as below:

Generator 1 : 10 MVA, reactance of 12%.
Generator 2 : 5 MVA, reactance of 8%.
Transformer: 15 MVA, reactance of 6%
Transmission line: impedance of $4 + j60\ \Omega$,

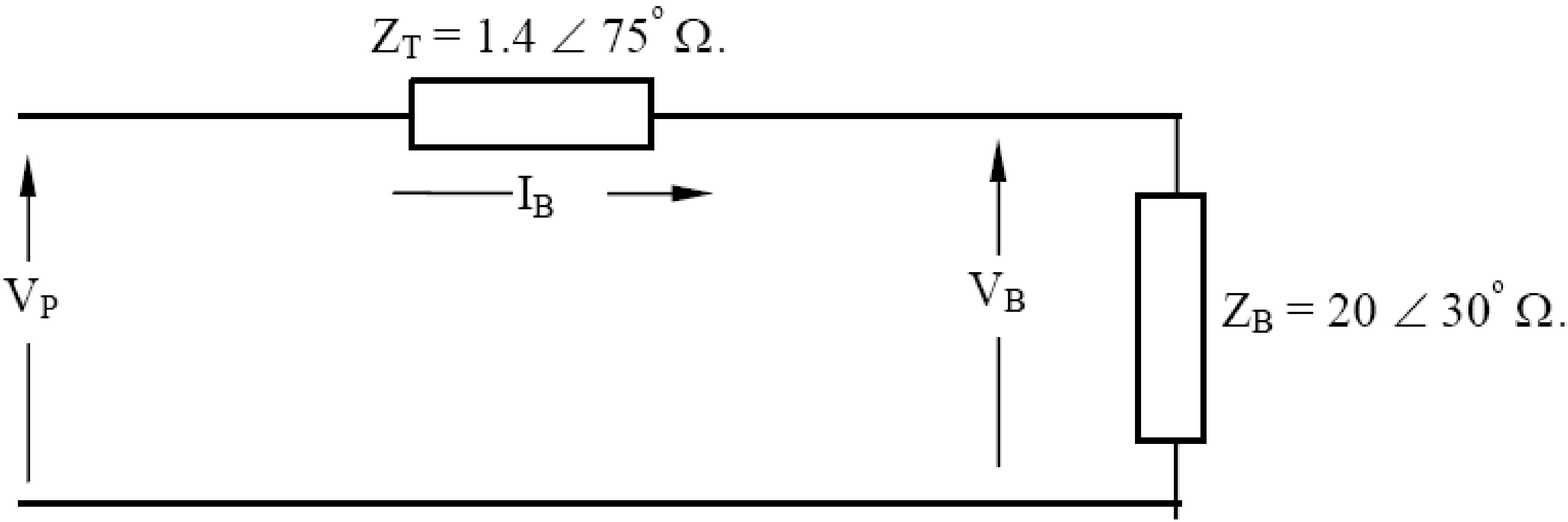
for which the reactances in percentage are based on the ratings of the respective components. Obtain the reactance and impedance values of the components in the system in percentage, based on 15 MVA and rated voltage of the components.

[X_{G1} = 18%, X_{G2}=24%, X_T=6%,X_{TP}=(0.113+j1.7)%]

A 3 phase Wye connected load consists of three impedances, each with a value of $20\angle 30^\circ\ \Omega$. The line voltage at the load terminal is 4.4 kV. The line impedance is $1.4 \angle 75^\circ\ \Omega$. Obtain the line voltage at the source end.



Impedance diagram for the system



Using base values of 4.4 kV (line voltage) and 127 A (phase current),

$$V_B = 1\angle 0^\circ \text{ p.u.}$$

$$Z_{base} = \frac{4.4\text{ kV}/\sqrt{3}}{127\text{ A}} = 20\ \Omega.$$

$$Z_B = \frac{20\angle 30^\circ}{20} = 1\angle 30^\circ \text{ p.u.}$$

$$Z_T = \frac{1.4\angle 75^\circ}{20} = 0.07\angle 75^\circ \text{ p.u.}$$

Phase voltage at the source end is

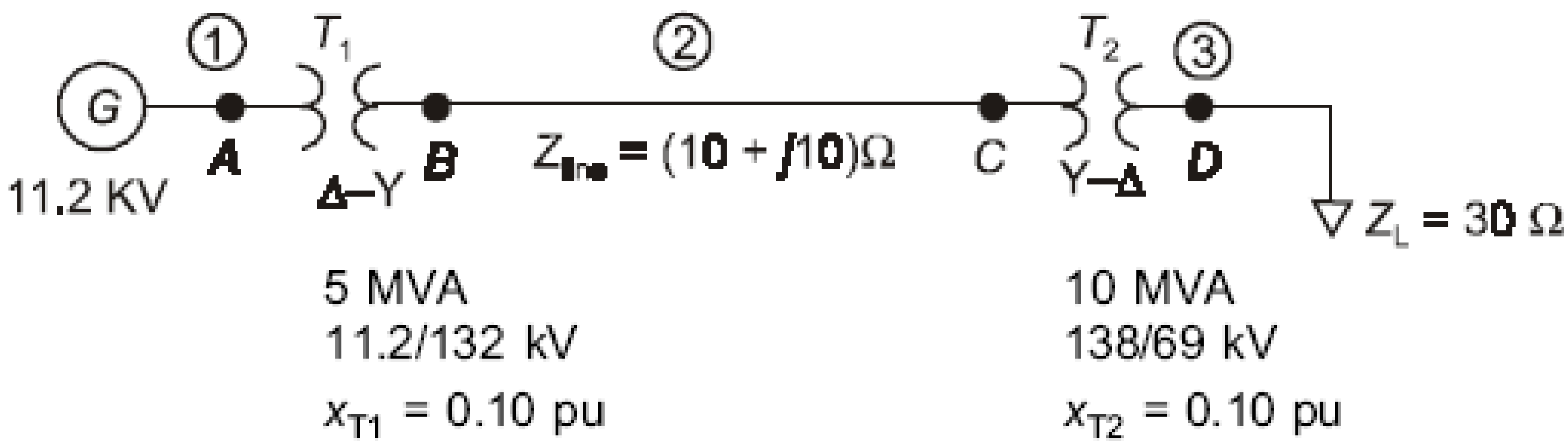
$$V_P = I_B Z_T + V_B = \frac{V_B}{Z_B} Z_T + V_B$$

$$= \frac{1}{1\angle 30^o} 0.07\angle 75^\circ + 1$$

$$= 1.05\angle 2.7^\circ \text{ p.u.} \quad (@ 1.05 \text{ x } 4.4 \text{ kV} = 4.62 \text{ kV line-to-line}).$$

Example :

This figure shows a sample of power system network. Find the current supplied by the generator, the transmission line current, the load current, the load voltage and the power consumed by the load. Choose base 100MVA and 138kV at the line.



Choose

$$(\mathrm{MVA})_{\mathrm{B}} = 100$$

$$(\mathrm{KV})_{\mathrm{B}} = 138$$

$$V_{\mathrm{B1}} = \left(\frac{138}{132}\right) \times 11.2 = 11.71 \text{ kV}$$

$$V_{\mathrm{B2}} = 138 \text{ kV}$$

$$V_{\mathrm{B3}} = 69 \text{ kV}$$

$$Z_{\mathrm{B2}} = \frac{(138)^2}{100} = 190.44\ \Omega$$

$$Z_{\mathrm{B3}} = \frac{(69)^2}{100} = 47.61\ \Omega$$

$$Z_{\mathrm{Line}}^{(\mathrm{pu})} = \frac{10 + j10}{Z_{\mathrm{B2}}} = \frac{10 + j10}{190.44} = 0.0525\ (1 + j1)\ \mathrm{pu}$$

$$Z_{\mathrm{Load}}^{(\mathrm{pu})} = \frac{30}{Z_{\mathrm{B3}}} = \frac{30}{47.61} = 0.63\ \mathrm{pu}$$

$$x_{\mathrm{T1,\,new}} = \frac{x_{\mathrm{T1,old}} \times (\mathrm{KV})^2_{\mathrm{B,\,old}}}{(\mathrm{MVA})_{\mathrm{B,\,old}}} \times \frac{(\mathrm{MVA})_{\mathrm{B,\,new}}}{(\mathrm{KV})^2_{\mathrm{B,\,new}}}$$

$$\mathrm{x}_{\mathrm{T1,\,old}} = 0.10 \text{ pu}, \ (\mathrm{KV})_{\mathrm{B,\,old}} = 11.2 \text{ kV},$$

$$(\mathrm{KV})_{\mathrm{B,\,new}} = 11.71 \text{ kV}, (\mathrm{MVA})_{\mathrm{B,\,old}} = 5, (\mathrm{MVA})_{\mathrm{B,\,new}} = 100$$

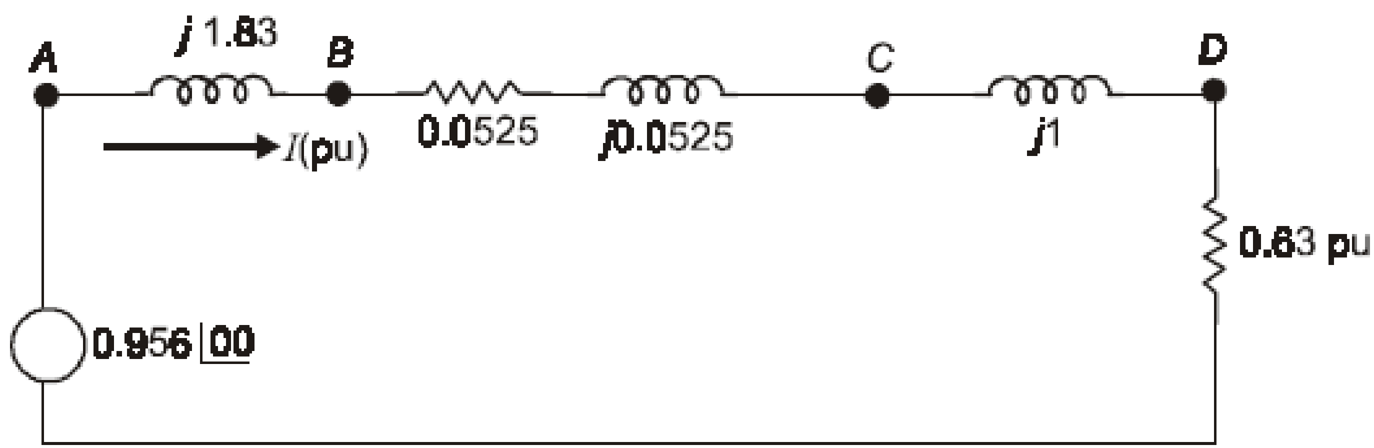
$$x_{\mathrm{T1,\,new}} = 0.10 \times \frac{(11.2)^2}{5} \times \frac{100}{(11.71)^2} \text{ pu} = 1.83 \text{ pu}.$$

$$x_{\mathrm{T2}} = 0.10 \times \frac{100}{10} = 1 \text{ pu}$$

Finally the source voltage in per-unit,

$$|E_S| = \frac{11.2}{11.71} = 0.956 \text{ pu}.$$

Figure 5.20 shows the impedance diagram of example 5.7.



$$I(\text{pu}) = \frac{0.956\angle 0^\circ}{j\,1.83 + 0.0525 + j\,0.0525 + j1 + 0.63}\ \text{pu}$$

$$I(\text{pu}) = \frac{0.956\angle 0^\circ}{(0.6825 + j\,2.8825)} = \frac{0.956}{2.962\angle 76.68^\circ}\ \text{pu}$$

$$I(\text{pu}) = 0.3227\angle -76.68^\circ\ \text{pu}$$

Load voltage $V_{\text{Load}}(\text{pu}) = 0.63 \times 0.3227\angle -76.68^\circ = 0.203\angle -76.68^\circ$ pu.

$\therefore$ $P_{\text{Load}}(\text{pu}) = Z_{\text{Load}}(\text{pu})\,|I(\text{pu})|^2 = 0.63 \times (0.3227)^2 = 0.0656$ pu

$= 0.0656 \times 100 = 6.56$ MW load

$$I_{\mathrm{B1}} = \frac{100 \times 10^{6}}{3} \times \frac{\sqrt{3}}{11.7 \times 10^{3}} = 4934.6 \text{ Amp}$$

$$I_{\mathrm{B2}} = \frac{11.2}{132} \times 4934.6 = 418.7 \text{ Amp}$$

$$I_{\mathrm{B3}} = \frac{138}{69} \times 418.7 = 837.4 \text{ Amp.}$$

Generator current

$$\therefore \qquad I_g = |I(\text{pu})| \times I_{B1} = 0.3227 \times 4934.6 = 1592.4 \text{ Amp}$$

Transmission line current

$$|I_2| = 0.3227 \times 418.7 \text{ Amp} = 135.11 \text{ Amp}$$

Load current

$$|I_3| = 0.3227 \times 837.4 = 270.23 \text{ Amp}$$

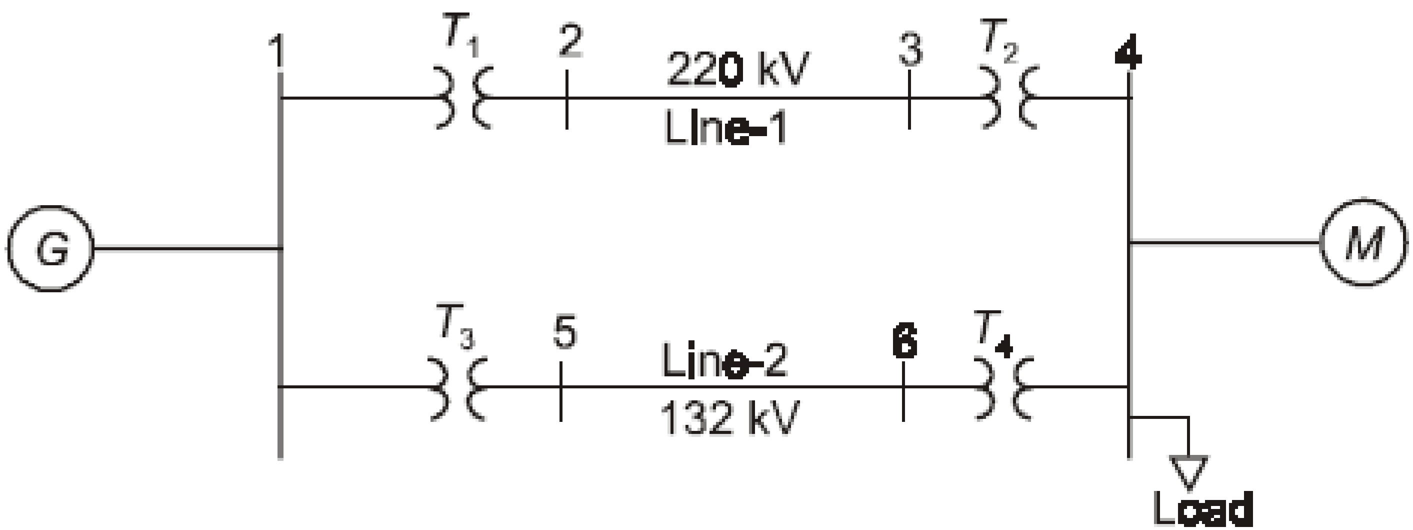
Load voltage,

$$V_L(\text{pu}) = I(\text{pu}) \times Z_L(\text{pu}) = 0.3227 \underline{|-76.68^\circ} \times 0.63 \text{ pu}$$

$$V_L(\text{pu}) = 0.2033 \underline{|-76.68^\circ} \text{ pu}$$

$$|V_L| = 0.2033 \times 69 \text{ KV} = 14.02 \text{ KV (Line-to-line)}$$

This figure shows a single line diagram of a network. Select a common base of 100MVA and 13.8kV on the generator side. Draw the per unit impedance diagram.



G : 90 MVA, 13.8 kV, $x_g = 18\%$

T_1 : 50 MVA, 13.8/220 kV, $x_{T1} = 10\%$

T_2 : 50 MVA, 220/11 kV, $x_{T2} = 10\%$

T_3 : 50 MVA, 13.8/132 kV, $x_{T3} = 10\%$

T_4 : 50 MVA, 132/11 kV, $x_{T4} = 10\%$

M : 80 MVA, 10.45 kV, $x_m = 20\%$

Load : 57 MVA, 0.8 pf (lagging) at 10.45 kV.

$$x_{\text{line1}} = 50\ \Omega$$

$$x_{\text{line2}} = 70\ \Omega$$

$$x_{g1} = 0.18 \times \frac{100}{90} = 0.20 \text{ pu}$$

$$x_{T1} = 0.10\left(\frac{100}{50}\right) = 0.20 \text{ pu}$$

$$x_{T2} = 0.10\left(\frac{100}{50}\right) = 0.20 \text{ pu}$$

$$x_{T3} = 0.10\left(\frac{100}{50}\right) = 0.20 \text{ pu}$$

$$x_{T4} = 0.10\left(\frac{100}{50}\right) = 0.20 \text{ pu}$$

$$x_{\text{m, new}} \text{ (pu)} = 0.2 \times \frac{100}{80} \times \left(\frac{10.45}{11}\right)^2 = 0.2256 \text{ pu.}$$

Base impedance for lines

$$Z_{\mathrm{B},\,2\text{–}3} = \frac{(\mathrm{V_{B2}})^2}{(\mathrm{MVA})_\mathrm{B}} = \frac{(220)^2}{100} = 484\ \Omega$$

$$Z_{\mathrm{B},\,5\text{–}6} = \frac{(V_{\mathrm{B5}})^2}{(\mathrm{MVA})_\mathrm{B}} = \frac{(132)^2}{100} = 174.24\ \Omega\,.$$

$\therefore$

$$x_{\text{line-1}}(\mathrm{pu}) = \frac{50}{484}\ \mathrm{pu} = 0.1033\ \mathrm{pu}$$

$$x_{\text{line-2}}(\mathrm{pu}) = \frac{70}{174.24}\ \mathrm{pu} = 0.4017\ \mathrm{pu}.$$

The load is at 0.8 pf lagging is given by

$$S_{\mathrm{L}}(3\phi) = 57\angle 36.87^\circ$$

Load impedance is given by

$$Z_{\mathrm{L}} = \frac{(V_{\mathrm{L-L}})^2}{S^*_{\mathrm{L}(3\phi)}} = \frac{(10.45)^2}{57\angle -36.87^\circ}$$

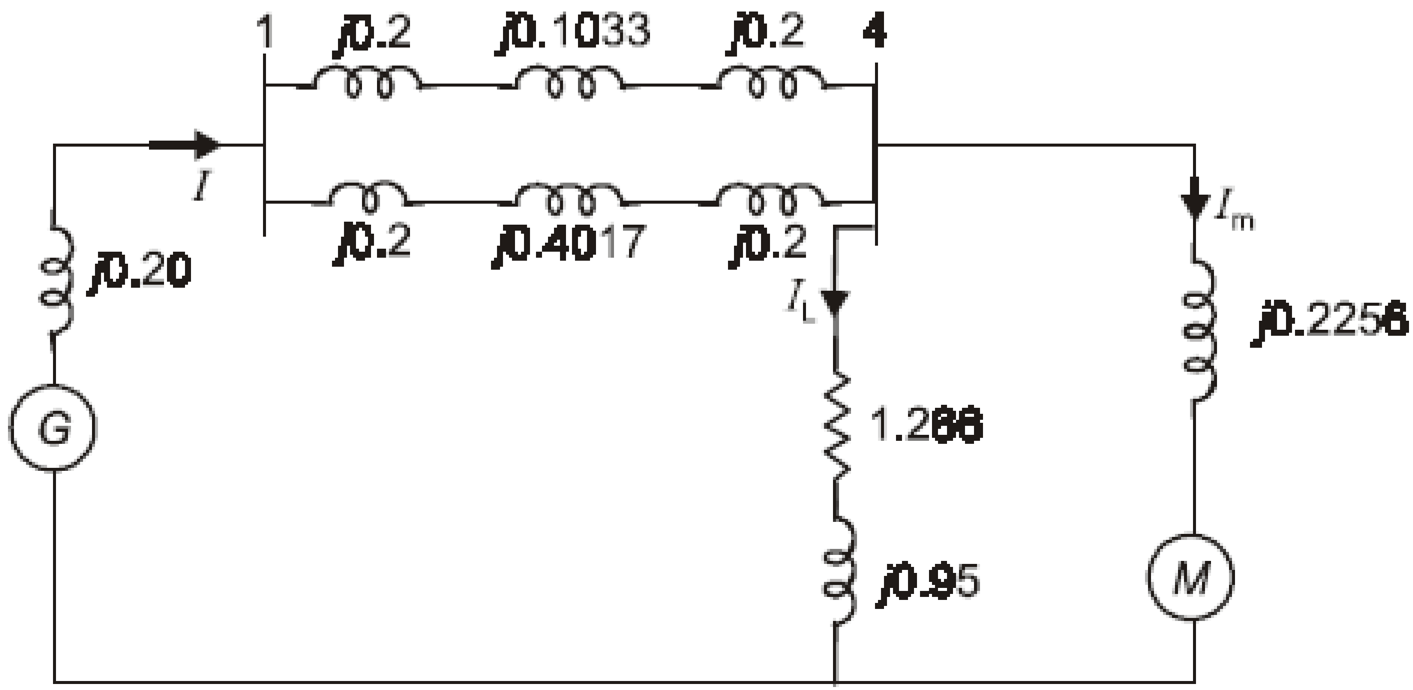
$$\therefore \quad Z_{\mathrm{L}} = (1.532 + j1.1495)\Omega.$$

Base impedance for the load is

$$Z_{\mathrm{B,\ load}} = \frac{(11)^2}{100}\Omega = 1.21\ \Omega$$

$$\therefore \quad Z_{\mathrm{L}}(\mathrm{pu}) = \frac{(1.532 + j\,1.1495)}{1.21}\ \mathrm{pu} = (1.266 + j\,0.95)\ \mathrm{pu}$$

The per-unit equivalent circuit diagram

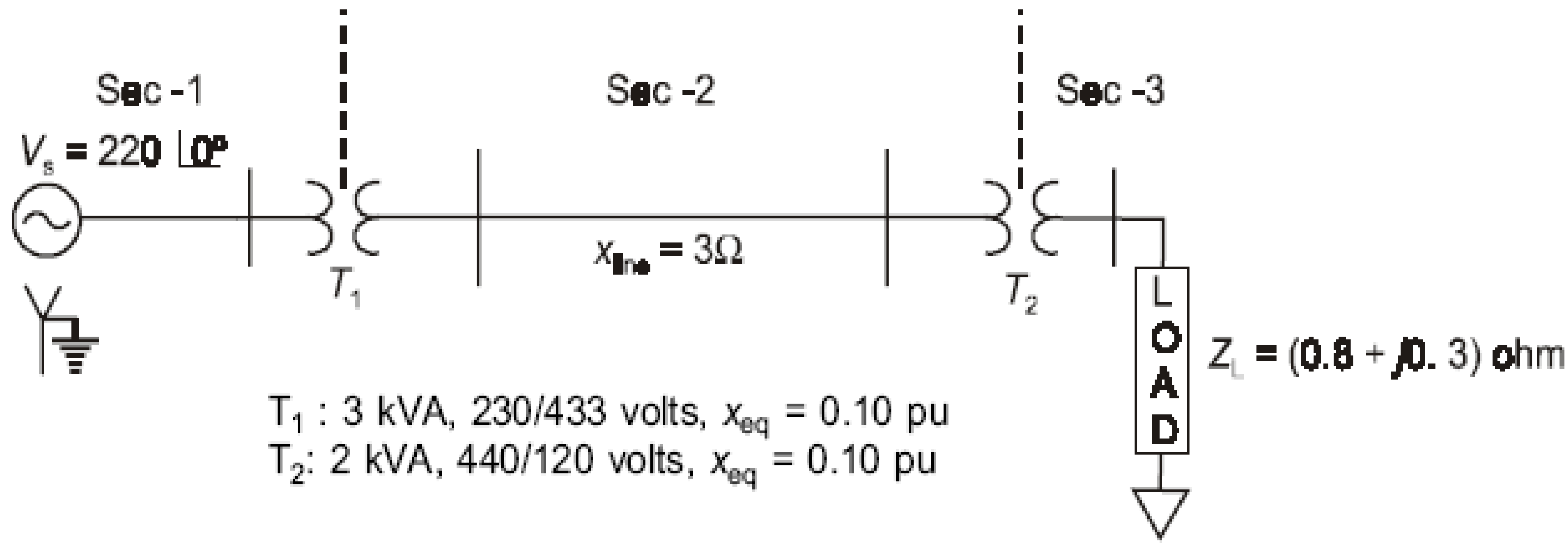


The figure shows a single line diagram of a **single phase circuit.**

Using the base value of 3 kVA and 230V.

Draw the per unit circuit diagram.

Also calculate the load current both in per unit and amperes.



for transformer-T_1, $x_{1,\,\mathrm{old}} = x_{\mathrm{eq}} = 0.10$ pu

For transformer, T_2, $x_{2,new} = 0.1\left(\frac{440}{433}\right)^2\left(\frac{3k}{2k}\right) = 0.1548\ pu$

For generator $v_{g1} = \left(\frac{220}{230}\right) = 0.956\ pu$

For Line

$$Z_{\mathrm{B2}} = \frac{(V_{\mathrm{B2}})^2}{(\mathrm{MVA})_{\mathrm{B}}} = \frac{(0.433)^2}{0.003} = 62.5\ \mathrm{ohm}$$

$$x_{\mathrm{line}}\ (\mathrm{pu}) = \frac{x_{\mathrm{line}}\,(\mathrm{ohm})}{Z_{\mathrm{B2}}} = \frac{3}{62.5} = 0.048\ \mathrm{pu}$$

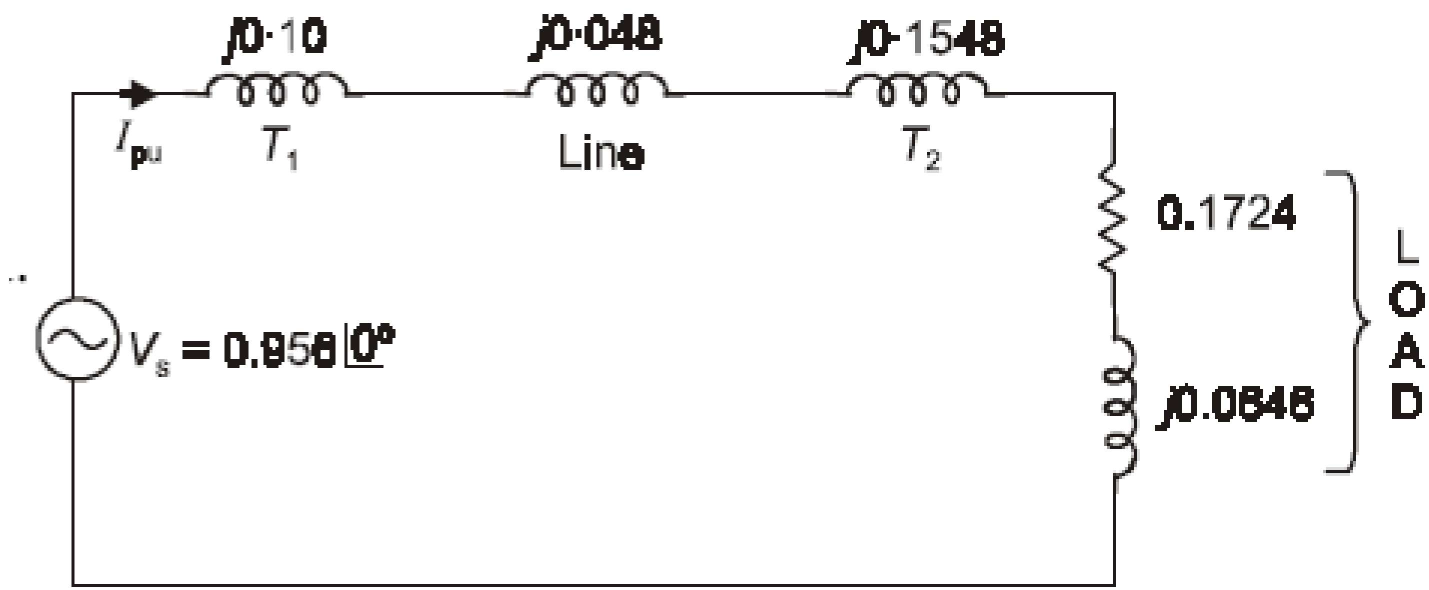
For load $$V_{B3} = \left(\frac{120}{440}\right) \times 433 \text{ volts} = 118.09 \text{ volts} = 0.11809 \text{ kV}.$$

$$Z_{B3} = \frac{(V_{B3})^2}{(\mathrm{MVA})_B} = \frac{(0.11809)^2}{0.003} = 4.64\ \Omega$$

$$Z_L(\mathrm{pu}) = \frac{Z_L\,(\mathrm{ohm})}{Z_{B3}} = \frac{(0.8 + j\,0.3)}{4.64}$$

$$= (0.1724 + j\ 0.0646) \text{ pu}.$$

Per-unit circuit



$$I_{\mathrm{L}}(\mathrm{pu}) = \frac{V_S(\mathrm{pu})}{Z_T(\mathrm{pu})}$$

$$Z_{\mathrm{T}}(\mathrm{pu}) = j\,0.10 + j0.048 + j0.1548 + 0.1724 + j0.0646$$

$$= 0.4058\,\underline{\big|\,64.86^\circ}$$

Base current in Section-3 is

$$I_{\mathrm{B3}} = \frac{(\mathrm{MVA})_{\mathrm{B}}}{(V_{\mathrm{B3}})} = \frac{0.003}{0.11809}\ \mathrm{kA} = 25.4\ \mathrm{Amp}$$

$$\therefore \quad I_{\mathrm{L}}(\mathrm{pu}) = \frac{0.956\,\underline{\big|\,0^\circ}}{0.4058\,\underline{\big|\,64.86^\circ}} = 2.355\,\underline{\big|\,-64.86^\circ}\ \mathrm{pu}$$

$$I_{\mathrm{L}}(\mathrm{Amp}) = I_{\mathrm{L}}(\mathrm{pu}) \times I_{\mathrm{B3}} = 2.355\,\underline{\big|\,-64.86^\circ} \times 25.4$$

$$= 59.83\,\underline{\big|\,-64.86^\circ}\ \mathrm{Amp.}$$

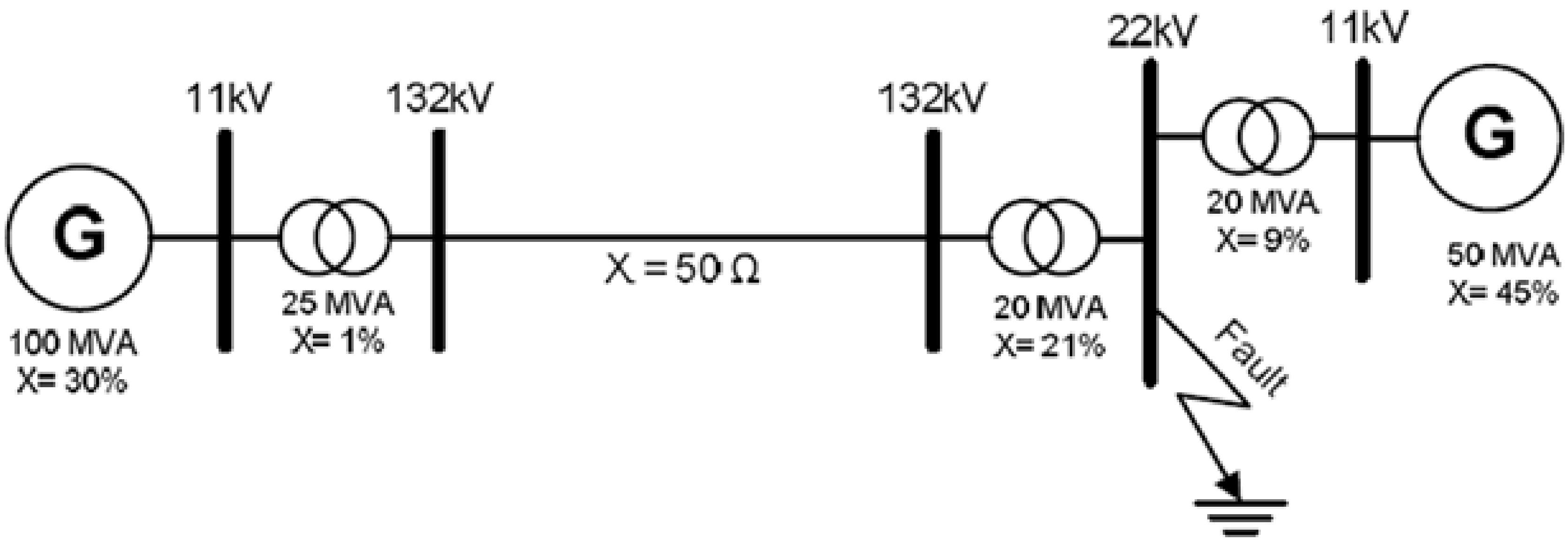
Fault calculation

Per unit calculation is used extensively in fault calculation. The idea is to convert all quantities to per unit.

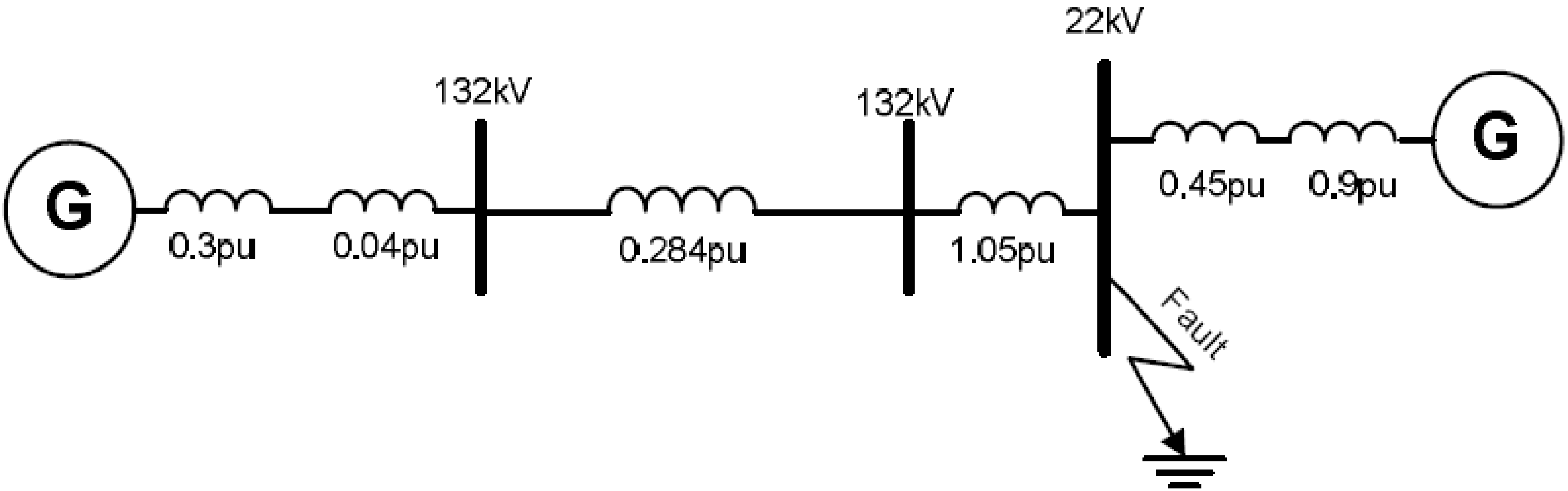
The normal circuit analysis is used to calculate current and voltage under short-circuit condition.

Xpu= Base MVA/ SC MVA

As an example, consider the following network



Changing impedances into pu with 100MVA base,



What is the fault current in p.u and absolute value?