SKEE 4443
POWER SYSTEM ANALYSIS

CHAPTER 2

## Power Flow (PF) Studies

Learning outcome

- Understand the p.f. problem and methods of solution.

Enabling objectives:

- You should be able to establish p.f. equations for general power system network

- You should be able to solve the p.f. equation. Using numerical methods, namely, Gauss-Seidel method and Newton-Raphson (N-R)

- You should be able to calculate branch power flows and power losses

- You should have an appreciation of the relevance of p.f. studies in power system planning and operation
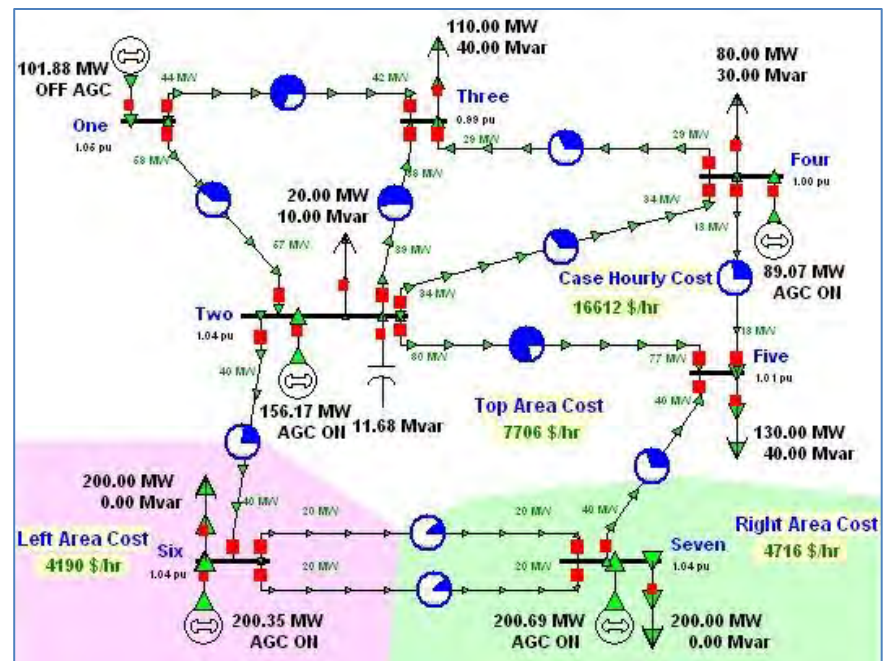
Introduction

- Power flow study is the analysis of a power system in normal steady-state operation.

- One of the most common computational procedures used in power system analysis is the power flow or sometimes called as load flow calculation.

- Under a given set of loads power flow study will determine:
    - Voltages
    - Currents
    - Real power
    - Reactive power

- It is the fundamental network analysis which provides a snapshot of the network.
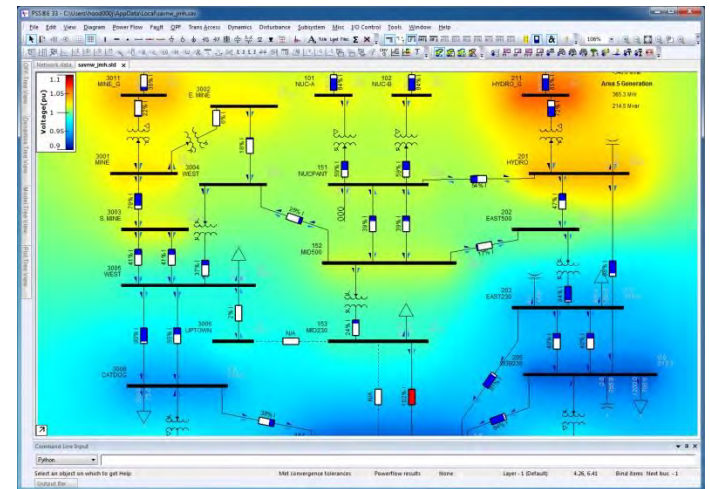
Introduction

- Why do we need power flow study?

- The planning, design, and operation of power systems require power flow calculations to analyze the steady-state performance of the power system under various operating conditions and to study the effects of changes in equipment configuration.

- Determine
  - Equipment loading
  - System security
  - Transfer limits
  - Stability limits
  - Network development
  - Reactive compensation
- Subject to
  - Equipment ratings
  - Reliability standards

Introduction

- Who Uses Power Flow?

- Power System Consultants
  - System Studies
  - System Design

- Electrical Utilities
  - System Planning
  - Operation Planning
  - Real-time Network Control

- The power flow problem was originally motivated within planning environments where engineers considered different network configurations necessary to serve an expected future load.

- Later, it became an operational problem as operators and operating engineers were required to monitor the real-time status of the network in terms of voltage magnitudes and circuit flows.

Power Flow Problem Formulation

- The power flow solution contains the voltages and angles at all buses, and from this information, we may compute the real and reactive generation and load levels at all buses and the real and reactive flows across all circuits.

- The power flow calculation is a network solution problem. The voltages and currents are related by the following equation:

$$
\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{i1} & Y_{i2} & \cdots & Y_{ii} & \cdots & Y_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{ni} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \qquad (1)
$$

- where,
- [I] is the vector of total positive sequence currents flowing into the network buses
- [V] is the vector of positive sequence voltages at the network buses
- [Y] is the network admittance matrix

Power Flow Problem Formulation

- Equation (1) is a linear algebraic equation with complex coefficients. If either [I] or [V] were known, the solution for the unknown quantities could be obtained by application of widely used numerical solution techniques for linear equations.

- The terminal conditions at each bus are normally described in terms of active and reactive powers (P and Q). Thus, the bus current at bus i is related to these quantities as follows:

$$I_i = \frac{(P_i + jQ_i)^*}{V_i^*} \quad (2)$$

$$\left[\frac{(P - jQ)}{V^*} \right] = [Y] \times [V] \quad (3)$$

- where * designates the complex conjugate.
- Combining Equations (1) and (2) yields,

- Equation (3) is nonlinear and cannot be readily solved by closed-form matrix techniques. Because of this, power flow solutions are obtained by procedures involving iterative techniques.

Power Flow Problem Formulation: Bus injection

▪ An injection is the power (P or Q), that is being injected into or withdrawn from a bus by an element having its other terminal (in the per-phase equivalent circuit) connected to ground. Such an element would be either a generator or a load.

▪ Positive injection is defined as one where power is flowing from the element into the bus.

▪ Negative injection is then when power is flowing from the bus, into the element.

G1

P_G=50 MW
Q_G=30 MVar
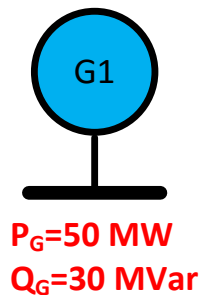
Generators may have either positive or negative reactive power injections

P_D=-50 MW
Q_D=-30 MVar
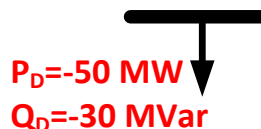
Loads normally have negative real and reactive power injections.

G1

P_{NET}=P_G-P_D
Q_{NET}=Q_G-Q_D
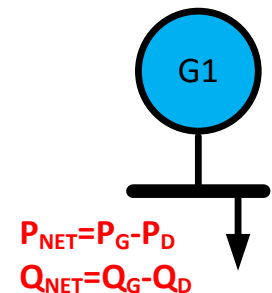
Bus Admittance Matrix or Y_{bus}

- First step in solving the power flow is to create what is known as the bus admittance matrix, often call the Ybus.

- The Ybus gives the relationships between all the bus current injections, I, and all the bus voltages, V.

$$\left[I_{inj}\right] = \left[Y_{bus}\right] \times \left[V_{node}\right]$$

- The Ybus is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances
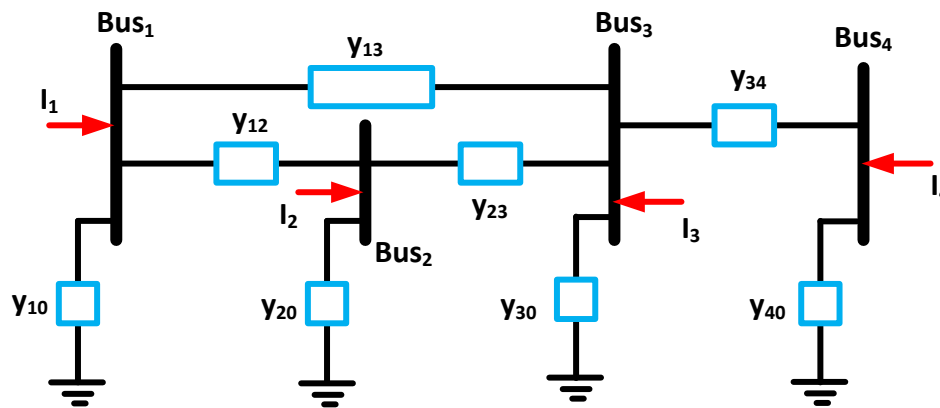
Formation of Bus Admittance Matrix (Y_{bus})

From Kirchoff's Current Law (KCL) –

- the current injections be equal to the sum of the currents flowing out of the bus and into the lines connecting the bus to other buses, or to the ground.
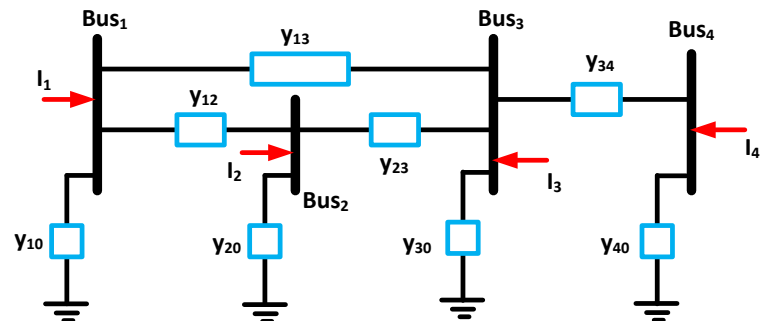- Therefore, recalling Ohm's Law, I=V/Z=VY, the current injected into bus 1 may be written as:

$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$

- Similarly for other buses we can write

$$I_2 = y_{20}V_2 + y_{21}(V_2 - V_1) + y_{23}(V_2 - V_3)$$

$$I_3 = y_{30}V_3 + y_{31}(V_3 - V_1) + y_{32}(V_3 - V_2) + y_{34}(V_3 - V_4)$$

$$I_4 = y_{40}V_4 + y_{43}(V_4 - V_3)$$

Formation of Bus Admittance Matrix (Y_{bus})

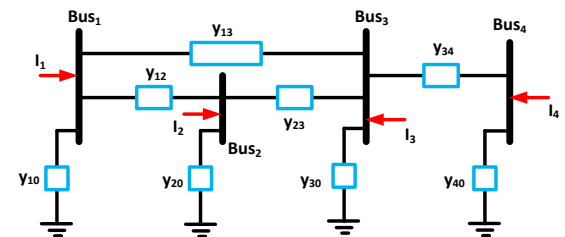- Rearranging the equations we have

$$I_1 = V_1(y_{10} + y_{12} + y_{23}) + V_2(-y_{12}) + V_3(-y_{13})$$

$$I_2 = V_1(-y_{21}) + V_2(y_{20} + y_{21} + y_{23}) + V_3(-y_{23})$$

$$I_3 = V_1(-y_{31}) + V_3(-y_{32}) + V_3(y_{30} + y_{31} + y_{32} + y_{34}) + V_4(-y_{34})$$

$$I_4 = V_3(-y_{43}) + V_4(y_{40} + y_{43})$$

$$Y_{bus} = \begin{bmatrix} y_{10} + y_{12} + y_{23} & -y_{12} & -y_{13} & 0 \\ -y_{21} & y_{20} + y_{21} + y_{23} & -y_{23} & 0 \\ -y_{31} & -y_{32} & y_{30} + y_{31} + y_{32} + y_{34} & -y_{34} \\ 0 & 0 & -y_{43} & y_{40} + y_{43} \end{bmatrix}$$

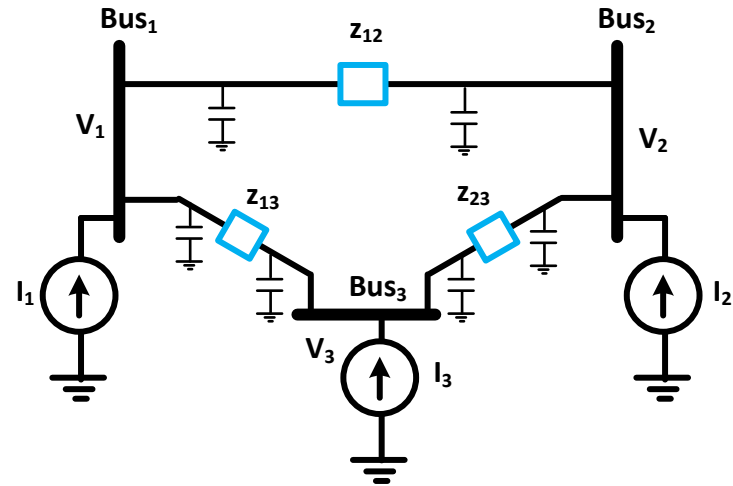# Formation of Bus Admittance Matrix (Y_{bus})

- General formula for building Ybus

$$I_i = V_i y_{i0} + \sum_{\substack{j \\ j \neq i}} \frac{V_i - V_j}{z_{ij}}$$

$$I_i = V_i \left(y_{i0} + \sum_{\substack{j \\ j \neq i}} \frac{1}{z_{ij}} \right) - \sum_{\substack{j \\ j \neq i}} \frac{V_j}{z_{ij}}$$

$$Y_{ii} = y_{i0} + \sum_{\substack{j \\ j \neq i}} \frac{1}{z_{ij}}$$

$$Y_{ij} = -\frac{1}{z_{ij}}$$

Formation of Bus Admittance Matrix (Y_{bus})

- The matrix is symmetric, i.e., Yij=Yji.
- A diagonal element Yii = Self Admittance
- is obtained as the sum of admittances for all branches connected to bus i, including the shunt branch

$$Y_{ii} = y_{i0} + \sum_{\substack{j \\ j \neq i}} \frac{1}{z_{ij}}$$

- The off-diagonal elements are the negative of the admittances connecting buses i and j, i.e., Yij=-yji = mutual admittance.

$$Y_{ij} = -\frac{1}{z_{ij}}$$

E.g. for a 4 bus system

$$\underline{Y} = \begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix}$$

Modeling Shunts in the Ybus
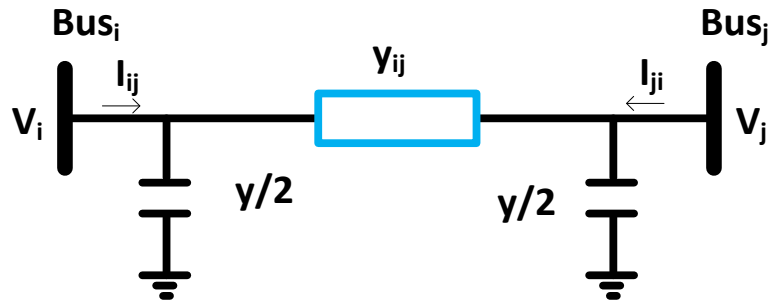
- Since

$$I_{ij} = (V_i - V_i) y_{ij} + V_i \frac{y}{2}$$

$$Y_{ii} = Y_{ii}^{\text{from other lines}} + y_{ij} + \frac{y}{2}$$

2 Bus example
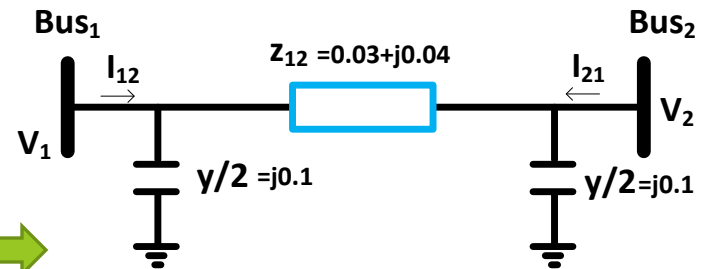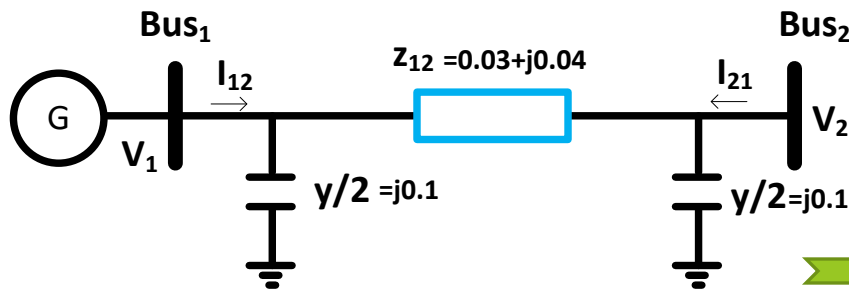
Ybus

2 Bus example: Solution
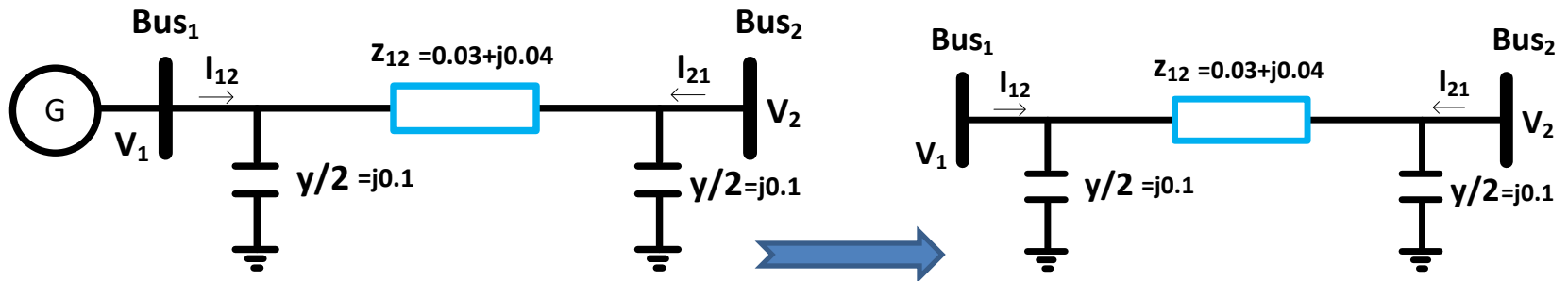
$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}} = \frac{1}{0.03 + j0.04} = 12 - j16$$

$$Y_{ii} = y_{ij} + \frac{y}{2} = 12 - j16 + j0.1 = 12 - j15.9$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & 12 - j16 \\ 12 - j16 & 12 - j15.9 \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Ybus

2 Bus example: Solution

- If the current injection at bus 1 and bus 2 are

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix}$$
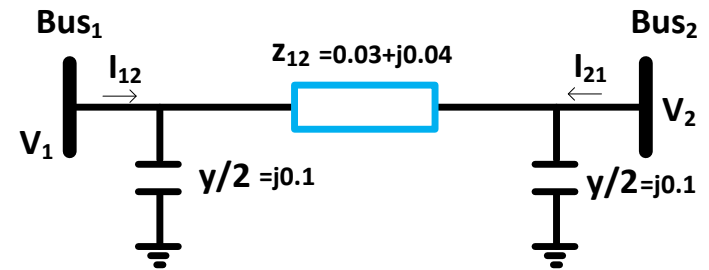
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & 12 - j16 \\ 12 - j16 & 12 - j15.9 \end{bmatrix}^{-1} \times \begin{bmatrix} 5 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j0.1098 \end{bmatrix}$$

- Then the power flow from bus1 and bus 2 are

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} V_1 I_1^* \\ V_2 I_2^* \end{bmatrix} = \begin{bmatrix} (0.0738 - j0.902)(5) \\ (-0.0738 - j0.1098)(-4.8) \end{bmatrix} = \begin{bmatrix} 0.37 - j4.51 \\ 0.35 + j5.27 \end{bmatrix}$$

Ybus

3 Bus example

- Convert impedance to admittance and draw the diagram
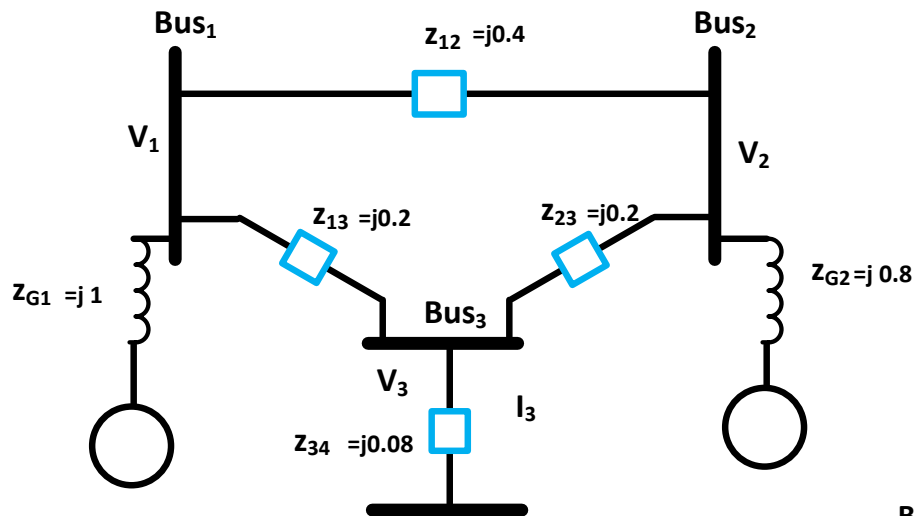
$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$
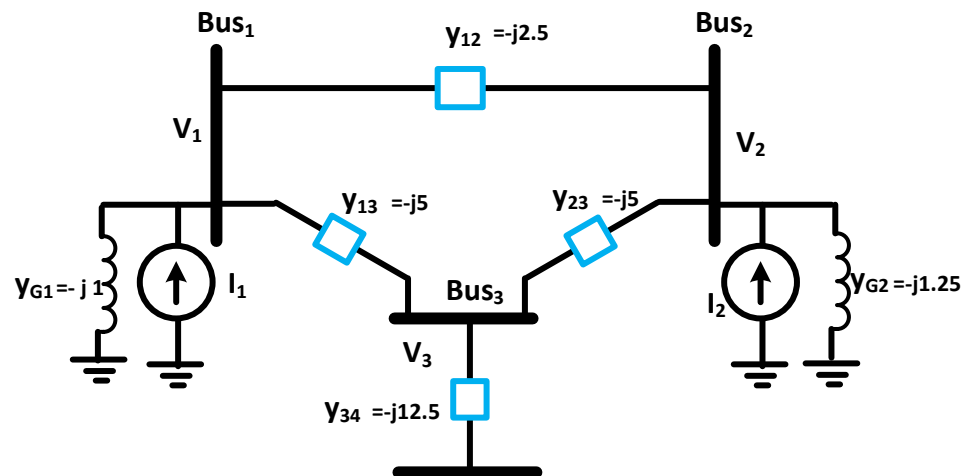
Ybus

3 Bus example

Applying KCL at each bus
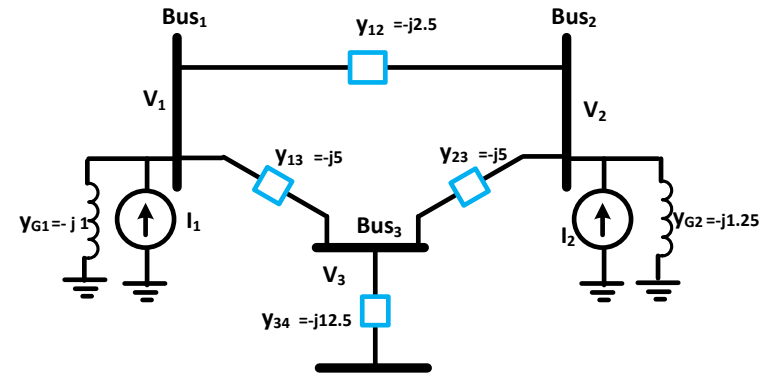
Bus 1 $\quad I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$

Bus 2 $\quad I_2 = y_{20}V_2 + y_{21}(V_2 - V_1) + y_{23}(V_2 - V_3)$

Bus 3 $\quad I_3 = y_{32}(V_3 - V_2) + y_{31}(V_3 - V_1) + y_{34}(V_3 - V_4)$

Bus 4 $\quad I_4 = y_{43}(V_4 - V_3)$

Rearranging

$$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3$$

$$I_3 = -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4$$

$$I_4 = -y_{43}V_3 + y_{43}V_4$$

Ybus

3 Bus example

Rearranging

$$Y_{11} = y_{10} + y_{12} + y_{13}$$

$$Y_{22} = y_{20} + y_{12} + y_{23}$$

$$Y_{33} = y_{13} + y_{23} + y_{34}$$

$$Y_{44} = y_{34}$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$
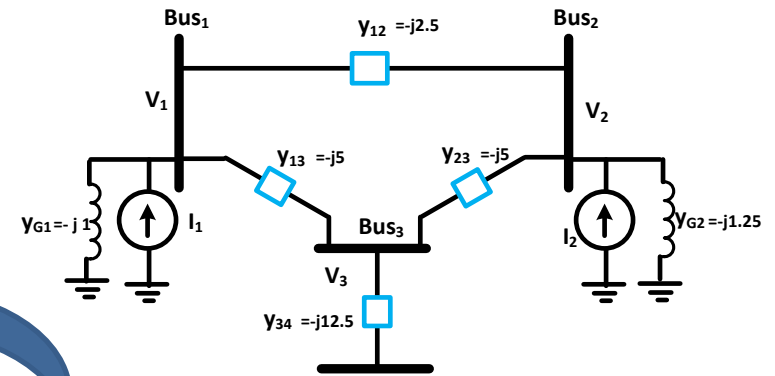
$$Y_{34} = Y_{43} = -y_{34}$$

$$
Y_{bus} = \begin{vmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{vmatrix} = \begin{bmatrix} -j8.5 & j2.5 & j5 & 0 \\ j2.5 & -j8.75 & j5 & 0 \\ j5.0 & j5.0 & -j22.5 & j12.5 \\ 0 & 0 & j12.5 & -j12.5 \end{bmatrix}
$$

Input System Data for Power Flow Program

SYSTEM DATA
- Most power flow programs perform their calculations using a per unit representation of the system rather than working with volts, amperes, and ohms.

- Converting the system data to a per unit representation requires the selection of a base kVA and base voltage. current.

- The system data specifies the base kVA (or MVA) for the entire system. A base kVA of 10000 kVA (10 MVA) is often used for industrial studies. For utility systems, the accepted convention is a base of 100 MVA.

- The base kV is chosen for each voltage level.

Input System Data for Power Flow Program

SYSTEM DATA: Bus Data
- The bus data describes each bus and the load and shunts connected to that bus. The data includes the following:
  - bus number;
  - bus name;
  - bus type;
  - load;
  - shunt;
  - per unit voltage and angle; and bus base kV.

- Load is normally entered in MW and MVar at nominal voltage. Normally, the load is treated as a constant MVA, that is, independent of voltage.

Input System Data for Power Flow Program

SYSTEM DATA: Generator Data

- Generator data is entered for each generator in the system including the system swing generator. The data defines the generator power output and how voltage is controlled by the generator. The data items normally entered are as follows:

  - real power output in MW;
  - maximum reactive power output in MVar (machine maximum reactive limit);
  - minimum reactive power output in MVar (machine minimum reactive limit);
  - scheduled voltage in per unit; and
  - generator in-service/out-of-service code.

Input System Data for Power Flow Program

SYSTEM DATA: Branch/Line Data

- The term "branch" refers to all elements that connect two buses including transmission lines, cables, series reactors, series capacitors, and transformers. The data items include the following:
  - resistance;
  - reactance;
  - charging susceptance (shunt capacitance);
  - line ratings;
  - line in-service/out-of-service code; and
  - line-connected shunts.
- Lines are represented by a model with series resistance and reactance and one-half of the charging susceptance placed on each end of the line. The resistance, reactance, and susceptance are usually input in either per unit or per cent, depending on program convention. Line ratings are normally input in amperes or MVA.

Input System Data for Power Flow Program

SYSTEM DATA: Transformer data

- Additional data is required for transformers. This can either be entered as part of the branch data or as a separate data category depending on the particular power flow program being used. This additional data usually includes the following:

- tap setting in per unit;
- tap angle in degrees;
- maximum tap position;
- minimum tap position; and
- scheduled voltage range with tap step size or a fixed scheduled voltage using a continuous tap approximation.

- The last three data items are needed only for load tap changing (LTC) transformers that automatically vary their tap setting to control voltage on one side of the transformer.

Solving Power Flow Problem

Assumptions:

- At generator or PV bus, the active power P_G is controlled by speed governor and the voltage magnitude is controlled by a voltage regulator. Thus, real power, P and voltage magnitude, V are treated as known parameters.

- At load bus, a reasonable approximation is that the load active and reactive power demand, P_D and Q_D are considered as known parameters.

- At one generator bus, treat it as a slack or reference bus in which the active and reactive powers are variables to make up system losses.

Power Flow Input and Output and Control

Input
- Load (P,Q)
- Active Power Generation
- Network model
- Controls (transformer, interchange, var)

Output
- P, Q at the slack bus
- V at the load bus
- Relative power angles at the generator and load buses

Generator Voltage Control
- PV bus

Transformer Tap Control
- Tap positions

Interchange Control
- Generation MW dispatch

Classification of Buses in a Power System

- The power system buses (busbars) are classified as follows:

Classification	Knowns	Unknowns
PQ (Load Bus)	P, Q	V, δ
PV (Generator Bus)	P, V	Q, δ
Vδ (Swing Bus)	V, δ	P, Q

Load Bus

- Also called the P-Q bus
- Real power Pi and Qi are specified
- All load buses fall into this category, including buses that have not either load or generation.

Classification of Buses in a Power System

Generation Bus

- Also called the P-V bus or voltage-controlled buses
- Voltage magnitude |Vi| and real power Pi are specified
- Able to specify (and therefore to know) the voltage magnitude of this bus.
- Most generator buses fall into this category, independent of whether it also has load

Slack or Swing Bus

- Known as reference bus
- Voltage magnitude |Vi| and phase angle δ_i are specified
- There is only one swing bus, and it can be designated by the engineer to be any generator bus in the system.
- This generator "swings" to compensate for the network losses, or, one may say that it "takes up the slack."

THREE WINDING TRANSFORMER EQUIVALENTS
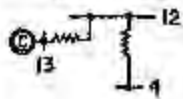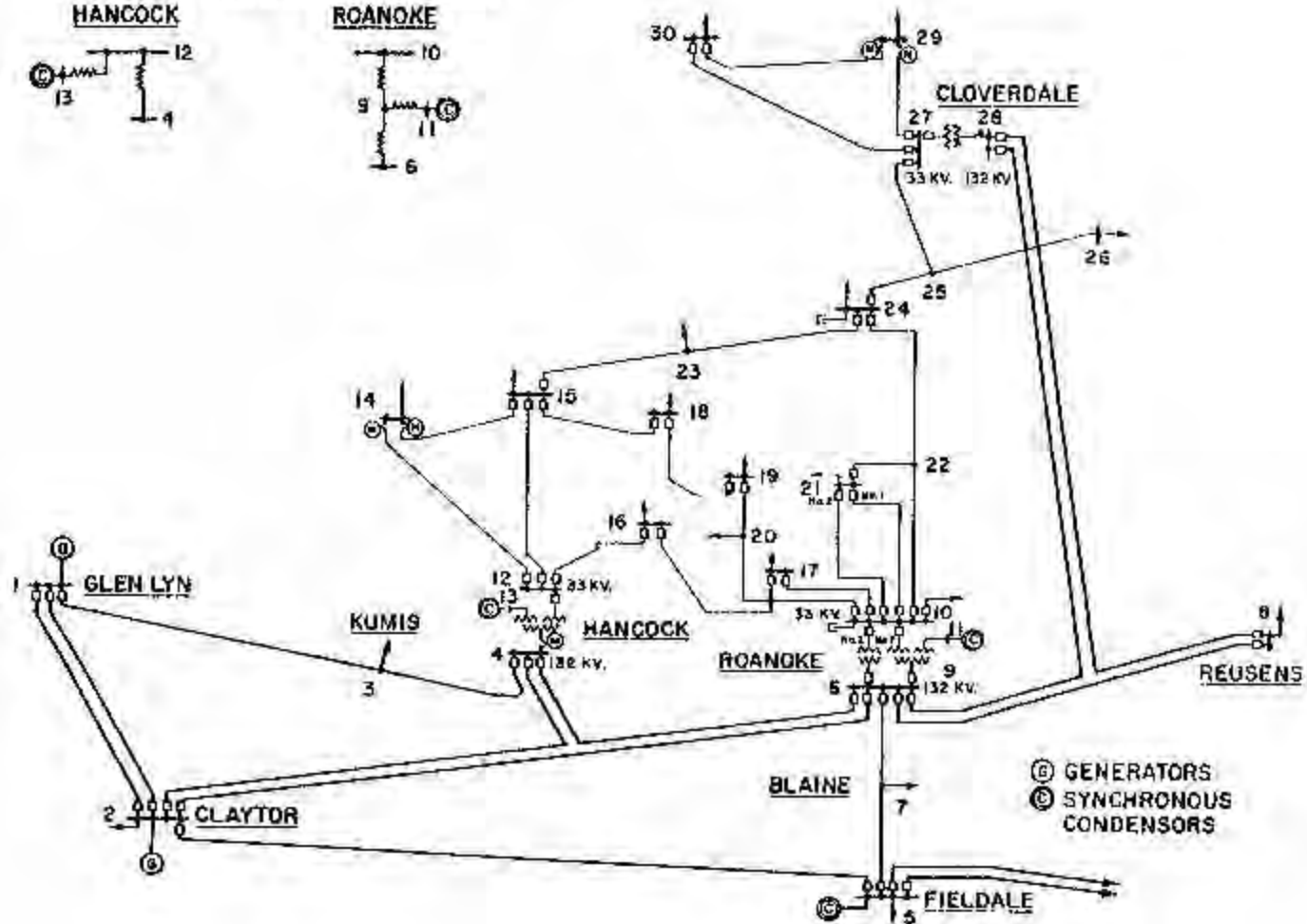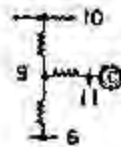
Introduction

- When analyzing power systems we know neither the complex bus voltages nor the complex current injections

- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes

- Therefore, we cannot directly use the Ybus equations, but rather must use the power balance equations
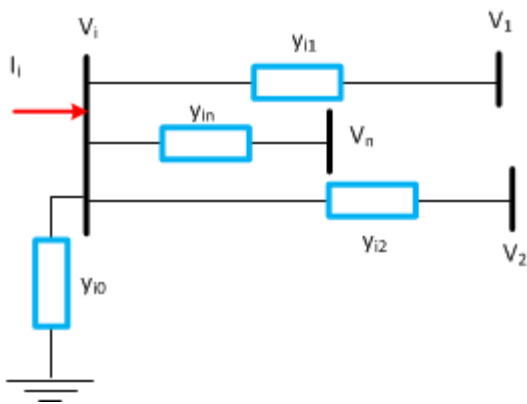
$$I_i = y_{10}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n)$$

$$I_i = (y_{10} + y_{i1} + y_{i2} \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n$$

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \quad j \neq i$$

$$I_i = V_i Y_{ii} + \sum_{j=1}^{n} Y_{ij}V_j \quad j \neq i$$

Power flow problem

- Basic equation for power-flow analysis is derived from the nodal analysis equation for the power system:

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{i1} & Y_{i2} & \cdots & Y_{ii} & \cdots & Y_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{ni} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix}$$

- the typical element Y_{ij} is

$$\boldsymbol{Y}_{ij} = \left|\boldsymbol{Y}_{ij}\right| \angle \theta_{ij} = \left|\boldsymbol{Y}_{ij}\right| \boldsymbol{e}^{j\theta_{ij}} = \left|\boldsymbol{Y}_{ij}\right|(\cos\theta_{ij} + \boldsymbol{j}\sin\theta_{ij}) = \boldsymbol{G}_{ij} + \boldsymbol{jB}_{ij}$$

- In advance of each power-flow study certain bus voltages and power injections must be given known values ,

$$\boldsymbol{V}_i = \left|\boldsymbol{V}_i\right| \boldsymbol{e}^{j\delta_i} = \left|\boldsymbol{V}_i\right| \angle \delta_i = \boldsymbol{V}_i(\cos\delta_i + \boldsymbol{j}\sin\delta_i)$$

Note:
Other bus voltages can be represented similarly by changing the subscript from i to j

Power flow problem

- The net current injected into the network at bus i in terms of the elements Y_{ij} of Y_{bus} is given by the summation

$$I_i = Y_{i1}V_1 + Y_{i2}V_{21} + \cdots + Y_{iN}V_N = \sum_n^N Y_{in}V_n$$

- Let P_i and Q_i denote the net real and reactive power entering the network at the bus i .

$$P_i - jQ_i = V_i^* \sum_n^N Y_{in}V_n = |Y_{in}V_iV_n| \angle \theta_{in} + \delta_n - \delta_i$$

- Expanding this equation and equating real and reactive parts, we obtain

$$P_i = \mathrm{Re}\left[V_i^* \sum_n^N Y_{in}V_n \right] = \sum_n^N |Y_{in}V_iV_n| \cos(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = -\mathrm{Im}\left[V_i^* \sum_n^N Y_{in}V_n \right] = -\sum_n^N |Y_{in}V_iV_n| \sin(\theta_{in} + \delta_n - \delta_i)$$

- The above constitute the rectangular and polar form of the power-flow equations; . They provide calculated values for the net real power P_i and reactive power Q_i entering the network at typical bus i .

Power flow problem

- The net scheduled power being injected into the network at bus i

$$P_{i\_sch} = P_{Gi} - P_{Di}$$
$$Q_{i\_sch} = Q_{Gi} - Q_{Di}$$
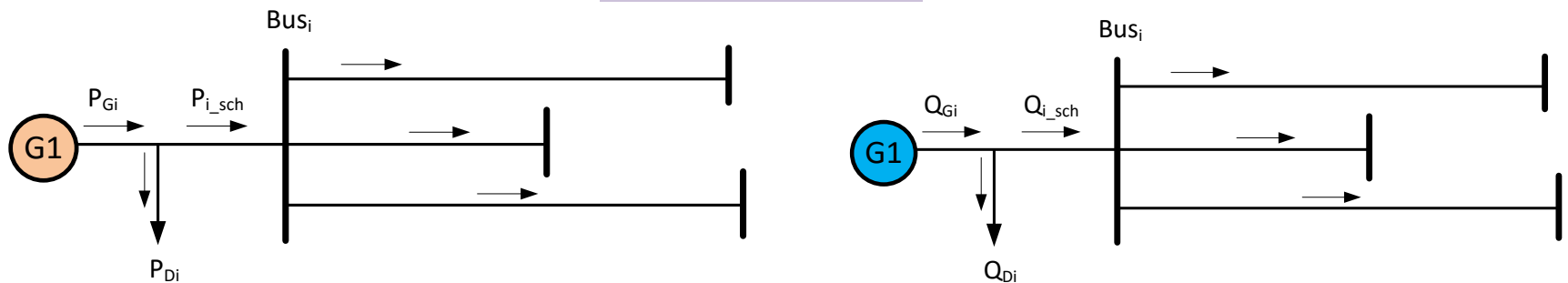
- Then the mismatch powers ΔP_i and ΔQ_i can be obtained as

$$\Delta P_i = P_{i\_sch} - P_{i\_calc} = (P_{Gi} - P_{Di}) - P_{i\_calc}$$
$$\Delta Q_i = Q_{i\_sch} - Q_{i\_calc} = (Q_{Gi} - Q_{Di}) - Q_{i\_calc}$$

Note:
When ΔP_i and ΔQ_i are zero it reaches power balance equations

- Mismatches occur in the course of solving a power-flow problem when calculated values of P_i and Q_i do not coincide with the scheduled values.

Power flow problem

- The complete definition of power flow problem requires knowledge of four variables of each k-bus in the system:
  - P_i- Real or active power
  - Q_i- reactive or quadrature power
  - V_i- voltage magnitude
  - δ_i- voltage phase angle

- Only two are known a priori to solve the problem, and the aim of power flow is to solve the remaining two variables at a bus.

- Depending upon which two variables are specified a priori, the buses are classified into three categories

Bus type	Specified	Non Specified	No. Equations
PQ bus	P_i, Q_i	$|V_i|?, \delta_i?$	$2(N-N_G-1)$
PV bus	$P_i, |V_i|$	$\delta_i?, Q_i?$	N_G
Slack bus	$|V_i|, \delta_i = 0$	$P_i?, Q_i?$	0

Power flow problem

Example 1

- Suppose that the P-Q load is known at each of the nine buses of a small power system and that synchronous generators are connected to buses 1,2, 5 and 7. For a power-flow study, identify the ΔP and ΔQ mismatches and the state variables associated with each bus. Choose bus 1 as the slack bus.

Solution

The nine buses of the system a re categorize as follows:

Bus type	Bus number
PQ bus	3, 4, 6, 8, 9
PV bus	2, 5 7
Slack bus	1

The state variables are

@ PQ buses $|V_3|, \delta_3$; $|V_4|, \delta_4$; $|V_6|, \delta_6$; $|V_8|, \delta_8$ $|V_9|, \delta_9$

@ PV buses δ_2 ; δ_5 ; δ_7

Therefore there are 13 equations to solve

The mismatches corresponding to the specified P and Q are

@ PQ buses $\Delta P_3, \Delta Q_3$; $\Delta P_4, \Delta Q_4$; $\Delta P_6, \Delta Q_6$; $\Delta P_8, \Delta Q_8$; $\Delta P_9, \Delta Q_9$

@ PV buses ΔP_2 ; ΔP_5 ; ΔP_7

The Gauss-Seidel Method

- The complexity of obtaining a formal solution for power flow in a power system arises because of the differences in the type of data specified for the different kinds of buses.

- Consider a 4 bus power system with bus 1 designated as slack bus

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

- For bus 2 in the four-bus system,

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 \qquad (1)$$

- The loads on real p.s. are specified in terms of real and reactive power, not as currents. Relationship between power and current at bus i can be expressed as:

$$S_2 = V_2 I_2^* = P_2 + jQ_2$$
$$S_2^* = V_2^* I = P_2 - jQ_2$$

The Gauss-Seidel Method

- Current injected at bus 2 can be found as:

$$I_2 = \frac{P_{2\_sch} - jQ_{2\_sch}}{V_2^*} \qquad (2)$$

- If $P_{2\_spec} - jQ_{2\_spec}$ are the scheduled real and reactive power, respectively, entering the network at bus 2 can be obtained by equating Equations 1 and 2 as

$$\frac{P_{2\_sch} - jQ_{2\_sch}}{V_2^*} = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 \qquad (3)$$

- Solving for V$_2$ gives
$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_{2\_sch} - jQ_{2\_sch}}{V_2^*} - \left(Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4\right)\right] \qquad (4)$$

- If we assume that buses 3 and 4 are also load buses with real and reactive power specified, we can get similar expressions as

$$V_3 = \frac{1}{Y_{33}}\left[\frac{P_{3\_sch} - jQ_{3\_sch}}{V_3^*} - \left(Y_{31}V_1 + Y_{32}V_2 + Y_{34}V_4\right)\right]$$

$$V_4 = \frac{1}{Y_{44}}\left[\frac{P_{4\_sch} - jQ_{4\_sch}}{V_4^*} - \left(Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3\right)\right]$$

The Gauss-Seidel Method

- If the V_i^* was not present, then it could have been solved using the usual elimination methods. However, the Eq. is now a non-linear Eq., since it contain products of unknown variables. The only way it can be solved is by means of iteration.

- The solution proceeds by iteration based on the scheduled real and reactive power at buses 2, 3, and 4, the scheduled slack bus voltage , $V_1 = |V_1| \angle \delta_1$ and initial voltage estimates $V_2^{(0)}$; $V_3^{(0)}$; $V_4^{(0)}$ the other buses.

- Solution of Eq. (3) gives the corrected voltage $V_2^{(1)}$ calculated from

$$V_2^{(1)} = \frac{1}{Y_{22}} \left[\frac{P_{2\_sch} - jQ_{2\_sch}}{V_2^{(0)*}} - \left(Y_{21}V_1 + Y_{23}V_3^{(0)} + Y_{24}V_4^{(0)} \right) \right]$$

- As the corrected voltage is found at each bus, it is used to calculate the corrected voltage at the next bus.

$$V_3^{(1)} = \frac{1}{Y_{33}} \left[\frac{P_{3\_sch} - jQ_{3\_sch}}{V_3^{(0)*}} - \left(Y_{21}V_1 + Y_{32}V_2^{(1)} + Y_{34}V_4^{(0)} \right) \right]$$

$$V_4^{(1)} = \frac{1}{Y_{44}} \left[\frac{P_{4\_sch} - jQ_{4\_sch}}{V_4^{(0)*}} - \left(Y_{41}V_1 + Y_{42}V_2^{(1)} + Y_{43}V_3^{(3)} \right) \right]$$

- This complete the first iteration in which calculated values are found for each state variable.

The Gauss-Seidel Method

- This process of solving the power-flow equations is known as the Gauss-Seidel iterative method.

- The number of iterations required may be reduced if the correction in voltage at each bus is multiplied by some constant known as acceleration factor given by

- For example, at bus 2 in the first iteration we have the accelerated value $V_{2\_acc}^{(1)}$ defined by the straight-line formula.

$$V_{2\_acc}^{(1)} = (1-\alpha)V_2^{(0)} + \alpha V_2^{(1)} = V_2^{(0)} + \alpha(V_2^{(1)} - V_2^{(0)})$$

- For a system of N buses the general equation for the calculated voltage at any bus i where P and Q are scheduled is

$$V_i^{(k)} = \frac{1}{Y_{ii}}\left[\frac{P_{i\_sch} - jQ_{i\_sch}}{V_i^{(k-1)*}} - \sum_{j=1}^{i-1}Y_{ij}V_j^{(k)} - \sum_{j=i+1}^{N}Y_{ij}V_j^{(k-1)} \right]$$

Note:
In power-flow studies α is generally set a t about 1 .6 and cannot exceed 2

- Then updated with

$$V_{i\_acc}^{(k)} = (1-\alpha)V_{k\_acc}^{(k-1)} + \alpha V_i^{(k)} = V_{i\_acc}^{(k-1)} + \alpha(V_i^{(k)} + V_{i\_acc}^{(k-1)})$$

The Gauss-Seidel Method:

Handling PV buses

- When voltage magnitude rather than reactive power is specified at bus i , the real and imaginary components of the voltage for each iteration are found by first computing a value for the reactive power.

- In the four-bus example if bus 4 is a PV bus,

$$Q_4^1 = -\operatorname{Im}\left[V_4^{(0)*}\left(Y_{41}V_1 + Y_{42}V_{2\_acc}^{(1)} + Y_{43}V_{3\_acc}^{(1)} + Y_{44}V_4^{(0)}\right)\right]$$

- Substituting Q_4^1 for Q_{4_sch}

$$V_4^{(1)} = \frac{1}{Y_{44}}\left[\frac{P_{4\_sch} - jQ_4^1}{V_4^{(0)*}} - \left(Y_{41}V_1 + Y_{42}V_{2\_acc}^{(1)} + Y_{43}V_{3\_acc}^{(3)}\right)\right]$$

- Since $\left|V_4\right|$ is specified, we correct the magnitude of V_4^1 as follows

$$V_{4\_corr}^{(1)} = \left|V_4\right|\frac{V_4^{(1)}}{\left|V_4^{(1)}\right|}$$

The Gauss-Seidel Method:
Handling PV buses

- General Equations used in the algorithm for the reactive power is.

$$Q_i^{(k)} = -\text{Im}\left[V_i^{(k-1)*}\left\{\sum_{j=1}^{i-1}Y_{ij}V_j^{(k)} + \sum_{j=i}^{N}Y_{ij}V_j^{(k-1)} + \right\}\right]$$

- From a practical viewpoint the P_G and Q_G output of the generator must be within definite limits given by the inequality

$$\left|Q_{Gi}\right|_{min} \leq \left|Q_{Gi}\right| \leq \left|Q_{Gi}\right|_{max} \qquad \left|P_{Gi}\right|_{min} \leq \left|P_{Gi}\right| \leq \left|P_{Gi}\right|_{max}$$

- Voltage is calculated as follows

$$V_i^{(k)} = \frac{1}{Y_{ii}}\left[\frac{P_{i\_sch} - jQ_i^{(k)}}{V_i^{(k-1)*}} - \sum_{j=1}^{i-1}Y_{ij}V_j^{(k)} - \sum_{j=i+1}^{N}Y_{ij}V_j^{(k-1)}\right] \qquad V_{i\_corr}^{(k)} = \left|V_i\right|\frac{V_i^{(k)}}{\left|V_i^{(k)}\right|}$$

The Gauss-Seidel Method:

The advantages of the G-S method can be listed below:

- The iteration eq. are quite simple and easy to program. In fact, with computing tools, such as MATLAB, it is fairly straightforward to perform power flow calculations   using iterative eqs. which have been developed.

- Since Y-bus is a sparse matrix, the memory requirements can be reduced substantially by storing only the non-zero elements of the matrix.

- The iterations are numerically stable and typically a fairly reliable convergence is obtained, provided a feasible solution exists. This is due to the fact that the Y-bus matrix has predominant diagonal element values

The G-S method has some disadvantages.

- The number of iterations increases as the system size increases.

- The main reason of large number of iterations is that the convergence of the G-S method is asymptotic. In other words, the convergence slows down as the values get closer to the final values. Hence, in general, it is difficult to obtain higher precision results.

- This method may not converge for systems with negative impedance branches.

The Gauss-Seidel Method: Summary

Step 1

- **Calculate the bus admittance matrix Y_{bus}.**
- Include the admittances of all transmission lines, transformers, between lines, but exclude the admittances of the loads or generators themselves.

Step 2

- **Select a slack bus.**
  - One of the busses in the power system should be chosen as the slack bus. Its voltage will be assumed to be $1.0\angle 0$.

Step 3

- **Select initial estimates for all bus voltages.**
  - The estimate should be reasonable as poor choice may result in convergence to incorrect values. Usually we start with $1.0\angle 0$ (flat start).

Step 4

- **Write voltage equations for every other bus in the system**
- The voltage equation will have generic form

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ (k\neq i)}}^{N} Y_{ik}V_k \right)$$

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{k=1}^{i-1} Y_{ik}V_k - \sum_{k=i+1}^{N} Y_{ik}V_k \right)$$

The Gauss-Seidel Method: Summary

Step 5

- **Calculate and update estimate of the voltage at each load bus in succession using the voltage equation**

Step 6

- **Compare the difference between the old voltage estimates and the estimates**
  - If the differences between the estimates less than specified tolerance for all busses, we are done.

Step 7

- **Confirm that the resulting solution is reasonable.**
  - Typical value of phase range is less than 45°.
  - Larger ranges may indicate the system converged to an incorrect solution.
  - Change the initial condition. Try again.

The Gauss-Seidel Method: 2 bus example

- Consider a simple two bus power system and calculate the voltage at bus 2. Apply Gauss-Seidel method

Bus 1   Bus 2

G

Load:
P=0.30 p.u.
Q=0.2 p.u.

$Z_{12}=0.1+j0.5$
Shunt admittance is neglected
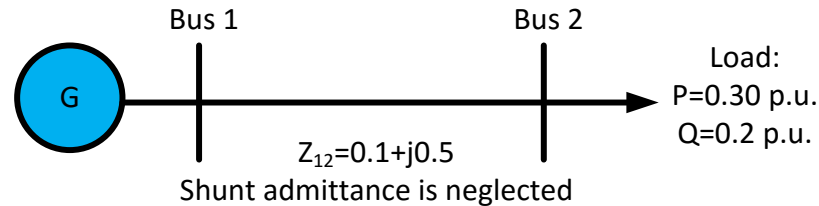
Solution

STEP 1

Calculate the bus admittance matrix

$$y_{line} = \frac{1}{z_{line}} = \frac{1}{0.10 + j0.50} = 0.3846 - j1.9231$$

$$Y_{11} = 0.3846 - j1.9231$$

$$Y_{22} = 0.3846 - j1.9231$$

$$Y_{12} = -0.3846 + j1.9231$$

$$Y_{21} = -0.3846 + j1.9231$$

$$Y_{bus} = \begin{bmatrix} 0.3846 - j1.9231 & -0.3846 + j1.9231 \\ -0.3846 + j1.9231 & 0.3846 - j1.9231 \end{bmatrix}$$

The Gauss-Seidel Method: 2 bus example

Determine the voltage at each bus for the specified load condition

STEP 2

Bus 1 : Slack bus. $V_1 = 1.0 \angle 0$ pu

STEP 3

Select initial values **(0)** of all bus voltages:

Bus 2 is a Load bus. Choose $V_2^{(0)} = 1.0 \angle 0$ pu as initial estimate

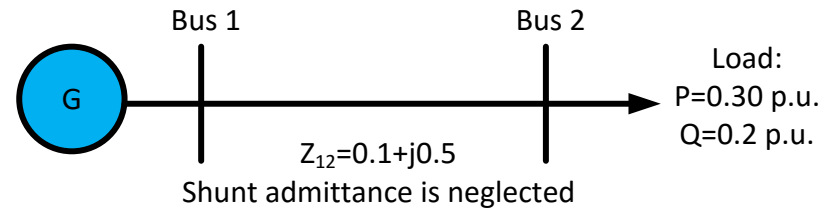Determine the voltage at each bus for the specified load condition

STEP 4

Write voltage equations for every other bus in the system

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{(k)*}} - \left(Y_{21}V_1\right)\right]$$

$$V_2^{(1)} = \frac{1}{0.3846 - j1.9231}\left[\frac{-0.30 + j0.2}{V_2^{(0)*}} - \left[\left(-0.3846 + j1.9231\right)V_1\right]\right]$$

$$V_2^{(1)} = \frac{1}{1.9612 \angle -78.8}\left[\frac{0.3603 \angle 146.3}{V_2^{(0)*}} - \left[\left(1.9612 \angle 101.3\right)1 \angle 0\right]\right]$$

Bus 1   Bus 2

G

Load:
P=0.30 p.u.
Q=0.2 p.u.

$Z_{12} = 0.1 + j0.5$

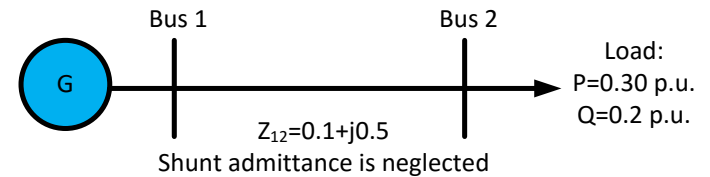Shunt admittance is neglected

Note:
$P_2 = -0.3$ p.u; $Q_2 = -0.2$ p.u

The Gauss-Seidel Method: 2 bus example

STEP 5: Calculate an updated estimate of the voltage at each load bus

Initial estimate $V_2^0 = 1.0\angle 0$ pu

Bus 1　　　　　　　Bus 2

G

Load:
P=0.30 p.u.
Q=0.2 p.u.

$Z_{12}=0.1+j0.5$

Shunt admittance is neglected

$$V_2^1 = \frac{1}{1.9612\angle -78.8}\left[\frac{0.3603\angle 146.3}{V_2^{0*}} - \left[(1.9612\angle 101.3)1\angle 0\right]\right]$$

$$= \frac{1}{1.9612\angle -78.8}\left[\frac{0.3603\angle 146.3}{1\angle 0} - \left[(1.9612\angle 101.3)1\angle 0\right]\right]$$

$$= ?$$

Second estimate

$$V_2^2 = \frac{1}{1.9612\angle -78.8}\left[\frac{0.3603\angle 146.3}{V_2^{1*}} - \left[(1.9612\angle 101.3)1\angle 0\right]\right]$$

$$= ?$$

Third iteration

$$V_2^3 = \frac{1}{1.9612\angle -78.8}\left[\frac{0.3603\angle 146.3}{V_2^{2*}} - \left[(1.9612\angle 101.3)1\angle 0\right]\right] = ?$$

Calculate for Fourth and fifth iteration etc…

The Gauss-Seidel Method: 2 bus example

STEP 6: compare the differences between the old and new estimates

If the magnitude of the voltage is barely changing, we consider this value is close enough to the correct answer. Iteration stops.

STEP 7: Confirm that the resulting solution is reasonable

$V_1 = 1.0 \angle 0$ pu

$V_2 = 0.8315 \angle -8.994$ pu   ⬅   assumed value for discussions

The phase angles differ by only 9°,

these result appear reasonable

Calculate current flowing in the transmission

 line from Bus 1 to Bus 2

$$I_{1,2} = \frac{(V_1 - V_2)}{Z_{Line}}$$

$$= \frac{1\angle 0 - 0.8315 \angle -8.994}{0.10 + j0.50}$$

$$= 0.4333 \angle -42.65$$

Calculate power supplied by the transmission line to Bus 2

$$S_2 = V_2 I_{12}^*$$

$$= (0.8315 \angle -8.994)(0.4333 \angle -42.65)^*$$

$$= 0.2999 + j0.1997$$

This is almost equal to the power being consumed by the loads. Thus the solution appears to be correct.

The Gauss-Seidel Method: 3 bus example

A simple three-bus power system with generation at buses 1 and 3. The voltage at bus 1 is $V_1 = 1.025 \angle 0°$. Voltage magnitude at bus 3 is fixed at 1.03 pu with real power generation of 300 MW. A load consisting of 400 MW and 200 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base. Line resistances and line charging suceptances are neglected.

Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.03 + j0$ and keeping $|V_3| = 1.03$ pu, determine the phasor values of V_2 and V_3. Stop after **two (2)** iterations.
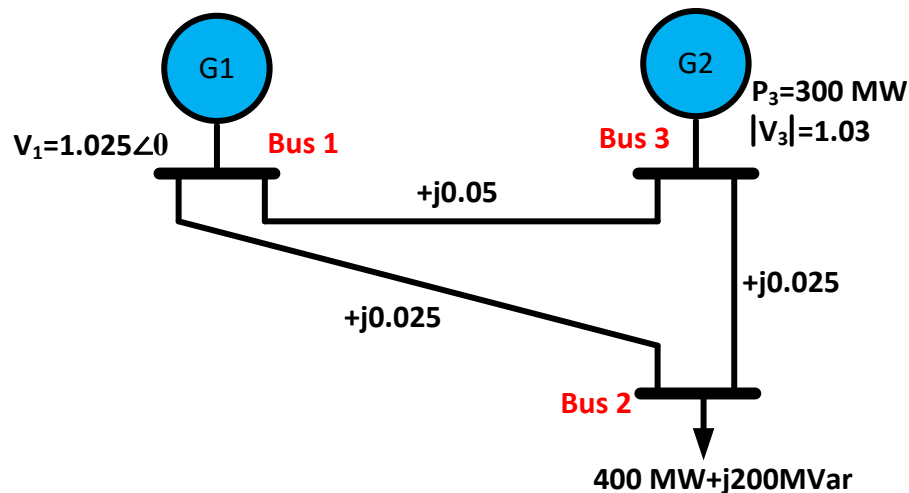
The Gauss-Seidel Method: 3 bus example

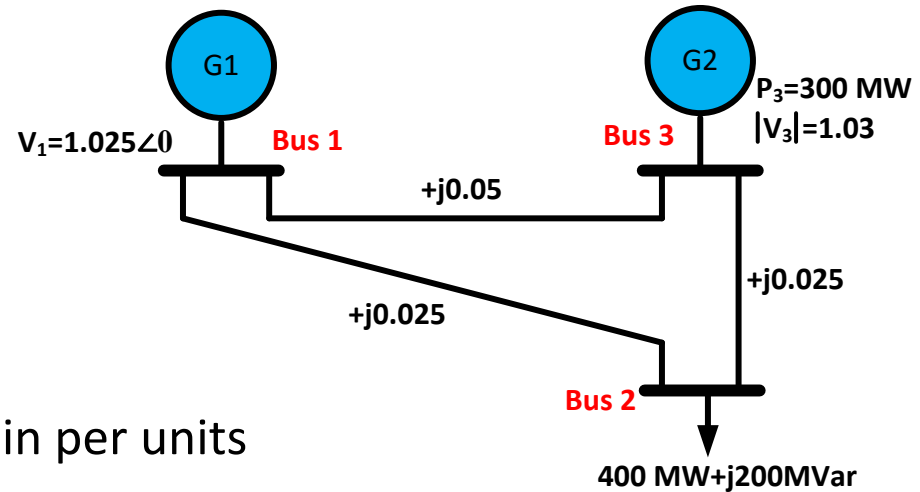1. Calculate Line admittances

$$y_{12} = -j40,$$

$$y_{13} = -j20$$

$$y_{23} = -j40$$

$V_1 = 1.025 \angle 0$ **Bus 1**   **Bus 3** $P_3 = 300$ MW $|V_3| = 1.03$

+j0.05

+j0.025

+j0.025

Bus 2

400 MW+j200MVar

2. Expressed load and generation in per units

$$S_{2\_sch} = -\frac{(400 + j200)}{100} = -4.0 - j2.0 \quad p.u$$

$$P_{3\_sch} = \frac{300}{100} = 3.0 \quad p.u$$

3. Bus 1 is taken as slack bus. Starting from an initial estimate V_2 is computed from equation:

$$V_2^{(1)} = \frac{\dfrac{S_{2\_sch}^*}{V_2^{(0)*}} + y_{21}V_1 + y_{23}V_3^{(0)}}{y_{21} + y_{23}}$$

$$\boxed{V_2^{(0)} = 1.0 + j0.0}$$

$$\boxed{V_3^{(0)} = 1.03 + j0.0}$$

$$V_2^{(1)} = 1.0025 - j0.05$$

The Gauss-Seidel Method: 3 bus example

4. Bus 3 is a PV bus or voltage controlled bus therefore Q and δ are unknown.

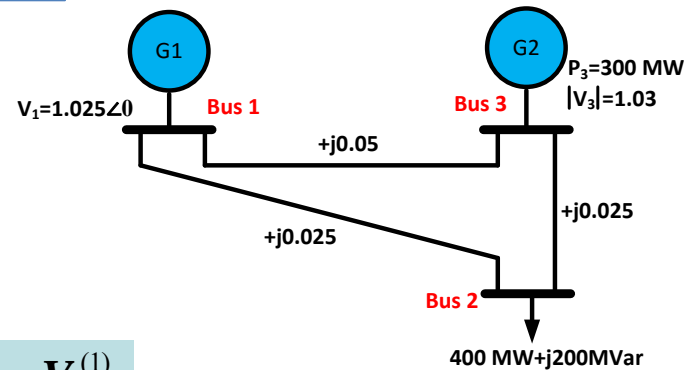Determine Q first using $\boxed{V_2^{(1)} = 1.0025 + j0.05}$

$$Q_3^{(1)} = -\Im\left\{V_3^{(0)^*}\left[V_3^0\left(y_{13} + y_{23}\right) - y_{13}V_1 - y_{23}V_2^{(1)}\right]\right\}$$

4. Compute V_3 are as follows

$$V_3^{(1)} = \frac{\dfrac{S_{2\_sch}^*}{V_3^{(0)^*}} + y_{31}V_1 + y_{32}V_2^{(1)}}{y_{21} + y_{32}}$$

$$V_{3\_corr}^{(1)} = |V_3|\frac{V_3^{(1)}}{|V_3^{(1)}|}$$

$$V_{3\_corr}^{(1)} = 1.03 - j0.0152$$

Applying the same procedure for the second iteration we obtain

$$V_2^{(2)} = 1.0001 - j0.0409 \qquad\qquad V_{c3}^{(2)} = 1.0298 + j0.0216$$

$$Q_3^{(2)} = 1.3671 \qquad\qquad V_3^{(2)} = 1.0298 + j0.0216$$
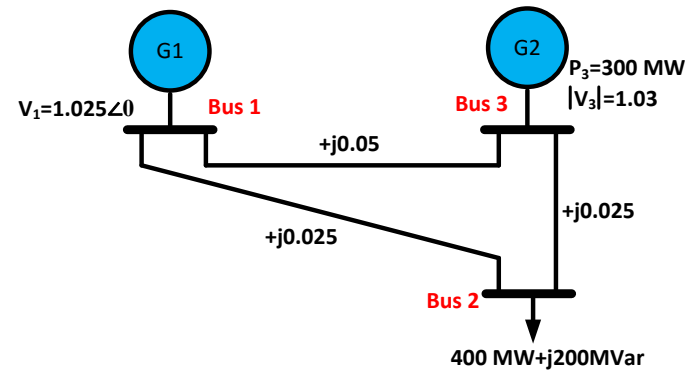
The Gauss-Seidel Method: 3 bus example

If after several iterations the bus voltages converge to V_2=1.000571-j0.0366898 pu

V_3=1.029706+j0.0246 pu

Determine the line flows and line losses and

the slack bus real and reactive power.

$$S_{Loss12} = 0.0 + j7.77$$

$$S_{Loss13} = 0.0 + j1.25$$

$$S_{Loss23} = 0.0 + j18.42$$

$$S_1 = 100\ MW + j90.51\ MVAR$$

The Gauss-Seidel Method: 5 bus example

An illustrative five-bus system has the following bus power and voltage

specifications:

Bus	P_D pu	$Q_D$pu	$P_G$pu	$Q_G$pu	Vpu	Bus type
1	0.5	0.25	unspecified	Unspecified	1.02+j0.0	Slack
2	0.0	0.0	1.0	Unspecified	1.02∠δ=?	P-V
3	0.25	0.1	0.0	0.0	Unspecified	P-Q
4	0.25	0.1	0.0	0.0	Unspecified	P-Q
5	0.25	0.1	0.0	0.0	Unspecified	P-Q

$$Y_{bus} = \begin{bmatrix} 2-j6 & -1+j3 & 0+j0 & 0+j0 & -1+j3 \\ -1+j3 & 3-j9 & -1+j3 & 0+j0 & -1+j3 \\ 0+j0 & -1+j3 & 2-j6 & -1+j3 & 0+j0 \\ 0+j0 & 0+j0 & -1+j3 & 2-j6 & -1+j3 \\ -1+j3 & -1+j3 & 0+j0 & -1+j3 & 3-j6 \end{bmatrix}$$

Show calculation for one iteration to solve the five PFEs using Gauss-Seidel

method.

The Gauss-Seidel Method: 5 bus example

From system data, $V_1 = 1.02 \angle 0$,

$$P_{G2} = 1.0 ;$$

$$V_2 = 1.02$$

$$S_3 = P_3 + jQ_3 = -0.25 - j0.1;$$

$$S_4 = P_4 + jQ_4 = -0.25 - j0.1 ;$$

$$S_5 = P_5 + jQ_5 = -0.25 - j0.1 .$$

Let us assume the following initial estimates:

$$V_3^0 = V_4^0 = V_5^0 = 1.0\angle 0 ; \, \delta_2 = 0$$
$$V_2^0 = 1.02\angle 0$$

Bus 2 is the only P-V bus. Let us calculate Q_2^1

$$Q_2^1 = -\mathrm{Im}\left(V_2^{0*}\left(Y_{21}V_1 + Y_{22}V_2^0 + Y_{23}V_3^0 + Y_{24}V_4^0 + Y_{25}V_5^0\right)\right)$$
$$= 0.1224$$

The Gauss-Seidel Method: 5 bus example

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2^1}{V_2^{0*}} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= 1.016 + j0.0908 = 1.019\angle 5.1°$$

We fix **V_2^1** to **1.02 ∠5.1°**, the magnitude being set at the specified value with the phase angle kept as the calculated value.

The voltages at the load buses after the first iteration are:

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= 0.9804 + j0.0129 = 0.9805\angle 0.75°$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= 0.9804 + j0.0129 = 0.9805\angle 0.75°$$

The Gauss-Seidel Method: 5 bus example

$$V_5^1 = \frac{1}{Y_{55}} \left[\frac{P_5 - jQ_5}{V_5^{0*}} - Y_{51}V_1 - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1 \right]$$

$$= 0.9812 + j0.000005 = 0.9812\angle -0.003°$$

You may apply acceleration factor (α), (about 1.6 and cannot exceed 2) to speed up the convergence

$$V_{i\_acc}^{(k)} = (1-\alpha)V_{k\_acc}^{(k-1)} + \alpha V_i^{(k)} = V_{i\_acc}^{(k-1)} + \alpha(V_i^{(k)} + V_{i\_acc}^{(k-1)})$$

THE Newton-Raphson Method

- Taylor's series expansion for a function of two or more variables is the basis for the Newton-Raphson method of solving the power-flow problem.

- Let us consider the equation of a function h_1 of two variables x_1 and x_2 equal to a constant b_1 expressed as

$$h_1(x_1, x_2, u) = b_1$$
$$g_1(x_1, x_2, u) = h_1(x_1, x_2, u) - b_1 = 0$$

- and a second equation involving another function h_2 such that

$$h_2(x_1, x_2, u) = b_2$$
$$g_2(x_1, x_2, u) = h_2(x_1, x_2, u) - b_2 = 0$$

Difference between calculated and specified

Indipendent control variable (a constant)

Specified function value

Note:
In [mathematics](#), a **Taylor series** is a representation of a [function](#) as an [infinite sum](#) of terms that are calculated from the values of the function's [derivatives](#) at a single point

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots.$$

THE Newton-Raphson Method

- For a specified value of u let us estimate the solutions, (initial guess)

$$g_1(x_1^{(0)}, x_2^{(0)}, u) = h_1(x_1^{(0)}, x_2^{(0)}, u) - b_1 \neq 0$$

$$g_2(x_1^{(0)}, x_2^{(0)}, u) = h_2(x_1^{(0)}, x_2^{(0)}, u) - b_2 \neq 0$$

- If we designate the correction values $\Delta x_1^{(0)}, \Delta x_2^{(0)}$ to get actual solution x_1^*, x_2^*, we can write

$$g_1(x_1^*, x_2^*, u) = h_1(x_1^{(0)} + \Delta x_1^{(0)}, x_2^{(0)} + \Delta x_2^{(0)}, u) - b_1 = 0$$

$$g_2(x_1^*, x_2^*, u) = h_2(x_1^{(0)} + \Delta x_1^{(0)}, x_2^{(0)} + \Delta x_2^{(0)}, u) - b_2 = 0$$

- Now we can solve this problem by Taylor's series expansion

$$g_1(x_1^*, x_2^*, u) = g_1(x_1^{(0)}, x_2^{(0)}, u) + \Delta x_1^{(0)} \frac{\partial g_1}{\partial x_1}\bigg|^{(0)} + \Delta x_2^{(0)} \frac{\partial g_1}{\partial x_2}\bigg|^{(0)} + \cdots = 0$$

$$g_2(x_1^*, x_2^*, u) = g_2(x_1^{(0)}, x_2^{(0)}, u) + \Delta x_1^{(0)} \frac{\partial g_2}{\partial x_1}\bigg|^{(0)} + \Delta x_2^{(0)} \frac{\partial g_2}{\partial x_2}\bigg|^{(0)} + \cdots = 0$$

THE Newton-Raphson Method

- If we neglect the partial derivatives of order greater than 1 , we can rewrite

$$\begin{bmatrix} \dfrac{\partial g_1}{\partial x_1} & \dfrac{\partial g_1}{\partial x_2} \\[2ex] \dfrac{\partial g_2}{\partial x_1} & \dfrac{\partial g_2}{\partial x_2} \end{bmatrix}^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\[1ex] \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} 0 - g_1(x_1^{(0)}, x_2^{(0)}, u) \\[1ex] 0 - g_2(x_1^{(0)}, x_2^{(0)}, u) \end{bmatrix} = \begin{bmatrix} b_1 - h_1(x_1^{(0)}, x_2^{(0)}, u) \\[1ex] b_2 - h_2(x_1^{(0)}, x_2^{(0)}, u) \end{bmatrix}$$

Jacobian matrix (J)

- square matrix of partial derivatives is called the jacobian J

- By solving the mismatch equations we can determine $\Delta x_1^{(0)}, \Delta x_2^{(0)}$ as

$$\begin{bmatrix} \Delta x_1^{(0)} \\[1ex] \Delta x_2^{(0)} \end{bmatrix} = \left[J^{(0)} \right]^{-1} \begin{bmatrix} \Delta g^{(0)}{}_1 \\[1ex] \Delta g^{(0)}{}_2 \end{bmatrix}$$

- Since initial guess do not determine the correct solution, we must try to get new estimate as

$$x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)}$$
$$x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)}$$

- We repeat the process until the corrections become so small in magnitude that they satisfy a chosen precision index ε > 0

THE Newton-Raphson Method   **Example**

- Use Newton-Raphson Mehtod to solve $h(x) = x^2 = 2$

- We can rewrite this equation as $g(x) = x^2 - 2 = 0$

- Now applying Taylor's series expansion we have

$$0 = g(x^{(0)}) + \Delta x^{(0)} \left. \frac{\partial g(x)}{\partial x} \right|^{(0)}$$

- By rearranging the last equation we have

$$\Delta x^{(0)} = \frac{1}{\left. \frac{\partial g(x)}{\partial x} \right|^{(0)}} \left(0 - g(x^{(0)})\right)$$

- Substituting the values given in the function we have

$$\Delta x^{(0)} = \frac{1}{2x^{(0)}} \times \left(-(x^{(0)} - 2)\right)$$

- Next we can get updated value of x as

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = x^{(0)} + \frac{1}{2x^{(0)}} \times \left(-(x^{(0)} - 2)\right)$$

THE Newton-Raphson Method   **Example**

- General formula for this example

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)} = x^{(k)} + \frac{1}{2x^{(k)}} \times \left(-(x^{(k)} - 2)\right)$$

- Starting from initial guess x(0)=1, Iteratively solving we get

k	$x^{(k)}$	$\Delta g(x^{(k)})$	$\Delta x^{(k)}$
0	1	-1	0.5
1	1.5	0.25	-0.0833
2	1.41667	6.953×10^{-3}	-2.454×10^{-3}
3	1.41422	6.024×10^{-3}	

THE Newton-Raphson Method Example 2

- Using the Newton-Raphson method, solve for x1 and x2 of the nonlinear equations

$$g_1(x_1, x_2, u) = h_1(x_1, x_2, u) - b_1 = 4ux_2 \sin x_1 + 0.6 = 0$$

$$g_2(x_1, x_2, u) = h_2(x_1, x_2, u) - b_2 = 4x_2^2 - 4ux_2 \cos x_1 + 0.3 = 0$$

- Treat the parameter u as a fixed number equal to 1, and choose the initial conditions for x1 and x2. Solve until the error is reasonably small. Let initial x1=0 and x2=1

Solution

- Partial differentiation with respect to the x 's yields

$$J = \begin{vmatrix} \dfrac{\partial g_1}{\partial x_1} & \dfrac{\partial g_1}{\partial x_2} \\ \dfrac{\partial g_2}{\partial x_1} & \dfrac{\partial g_2}{\partial x_2} \end{vmatrix} = \begin{bmatrix} 4ux_2 \cos x_1 & 4u \sin x_1 \\ 4ux_2 \sin x_1 & 8x_2 - 4u \cos x_1 \end{bmatrix}$$

$$J = \begin{vmatrix} \dfrac{\partial g_1}{\partial x_1} & \dfrac{\partial g_1}{\partial x_2} \\ \dfrac{\partial g_2}{\partial x_1} & \dfrac{\partial g_2}{\partial x_2} \end{vmatrix} = \begin{bmatrix} 4x_2 \cos x_1 & 4 \sin x_1 \\ 4x_2 \sin x_1 & 8x_2 - 4 \cos x_1 \end{bmatrix} \quad \textbf{If u=1}$$

THE Newton-Raphson Method   **Example 2**

First iteration

- Find mismatch $\quad \Delta g_1^{(0)}, \Delta g_2^{(0)}$

$$\begin{bmatrix} \Delta g_1^{(0)} \\ \Delta g_2^{(0)} \end{bmatrix} = \begin{bmatrix} 0 - g^{(0)}{}_1 \\ 0 - g^{(0)}{}_2 \end{bmatrix} = \begin{bmatrix} -0.6 - 4\sin(0) \\ -0.3 - 4\times(1)^2 + 4\cos(0) \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.3 \end{bmatrix}$$

- Substitute in mismatch equation

$$\begin{bmatrix} \dfrac{\partial g_1}{\partial x_1} & \dfrac{\partial g_1}{\partial x_2} \\[2ex] \dfrac{\partial g_2}{\partial x_1} & \dfrac{\partial g_2}{\partial x_2} \end{bmatrix}^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} 0 - g_1(x_1^{(0)}, x_2^{(0)}, u) \\ 0 - g_2(x_1^{(0)}, x_2^{(0)}, u) \end{bmatrix}$$

$$\begin{bmatrix} 4\cos(0) & 4\sin(0) \\ 4\sin(0) & 8x_2 - 4\cos(0) \end{bmatrix}^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.3 \end{bmatrix}$$

- Inverting this simple 2 X 2 matrix, we determine the initial corrections

$$\begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -0.6 \\ -0.3 \end{bmatrix} = \begin{bmatrix} -0.150 \\ -0.075 \end{bmatrix}$$

THE Newton-Raphson Method   **Example 2**

First iteration
- Therefore $x_1^{(1)}, x_2^{(1)}$ are

$$x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)} = 0 + (-0.15) = -0.15\, rad$$

$$x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)} = 1 + (-0.075) = 0.925\, rad$$

- Second Iteration

$$\begin{bmatrix} \Delta g_1^{(1)} \\ \Delta g_2^{(1)} \end{bmatrix} = \begin{bmatrix} 0 - g_1^{(1)} \\ 0 - g_2^{(1)} \end{bmatrix} = \begin{bmatrix} -0.6 - 4(0.925)\sin(-0.15) \\ -0.3 - 4\times(0.925)^2 + 4(0.925)\cos(-0.15) \end{bmatrix} = \begin{bmatrix} -0.047079 \\ -0.064047 \end{bmatrix}$$

- Substitute in mismatch equation

$$J = \begin{bmatrix} 4x_2 \cos x_1 & 4\sin x_1 \\ 4x_2 \sin x_1 & 8x_2 - 4\cos x_1 \end{bmatrix}$$

$$\begin{bmatrix} 4(0.925)\cos(-0.15) & 4\sin(-0.15) \\ 4(0.925)\sin(-0.15) & 8(0.925) - 4\cos(-0.15) \end{bmatrix}^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} -0.047079 \\ -0.064047 \end{bmatrix}$$

THE Newton-Raphson Method   **Example 2**

- And updating the Jacobian, we compute the new corrections

$$\begin{bmatrix} \Delta x_1^{(1)} \\ \Delta x_2^{(1)} \end{bmatrix} = \begin{bmatrix} 3.658453 & -0.597753 \\ -0.552921 & 3.444916 \end{bmatrix}^{-1} \begin{bmatrix} -0.047079 \\ -0.064047 \end{bmatrix} = \begin{bmatrix} -0.016335 \\ -0.021214 \end{bmatrix}$$

- Therefore $x_1^{(2)}, x_2^{(2)}$ are

$$x_1^{(2)} = x_1^{(1)} + \Delta x_1^{(1)} = -0.15 + (-0.016335) = -0.166335 \textbf{rad}$$

$$x_2^{(2)} = x_2^{(1)} + \Delta x_2^{(1)} = 0.925 + (-0.021214) = 0.903768 \textbf{rad}$$

- Continuing the 3rd iteration $\Delta x_1^{(3)}, \Delta x_2^{(3)}$ found to be very small which is less than 10^{-5}

$$x_1^{(4)} = -0.166876 \textbf{rad}$$

$$x_2^{(4)} = 0.903057 \textbf{rad}$$

The Newton-Raphson Method
Application to power flow

▪ To apply the Newton- Raphson method to the solution of the power-flow equations, we express bus voltages and line admittances in polar form.

$$P_i = \text{Re}\left[V_i^* \sum_n^N Y_{in} V_n \right] = \sum_n^N |Y_{in} V_i V_n| \cos(\theta_{in} + \delta_n - \delta_i)$$

$$P_i = |V_i|^2 G_{ii} + \sum_{n=1,\ n \neq i}^N |Y_{in} V_i V_n| \cos(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = -\text{Im}\left[V_i^* \sum_n^N Y_{in} V_n \right] = -\sum_n^N |Y_{in} V_i V_n| \sin(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = [|V_{ii}|^2 B_{ii} - \sum_{n=1,\ n \neq i}^N |Y_{in} V_i V_n| \sin(\theta_{in} + \delta_n - \delta_i)$$

▪ Let us compare the nodal power equations with the general one as below

$$b_1 = h_1(x_1, x_2, u)$$
$$h_1(x_1, x_2, u) = b_1$$

The Newton-Raphson Method
Application to power flow

- Correspondence to the Δg mismatches, the power mismatches for the typical load bus can be written as

$$\begin{bmatrix} \Delta \boldsymbol{x}_1^{(0)} \\ \Delta \boldsymbol{x}_2^{(0)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}^{(0)} \end{bmatrix}^{-1} \begin{bmatrix} \Delta \boldsymbol{g}_1^{(0)} \\ \Delta \boldsymbol{g}_2^{(0)} \end{bmatrix}$$

$$\Delta \boldsymbol{P}_i = \boldsymbol{P}_{i\_sch} - \boldsymbol{P}_{i\_calc} = (\boldsymbol{P}_{Gi} - \boldsymbol{P}_{Di}) - \boldsymbol{P}_{i\_calc}$$

$$\Delta \boldsymbol{Q}_i = \boldsymbol{Q}_{i\_sch} - \boldsymbol{Q}_{i\_calc} = (\boldsymbol{Q}_{Gi} - \boldsymbol{Q}_{Di}) - \boldsymbol{Q}_{i\_calc}$$

- For simplicity sake, we now write mismatch equations for a four-bus system,

$$\Delta \boldsymbol{P}_i = \frac{\partial \boldsymbol{P}_i}{\partial \delta_2} \Delta \delta_2 + \frac{\partial \boldsymbol{P}_i}{\partial \delta_3} \Delta \delta_3 + \frac{\partial \boldsymbol{P}_i}{\partial \delta_4} \Delta \delta_4 + \frac{\partial \boldsymbol{P}_i}{\partial |\boldsymbol{V}_2|} \Delta |\boldsymbol{V}_2| + \frac{\partial \boldsymbol{P}_i}{\partial |\boldsymbol{V}_3|} \Delta |\boldsymbol{V}_3| + \frac{\partial \boldsymbol{P}_i}{\partial |\boldsymbol{V}_4|} \Delta |\boldsymbol{V}_4|$$

$$\Delta \boldsymbol{Q}_i = \frac{\partial \boldsymbol{Q}_i}{\partial \delta_2} \Delta \delta_2 + \frac{\partial \boldsymbol{Q}_i}{\partial \delta_3} \Delta \delta_3 + \frac{\partial \boldsymbol{Q}_i}{\partial \delta_4} \Delta \delta_4 + \frac{\partial \boldsymbol{Q}_i}{\partial |\boldsymbol{V}_2|} \Delta |\boldsymbol{V}_2| + \frac{\partial \boldsymbol{Q}_i}{\partial |\boldsymbol{V}_3|} \Delta |\boldsymbol{V}_3| + \frac{\partial \boldsymbol{Q}_i}{\partial |\boldsymbol{V}_4|} \Delta |\boldsymbol{V}_4|$$

- By multiplying the last 3 terms in each equation we can also write

$$\Delta \boldsymbol{P}_i = \frac{\partial \boldsymbol{P}_i}{\partial \delta_2} \Delta \delta_2 + \frac{\partial \boldsymbol{P}_i}{\partial \delta_3} \Delta \delta_3 + \frac{\partial \boldsymbol{P}_i}{\partial \delta_4} \Delta \delta_4 + |\boldsymbol{V}_2| \frac{\partial \boldsymbol{P}_i}{\partial |\boldsymbol{V}_2|} \frac{\Delta |\boldsymbol{V}_2|}{|\boldsymbol{V}_2|} + |\boldsymbol{V}_3| \frac{\partial \boldsymbol{P}_i}{\partial |\boldsymbol{V}_3|} \frac{\Delta |\boldsymbol{V}_3|}{|\boldsymbol{V}_3|} + |\boldsymbol{V}_4| \frac{\partial \boldsymbol{P}_i}{\partial |\boldsymbol{V}_4|} \frac{\Delta |\boldsymbol{V}_4|}{|\boldsymbol{V}_4|}$$

$$\Delta \boldsymbol{Q}_i = \frac{\partial \boldsymbol{Q}_i}{\partial \delta_2} \Delta \delta_2 + \frac{\partial \boldsymbol{Q}_i}{\partial \delta_3} \Delta \delta_3 + \frac{\partial \boldsymbol{Q}_i}{\partial \delta_4} \Delta \delta_4 + |\boldsymbol{V}_2| \frac{\partial \boldsymbol{Q}_i}{\partial |\boldsymbol{V}_2|} \frac{\Delta |\boldsymbol{V}_2|}{|\boldsymbol{V}_2|} + |\boldsymbol{V}_3| \frac{\partial \boldsymbol{Q}_i}{\partial |\boldsymbol{V}_3|} \frac{\Delta |\boldsymbol{V}_3|}{|\boldsymbol{V}_3|} + |\boldsymbol{V}_4| \frac{\partial \boldsymbol{Q}_i}{\partial |\boldsymbol{V}_4|} \frac{\Delta |\boldsymbol{V}_4|}{|\boldsymbol{V}_4|}$$

The Newton-Raphson Method
Application to power flow

▪ Collecting all the mismatch equations into vector-matrix form yields

Jacobian Matrix

▪ In the general case if there are Ng voltage-controlled buses besides the slack bus, a row and column for each such bus is omitted from the polar form of the system jacobian,

The Newton-Raphson Method
Application to power flow

- Collecting all the mismatch equations into vector-matrix form from the second equation yields

Jacobian Matrix

- The solution can be obtained by iterative process

The Newton-Raphson Method
Application to power flow

- In this equation, Bus 1 is assumed to be the slack bus

- The Jacobian matrix gives the linearized relationship between small changes in voltage angle $\Delta\delta i(k)$ and voltage magnitude $\Delta|Vi(k)|$ with the small changes in real and reactive power $\Delta Pi(k)$ and $\Delta Qi(k)$.

- Elements of the Jacobian matrix are the partial derivatives of P and Q, evaluated at $\Delta\delta i(k)$ and $\Delta|Vi(k)|$

$$\mathbf{J_1} = (n-1)\times(n-1) \qquad \mathbf{J_2} = (n-1)\times(n-1-m)$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

$$\mathbf{J_3} = (n-1-m)\times(n-1) \qquad \mathbf{J_4} = (n-1-m)\times(n-1-m)$$

If **m** buses are **voltage-controlled**, **m equations** involving Δ**Q and** Δ**V** and corresponding **columns of the Jacobian matrix are eliminated**

n-1 real power constraints and **n-1-m reactive power constraints** $\therefore$ **Jacobian matrix** is of order **(2n-2-m)×(2n-2-m)**

The Newton-Raphson Method
Application to power flow

- Expressions for the elements of Jacobian can be easily found by differentiating the appropriate number of terms

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

Diagonal and off-diagonal elements of J_1:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$$

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

The Newton-Raphson Method
Application to power flow

- Expressions for the elements of Jacobian can be easily found by differentiating the appropriate number of terms

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

Diagonal and off-diagonal elements of J_2:

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\theta_{ii} + \sum_{j \neq i}^{n} |V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$$

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

The Newton-Raphson Method
Application to power flow

- Expressions for the elements of Jacobian can be easily found by differentiating the appropriate number of terms

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

Diagonal and off-diagonal elements of J_3:

$$\frac{\partial \boldsymbol{Q_i}}{\partial |\delta_i|} = \sum_{j \neq i}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial \boldsymbol{Q_i}}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

The Newton-Raphson Method
Application to power flow

- Expressions for the elements of Jacobian can be easily found by differentiating the appropriate number of terms

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

Diagonal and off-diagonal elements of J_4:

$$\frac{\partial \boldsymbol{Q}_i}{\partial |\boldsymbol{V}_i|} = -2|\boldsymbol{V}_i||\boldsymbol{Y}_{ii}|\sin\theta_{ii} - \sum_{j\neq i}^{n} |\boldsymbol{V}_j||\boldsymbol{Y}_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial \boldsymbol{Q}_i}{\partial |\boldsymbol{V}_j|} = -|\boldsymbol{V}_i||\boldsymbol{Y}_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \qquad \boldsymbol{j \neq i}$$

$$\boldsymbol{Q}_i = -\sum_{j=1}^{n} |\boldsymbol{V}_i||\boldsymbol{V}_j||\boldsymbol{Y}_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

The Newton-Raphson Method
Application to power flow

- Expressions for the elements of Jacobian can be easily found by in a different form as

$$P_k = G_{kk}V_k^2 + V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km} \right)$$

$$Q_k = -B_{kk}V_k^2 + V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km} \right)$$

$$\frac{\partial P_k}{\partial \theta_k} = V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(-G_{km} \sin\theta_{km} + B_{km} \cos\theta_{km} \right) \qquad \frac{\partial P_k}{\partial \theta_j} = V_k V_j \left(G_{kj} \sin\theta_{kj} - B_{kj} \cos\theta_{kj} \right) \qquad j \neq k$$

$$\frac{\partial P_k}{\partial V_k} = 2G_{kk}V_k + \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km} \right) \qquad \frac{\partial P_k}{\partial V_j} = V_k \left(G_{kj} \cos\theta_{kj} + B_{kj} \sin\theta_{kj} \right) \qquad j \neq k$$

$$\frac{\partial Q_k}{\partial \theta_k} = V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km} \right) \qquad \frac{\partial Q_k}{\partial \theta_j} = V_k V_j \left(-G_{kj} \cos\theta_{kj} - B_{kj} \sin\theta_{kj} \right) \qquad j \neq k$$

$$\frac{\partial Q_k}{\partial V_k} = -2B_{kk}V_k + \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km} \right) \qquad \frac{\partial Q_k}{\partial V_j} = V_k \left(G_{kj} \sin\theta_{kj} - B_{kj} \cos\theta_{kj} \right) \qquad j \neq k$$

The Newton-Raphson Method
Power flow Solution Procedure

▪ Collecting all the mismatch equations into vector-matrix form from the second equation yields

1 • Estimate values of $\Delta\delta_i^{(k)}$ and $\left|\Delta V_i\right|^{(k)}$ for the state variables,

2 • Use the estimates to calculate: $P_{i\_calc}^{(k)}$; $Q_{i\_calc}^{(k)}$ and mismatches $\Delta P_i^{(k)}$; $\Delta Q_i^{(k)}$

3 • Calculate the partial derivative elements of the jacobian $\begin{bmatrix} J_{11}^{(0)} & J_{12}^{(0)} \\ J_{21}^{(0)} & J_{22}^{(0)} \end{bmatrix}$

4 • Solve for the initial corrections $\begin{bmatrix} \Delta\delta_i^{(k)} \\ \Delta\left|V\right|_i^{(k)} \end{bmatrix} = \left[J^{(k)}\right]^{-1}\begin{bmatrix} \Delta P_i^{(k)} \\ \Delta Q_i^{(k)} \end{bmatrix}$

5 • Add the solved corrections to the initial estimates to obtain $\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)}$

$\left|V\right|_i^{(k+1)} = \left|V\right|_i^{(k)} + \Delta\left|V\right|_i^{(k)}$

6 • repeat the process until the corrections become so small in magnitude that they satisfy a chosen precision index $\varepsilon > 0$

The Newton-Raphson Method 2 Bus Example

In a two bus system, bus 1 is a slack bus with $V_1=1.0\angle 0$ pu. A load of 100 MW and 50 Mvar is taken from bus 2. The line impedance is $z_{12} = 0.12+j0.16$ pu on a base of 100 MVA. Using Newton-Raphson method determine V_2. Continue until converge.

$V_1 = 1.0\angle 0$

100 MW
50 Mvar

1

$z_{12} = 0.12+j0.16$

2

$y_{12} = 3\text{-}j4$

The Newton-Raphson Method **2 Bus Example**

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13 & 5\angle 126.87 \\ 5\angle 126.87 & 5\angle -53.13 \end{bmatrix}$$

$y_{12} = 3\text{-}j4$

100 MW

50 Mvar

V1=1.0∠0

1   2

Power flow equations in polar forms:

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

The Newton-Raphson Method **2 Bus Example**

Solution

$$y_{12} = 3-j4$$

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13 & 5\angle 126.87 \\ 5\angle 126.87 & 5\angle -53.13 \end{bmatrix}$$

At bus 2

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$P_2 = \sum_{j=1}^{n} |V_2||V_j||Y_{2j}| \cos(\theta_{2j} - \delta_2 + \delta_j)$$

$$= |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_2||Y_{22}| \cos(\theta_{22} - \delta_2 + \delta_2)$$

$$= |V_2||V_1| 5\cos(126.87 - \delta_2 + \delta_1) + |V_2||V_2| 5\cos(-53.13)$$

$$= |V_2||V_1| 5\cos(126.87 - \delta_2 + \delta_1) + |V_2|^2 5\cos(-53.13)$$

The Newton-Raphson Method 2 Bus Example

Solution

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13 & 5\angle 126.87 \\ 5\angle 126.87 & 5\angle -53.13 \end{bmatrix}$$

At bus 2

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_2 = -\sum_{j=1}^{n} |V_2||V_j||Y_{2j}|\sin(\theta_{2j} - \delta_2 + \delta_j)$$

$$= -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2||V_2||Y_{22}|\sin(\theta_{22} - \delta_2 + \delta_2)$$

$$= -|V_2||V_1|5\sin(126.87 - \delta_2 + \delta_1) - |V_2||V_2|5\cos(-53.13)$$

$$= -|V_2||V_1|5\sin(126.87 - \delta_2 + \delta_1) - |V_2|^2 5\sin(-53.13)$$

The Newton-Raphson Method **2 Bus Example**

Solution

Partial derivatives of P_2 w.r.t. δ_2 and $|V_2|$

$$P_2 = |V_2||V_1|5\cos(126.87 - \delta_2 + \delta_1) + |V_2|^2 5\cos(-53.13)$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1|5\sin(126.87 - \delta_2 + \delta_1)$$

$$\frac{\partial P_2}{\partial |V_2|} = 5|V_1|\cos(126.87 - \delta_2 + \delta_1) + 2|V_2|5\cos(-53.13)$$

Partial derivatives Q_2 w.r.t. δ_2 and $|V_2|$

$$Q_2 = -|V_2||V_1|5\sin(126.87 - \delta_2 + \delta_1) - |V_2|^2 5\sin(-53.13)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1|5\cos(126.87 - \delta_2 + \delta_1)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -5|V_1|\sin(126.87 - \delta_2 + \delta_1) - 10|V_2|\sin(-53.13)$$

The Newton-Raphson Method **2 Bus Example**

Solution

Expressed Load in p.u.
$$S_2 = -\frac{(100 + j50)}{100} = -1.0 - j0.5$$

Initial values? Slack bus voltage is V_1 = 1.0 $\angle$0 pu.

Initial estimates : $\left|V_2^0\right| = 1.0 \quad \delta_2^0 = 0.0$

P$_2$ = ?; Q$_2$ =?

$$P_2^0 = \left|V_2^0\right|\left|V_1\right|5\cos(126.87 - \delta_2^0 + \delta_1) + \left|V_2^0\right|^2 5\cos(-53.13)$$
$$= 1 \times 1 \times 5\cos(126.87 - 0 + 0) + 1^2 \times 5\cos(-53.13)$$
$$= -3 + 3 = 0$$
$$\Delta P_2^0 = P_2 - P_2^0 = -1.0 - 0 = -1$$

$$Q_2^0 = -\left|V_2^0\right|\left|V_1\right|5\sin(126.87 - \delta_2^0 + \delta_1) - \left|V_2^0\right|^2 5\sin(-53.13)$$
$$= -1 \times 1 \times 5\sin(126.87 - 0 + 0) - 1^2 \times 5\sin(-53.13)$$
$$= -4 + 4 = 0$$
$$\Delta Q_2^0 = Q_2 - Q_2^0 = -0.50 - 0 = -0.5$$

The Newton-Raphson Method **2 Bus Example**

Solution

The elements of Jacobian matrix

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1|5\sin(126.87 - \delta_2 + \delta_1)$$

$$J_1^{\ 0} = \frac{\partial P_2}{\partial \delta_2} = 1 \times 1 \times 5\sin(126.87) = 4$$

$$\frac{\partial P_2}{\partial |V_2|} = 5|V_1|\cos(126.87 - \delta_2 + \delta_1) + 2|V_2|5\cos(-53.13)$$

$$J_2 = \frac{\partial P_2}{\partial |V_2|} = 5 \times 1\cos(126.87) + 2 \times 1 \times 5\cos(-53.13) = 3$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1|5\cos(126.87 - \delta_2 + \delta_1)$$

$$J_3^{\ 0} = \frac{\partial Q_2}{\partial \delta_2} = 1 \times 1 \times 5\cos(126.87) = -3$$

The Newton-Raphson Method **2 Bus Example**

Solution

The elements of Jacobian matrix

$$\frac{\partial Q_2}{\partial |V_2|} = -5|V_1|\sin(126.87 - \delta_2 + \delta_1) - 10|V_2|\sin(-53.13)$$

$$J_4^0 = \frac{\partial Q_2}{\partial |V_2|} = -5 \times 1 \times \sin(126.87) - 10 \times 1 \sin(-53.13) = 4$$

The set of linear equations in the first iteration becomes

$$\begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^0 \\ \Delta|V_2^0| \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 0.16 & -0.12 \\ 0.12 & 0.16 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} \Delta\delta_2^0 \\ \Delta|V_2^0| \end{bmatrix}$$

The Newton-Raphson Method 2 Bus Example

Solution

Solution of the matrix gives:

$$\Delta\delta_2^0 = -0.10$$

$$\Delta\left|V_2^0\right| = -0.2$$

$$\delta_2^1 = 0 + (-0.10) = -0.10 \, radian = -5.7296°$$

$$\left|V_2^1\right| = 1 + (-0.2) = 0.8$$

For the second iteration

$$P_2^1 = \left|V_2^1\right|\left|V_1\right|5\cos(126.87 - \delta_2^1 + \delta_1) + \left|V_2^1\right|^2 5\cos(-53.13)$$

$$= 0.8 \times 1 \times 5\cos(126.87 - (-5.7296)) + 0.8^2 \times 5\cos(-53.13)$$

$$= -2.7075 + 1.92 = -0.7875$$

$$\Delta P_2^1 = P_2 - P_2^1 = -1.0 - (-0.7875) = -0.2125$$

$$Q_2^1 = -\left|V_2^1\right|\left|V_1\right|5\sin(126.87 - \delta_2^1 + \delta_1) - \left|V_2^1\right|^2 5\sin(-53.13)$$

$$= -0.8 \times 1 \times 5\sin(126.87 - (-5.7296)) + 0.8^2 \times 5\sin(-53.13)$$

$$= -2.9444 + 2.56 = -0.3844$$

$$\Delta Q_2^1 = Q_2 - Q_2^1 = -0.50 - (-0.3844) = -0.1156$$

The Newton-Raphson Method **2 Bus Example**

Solution

The set of linear equations in the 2nd iteration becomes

$$\begin{bmatrix} -0.2125 \\ -0.1156 \end{bmatrix} = \begin{bmatrix} 2.9444 & 1.4157 \\ -2.7075 & 2.7195 \end{bmatrix} \begin{bmatrix} \Delta \delta_2^1 \\ \Delta |V_2^1| \end{bmatrix}$$

Solution of the matrix gives:

$$\Delta \delta_2^1 = -0.0350 \quad \delta_2^2 = -0.10 + (-0.0350) = -0.135 \, radian$$

$$\Delta |V_2^1| = -0.0773 \quad |V_2^2| = 0.8 + (-0.0773) = 0.7227$$

CONTINUE WITH THE 3RD ITERATION!
Good Luck......

Power Flow (PF) Studies

The general power flow problem involves evaluation of node (bus) voltages for given injected powers at the buses. The injected powers essentially represent the generations and loads at the buses. It is assumed that the network configuration and the branch impedances are known which, of course, is true in the case of an existing power system. To address future loads and generations, various alternative configurations are evaluated to establish the best configuration. The concept of accepting generations as injected power is quite straightforward. However, it may cause some confusion in the case of loads. Note that a load can be mathematically treated as negative injected power.

In fact, when both loads and generators are present at the same bus, we even talk in terms of net injected power. In other words, the net injected power is the generation minus the load at that bus.

Typical objectives of the power flow studies are given below:

- Buses with voltage level violations i.e. voltage beyond ±5% of rated value

- Current and power flows in all branches

- Equipment over load conditions

- Power losses in the system

- Transformer tap specifications to improve voltage levels

- Capacitive compensation (reactive support)

It is obvious that power flow studies are an important aspect of power system operation and planning. In the case of operation, pf studies are used to arrive at a feasible and economic operating condition for given load and system configurations. However, in the case of power system planning, the objective will be to arrive at the equipment ratings and the best configuration for various future load conditions and contingencies.

In practice, there is a strong incentive to operate power system as close to balanced conditions as possible, or else we are essentially wasting the system capacity. Balanced operation can generally be achieved by proper planning. The industrial and utility power system networks can normally be treated as balanced systems. However, there could be some unbalance present in the LV distribution networks. It is common to assume a balanced power system for power flow (pf) studies. The solution of the pf problem is not so straight forward. The simples way to perform power-flow calculations is by iteration. Whenever iterative methods are used, there is no guarantee that the method will converge and provide a solution. After the advent of computers, various methods to solve pf problem have been developed and researched extensively .Two methods have survived the test of time and they do provide reliable power flow solutions.

- The first one is a simple and reliable method called the Gauss-Siedal method of

iteration using Y-bus. Though quite reliable, it is not very efficient for large systems, as it requires a very large number (hundreds) of iterations.

- The second method, which is more complex to formulate and solve, is called the fast decoupled (Newton Raphson) method.

The fast decoupled method uses the imaginary part of the Y-bus matrix for iterations and is computationally very efficient. Typically, it provides the final solution within a few iterations. If a convergence is not obtained with fast decoupled method, it is reasonably safe to assume that the specified gen/load conditions do not constitute a feasible operating condition i.e. bus voltages may be too high or too low.

The difficulty of solving a power flow problem is best illustrated by considering the following numerical examples.

Given: $V_2 = 1+j0$ pu and $S_2 = 4+j3$ pu

To find: Source bus voltage and pf ans:(4+j4.25)

Given: $V_1 = 1+j0$ pu and $S_2 = 4+j3$ pu

To find: Load bus voltage

$$I_2 = \left(\frac{S_2}{V_2}\right)^* \qquad (1)$$

$$V_2 = V_1 - I_2 Z \qquad (2)$$
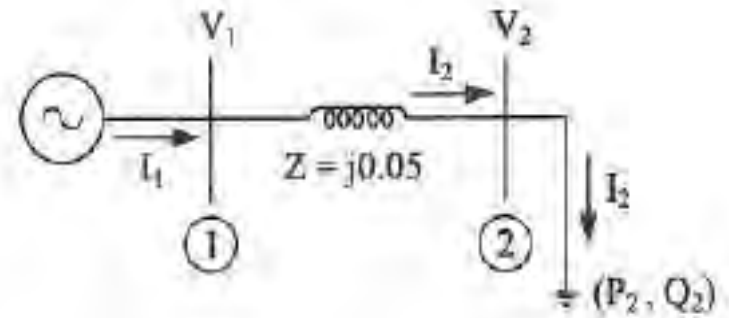
The main assumptions in the case of power flow modeling is that the system is balanced.

For the purpose of developing the p.f. equation, the power system network is generally treated as a black box as shown beside.

Network model for p.f. problem
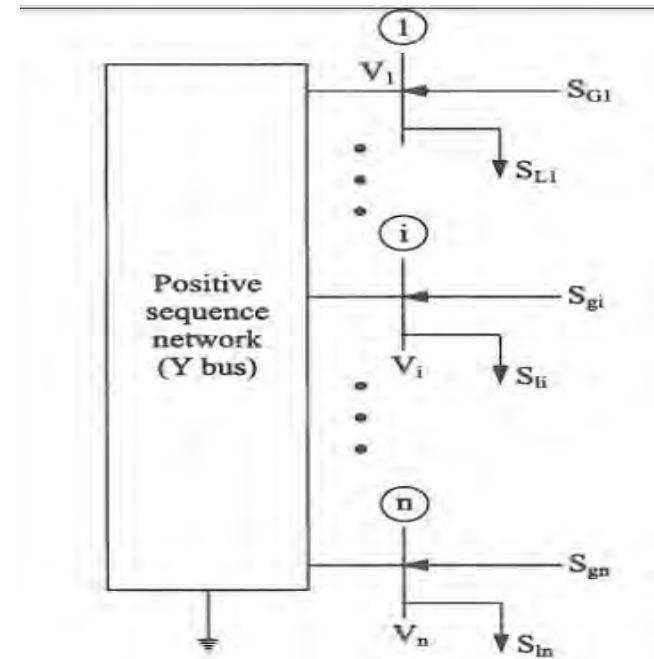
The black box essentially represents the positive sequence equivalent network of the power system. From the p.s. point of view, the black box essentially contains all the lines, transformers and capacitors in the p.s. However, in network theory, we normally say that it contains linear passive elements. The nodes in the network are brought out as shown in Fig. and numbered as 1,2,…i,…n
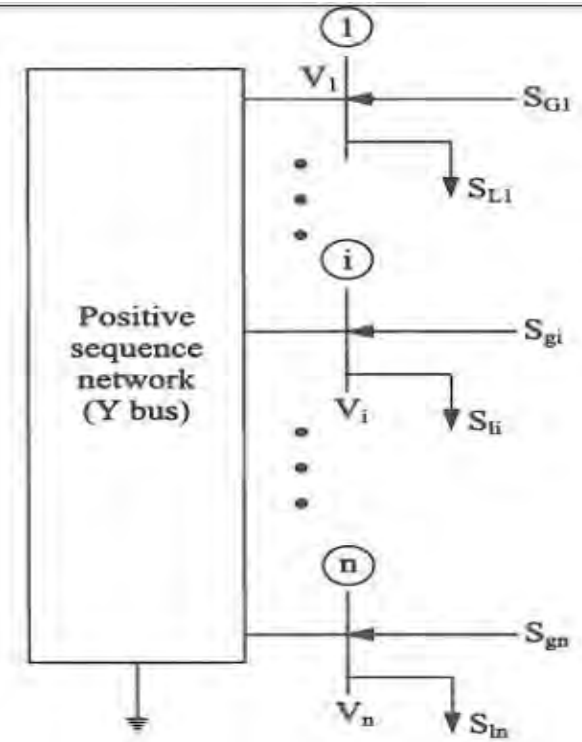
Network model for pf problem

The generation and load at the buses are indicated by S_{Gi} and S_{Li}, respectively.

Hence, the net injected power at a given i^{th} bus can be written as:

$S_i = S_{Gi} - S_{Li}$ for i = 1 to n

Or $P_i = P_{Gi} - P_{Li}$ for i = 1 to n

$Q_i = Q_{Gi} - Q_{Li}$ for i = 1 to n

We use the subscript 'i', to emphasize that they are the specified or known values at the bus in the case of power flow problems. This is generally true, but there are some exceptions. We will discuss the exceptions later. We can recall that the power flow problem essentially involves solving for the node voltages for given injected powers. In other words, we need to solve for the voltages, V_i (for i = 1 to n), at all nodes.

The complex injected power at a given node can also be written in the following form:

Network model for p.f. problem

$$S_i = (P_i + jQ_i) = V_i(I_i)* \text{ for } i = 1 \text{ to } n \qquad (3)$$

Where, I_i is the corresponding injected current at the bus and V_i is the bus voltage.

We can write the above eq. in the following form: $\quad I_i = \left(\dfrac{S_i}{V_i}\right)^* \quad$ for i = 1 to n $\quad$ (4)
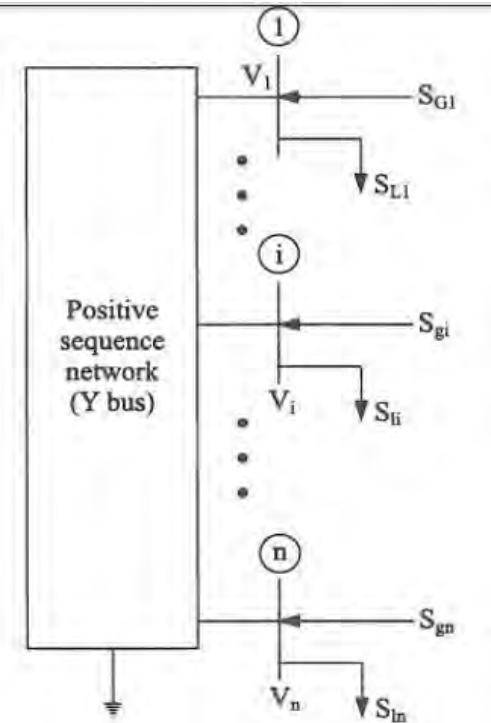
We also have the network equation which relate injected currents and node voltages, namely:

$$\overline{I_{bus}} = [Y_{bus}]\overline{V_{bus}} \qquad (5)$$

Where

$\overline{I_{bus}}$ is the injected currents at the nodes (nx1)

Y_{bus} is the node admittance matrix (nxn)

$\overline{V_{bus}}$ is the node voltages (nx1)
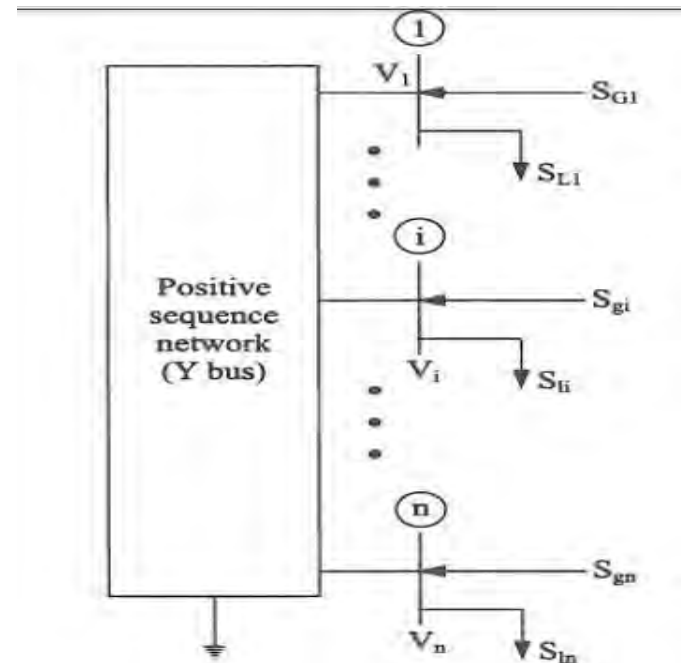
You should note the network inside the black box is essentially represented by the node admittance (Y-bus) matrix. We also know how to establish the admittance matrix for a given network. It is probably a good idea at this stage to compare equations 4 and 5 with the equation used for iterations in the sample example, namely, equation1 and 2. You should note that the equation 4 and 5 are more versatile, since they can be applied to any general network.

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In fact, it is possible to use the same iterative procedure to solve the general power flow problem. The steps can be written as below:

1. Assume initial voltages for at all nodes

2. Calculate the injected currents at all nodes using equation 4

3. Obtain new estimates of the node voltages using equation 5

4. Repeat steps 3 and 4, until convergence is obtained for all node voltages

In theory, the above procedure is OK. However, it is considered computationally inefficient. Figure below gives a simple network showing the nodal currents, voltages and powers. It is convenient to use branch admittances rather than impedances.

$I_1(S_1=E_1I_1^{*})$ $I_2(S_2=E_2I_2^{*})$ $I_3(S_3=E_3I_3^{*})$
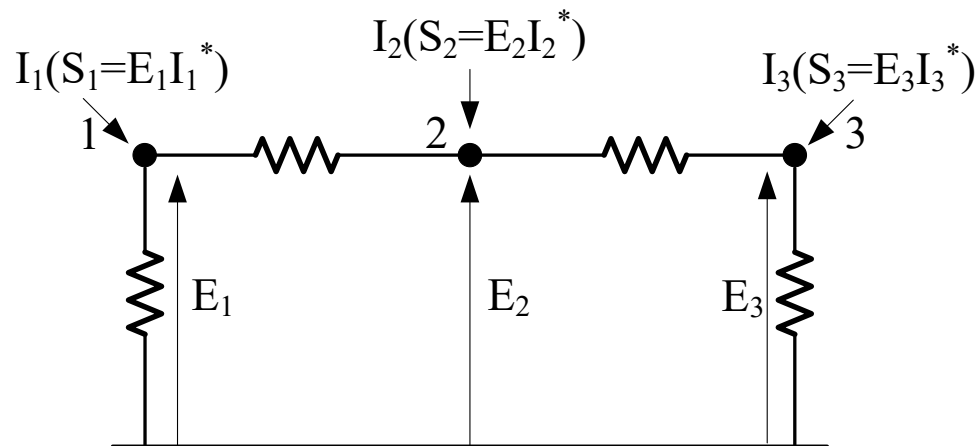
Denoting the voltages of nodes k and i as E_k and E_i respectively, and the admittance of the branch between them as y_{ki}, then the current flowing in this branch from node k to node i is given by

$$I_{ki} = y_{ki}(E_k - E_i)$$

Let the nodes in the network be numbered 0,1,...,n, where 0 designates the reference node. By KCL, the injected current I_k must be equal to the sum of the currents leaving node k, hence,

$$I_k = \sum_{i=0}^{n} I_{ki} = \sum_{i=0}^{n} y_{ki}(E_k - E_i)$$

Since $E_0 = 0$, and if the system is linear,

$$I_k = \sum_{i=0 \neq k}^{n} y_{ki} E_k - \sum_{i=1 \neq k}^{n} y_{ki} E_i$$

If this equation is written for all the nodes except the reference, i.e. for all busbar in the case of a power system network, then a complete set of equations defining the network is obtained in matrix form as,

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

where $Y_{kk} = \sum_{i=0 \neq k}^{n} y_{ki}$ = self-admittance of node k,

$\quad Y_{ki} = -y_{ki}$ = mutual admittance between nodes k and i

in summation notation $I_k = \sum_{i=1}^{n} Y_{ki} E_i$ for $i = 1, \ldots, n$.

The nodal admittance matrix has a well defined structure, which makes it easy to construct automatically. Tiney et.al were the first to exploit the sparsity feature of Y_{BUS} in greatly reducing numerical computations in LF studies and in minimizing the memory required as only non-zero terms need be stored.

Its properties are as follows.

i)   Square of order n x n

ii)   Symmetrical, since $y_{ki} = y_{ik}$

iii) Complex

iv)   Each off diagonal element y_{ki} is the negative of the branch admittance between nodes k and i, and is frequently of value zero

v)   Each diagonal element y_{kk} is the sum of the admittance of the branches which terminate on node k, including branches to ground.

vi)   Because in all but the smallest practical networks very few nonzero mutual admittances exist, matrix Y is highly sparse. In a system containing hundreds of buses, the sparsity may be as high as 90%

The complete definition of power flow problem requires knowledge of four variables of each k-bus in the system:

i) P_k- Real or active power

ii) Q_k- reactive or quadrature power

iii) V_k- voltage magnitude

iv) δ_k- voltage phase angle

Only two are known a priori to solve the problem, and the aim of LF is to solve the remaining two variables at a bus. We define three different bus conditions based on the steady state assumptions of constant system frequency and constant voltage, where there are controlled.

i) Voltage controlled bus.

The total injected active power P_k is specified, and the voltage magnitude V_k is maintained at a specified value by reactive power injection. This type of bus generally corresponds to either a generator where P_k is fixed by turbine governor setting and V_k is fixed by automatic voltage regulators acting on the machine excitation, or a bus where the voltage is fixed by supplying reactive power from static shunt capacitors or rotating synchronous compensators i.e. at substation

In practice, it is more convenient to specify the injected active power and voltage magnitude at the generator buses, rather than injected active and reactive power.

In other words, the specified values are the active power, voltage magnitude, and we will need to solve for the reactive power and voltage angle at the generator bus.

The active power of a generator is controlled by the governor and is directly related to the governor setting.

Also, the voltage magnitude is controlled by the voltage regulator and is directly related to voltage regulator setting.

Hence, the specification of active power and voltage magnitude is more convenient from the point of view of generator operation. Such a specification is often referred to as P-V bus in pf problems. Consequently the buses where injected powers are specified can be referred to as P-Q buses.

In general, the generator buses are referred to as P-V buses and the load buses are referred to P-Q buses. For the purpose of iterations, we use the eq.

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ (k \neq i)}}^{N} Y_{ik} V_k \right)$$

However, we will need to use as estimated value for the injected reactive power for the P-V bus, since the value of reactive power is no longer specified. We know that injected power at a bus is related to the injected current and the bus voltage by the following eq. $S_i = P_i + jQ_i = V_i I_i^*$

The injected current at a bus can be calculated using equation, $I_i = \sum\limits_{k=1}^{n} Y_{ik} V_k$

Combining the two above equations, we can write the equation for the estimated value of the injected reactive power as given below: $Q_{est-i} = -\text{Im}\left\{ V_i^* \sum\limits_{k=1}^{n} Y_{ik} V_k \right\}$

We can now write the modified version of voltage equation for P-V buses as below:

$$V_{i(est)} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_{est-i}}{V_i^*} - \sum\limits_{\substack{k=1 \\ (k \neq i)}}^{N} Y_{ik} V_k \right)$$

Note that the above eq. is used only when the subscript 'i' corresponds to P-V buses. Of course, we will still use the equation for P-Q buses. As before the latest available voltage values are used on the RHS of equations Q_{est-i} and $V_{i(est)}$

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum\limits_{\substack{k=1 \\ (k \neq i)}}^{N} Y_{ik} V_k \right)$$

Finally, the equation gives the new estimated value of the voltage $V_{i(est)}$ in complex form.

$$V_{i(est)} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_{est-i}}{V_i^*} - \sum_{\substack{k=1 \\ (k \neq i)}}^{N} Y_{ik} V_k \right)$$

However, we are really only interested in the voltage angle, since the voltage magnitude has already been specified at the P-V bus. In other words, we just ignore the magnitude obtained by above equation.

Mathematically, we can write the corresponding eq. as below:

$$V_{i(new)} = V_{sp-i} \angle \delta_i \quad \text{where } \delta_i = angle \text{ of } \left(V_{i(est)} \right)$$

The rest of the iteration process remains the same.

ii) Slack(swing) bus.

The injected powers in a pf problem essentially represent the loads and scheduled generation values for a given system condition. In practice, it is not possible to specify scheduled generation values at all the generators. The reason being that, the total generation and load should satisfy the following equations

$$\sum P_G = \sum P_L + P_{Losses}$$

$$\sum Q_G = \sum Q_L + Q_{Losses}$$

In other words, we will need to know the total system losses, before generations at all generator buses can be specified. A person familiar with the system may have some idea of the system losses. To get the exact value of losses for a given system condition, we will need to solve the power flow problem. In fact, calculation of power losses is one of the objectives of pf calculations. In view of the above, the injected power at one of the generation buses is not specified. Instead, the voltage magnitude and angle are specified at that bus. Such a bus is called the swing bus or the slack bus. Typically, the bus with the largest generation is specified as the swing bus. We can use the swing bus voltage as the reference vector, which means that its vectorial angle is zero. Hence, it is a common practice to specify the voltage magnitude at the swing bus and set the angle to zero, while solving pf problems. After solving the pf problem, it will be necessary to calculate the injected power at

the swing bus, and to ensure that the pf values are within the capability of the generator at the swing bus. This bus arises because the system losses are not known precisely in advance of the LF calculation. Therefore the total injected power cannot be specified at every single bus. It is usual to choose one of the available voltage controlled buses as slack, and to regard its active power as unknown. The slack bus voltage is usually assigned as the system phase reference, and its complex voltage is therefore specified. The analogy in a practical power system is the generating station which has the responsibility of system frequency control. $E_s = V_s \angle \theta_s$

iii) PQ Bus

At this type of bus, the net powers P_i and Q_i are known (P_{Di} and Q_{Di} are known) and P_{Gi} and Q_{Gi} are specified. The unknown are V_i and δ_i.

$$S_i = P_i + jQ_i = V_i I_i^* \quad i = 1,2,\ldots,n$$

$$I_i = source \text{ current injected into the bus}$$

$$P_i - jQ_i = V_i^* I_i \quad i = 1,2,\ldots,n$$

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k$$

$$P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

$$P_i = \mathrm{Re}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\}$$

$$Q_i = -\mathrm{Im}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\}$$

$$V_i = |V_i| e^{j\delta_i} = |V_i| \angle \delta_i = V_i (\cos \delta_i + j \sin \delta)$$

$$Y_{ik} = |Y_{ik}| e^{j\theta_{ik}}$$

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \quad i = 1,2,\ldots,n$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |V_k| |Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) \quad i = 1,2,\ldots,n$$

2n Power flow eqs

4n variables P_i, Q_i, $|V_i|$ and δ_i.

SLFE can be solved for 2n variables if remaining 2n variables can be specified.

Fixed a priori two variables at each bus.

Non linear algebraic equation difficult to solve

Use iterative numerical techniques.

Depending upon which two variables are specified a priori, the buses are classified into three categories

PQ bus	P_i, Q_i	$\lvert V_i \rvert, \delta_i$?
PV bus	$P_i, \lvert V_i \rvert$	Q_i, δ_i ?
Slack/Swing/Ref bus	$\lvert V_i \rvert, \delta_i = 0$	P_1, Q_1?

Why slack bus?

i.e. diff $\qquad \left(\sum S_{Di} + S_L - \sum S_{Gi} \right)$

This is known in the end only.

Bus connected to the largest

$\text{Gen}^R \longrightarrow \quad$ SLACK No. 1

$f(x,y) = 0$ dim 2n

Where, x = dependent or state vector dim $2n$ ($2n$ unspecified variables)

            y = vector of independent variables ($2n$ vari speci a priori)

$$y = \begin{bmatrix} u \\ p \end{bmatrix} \quad \begin{array}{l} u = \text{control variables} \\ \\ p = \text{fixed parameters (uncontrollable)} \end{array}$$

$u \dashrightarrow |V_{pv}|, P_i$

For SLFE solution to have practical significance, all the state and control variables must lie within specified practical limits. These limits, which are dictated by specifications of power system hardware and operating constraints are described below.

(i)   Voltage magnitude must satisfy the inequality $\left|V_i\right|_{\min} \leq \left|V_i\right| \leq \left|V_i\right|_{\max}$

(ii) $\left|\delta_i - \delta_k\right| \leq \left|\delta_i - \delta_{k\,\max}\right|$ System stability requirement

(iii) Owing to physical limitations of P and Q generation sources,

$$\left|P_{Gi}\right|_{\min} \leq \left|P_{Gi}\right| \leq \left|P_{Gi}\right|_{\max}, \quad \left|Q_{Gi}\right|_{\min} \leq \left|Q_{Gi}\right| \leq \left|Q_{Gi}\right|_{\max}$$

PV- V maintained if controllable

Q source available

Transformer taps

Phase shifting transformer

Specify variables (2n) at all buses (y)

2n SLFE solved iteratively for 2n (remaining) variables (x)

Compute line flows & losses

Approximate LFS $\qquad \theta_{ik} \approx 90°; \ \theta_{ii} \approx -90°$

R=0; P_L=0 $\qquad \sin(\delta_i - \delta_k) \approx (\delta_i - \delta_k)$

The LF problem can now be fully defined as follows:

Assume a certain nominal bus load configuration. Specify $P_{Gi} + jQ_{Gi}$ at all the PQ buses; specify P_{Gi} and $|V_i|$ at all the PV buses. Also specify $|V_1|$ and δ_i (=0) at the slack bus. Thus 2n variables of the vector **y** are specified. The 2n SLFE can now be solved iteratively to determine the values of the 2n variables of the vector **x** comprising voltages and angles at the PQ buses, reactive powers and angles at the PV buses and active and reactive powers at the slack bus. The next step is to compute line flows.

Lines	Z
1-2	Z_d
1-3	Z_c
1-4	Z_f
2-3	Z_b
2-4	Z_e

Generator	Impedance
G_a	Z_a
G_g	Z_g
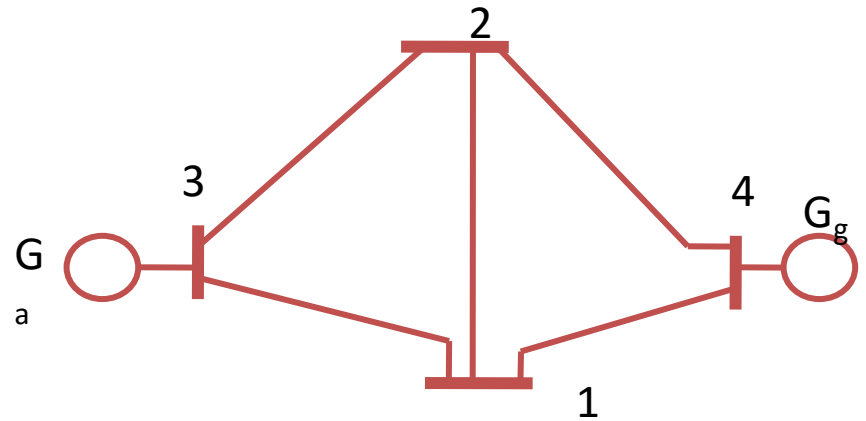
1. Develop **Bus Admittance Matrix**

2. Estimation bus voltages

3. Update voltages until converge

   ITERATION techniques

BUS ADMMITANCE MATRIX

Consider a simple power system:

Impedances are expressed in p.u. on a common

MVA base.

Impedance are converted to admittance:

$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$

$z_{12} = j0.4$

$j0.8$

1 **2**

$Z_{13} = j0.2$

$Z_{23} = j0.2$

$Z_{34} = j0.08$

3

4
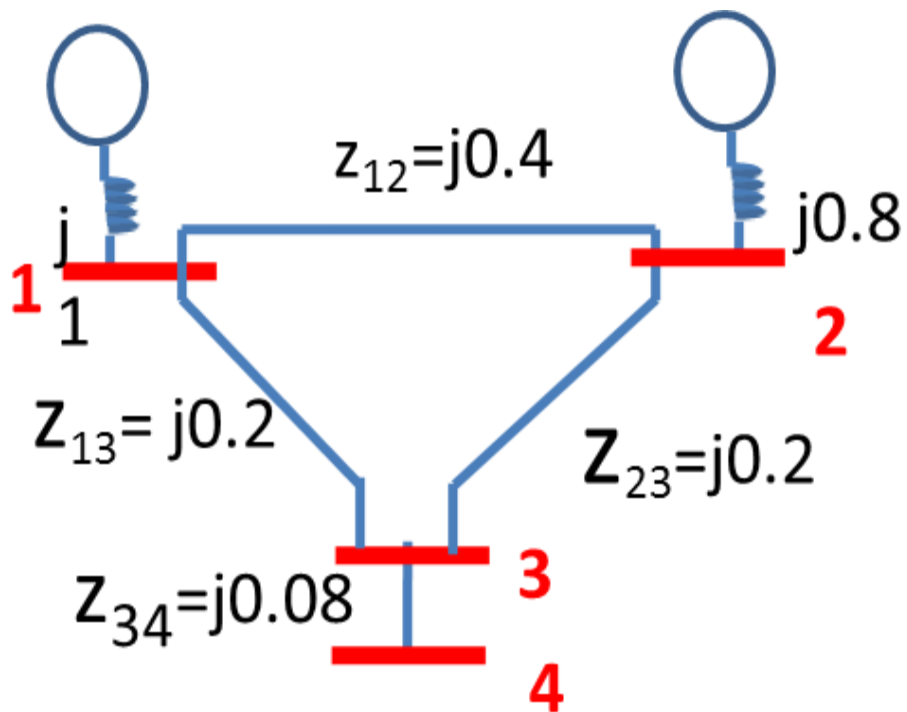
Convert impedance to admittance

Draw admittance diagram

Impedance	Admittance
$z_{10} = j1$	$y_{10} = -j1$
$z_{12} = j0.4$	$y_{12} = -j2.5$
$z_{13} = j0.2$	$y_{13} = -j5$
$z_{23} = j0.2$	$y_{23} = -j5$
$z_{34} = j0.08$	$y_{34} \quad -j12.5$

$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$
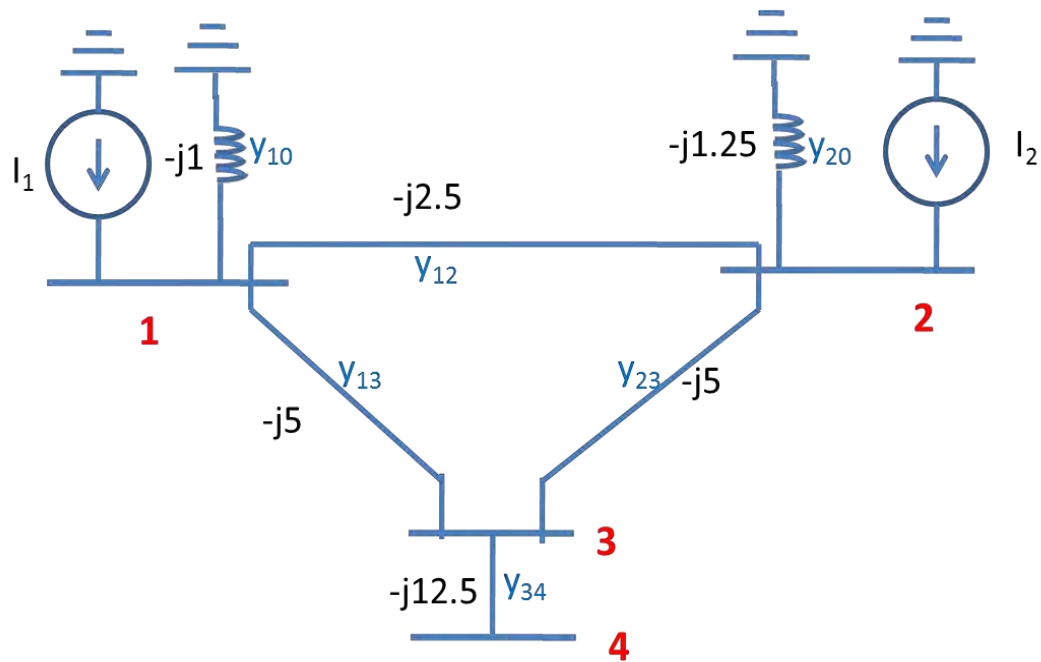
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Applying KCL at each bus

Bus 1 $\quad I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$

Bus 2 $\quad I_2 = y_{20}V_2 + y_{21}(V_2 - V_1) + y_{23}(V_2 - V_3)$

Bus 3 $\quad I_3 = y_{32}(V_3 - V_2) + y_{31}(V_3 - V_1) + y_{34}(V_3 - V_4)$

Bus 4 $\quad I_4 = y_{43}(V_4 - V_3)$

Rearranging,

$$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3$$

$$I_3 = -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4$$

$$I_4 = -y_{43}V_3 + y_{44}V_4$$

$$Y_{11} = y_{10} + y_{12} + y_{13}$$

$$Y_{22} = y_{20} + y_{12} + y_{23}$$

$$Y_{33} = y_{13} + y_{23} + y_{34}$$

$$Y_{44} = y_{34}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$$

$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$

$$Y_{34} = Y_{43} = -y_{34}$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$

$$I_3 = Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4$$

$$I_4 = Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4$$
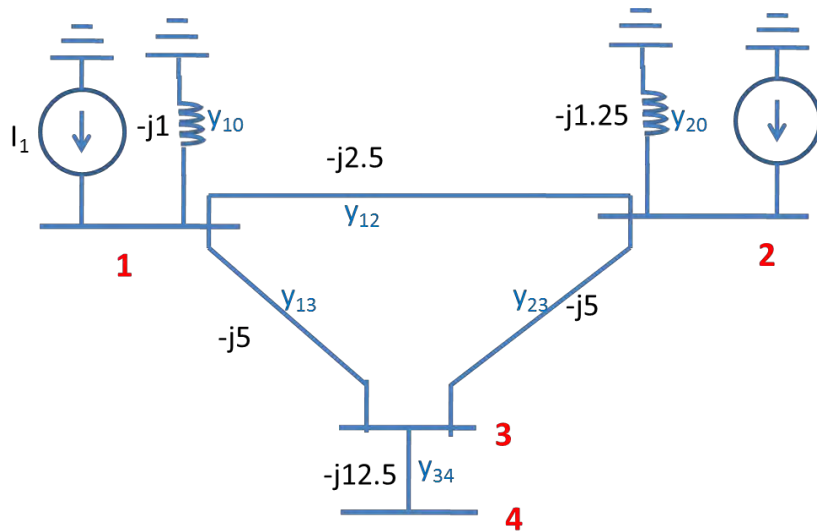
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$ **or**

$$I_i = \sum_{k=1}^{n} Y_{ik} \, V_k \,; i = 1, 2, ..., n \quad (6)$$

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{i1} & Y_{i2} & \cdots & Y_{ii} & \cdots & Y_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{ni} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix}$$

Write equations for I_1 and I_2

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$$Y_{11} = y_{10} + y_{12} + y_{13}$$

$$Y_{22} = y_{20} + y_{12} + y_{23}$$

$$Y_{33} = y_{13} + y_{23} + y_{34}$$

$$Y_{44} = y_{34}$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$
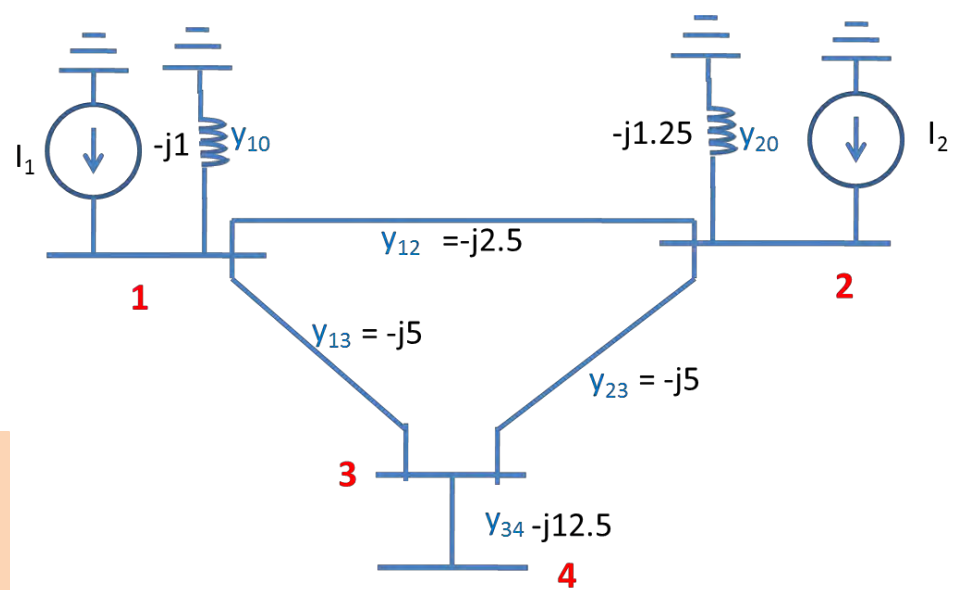
$$Y_{34} = Y_{43} = -y_{34}$$

$$Y_{12} = Y_{21} = -y_{12}$$

$$Y_{13} = Y_{31} = -y_{13}$$

$$Y_{23} = Y_{32} = -y_{23}$$

$$Y_{34} = Y_{43} = -y_{34}$$

$$Y_{11} = y_{10} + y_{12} + y_{13}$$

$$Y_{22} = y_{20} + y_{12} + y_{23}$$

$$Y_{33} = y_{13} + y_{23} + y_{34}$$

$$Y_{44} = y_{34}$$

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} -j8.5 & j2.5 & j5 & 0 \\ j2.5 & -j8.75 & j5 & 0 \\ j5.0 & j5.0 & -j22.5 & j12.5 \\ 0 & 0 & j12.5 & -j12.5 \end{bmatrix}$$

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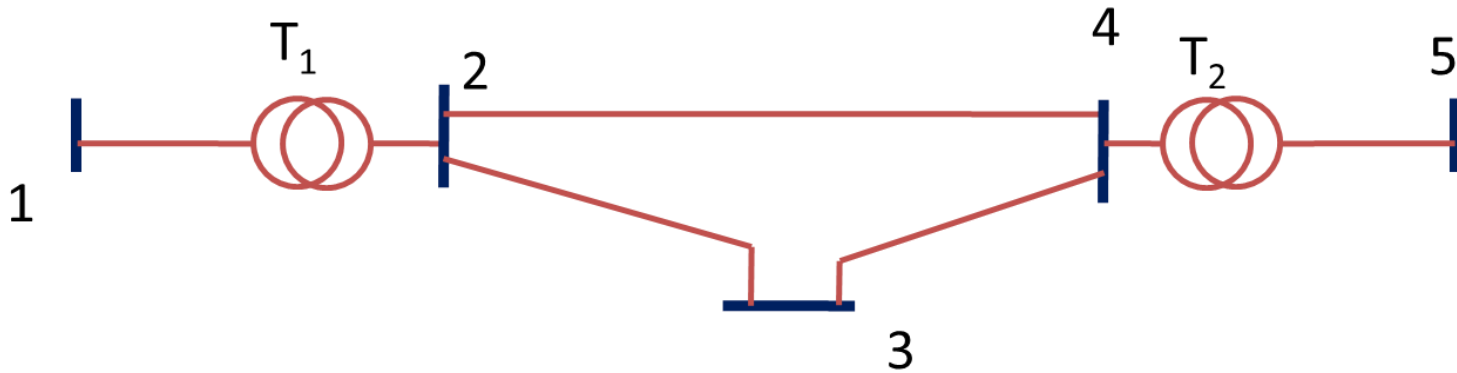
From Bus	To Bus	R(pu)	X(pu)	B(pu)
1	2	0.004	0.0533	0
2	3	0.02	0.25	0.22
3	4	0.02	0.25	0.22
2	4	0.01	0.15	0.11
4	5	0.006	0.08	0

Determine Y Admittance matrix of the system

The power system shown is operating at steady state. The reactance of each transmission line is X = 20 ohm. The generators and transformers are rated as follows:

- G1:   20 MVA,   12 kV,   X = 1.20 p.u.

- G2:   60 MVA,   13.8 kV,   X = 1.40 p.u.

- G3:   50 MVA,   13.2 kV,   X = 1.40 p.u.

- T1:   25 MVA,   12/69 kV,   X = 0.08 p.u.

- T2:   75 MVA,   13.8/69 kV,   X = 0.16 p.u.

- T3:   60 MVA,   69/13.2 kV,   X = 0.14 p.u.

- T4:   75 MVA,   69/13.8 kV,   X = 0.16 p.u.
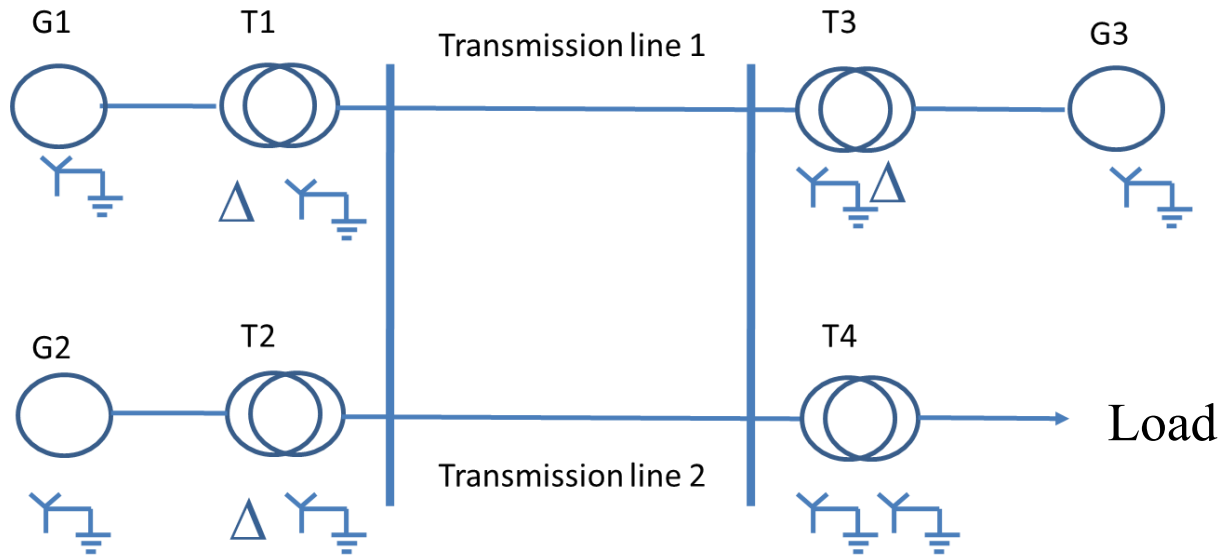
Use a power base of 100 MVA and a voltage base of 12 kV in the circuit of generator 1, form the bus admittance matrix $\mathbf{Y_{bus}}$ **.The three phase load absorbs 57 MVA, 0.6 pf lagging at 10.45kV.**
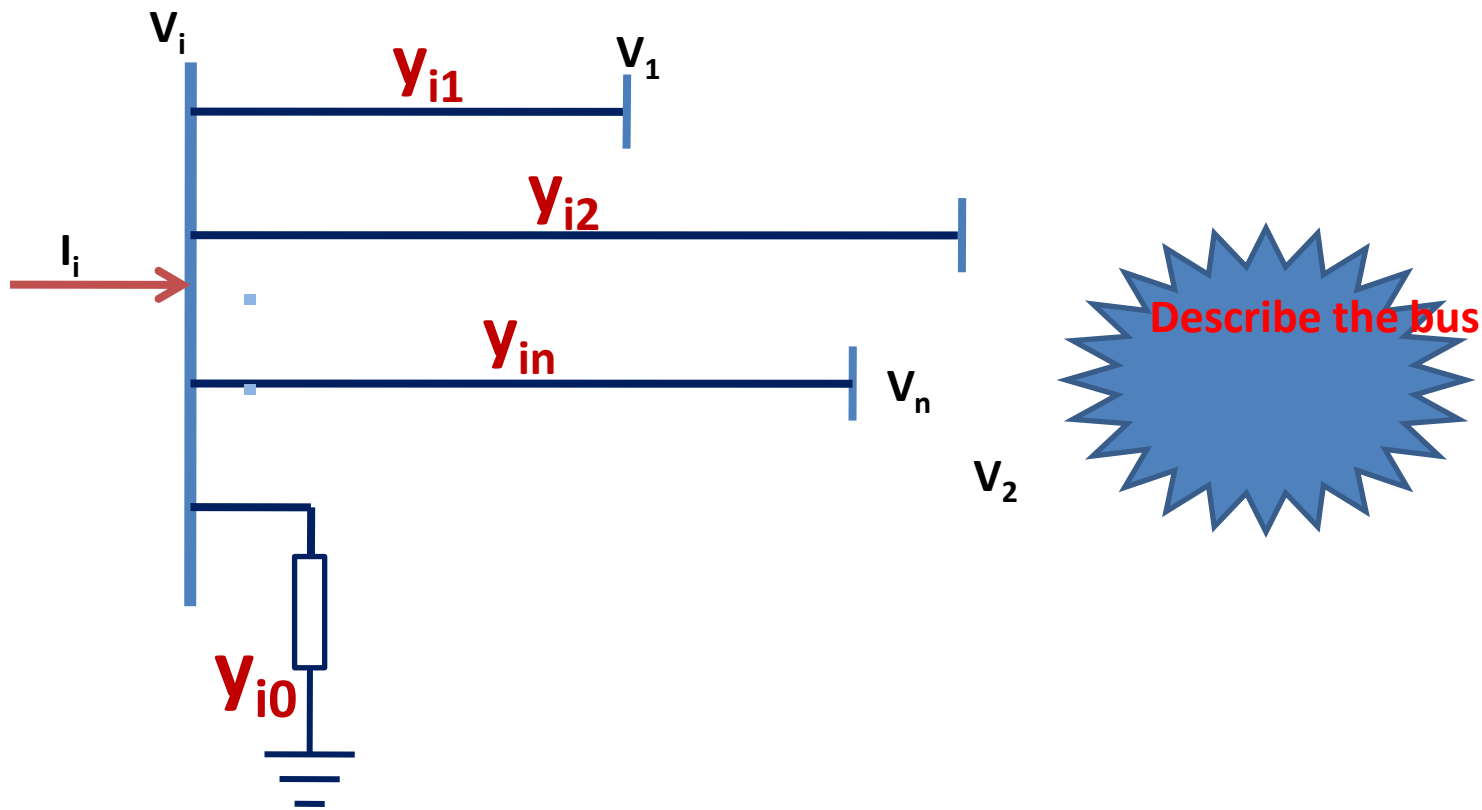
Describe the bus

$$I_i = y_{10}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n)$$

$$I_i = (y_{10} + y_{i1} + y_{i2} \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n$$

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \quad j \neq i$$

$$I_i = V_i Y_{ii} + \sum_{j=1}^{n} Y_{ij}V_j \quad j \neq i$$
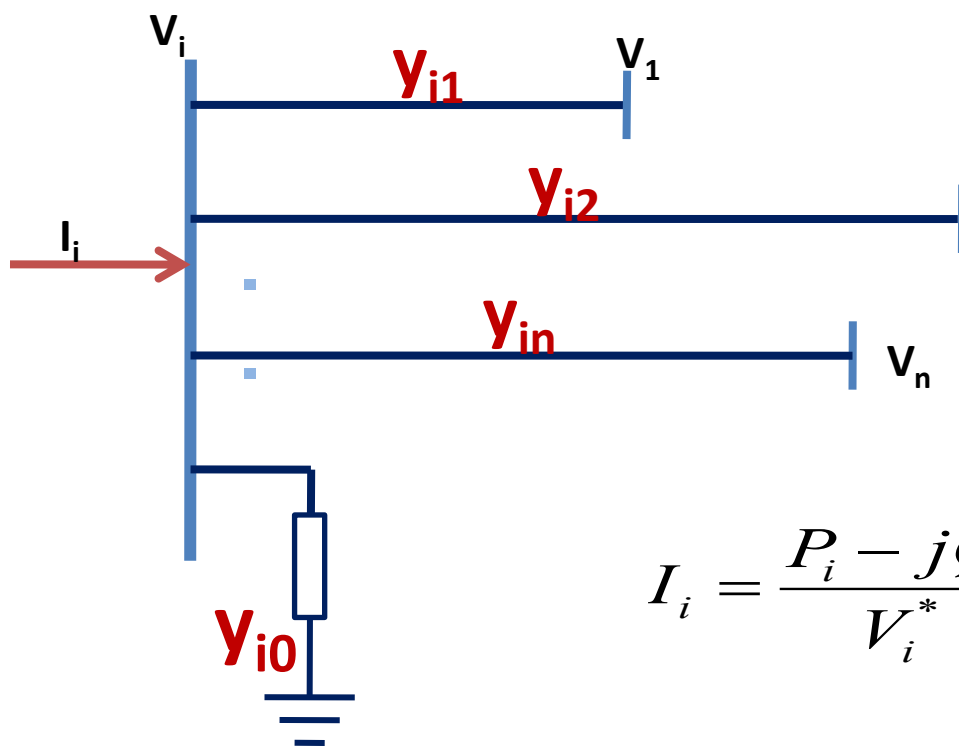
$$P_i + jQ_i = V_i I_i^*$$

$$I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i$$

$$I_i = \frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum y_{ij} V_j \qquad j \neq i$$

Power is given by the following equations:

$S = S_{generated} - S_{load}$

$S_i = S_{iG} - S_{iL}$

$S_1 = S_{1G} - S_{1L}$

$S_2 = S_{2G} - S_{2L}$

Basic equation for power-flow analysis is derived from the nodal analysis equation for the power system:

For a four-bus p.s.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

For bus 2 in the four-bus p.s.

$$I_2 = Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4$$

The loads on real p.s. are specified in terms of real and reactive power, not as currents. Relationship between power and current at bus i can be expressed as:

$$S = VI^* = P + jQ$$

Current injected at bus 2 can be found as:

$$V_2 I_2^* = P_2 + jQ_2$$

$$I_2^* = \frac{P_2 + jQ_2}{V_2}$$

$$I_2 = \frac{P_2 - jQ_2}{V_2^*}$$

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = I_2$$

$$I_2 = \frac{P_2 - jQ_2}{V_2^*}$$

Substituting gives

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = \frac{P_2 - jQ_2}{V_2^*}$$

Solving for V_2 gives

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - (Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4)\right]$$

Similar equations can be created for each load bus in the power system

The basic equation for pf studies are given in equations 4 & 5.

Main objective is to solve the same more efficiently.

Hence, we will be reorganising the equation to meet the objective.

Such a rearrangement will hopefully provide for computational efficiency and better convergence. One such method is known as the Gauss-Seidel method.

Such a terminology has been given, since the method is based on the general procedure suggested by Gauss to solve non-linear simultaneous equation. It was later modified by Seidel, which resulted in a slightly faster convergence. In other words, it resulted in fewer iterations. Expand the matrix equation given in eq.(5) to obtain an expression for injected current at any given node as shown below:

$$I_i = \sum_{k=1}^{n} Y_{ik}\, V_k ; i = 1, 2, \ldots, n \quad (6)$$

Substitute eq.6 into eq. 3, which results in the following eq:

$$(P_i + jQ_i) = V_i \left[\sum_{k=1}^{n} (Y_{ik} V_k) \right]^* \quad for \ i = 1 \ to \ n \quad (7)$$

Now move the conjugate to LHS and express the summation in the expanded form. The resulting eq. is as shown below:

$$(P_i + jQ_i)^* = V_i^* \left[Y_{i1} V_1 + ... + Y_{ii} V_i + ... + Y_{in} V_n \right] \quad for \ i = 1 \ to \ n \quad (8)$$

Eq. (8) is essentially a set of simultaneous eqs, where node voltages are the unknown variables.

In fact, we have 'n' unknown values, namely, V_1, V_2 ,... V_i ,... V_n and 'n' eqs.

If the first term in the RHS of eq.8, namely V_i^*, was not present, then it could have been solved using the usual elimination methods.

However, the eq. is now a non-linear eq., since it contain products of unknown variables. The only way it can be solved is by means of iteration.

There are 'n' eqs. in eq(8), one eq. corresponding to each node in the system.

We will use the first eq. to get the new estimate for V_1 and the second eq. to obtain the new estimate for V_2, and so on.

Hence, we can write the general eq. for iteration for the voltage at i^{th} bus as shown below:

$$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ (k \neq i)}}^{N} Y_{ik} V_k \right) \quad \text{for i=1 to n} \quad (9)$$

The above eq. looks very complex, but it is essentially obtained by rearranging the terms in equation (8).

In other words, by moving all the terms in the RHS to the LHS except for the term $Y_{ii}V_i$. Finally we can move Y_{ii} also to LHS and swap the eq. around. The eq.(9) is the general iteration eq. for the bus voltages. $V_{i(new)}$ provides a new estimate for the bus voltages after each iteration. Note, however, that evaluation of RHS requires the voltages at all buses, including the voltage at the bus where the new estimate is being calculated. Obviously, voltages obtained at the previous iteration are used for

the calculations.

Initial voltage estimates of $1\angle 0°$ pu is normally used at all buses to start the iterations. The method suggested by Gauss made use of the previous iteration voltage values at all buses for calculating new voltage estimates. However, Seidel suggested that the convergence is faster if the latest available voltage values are used. That is, when calculating the new estimate for voltage at bus 3, the latest estimates of the voltages at buses 1 & 2 (present iteration values) can be used, rather than previous iteration values. Of course, at other buses we will have to use the voltages from the previous iteration.

The above iterations are carried out for all buses, until the bus voltage at each bus converges to their final values. The convergence of iterations is checked as below:

$$V_{i(new)} - V_{i(prev)} \leq \varepsilon \qquad \text{for i =1 to n}$$

Note that convergence must be checked for real and imaginary parts.

The symbol ε is referred to as the convergence factor.

Typical values of the convergence factor used in practice are 0.0001 to 0.00001 pu. Such small values for voltage convergence are necessary to obtain power flow convergence in the range of about 0.01 (1%) to 0.001(0.1%).

If higher convergence factors are used, substantial errors could occur in pf and power loss calculations.

The advantages of the G-S method can be listed below:

1. The iteration eq. are quite simple and easy to program. In fact, with computing tools, such as MATLAB, it is fairly straightforward to perform pf calculations using iterative eqs. which have been developed.

2. Since Y-bus is a sparse matrix, the memory requirements can be reduced substantially by storing only the non-zero elements of the matrix.

3. The iterations are numerically stable and typically a fairly reliable convergence is obtained, provided a feasible solution exists. This is due to the fact that the Y-bus matrix has predominant diagonal element values.

The G-S method has some disadvantages, which becomes particularly significant while solving larger power systems.

1. The number of iterations increases as the system size increases.

The number of iterations can be excessive for large systems. i.e. 500-800 iterations are quite common for a system with 50 buses. In fact, a few thousand iterations are not uncommon in the case of large utility systems, where a few hundred buses are quite common. Some empirical methods have been suggested to reduce the number of iterations, by means of an acceleration factor. This does reduce the iterations slightly, but it still requires fairly large number of iterations. Also, the value of the acceleration factor depends on the system and needs experimentation.

2. The main reason of large number of iterations is that the convergence of the G-S method is asymptotic. In other words, the convergence slows down as the values get closer to the final values. Hence, in general, it is difficult to obtain higher precision results.

3. This method may not converge for systems with negative impedance branches.

Step 1

- **Calculate the bus admittance matrix Y_{bus}.**
- Include the admittances of all transmission lines, transformers, between lines, but exclude the admittances of the loads or generators themselves.

Step 2

- **Select a slack bus.**
  - One of the busses in the power system should be chosen as the slack bus. Its voltage will be assumed to be $1.0\angle 0$.

Step 3

- **Select initial estimates for all bus voltages.**
  - The estimate should be reasonable as poor choice may result in convergence to incorrect values. Usually we start with $1.0\angle 0$ (flat start).

Step 4

- Write voltage equations for every
- other bus in the system
- The voltage equation will have
- generic form

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ (k \neq i)}}^{N} Y_{ik}V_k \right)$$

or

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{k=1}^{i-1} Y_{ik}V_k - \sum_{k=i+1}^{N} Y_{ik}V_k \right)$$

Step 5

- **Calculate and update estimate of the voltage at each load bus in succession using the voltage equation**

Step 6

- **Compare the difference between the old voltage estimates and the estimates**
  - If the differences between the estimates less than specified tolerance for all busses, we are done.

Step 7

- **Confirm that the resulting solution is reasonable.**
  - Typical value of phase range is less than 45°.
  - Larger ranges may indicate the system converged to an incorrect solution.
  - Change the initial condition. Try again.

$$I_i = \frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i$$
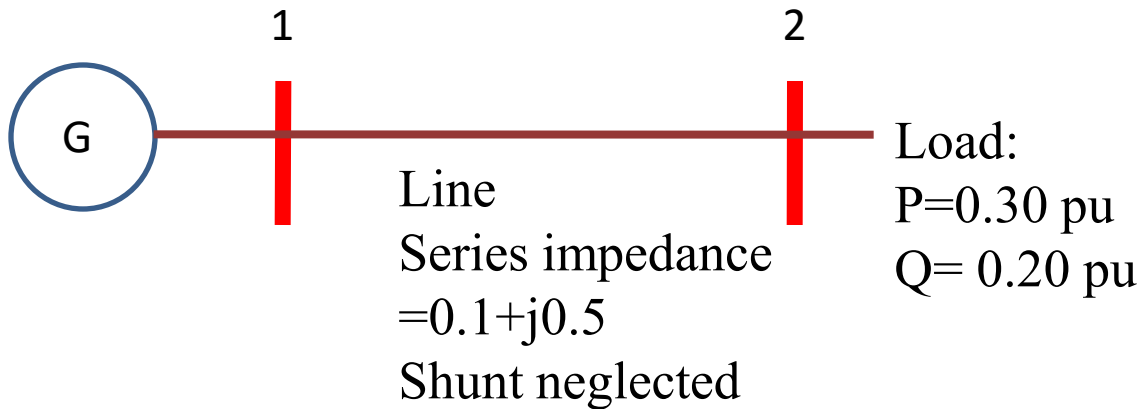
$$V_i^{(k+1)} = \frac{\dfrac{P_i - jQ_i}{V_i^{*(k)}} - \sum_{j=1}^{n} Y_{ij} V_j^{(k)}}{Y_{ii}} \qquad j \neq i$$

Write equations for I_2 and V_2 (for 2nd iteration) of a 5 bus system

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$$P_i^{(k+1)} = \Re\left\{V_i^{*(k)}\left[V_i^{(k)}\sum_{j=0}^{n}y_{ij} - \sum_{j=1}^{n}y_{ij}V_j^{(k)}\right]\right\} \quad j \neq i$$

$$Q_i^{(k+1)} = -\Im\left\{V_i^{*(k)}\left[V_i^{(k)}\sum_{j=0}^{n}y_{ij} - \sum_{j=1}^{n}y_{ij}V_j^{(k)}\right]\right\} \quad j \neq i$$

Write equations for P_2 and Q_2 (for 2nd iteration) of a 5 bus system

1

2

G

Line
Series impedance
=0.1+j0.5
Shunt neglected

Load:
P=0.30 pu
Q= 0.20 pu

STEP 1

Calculate the bus admittance matrix

$$y_{line} = \frac{1}{z_{line}} = \frac{1}{0.10 + j0.50} = 0.3846 - j1.9231$$

$$Y_{11} = 0.3846 - j1.9231$$

$$Y_{22} = 0.3846 - j1.9231$$

$$Y_{12} = -0.3846 + j1.9231$$

$$Y_{21} = -0.3846 + j1.9231$$

$$Y_{bus} = \begin{bmatrix} 0.3846 - j1.9231 & -0.3846 + j1.9231 \\ -0.3846 + j1.9231 & 0.3846 - j1.9231 \end{bmatrix}$$

Determine the voltage at each bus for the specified load condition

STEP 2

Bus 1 : Slack bus. $V_1 = 1.0 \angle 0$ pu

$$Y_{bus} = \begin{bmatrix} 0.3846 - j1.9231 & -0.3846 + j1.9231 \\ -0.3846 + j1.9231 & 0.3846 - j1.9231 \end{bmatrix}$$

STEP 3

Select initial values $^{(0)}$ of all bus voltages:

Bus 2 is a Load bus. Choose $V_2^{(0)} = 1.0 \angle 0$ pu as initial estimate

Determine the voltage at each bus for the specified load condition

$$Y_{bus} = \begin{bmatrix} 0.3846 - j1.9231 & -0.3846 + j1.9231 \\ -0.3846 + j1.9231 & 0.3846 - j1.9231 \end{bmatrix}$$

STEP 4

Write voltage equations for every other bus in the system

Real and reactive power at Bus 2:

$$S_i = S_{iG} - S_{iL}$$

$$P_2 = -0.3 \text{ pu}; Q_2 = -0.2 \text{pu}$$

Voltage equation for bus 2

$$V_2^{(k+1)} = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^{(k)*}} - \left(Y_{21}V_1 \right) \right]$$

$$V_2^{(1)} = \frac{1}{0.3846 - j1.9231} \left[\frac{-0.30 + j0.2}{V_2^{(0)*}} - \left[(-0.3846 + j1.9231)V_1 \right] \right]$$

$$V_2^{(1)} = \frac{1}{1.9612\angle -78.8} \left[\frac{0.3603\angle 146.3}{V_2^{(0)*}} - \left[(1.9612\angle 101.3)1\angle 0 \right] \right]$$

$$V_2^{k+1} = \frac{1}{1.9612\angle-78.8}\left[\frac{0.3603\angle146.3}{V_2^{k*}} - \left[(1.9612\angle101.3)1\angle0\right]\right]$$

STEP 5: Calculate an updated estimate of the voltage at each load bus

Initial estimate $V_2^0 = 1.0\angle0$ pu

$$V_2^1 = \frac{1}{1.9612\angle-78.8}\left[\frac{0.3603\angle146.3}{V_2^{0*}} - \left[(1.9612\angle101.3)1\angle0\right]\right]$$

$$= \frac{1}{1.9612\angle-78.8}\left[\frac{0.3603\angle146.3}{1\angle0} - \left[(1.9612\angle101.3)1\angle0\right]\right]$$

$$= ?$$

Second estimate

$$V_2^2 = \frac{1}{1.9612\angle-78.8}\left[\frac{0.3603\angle146.3}{V_2^{1*}} - \left[(1.9612\angle101.3)1\angle0\right]\right]$$

$$= ?$$

$$V_2^3 = \frac{1}{1.9612\angle-78.8}\left[\frac{0.3603\angle146.3}{V_2^{2*}} - \left[(1.9612\angle101.3)1\angle0\right]\right]$$

$$= ?$$

Calculate for Fourth and fifth iteration etc…

STEP 6: compare the differences between the old and new estimates

If the magnitude of the voltage is barely changing, we consider this value is close enough to the correct answer. Iteration stops.

STEP 7: Confirm that the resulting solution is reasonable

$V_1 = 1.0\angle0$ pu

$V_2 = 0.8315\angle-8.994$ pu   assumed value for discussions

The phase angles differ by only 10°, these result appear reasonable

Calculate current flowing in the transmission

 line from Bus 1 to Bus 2

$$I_{1,2} = \frac{(V_1 - V_2)}{Z_{Line}}$$

$$= \frac{1\angle0 - 0.8315\angle-8.994}{0.10 + j0.50}$$

$$= 0.4333\angle-42.65$$

$V_1 = 1.0\angle 0$ pu

$I_{1,2} = 0.4333\angle - 42.65$

$V_2 = 0.8315\angle -8.994$ pu

Calculate power supplied by the transmission line to Bus 2

$$S_2 = V_2 I_{12}^*$$
$$= (0.8315\angle - 8.994)(0.4333\angle - 42.65)^*$$
$$= 0.2999 + j0.1997$$

This is almost equal to the power being consumed by the loads. Thus the

solution appears to be correct.

If the real and reactive power supplied by bus 2 is 0.3+j0.2, and

If the voltage on the slack bus is $1.0\angle 0$ pu,

Then the voltage at bus 2 will be V_2=0.8315$\angle$-8.994 pu (get this from your

calculation)

This voltage is correct only for the assumed conditions; if a different amount

of power were supplied, a different voltage V_2 will be calculated.

A simple three-bus power system with generation at buses 1 and 3. The voltage at bus 1 is $V_1=1.025\angle0°$. Voltage magnitude at bus 3 is fixed at 1.03 pu with real power generation of 300 MW. A load consisting of 400 MW and 200 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base. Line resistances and line charging suceptances are neglected.
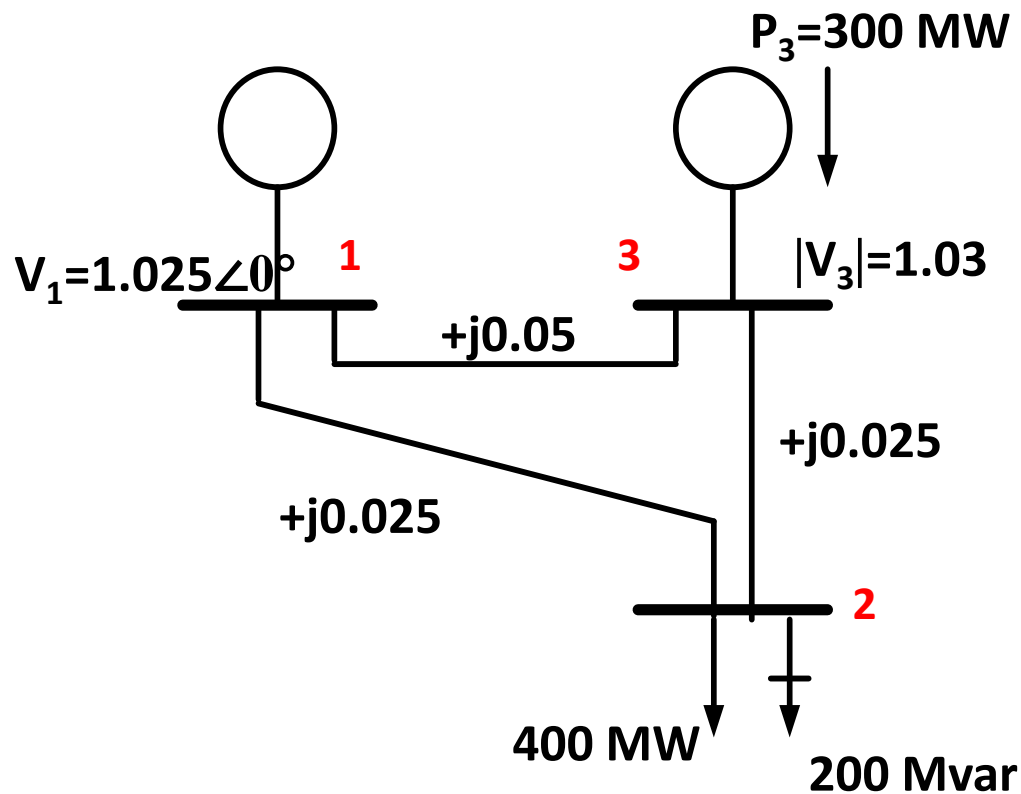
Using Gauss-Seidel method and initial estimates of $V_2^{(0)}=1.0+j0$ and $V_3^{(0)}=1.03+j0$ and keeping $|V_3|=1.03$ pu, determine the phasor values of V_2 and V_3. Stop after **two (2)** iterations.
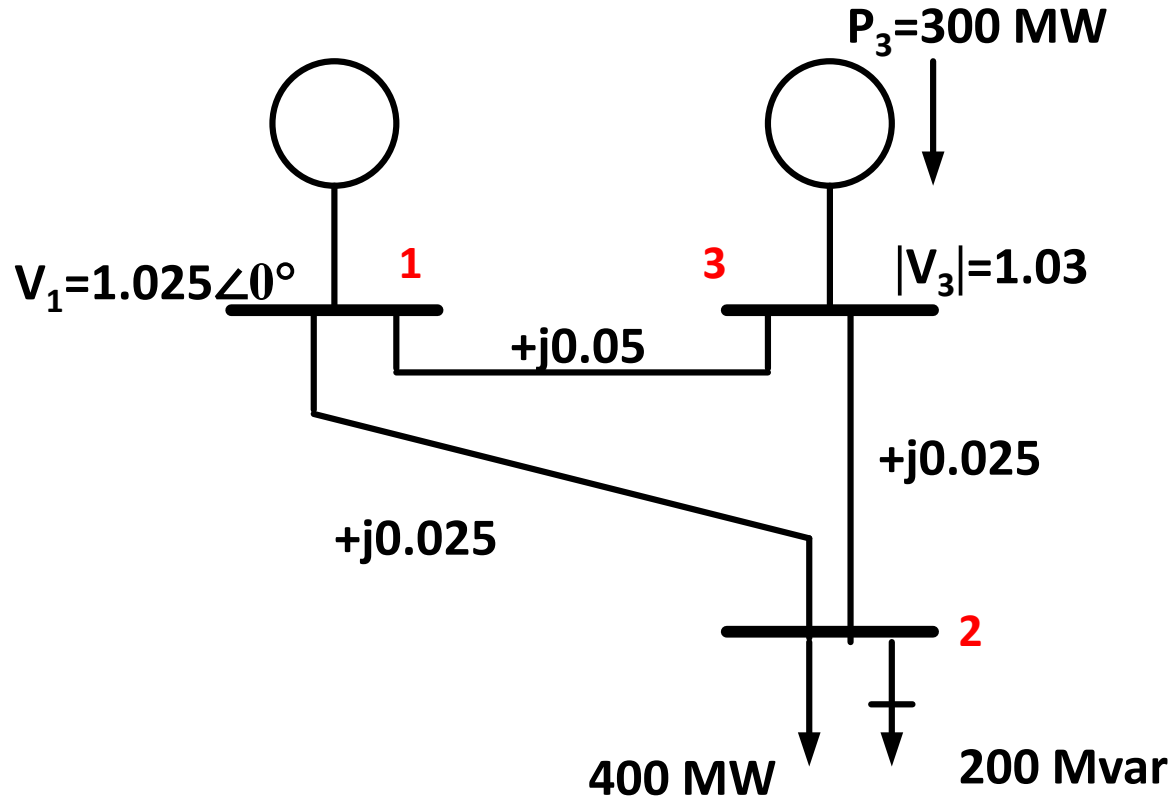
1. Calculate Line admittances

$$y_{12} = -j40,$$

$$y_{13} = -j20$$

$$y_{23} = -j40$$

2. Expressed load and generation in per units

$$S_2^{sch} = -\frac{(400 + j200)}{100} = -4.0 - j2.0 \quad pu$$

$$P_3^{sch} = \frac{300}{100} = 3.0 \quad pu$$

P₃=300 MW

V₁=1.025∠0°   **1**   **3**   **|V₃|=1.03**

+j0.05

+j0.025

+j0.025

2

400 MW   **200 Mvar**
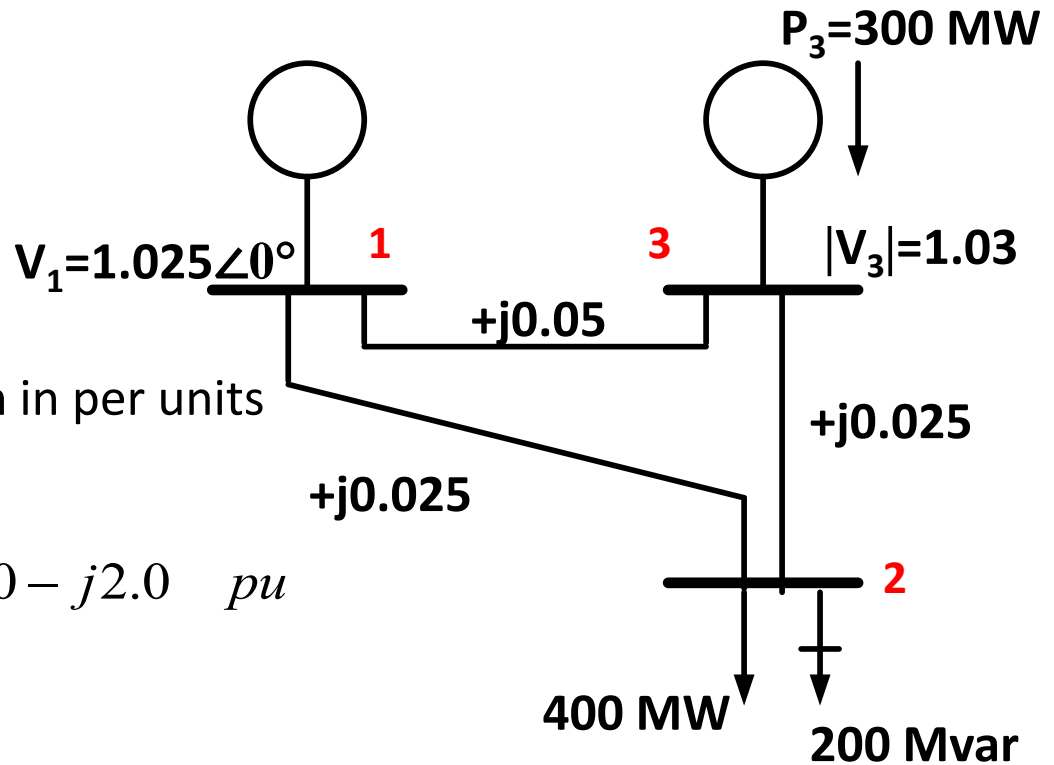
1. Bus 1 is taken as slack bus. Starting from an initial estimate V_2 and V_3 are

computed from equation:

$$V_2^{(1)} = 1.0025 + j0.05 \qquad V_2^{(0)} = 1.0 + j0.0$$

$$V_3^{(0)} = 1.03 + j0.0$$

Bus 3 is Voltage Control Bus

What you know about voltage control bus?

Known	Unknown
P, $\lvert V \rvert$, limits of Q	δ, Q

Determine Q

P$_3$=300 MW

V$_1$=1.025∠0°   **1**   **3**   **|V$_3$|=1.03**

+j0.05

+j0.025

+j0.025

2

400 MW
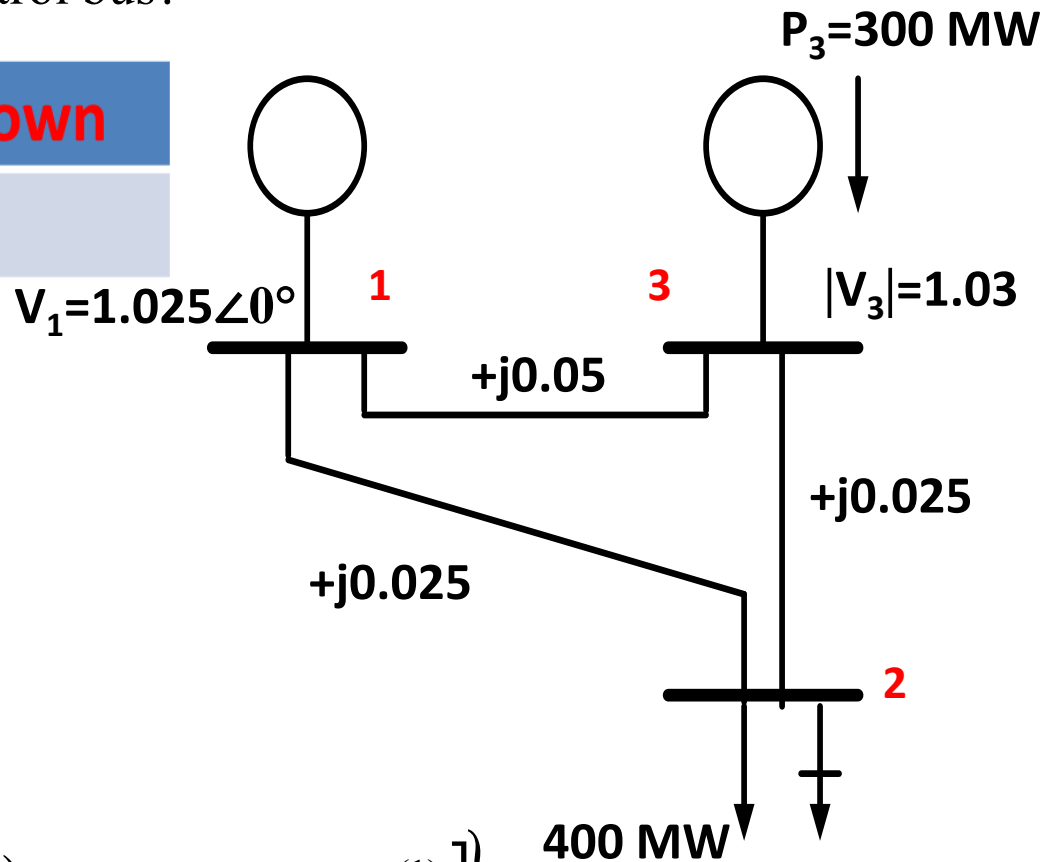
$$Q_3^{(1)} = -\Im\left\{V_3^{(0)^*}\left[V_3^0\left(y_{13}+y_{23}\right) - y_{13}V_1 - y_{23}V_2^{(1)}\right]\right\}$$

$$V_{c3}^{(1)} = \frac{\dfrac{S_3^{sch^*}}{V_3^{(0)^*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$V_{c3}^{(1)} = 1.03 - j0.0152$$

Since $|V_3|$ is held constant at 1.03 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained

$$V_2^{(2)} = 1.0001 - j0.0409$$

$$Q_3^{(2)} = 1.3671$$

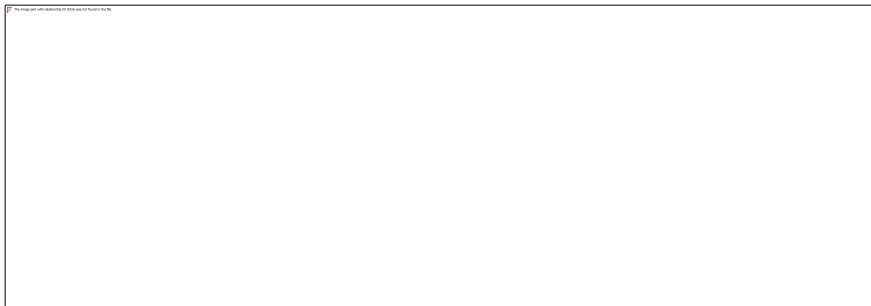$$V_{c3}^{(2)} = 1.0298 + j0.0216$$

$$V_3^{(2)} = 1.0298 + j0.0216$$

$$Q_3^{(1)} = -\Im\left\{V_3^{(0)^*}\left[V_3^0(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}\right]\right\}$$

$$V_{c3}^{(1)} = \frac{\dfrac{S_3^{sch^*}}{V_3^{(0)^*}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

If after several iterations the bus voltages converge to V_2=1.000571-j0.0366898 pu
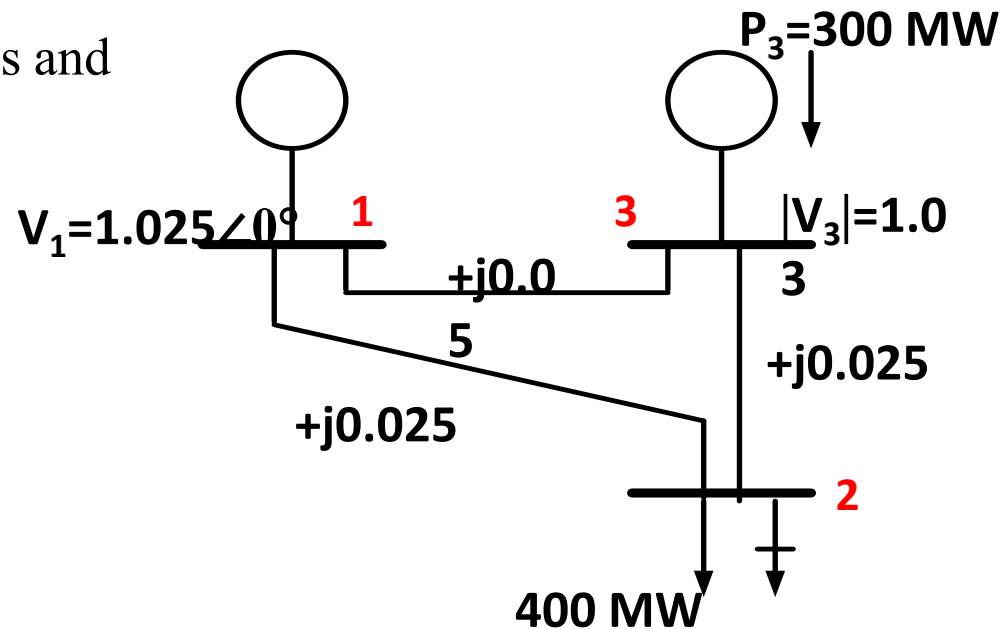
V_3=1.029706+j0.0246 pu

Determine the line flows and line losses and

the slack bus real and reactive power.

$$S_{Loss12} = 0.0 + j7.77$$

$$S_{Loss13} = 0.0 + j1.25$$

$$S_{Loss23} = 0.0 + j18.42$$

$$S_1 = 100\,MW + j90.51\,MVAR$$

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A major advantage of the Gauss-Seidel iterative method is that it is relatively stable and usually results in the right answer. The method however converges to a solution relatively slow and requiring a lot of computer time. Simple tricks that can be applied to speed up the convergence process:

- Applying updated bus voltage values immediately;

- Use of an acceleration factor

1. Applying updated bus voltage values immediately

Start using the new estimates of bus voltage as soon as they are calculated, (instead of waiting to the beginning of the next iteration). An illustrative five-bus system has the following bus power and voltage specifications:

Bus	P_D pu	$Q_D$pu	$P_G$pu	$Q_G$pu	Vpu	Bus type
1	0.5	0.25	unspecified	Unspecified	1.02+j0.0	Slack
2	0.0	0.0	1.0	Unspecified	1.02$\angle\delta$=?	P-V
3	0.25	0.1	0.0	0.0	Unspecified	P-Q
4	0.25	0.1	0.0	0.0	Unspecified	P-Q
5	0.25	0.1	0.0	0.0	Unspecified	P-Q

$$Y_{bus} = \begin{bmatrix} 2-j6 & -1+j3 & 0+j0 & 0+j0 & -1+j3 \\ -1+j3 & 3-j9 & -1+j3 & 0+j0 & -1+j3 \\ 0+j0 & -1+j3 & 2-j6 & -1+j3 & 0+j0 \\ 0+j0 & 0+j0 & -1+j3 & 2-j6 & -1+j3 \\ -1+j3 & -1+j3 & 0+j0 & -1+j3 & 3-j6 \end{bmatrix}$$

Show calculation for one iteration to solve the five PFEs using Gauss-Seidel method.

Bus	P_D pu	Q_D pu	P_G pu	Q_G pu	Vpu	Bus type
1	0.5	0.25	unspecified	Unspecified	1.02+j0.0	Slack
2	0.0	0.0	1.0	Unspecified	1.02∠δ=?	P-V
3	0.25	0.1	0.0	0.0	Unspecified	P-Q
4	0.25	0.1	0.0	0.0	Unspecified	P-Q
5	0.25	0.1	0.0	0.0	Unspecified	P-Q

$$Y_{bus} = \begin{bmatrix} 2-j6 & -1+j3 & 0+j0 & 0+j0 & -1+j3 \\ -1+j3 & 3-j9 & -1+j3 & 0+j0 & -1+j3 \\ 0+j0 & -1+j3 & 2-j6 & -1+j3 & 0+j0 \\ 0+j0 & 0+j0 & -1+j3 & 2-j6 & -1+j3 \\ -1+j3 & -1+j3 & 0+j0 & -1+j3 & 3-j6 \end{bmatrix}$$

From system data, $V_1 = 1.02 \angle 0$,

$$P_{G2} = 1.0 \;;$$

$$V_2 = 1.02$$

$$S_3 = P_3 + jQ_3 = -0.25 - j0.1;$$

$$S_4 = P_4 + jQ_4 = -0.25 - j0.1 \;;$$

$$S_5 = P_5 + jQ_5 = -0.25 - j0.1 \;.$$

Let us assume the following initial estimates:

$$V_3^0 = V_4^0 = V_5^0 = 1.0 \angle 0 \;;\; \delta_2 = 0$$

$$V_2^0 = 1.02 \angle 0$$

Bus 2 is the only P-V bus. Let us calculate Q_2^1

$$Q_2^1 = -\operatorname{Im}\left(V_2^{0*}\left(Y_{21}V_1 + Y_{22}V_2^0 + Y_{23}V_3^0 + Y_{24}V_4^0 + Y_{25}V_5^0\right)\right)$$
$$= 0.1224$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2^1}{V_2^{0*}} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= 1.016 + j0.0908 = 1.019\angle5.1°$$

We fix $\mathbf{V_2^1}$ to $\mathbf{1.02 \angle5.1°}$, the magnitude being set at the specified value with the phase angle kept as the calculated value.

The voltages at the load buses after the first iteration are:

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= 0.9804 + j0.0129 = 0.9805\angle0.75°$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1 - Y_{45}V_5^0\right]$$

$$= 0.9804 + j0.0129 = 0.9805\angle0.75°$$

$$V_5^1 = \frac{1}{Y_{55}} \left[\frac{P_5 - jQ_5}{V_5^{0*}} - Y_{51}V_1 - Y_{52}V_2^1 - Y_{53}V_3^1 - Y_{54}V_4^1 \right]$$

$$= 0.9812 + j0.000005 = 0.9812\angle -0.003°$$

Voltage at each bus is multiplied by some constant. The multiplier is called an acceleration factor (α), α is set at about 1.6 and cannot exceed 2.
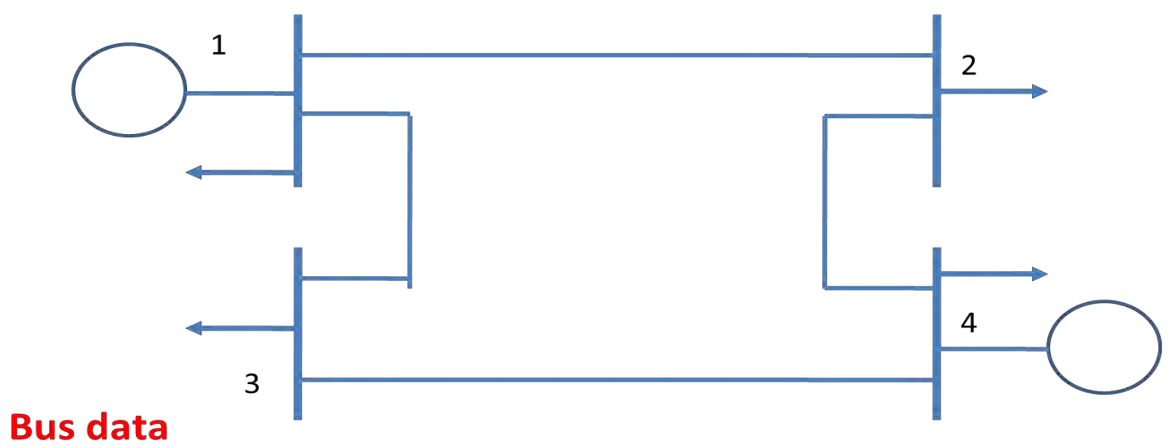
$$V_{2,acc}^1 = (1-\alpha)V_2^0 + \alpha V_2^1 = V_2^0 + \alpha\left(V_2^1 - V_2^0\right)$$

For bus i during iteration k

$$V_{i,acc}^k = (1-\alpha)V_{i,acc}^{k-1} + \alpha V_i^k = V_{i,acc}^{k-1} + \alpha\left(V_i^k - V_{i,acc}^{k-1}\right)$$

Line data

Base 100 MVA, 230 kV

bus to bus	Series Z (R+jX) (pu)	Series Y (G-jB) (pu)	Shunt Y (MVAr)	Y/2 (pu)
1-2	0.01008+j0.0504	3.8152629-j19.078144	10.25	0.05125
1-3	0.00744+j0.03720	5.169561-j25.847809	7.75	0.03875
2-4	0.00744+j0.03720	5.167561-j25847809	7.75	0.03875
3-4	0.01272+j0.06360	3.023705-j15.118528	12.75	0.06375

Bus data

bus	Generation (P,Q)(MVA)	Load (P,Q) MVA	V (pu)	Remarks
1	-, -	50, 30.99	$1\angle 0°$	Slack
2	0,0	170, 105.35	$1\angle 0°$	Load
3	0,0	200, 123.94	$1\angle 0°$	Load
4	318, -	80, 49.58	$1.02\angle 0°$	Voltage controlled

bus to bus	Series z (r+jx) (pu)	Series y (G-jB) (pu)	Shunt y (MVAr)	y/2 (pu)
1-2	0.01008+j0.0504	3.8152629-j19.078144	10.25	0.05125
1-3	0.00744+j0.03720	5.169561-j25.847809	7.75	0.03875
2-4	0.00744+j0.03720	5.167561-j25847809	7.75	0.03875
3-4	0.01272+j0.06360	3.023705-j15.118528	12.75	0.06375

Bus no.	1	2	3	4
1	$Y_{11} = ?$	$Y_{12}=?$	Y_{13}	Y_{14}
2	Y_{21}	Y_{22}	Y_{23}	Y_{24}
3	Y_{31}	Y_{32}	Y_{33}	Y_{34}
4	Y_{41}	Y_{42}	Y_{43}	Y_{44}

bus to bus	Series z (r+jx) (pu)	Series y (G-jB) (pu)	Shunt y (MVAr)	y/2 (pu)
1-2	0.01008+j0.0504	3.8152629-j19.078144	10.25	0.05125
1-3	0.00744+j0.03720	5.169561-j25.847809	7.75	0.03875
2-4	0.00744+j0.03720	5.167561-j25847809	7.75	0.03875
3-4	0.01272+j0.06360	3.023705-j15.118528	12.75	0.06375

Bus no.	1	2	3	4
1	**8.985190 - j44.835953**	**-3.815629 + j19.078144**	-5.169561 +j25.847809	0
2	-3.815629 + j19.078144	8.985190 – j44.835953	0	-5.169561 +j25.847809
3	-5.169561 +j25.847809	0	8.193267 – j40.863838	-3.023705 + j15.118528
4	0	-5.169561 +j25.847809	-3.023705 + j15.118528	8.193267 – j40.863838

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{j=1}^{i-j} Y_{ij}V_j - \sum_{j=i+1}^{N} Y_{ij}V_j \right]$$

$$V_2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_2^*} - \left(Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4 \right) \right]$$

$$V_3 = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{V_3^*} - \left(Y_{31}V_1 + Y_{32}V_2 + Y_{34}V_4 \right) \right]$$

$$V_4 = \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{V_4^*} - \left(Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 \right) \right]$$

$$V_2^{(1)} = \frac{1}{Y_{22}} \left| \frac{P_2 - jQ_2}{V_2^{(0)*}} - \left(Y_{21}V_1^{(0)} + Y_{23}V_3^{(0)} + Y_{24}V_4^{(0)} \right) \right|$$

$$= \frac{1}{Y_{22}} \left[\frac{-1.7 + j1.0535}{1.0} - 1.00(-3.815629 + j19.078144) - 1.02(-5.169561 + j25.847809) \right]$$

$$= \frac{1}{Y_{22}} \left[-1.7 + j1.0535 + 9.088581 - j45.442909 \right]$$

$$= 0.983564 - j0.032316$$

Using an acceleration factor of 1.6 gives

$$V_{2,acc}^1 = V_2^0 + \alpha\left(V_2^1 - V_2^0\right)$$
$$= 1 + 1.6[(0.983564 - j0.032316) - 1]$$
$$= 0.973703 - j0.051706 \quad \text{per unit}$$

Using $V_{2,acc}^1$ in similar calculations for bus 3 gives first iteration value

$$V_{3,acc}^1 = 0.953949 - j0.066708 \quad \text{per unit}$$

Next proceed with Voltage bus 4

bus	Generation (P,Q)(MVA)	Load (P,Q) MVA	V (pu)	Remarks
1	-, -	50, 30.99	1∠0°	Slack
2	0,0	170, 105.35	1∠0°	Load
3	0,0	200, 123.94	1∠0°	Load
4	318, -	80, 49.58	1.02∠0°	Voltage controlled

This is voltage control bus. What we know about voltage control buses?

Compute $Q_4{}^1$

Use this value to calculate $V_4{}^1$

Correct the magnitude of $V_4{}^1$ to be equal to 1.02 etc..

$$Q_4^1 = -\operatorname{Im}\left(V_4^{*0}\left(Y_{42}V_{2,acc}^1 + Y_{43}V_{3,acc}^1 + Y_{44}V_4^0\right)\right)$$

$$V_4^{(1)} = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4^1}{V_4^{0*}} - \left(Y_{42}V_{2,acc}^1 + Y_{43}V_{3,acc}^1\right)\right]$$

Exercise

In a two bus system, bus 1 is a slack bus with $V_1 = 1.0\angle 0$ pu. A load of 100 MW and 50 Mvar is taken from bus 2. The line impedance is $z_{12} = 0.12 + j0.16$ pu on a base of 100 MVA.

   a)   Using Gauss-Seidel method determine V_2. Perform 4 iteration.

   b)   Repeat using **Newton-Raphson**. Perform 2 iteration.

   c)   Explain how S_1 and real and reactive power loss in the line can be
          calculated.

NEWTON-RAPHSON POWER FLOW SOLUTION

- N-R method is mathematically superior to the Gauss-Seidel.

- Less prone to divergence with ill-conditioned problems.

- For large system it is more efficient and practical.

Current entering bus i is given by $I_i = \sum_{j=i}^{n} Y_{ij} V_j$

Expressing this equation in polar form gives

$$I_i = \sum_{j=i}^{n} |Y_{ij}||V_j| \angle (\theta_{ij} + \delta_j)$$

Complex power at bus i is

$$P_i - jQ_i = V_i^* I_i$$

$$P_i - jQ_i = (|V_i| \angle - \delta_i) \sum_{j=1}^{n} |Y_{ij}||V_j| \angle (\theta_{ij} + \delta_j)$$

Separating real and imaginary parts

1 equation per
voltage control
bus

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

2 equations
per load bus

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

We have two equations for each load bus (P and Q) and one equation for each voltage controlled bus (P).

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

Expanding these equations in **Taylor's series** and **neglecting all higher order** terms results in the following set of linear equations.

$$
\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P_2^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial P_2^{(k)}}{\partial \delta_n} & \dfrac{\partial P_2^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial P_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial P_n^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial P_n^{(k)}}{\partial \delta_n} & \dfrac{\partial P_n^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial P_n^{(k)}}{\partial |V_n|} \\ \hline \dfrac{\partial Q_2^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial Q_2^{(k)}}{\partial \delta_n} & \dfrac{\partial Q_2^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial Q_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial Q_n^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial Q_n^{(k)}}{\partial \delta_n} & \dfrac{\partial Q_n^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial Q_n^{(k)}}{\partial |V_n|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \hline \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix}
$$

Jacobian matrix

$$
\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix}
=
\begin{bmatrix}
\dfrac{\partial P_2^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial P_2^{(k)}}{\partial \delta_n} & \dfrac{\partial P_2^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial P_2^{(k)}}{\partial |V_n|} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\dfrac{\partial P_n^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial P_n^{(k)}}{\partial \delta_n} & \dfrac{\partial P_n^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial P_n^{(k)}}{\partial |V_n|} \\
\dfrac{\partial Q_2^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial Q_2^{(k)}}{\partial \delta_n} & \dfrac{\partial Q_2^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial Q_2^{(k)}}{\partial |V_n|} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\dfrac{\partial Q_n^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial Q_n^{(k)}}{\partial \delta_n} & \dfrac{\partial Q_n^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial Q_n^{(k)}}{\partial |V_n|}
\end{bmatrix}
\begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix}
$$

In this equation, Bus 1 is assumed to be the slack bus

$$
\begin{bmatrix}
\Delta P_2^{(k)} \\
\vdots \\
\dfrac{\Delta P_n^{(k)}}{\Delta Q_2^{(k)}} \\
\vdots \\
\Delta Q_n^{(k)}
\end{bmatrix}
=
\left[
\begin{array}{ccc|ccc}
\dfrac{\partial P_2^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial P_2^{(k)}}{\partial \delta_n} & \dfrac{\partial P_2^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial P_2^{(k)}}{\partial |V_n|} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\dfrac{\partial P_n^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial P_n^{(k)}}{\partial \delta_n} & \dfrac{\partial P_n^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial P_n^{(k)}}{\partial |V_n|} \\
\hline
\dfrac{\partial Q_2^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial Q_2^{(k)}}{\partial \delta_n} & \dfrac{\partial Q_2^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial Q_2^{(k)}}{\partial |V_n|} \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\dfrac{\partial Q_n^{(k)}}{\partial \delta_2} & \cdots & \dfrac{\partial Q_n^{(k)}}{\partial \delta_n} & \dfrac{\partial Q_n^{(k)}}{\partial |V_2|} & \cdots & \dfrac{\partial Q_n^{(k)}}{\partial |V_n|}
\end{array}
\right]
\begin{bmatrix}
\Delta \delta_2^{(k)} \\
\vdots \\
\dfrac{\Delta \delta_n^{(k)}}{\Delta |V_2^{(k)}|} \\
\vdots \\
\Delta |V_n^{(k)}|
\end{bmatrix}
$$

The Jacobian matrix gives the linearized relationship between small changes in voltage angle $\Delta \delta_i^{(k)}$ and voltage magnitude $\Delta |V_i^{(k)}|$ with the small changes in real and reactive power $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$.

Elements of the Jacobian matrix are the partial derivatives of P and Q, evaluated at $\Delta \delta_i^{(k)}$ and $\Delta |V_i^{(k)}|$

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

In short form

P mismatches for all PQ and PV busbars

Q mismatches for all PQ busbars

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

δ corrections for all PQ and PV busbars

V corrections for all PQ busbars

$\mathbf{J_1}$= (n-1)×(n-1) $\mathbf{J_2}$ =(n-1)×(n-1-m)

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$\mathbf{J_3}$ =(n-1-m)×(n-1) $\mathbf{J_4}$ =(n-1-m)×(n-1-m)

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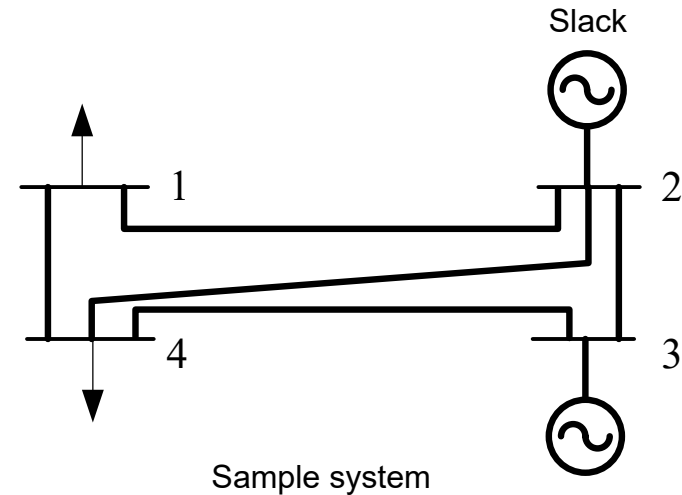For voltage-controlled buses the voltage magnitudes are known

If **m** buses are **voltage-controlled**, **m equations** involving **ΔQ and ΔV** and corresponding **columns of the Jacobian matrix are eliminated**
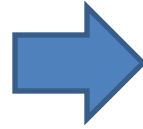
■ **n-1 real power constraints** and **n-1-m reactive power constraints**

∴ **Jacobian matrix** is of order **(2n-2-m)×(2n-2-m)**

As an i.e., the Jacobian matrix eq. for the four busbar system is given as eq. below

$$
\begin{bmatrix} \Delta P_1 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta Q_1 \\ \Delta Q_4 \end{bmatrix} = \begin{bmatrix} H_{11} & 0 & H_{14} & N_{11} & N_{14} \\ 0 & H_{33} & H_{34} & 0 & N_{34} \\ H_{41} & H_{43} & H_{44} & N_{41} & N_{44} \\ J_{11} & 0 & J_{14} & L_{11} & L_{14} \\ J_{41} & J_{43} & J_{44} & L_{41} & L_{44} \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta_1 \\ \Delta \delta_3 \\ \Delta \delta_4 \\ \Delta V_1 / V_1 \\ \Delta V_4 / V_4 \end{bmatrix}
$$

Slack

Sample system

$$
P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)
$$

Diagonal and off-diagonal elements of J_1:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$$

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

Diagonal and off-diagonal elements of J_2:

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\theta_{ii} + \sum_{j \neq i}^{n} |V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

Diagonal and off-diagonal elements of J$_3$:

$$\frac{\partial Q_i}{\partial |\delta_i|} = \sum_{j \neq i}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

Diagonal and off-diagonal elements of J$_4$:

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{j \neq i}^{n} |V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$$

The terms $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are the difference between the scheduled and calculated values (power residuals)

$$\Delta P_i^{(k)} = P_i - P_i^{(k)}$$

$$\Delta Q_i^{(k)} = Q_i - Q_i^{(k)}$$

The new estimates for bus voltages

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta \delta_i^{(k)}$$

$$\left| V_i^{(k+1)} \right| = \left| V_i^{(k)} \right| + \Delta \left| V_i^{(k)} \right|$$

Procedure for power flow solution by N-R method

For load buses:

P_i and Q_i are specified

Set voltage magnitude and δ equal to slack bus values

$$\left| V_i^{(0)} \right| = 1.0 \qquad \delta_i^{(0)} = 0.0$$

Calculate $P_i^{(k)}$, $Q_i^{(k)}$, $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ using the equations:

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad \Delta P_i^{(k)} = P_i - P_i^{(k)}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad \Delta Q_i^{(k)} = Q_i - Q_i^{(k)}$$

Procedure for power flow solution by N-R method

For voltage-controlled buses:

Calculate $P_i^{(k)}$ and $\Delta P_i^{(k)}$ using

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\Delta P_i^{(k)} = P_i - P_i^{(k)}$$

The Jacobian matrix (J_1, J_2, J_3 and J_4)

$$J_1 \begin{cases} \dfrac{\partial P_i}{\partial \delta_i} = \sum_{j \neq 1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \\[3mm] \dfrac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i \end{cases}$$

The Jacobian matrix (J_1, J_2, J_3 and J_4)

J_2
$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}|\cos\theta_{ii} + \sum_{j \neq 1}^{n}|V_j||V_{ij}||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

Calculate the Jacobian matrix (J_3 and J_4)

J_3
$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq 1}^{n}|V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

J_4
$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{j \neq 1}^{n}|V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \quad j \neq i$$

Solve the linear simultaneous equation

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad \therefore \quad \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Compute new voltage magnitudes and phase angles
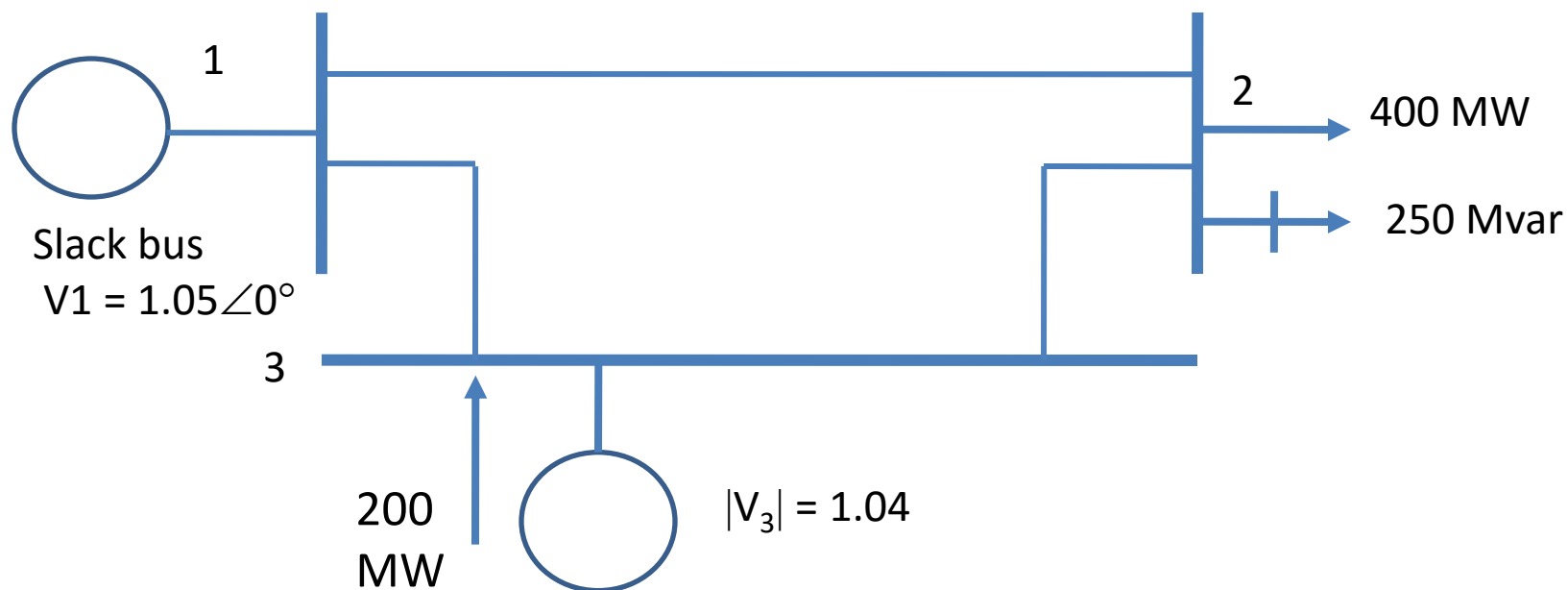
$$\delta_i^{(k+1)} = \delta_i^k + \Delta \delta_i^{(k)} \qquad \left| V_i^{(k+1)} \right| = \left| V_i^k \right| + \Delta \left| V_i^{(k)} \right|$$

Continue until the residuals $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are less than the specified accuracy

$$\left| \Delta P_i^{(k)} \right| = P_i^{sch} - P_i^{(k)}$$

$$\left| \Delta Q_i^{(k)} \right| = Q_i^{sch} - Q_i^{(k)}$$

$$\left| \Delta P_i^{(k)} \right| \leq \varepsilon$$

$$\left| \Delta Q_i^{(k)} \right| \leq \varepsilon$$

Consider a three bus power system

Line data

Base 100 MVA

| bus to bus	Series Z (R+jX) (pu)	Series Y (G-jB) (pu)
1-2	0.02+j0.04	10-j20
1-3	0.01+j0.03	10-j30
2-3	0.0125+j0.025	16-j32
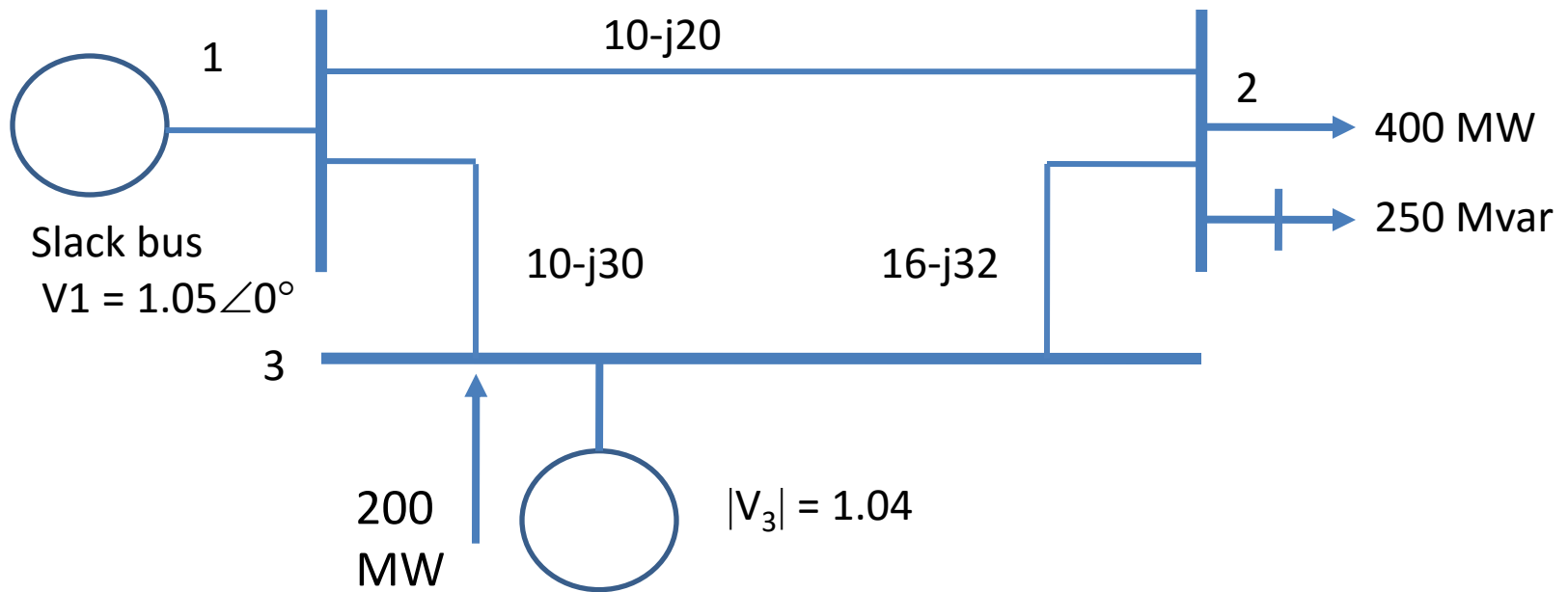
Y$_{bus}$ matrix

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

Convert to polar

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -68.1986 & 22.36068\angle 116.5651 & 31.62278\angle 108.4349 \\ 22.36068\angle 116.5651 & 58.13777\angle -63.4349 & 35.77709\angle 116.5651 \\ 31.62278\angle 108.4349 & 35.77709\angle 116.5651 & 67.23095\angle -67.4902 \end{bmatrix}$$
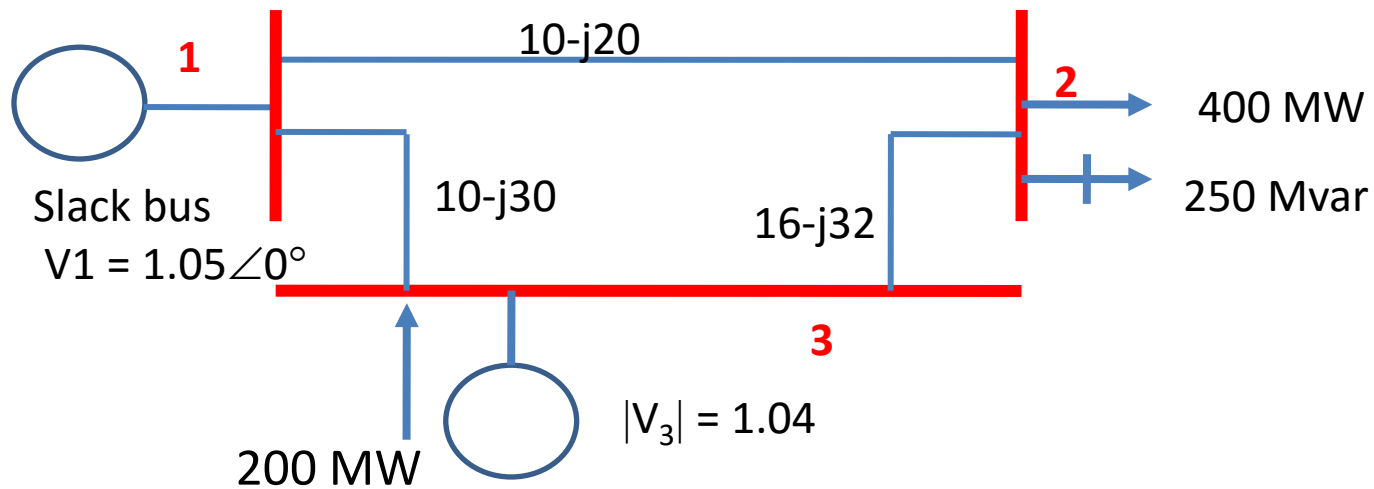
Slack bus
V1 = 1.05∠0°

10-j20

1

2

400 MW

250 Mvar

10-j30

16-j32

3

|V₃| = 1.04

200 MW
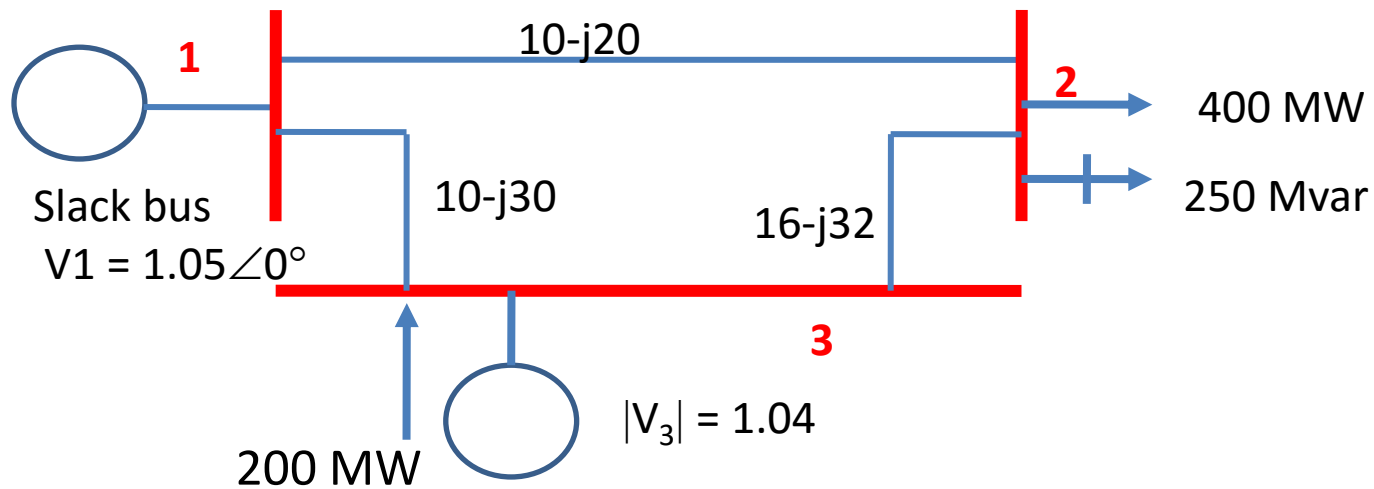
$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

For bus 2

$$P_2 = \sum_{j=1}^{3} |V_2||V_j||Y_{2j}| \cos(\theta_{2j} - \delta_2 + \delta_j)$$

$$P_2 = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1)$$

$$+ |V_2||V_2||Y_{22}| \cos(\theta_{22} - \delta_2 + \delta_2) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$
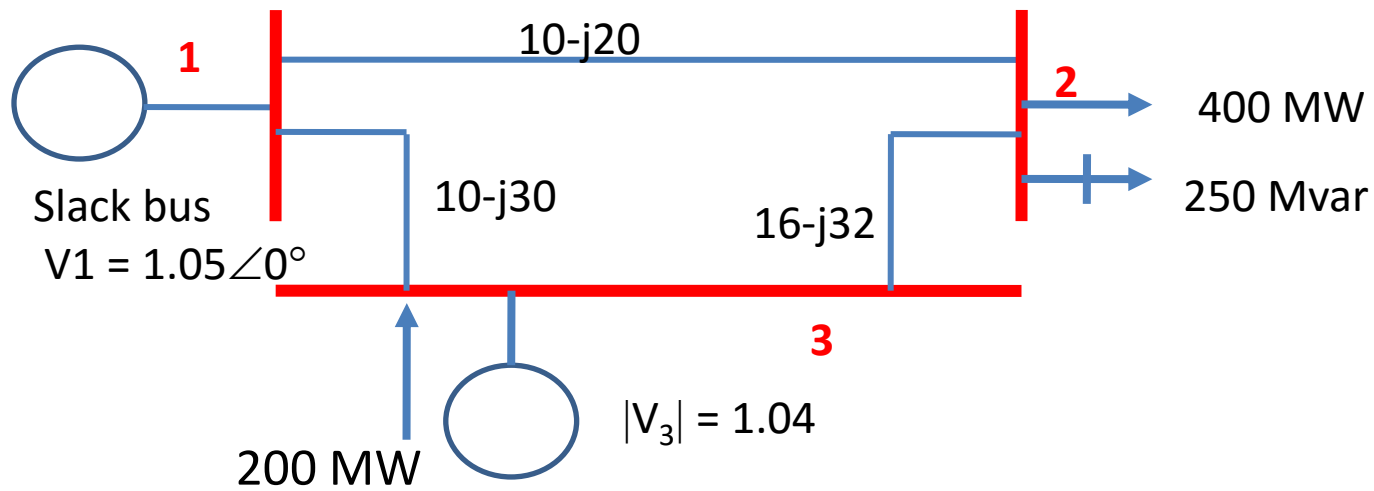
Slack bus
V1 = 1.05∠0°

|V₃| = 1.04

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

For bus 3

$$P_3 = \sum_{j=1}^{3} |V_3||V_j||Y_{3j}| \cos(\theta_{3j} - \delta_3 + \delta_j)$$

$$P_3 = |V_3||V_1||Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1)$$
$$+ |V_3||V_2||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3||V_3||Y_{33}| \cos(\theta_{33} - \delta_3 + \delta_3)$$

Slack bus
V1 = 1.05∠0°

10-j20

1

2

400 MW

250 Mvar

10-j30

16-j32

3

200 MW

$|V_3| = 1.04$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

For bus 2

$$Q_2 = \sum_{j=1}^{3} |V_2||V_j||Y_{2j}| \sin(\theta_{2j} - \delta_2 + \delta_j)$$

$$Q_2 = -|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1)$$
$$-|V_2||V_2||Y_{22}| \sin(\theta_{22} - \delta_2 + \delta_2) - |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

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Determine elements of Jacobian matrix

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1)$$
$$+ |V_2||V_2||Y_{22}|\cos(\theta_{22} - \delta_2 + \delta_2) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1)$$
$$+ |V_2|^2|Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1)$$
$$+ 2|V_2||Y_{22}|\cos\theta_{22} + |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

Determine elements of Jacobian matrix

$$P_3 = |V_3||V_1||Y_{21}|\cos(\theta_{31} - \delta_3 + \delta_1)$$
$$+ |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3||V_3||Y_{33}|\cos(\theta_{33} - \delta_3 + \delta_3)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1)$$
$$+ |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3|^2|Y_{33}|\cos\theta_{33}$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2)$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1)$$

$$-|V_2||V_2||Y_{22}|\sin(\theta_{22} - \delta_2 + \delta_2) - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$
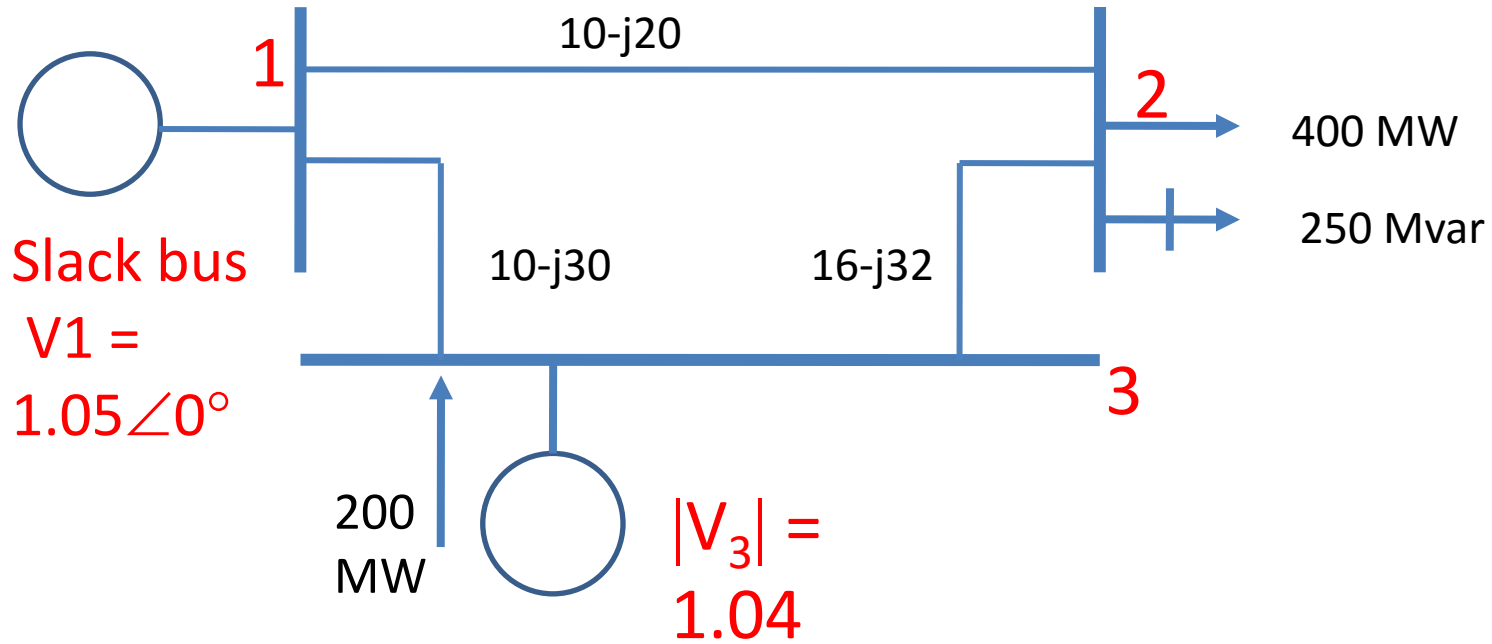
$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1)$$

$$-|V_2|^2|Y_{22}|\sin\theta_{22} - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1)$$

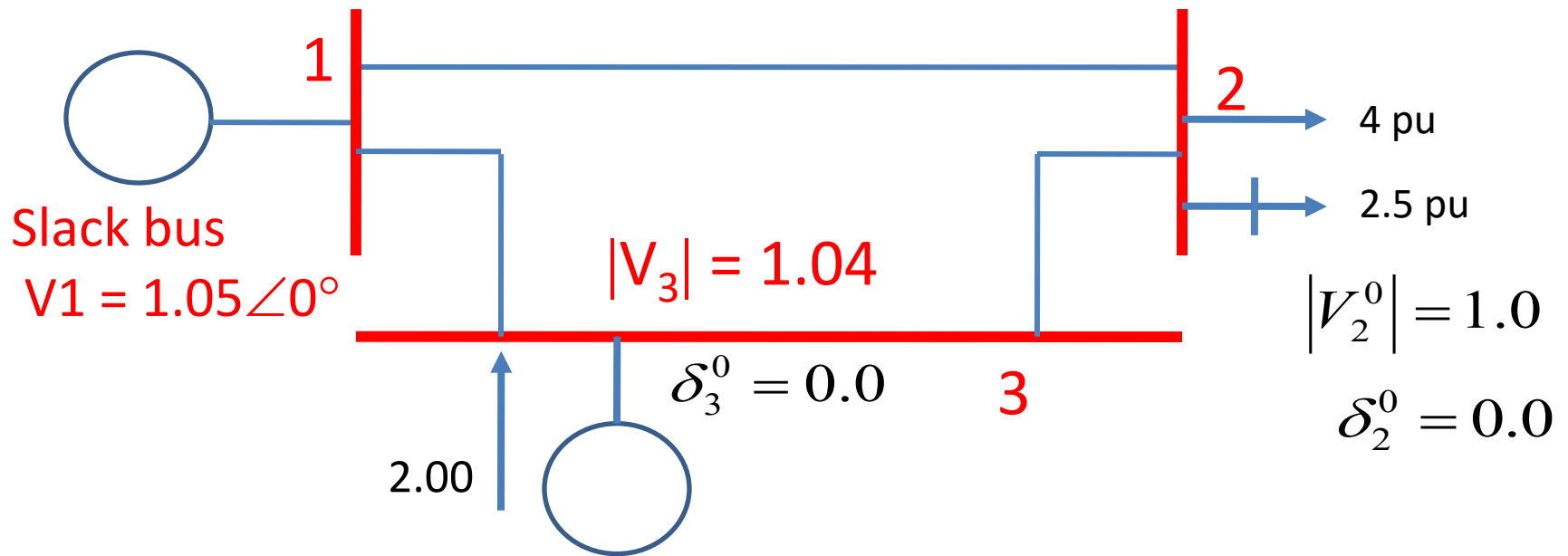$$-2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Initial values

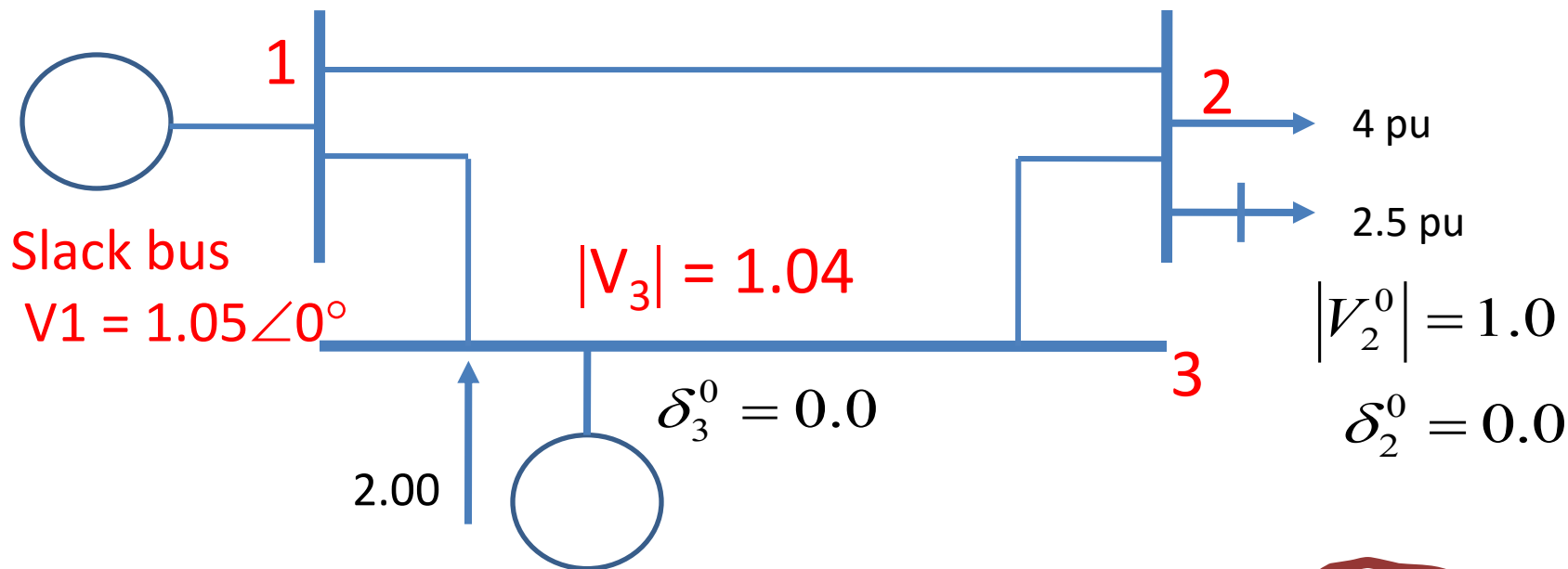$$S_2 = -\frac{(400 + j250)}{100} = -4.0 - j2.5$$

$$P_3 = \frac{200}{100} = 2.0$$

$$\left|V_2^0\right| = 1.0 \qquad \delta_2^0 = 0.0 \qquad \delta_3^0 = 0.0$$

Slack bus
V1 = 1.05∠0°

$|V_3| = 1.04$

$\delta_3^0 = 0.0$

3

1

2

4 pu

2.5 pu

$\left|V_2^0\right| = 1.0$

$\delta_2^0 = 0.0$

2.00

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -68.1986 & 22.36068\angle 116.5651 & 31.62278\angle 108.4349 \\ 22.36068\angle 116.5651 & 58.13777\angle -63.4349 & 35.77709\angle 116.5651 \\ 31.62278\angle 108.4349 & 35.77709\angle 116.5651 & 67.23095\angle -67.4902 \end{bmatrix}$$

$$P_2^0 = \left|V_2^0\right|\left|V_1\right|\left|Y_{21}\right|\cos(\theta_{21} - \delta_2^0 + \delta_1)$$

$$+ \left|V_2^0\right|^2\left|Y_{22}\right|\cos\theta_{22} + \left|V_2^0\right|\left|V_3\right|\left|Y_{23}\right|\cos(\theta_{23} - \delta_2^0 + \delta_3^0)$$

$$= -1.14$$

Evaluating the Jacobian Matrix

V1 = $1.05 \angle 0°$ $\left| V_2^0 \right| = 1.0$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$|V_3|$ = 1.04 $\delta_3^0 = 0.0$ $\delta_2^0 = 0.0$

$$Y_{bus} = \begin{bmatrix} 53.85165 \angle -68.1986 & 22.36068 \angle 116.5651 & 31.62278 \angle 108.4349 \\ 22.36068 \angle 116.5651 & 58.13777 \angle -63.4349 & 35.77709 \angle 116.5651 \\ 31.62278 \angle 108.4349 & 35.77709 \angle 116.5651 & 67.23095 \angle -67.4902 \end{bmatrix}$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$
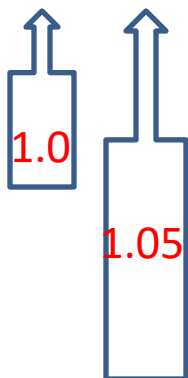
1.0

1.05

$$\frac{\partial P_2}{\partial \delta_2}^0 = 54.28$$

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Evaluating the Jacobian Matrix

$$
\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}
$$

$$
\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3) = -33.28
$$

$$
\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1)
$$

$$
+ 2|V_2||Y_{22}| \cos \theta_{22} + |V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)
$$

$$
= 24.86
$$

Evaluating the Jacobian Matrix

$$
\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}
$$

$$
\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) = -33.28
$$

$$
\frac{\partial P_3}{\partial \delta_3} = |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2)
$$

$$
= 66.04
$$

$$
\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) = -16
$$

Evaluating the Jacobian Matrix

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$= -27.14$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) = 66.04$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1)$$

$$- 2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$= 49.72$$

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad \boxed{\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} = \,?}$$

$$\frac{\partial P_2}{\partial \delta_2} = 54.28 \qquad \frac{\partial P_2}{\partial \delta_3} = -33.28 \qquad \frac{\partial P_2}{\partial |V_2|} = 24.86$$

$$\frac{\partial P_3}{\partial \delta_2} = -33.28 \qquad \frac{\partial P_3}{\partial \delta_3} = 66.04 \qquad \frac{\partial P_3}{\partial |V_2|} = -16$$

$$\frac{\partial Q_2}{\partial \delta_2} = -27.14 \qquad \frac{\partial Q_2}{\partial \delta_3} = 66.04 \qquad \frac{\partial Q_2}{\partial |V_2|} = 49.72$$

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} = \left[\begin{array}{cc|c} 54.28 & -33.28 & 24.86 \\ -33.28 & 66.04 & -16.64 \\ \hline -27.14 & 16.64 & 49.72 \end{array} \right]$$

Solve the linear simultaneous equation

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad \therefore \quad \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Compute new voltage magnitudes and phase angles
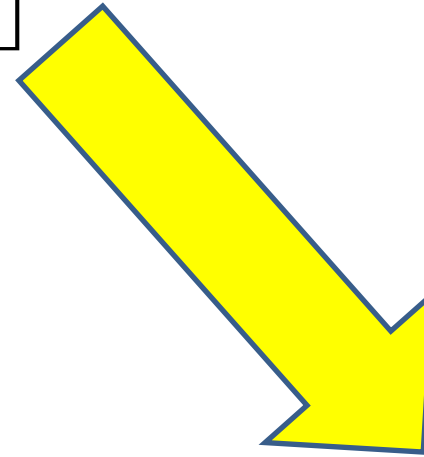
$$\delta_i^{(k+1)} = \delta_i^k + \Delta \delta_i^{(k)} \qquad \left| V_i^{(k+1)} \right| = \left| V_i^k \right| + \Delta \left| V_i^{(k)} \right|$$
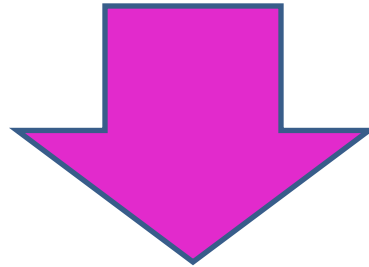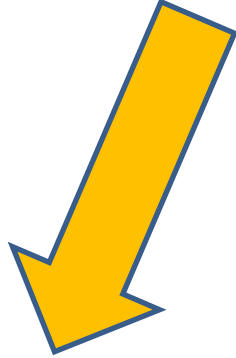
Continue until the residuals $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are less than the specified accuracy

$$\left| \Delta P_i^{(k)} \right| = P_i^{sch} - P_i^{(k)} \qquad \left| \Delta P_i^{(k)} \right| \le \varepsilon$$

$$\left| \Delta Q_i^{(k)} \right| = Q_i^{sch} - Q_i^{(k)} \qquad \left| \Delta Q_i^{(k)} \right| \le \varepsilon$$

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$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$

$$\begin{bmatrix} -2.86 \\ 1.4384 \\ -0.22 \end{bmatrix} = \begin{bmatrix} 54.28 & -33.28 & 24.86 \\ -33.28 & 66.04 & -16.64 \\ -27.14 & 16.64 & 49.72 \end{bmatrix} \begin{bmatrix} \Delta \delta_2^0 \\ \Delta \delta_3^0 \\ \Delta |V_2^0| \end{bmatrix}$$

?
How?

$$\Delta \delta_2^0 = -0.0453$$

$$\Delta \delta_3^0 = -0.0077$$

$$\Delta |V_2^0| = -0.026548$$

$$\Delta\delta_2^0 = -2.5934°$$

$$\Delta\delta_3^0 = -0.44221°$$

$$\Delta\left|V_2^0\right| = -0.026548$$

$$\boxed{\delta_i^{(k+1)} = \delta_i^k + \Delta\delta_i^{(k)}}$$

$$\boxed{\left|V_i^{(k+1)}\right| = \left|V_i^k\right| + \Delta\left|V_i^{(k)}\right|}$$

The new bus voltages in the first iteration

$$\delta_2^1 = 0 + (-2.5934) = -2.5934°$$

$$\delta_3^1 = 0 + (-0.44221) = -0.44221°$$

$$\left|V_2^1\right| = 1 + (-0.026548) = 0.97345$$

For the second iteration

$$\begin{bmatrix} -0.099218 \\ 0.021715 \\ -0.050914 \end{bmatrix} = \begin{bmatrix} 51.724675 & -31.765618 & 21.302567 \\ -32.981642 & 65.656383 & -15.379086 \\ -28.538577 & 17.402838 & 48.103589 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2^1| \end{bmatrix}$$

$$\Delta\delta_2^1 = -0.10023$$

$$\Delta\delta_3^1 = -0.05641$$

$$\Delta|V_2^0| = -0.001767$$

The new bus voltages in the 2nd iteration

$$\delta_2^2 = -2.5934 + (-0.10023) = -2.69363°$$

$$\delta_3^2 = -0.44221 + (-0.05641) = -0.49862°$$

$$|V_2^2| = 0.97345 + (-0.001767) = 0.971684$$

For the third iteration

$$\begin{bmatrix} -0.000216 \\ 0.000038 \\ -0.000143 \end{bmatrix} = \begin{bmatrix} 51.596701 & -31.693866 & 21.1474477 \\ -32.933865 & 65.597585 & -15.351628 \\ -28.548205 & 17.396932 & 47.954870 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^2 \\ \Delta\delta_3^2 \\ \Delta|V_2^2| \end{bmatrix}$$

$$\Delta\delta_2^2 = -0.00217724°$$
$$\Delta\delta_3^2 = -0.00013751°$$
$$\Delta|V_2^2| = -0.0000044$$

The new bus voltages in the 3rd iteration

$$\delta_2^3 = -2.6963°$$
$$\delta_3^3 = 0.49876°$$
$$|V_2^3| = 0.97168$$

The solution is assumed to converge after 3 iterations with a maximum power mismatched of 2.5×10^{-4}

$$Q_3 = 1.4617 \quad pu$$

$$V_2 = 0.97168 \angle -2.696°$$

$$P_1 = 2.1842 \quad pu$$

$$V_3 = 1.04 \angle -0.4988°$$

$$Q_1 = 1.4085 \quad pu$$

Exercise

In a two bus system, bus 1 is a slack bus with V_1=1.0∠0 pu. A load of 100 MW and 50 Mvar is taken from bus 2. The line impedance is z_{12} = 0.12+j0.16 pu on a base of 100 MVA. Using Gauss-Seidel method determine V_2. Continue until converge. How many iteration? Repeat using Newton-Raphson. Perform until convergence. How many iteration? Discuss/compare between the two solutions. Explain how S_1 and real and reactive power loss in the line can be calculated.

Solution with N-R

$V_1 = 1.0 \angle 0$

1 $z_{12} = 0.12 + j0.16$ **2**

100 MW
50 Mvar

$y_{12} = 3 - j4$

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13 & 5\angle 126.87 \\ 5\angle 126.87 & 5\angle -53.13 \end{bmatrix}$$

$y_{12} = ?$

$[Y_{bus}] = ?$

$V1 = 1.0 \angle 0$

$y_{12} = 3 - j4$

100 MW
50 Mvar

1   2

Power flow equations in polar forms:

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$
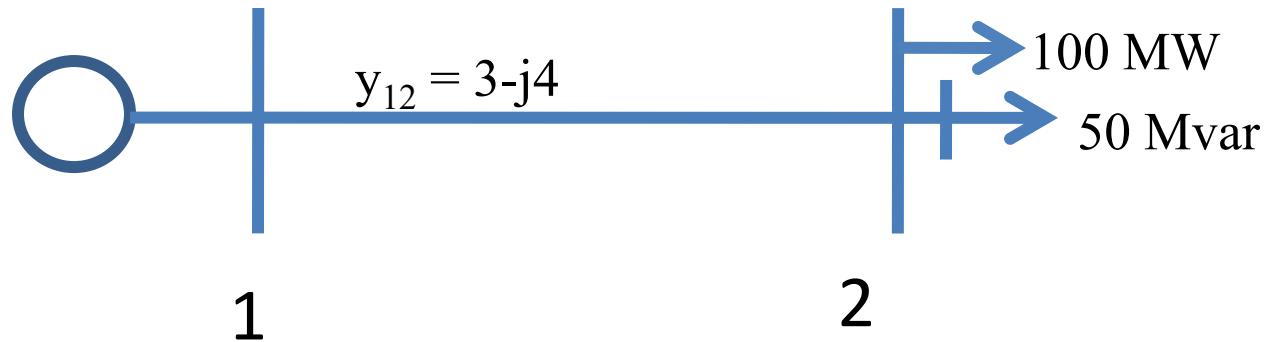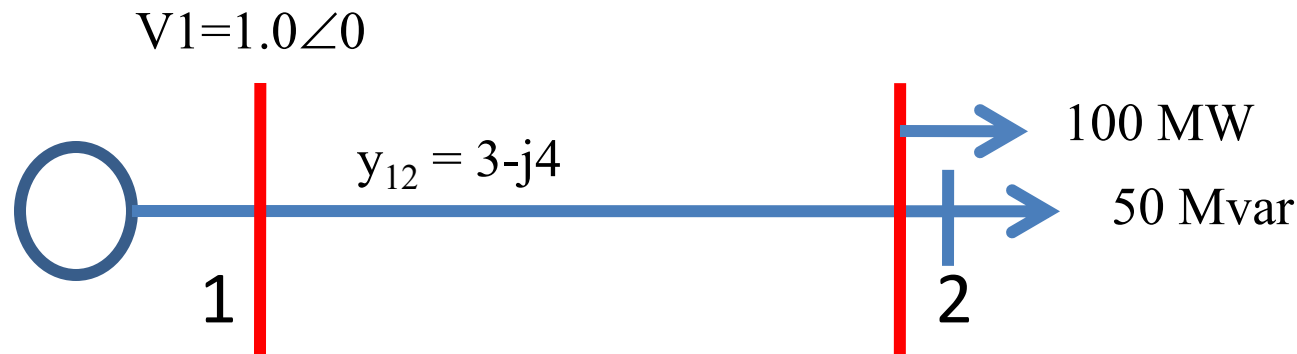
V1=1.0∠0

y₁₂ = 3-j4 → $y_{12} = 3 - j4$

1

2

100 MW

50 Mvar

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13 & 5\angle 126.87 \\ 5\angle 126.87 & 5\angle -53.13 \end{bmatrix}$$

At bus 2

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$P_2 = \sum_{j=1}^{n} |V_2||V_j||Y_{2j}| \cos(\theta_{2j} - \delta_2 + \delta_j)$$

$$= |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_2||Y_{22}| \cos(\theta_{22} - \delta_2 + \delta_2)$$

$$= |V_2||V_1| 5 \cos(126.87 - \delta_2 + \delta_1) + |V_2||V_2| 5 \cos(-53.13)$$

$$= |V_2||V_1| 5 \cos(126.87 - \delta_2 + \delta_1) + |V_2|^2 5 \cos(-53.13)$$

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V1=1.0∠0

y₁₂ = 3-j4
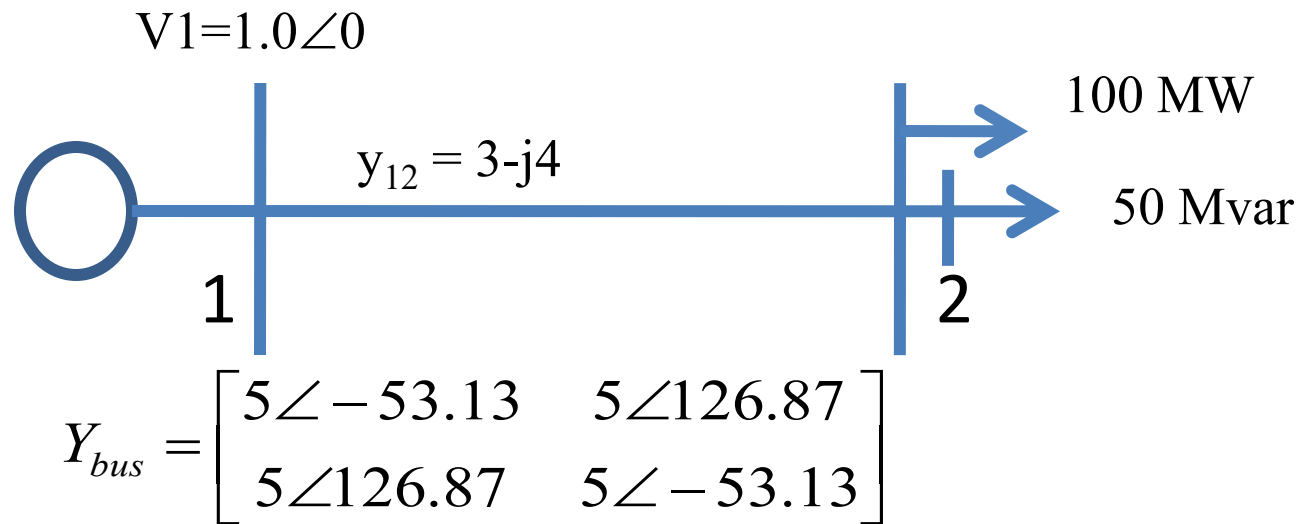
100 MW

50 Mvar

1   2

$$Y_{bus} = \begin{bmatrix} 5\angle -53.13 & 5\angle 126.87 \\ 5\angle 126.87 & 5\angle -53.13 \end{bmatrix}$$

At bus 2

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

$$Q_2 = -\sum_{j=1}^{n} |V_2||V_j||Y_{2j}| \sin(\theta_{2j} - \delta_2 + \delta_j)$$

$$= -|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2||V_2||Y_{22}| \sin(\theta_{22} - \delta_2 + \delta_2)$$

$$= -|V_2||V_1| 5\sin(126.87 - \delta_2 + \delta_1) - |V_2||V_2| 5\cos(-53.13)$$

$$= -|V_2||V_1| 5\sin(126.87 - \delta_2 + \delta_1) - |V_2|^2 5\sin(-53.13)$$

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Partial derivatives of P_2 w.r.t. δ_2 and $|V_2|$

$$P_2 = |V_2||V_1|5\cos(126.87 - \delta_2 + \delta_1) + |V_2|^2 5\cos(-53.13)$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1|5\sin(126.87 - \delta_2 + \delta_1)$$

$$\frac{\partial P_2}{\partial |V_2|} = 5|V_1|\cos(126.87 - \delta_2 + \delta_1) + 2|V_2|5\cos(-53.13)$$

Partial derivatives Q_2 w.r.t. δ_2 and $|V_2|$

$$Q_2 = -|V_2||V_1|5\sin(126.87 - \delta_2 + \delta_1) - |V_2|^2 5\sin(-53.13)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1|5\cos(126.87 - \delta_2 + \delta_1)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -5|V_1|\sin(126.87 - \delta_2 + \delta_1) - 10|V_2|\sin(-53.13)$$

Expressed Load in p.u.

$$S_2 = -\frac{(100 + j50)}{100} = -1.0 - j0.5$$

Initial values? Slack bus voltage is $V_1 = 1.0 \angle 0$ pu.

Initial estimates :

$P_2 = ?; Q_2 =?$ $\left|V_2^0\right| = 1.0$ $\delta_2^0 = 0.0$

$$P_2^0 = \left|V_2^0\right|\left|V_1\right|5\cos(126.87 - \delta_2^0 + \delta_1) + \left|V_2^0\right|^2 5\cos(-53.13)$$

$$= 1 \times 1 \times 5\cos(126.87 - 0 + 0) + 1^2 \times 5\cos(-53.13)$$

$$= -3 + 3 = 0$$

$$\Delta P_2^0 = P_2 - P_2^0 = -1.0 - 0 = -1$$

Load $S_2 = -1.0 - j0.5$

Slack bus voltage is $V_1 = 1.0 \angle 0$ pu.

Initial estimates : $\left|V_2^0\right| = 1.0$ $\delta_2^0 = 0.0$

$$Q_2^0 = -\left|V_2^0\right|\left|V_1\right|5\sin(126.87 - \delta_2^{\,0} + \delta_1) - \left|V_2^0\right|^2 5\sin(-53.13)$$

$$= -1 \times 1 \times 5\sin(126.87 - 0 + 0) - 1^2 \times 5\sin(-53.13)$$

$$= -4 + 4 = 0$$

$$\Delta Q_2^0 = Q_2 - Q_2^0 = -0.50 - 0 = -0.5$$

The elements of Jacobian matrix

$$\frac{\partial P_2}{\partial \delta_2} = \left|V_2\right|\left|V_1\right|5\sin(126.87 - \delta_2 + \delta_1)$$

$$J_1^{\,0} = \frac{\partial P_2}{\partial \delta_2} = 1 \times 1 \times 5\sin(126.87) = 4$$

$$\frac{\partial P_2}{\partial \left|V_2\right|} = 5\left|V_1\right|\cos(126.87 - \delta_2 + \delta_1) + 2\left|V_2\right|5\cos(-53.13)$$

$$J_2 = \frac{\partial P_2}{\partial \left|V_2\right|} = 5 \times 1\cos(126.87) + 2 \times 1 \times 5\cos(-53.13) = 3$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1|5\cos(126.87 - \delta_2 + \delta_1)$$

$$J_3^0 = \frac{\partial Q_2}{\partial \delta_2} = 1 \times 1 \times 5\cos(126.87) = -3$$

$$\frac{\partial Q_2}{\partial |V_2|} = -5|V_1|\sin(126.87 - \delta_2 + \delta_1) - 10|V_2|\sin(-53.13)$$

$$J_4^0 = \frac{\partial Q_2}{\partial |V_2|} = -5 \times 1 \times \sin(126.87) - 10 \times 1\sin(-53.13) = 4$$

The set of linear equations in the first iteration becomes

$$\begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^0 \\ \Delta|V_2^0| \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 0.16 & -0.12 \\ 0.12 & 0.16 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -1.0 \\ -0.5 \end{bmatrix} = \begin{bmatrix} \Delta\delta_2^0 \\ \Delta|V_2^0| \end{bmatrix}$$

Solution of the matrix gives:

$$\Delta \delta_2^0 = -0.10 \qquad \delta_2^1 = 0 + (-0.10) = -0.10 \, radian = -5.7296°$$

$$\Delta \left| V_2^0 \right| = -0.2 \qquad \left| V_2^1 \right| = 1 + (-0.2) = 0.8$$

For the second iteration

$$P_2^1 = \left| V_2^1 \right| \left| V_1 \right| 5 \cos(126.87 - \delta_2^1 + \delta_1) + \left| V_2^1 \right|^2 5 \cos(-53.13)$$

$$= 0.8 \times 1 \times 5 \cos(126.87 - (-5.7296)) + 0.8^2 \times 5 \cos(-53.13)$$

$$= -2.7075 + 1.92 = -0.7875$$

$$\Delta P_2^1 = P_2 - P_2^1 = -1.0 - (-0.7875) = -0.2125$$

$$Q_2^1 = -\left| V_2^1 \right| \left| V_1 \right| 5 \sin(126.87 - \delta_2^1 + \delta_1) - \left| V_2^1 \right|^2 5 \sin(-53.13)$$

$$= -0.8 \times 1 \times 5 \sin(126.87 - (-5.7296)) + 0.8^2 \times 5 \sin(-53.13)$$

$$= -2.9444 + 2.56 = -0.3844$$

$$\Delta Q_2^1 = Q_2 - Q_2^1 = -0.50 - (-0.3844) = -0.1156$$

The set of linear equations in the 2nd iteration becomes

$$\begin{bmatrix} -0.2125 \\ -0.1156 \end{bmatrix} = \begin{bmatrix} 2.9444 & 1.4157 \\ -2.7075 & 2.7195 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^1 \\ \Delta|V_2^1| \end{bmatrix}$$

Solution of the matrix gives:

$$\Delta\delta_2^1 = -0.0350 \quad \delta_2^2 = -0.10 + (-0.0350) = -0.135 \; radian$$

$$\Delta|V_2^1| = -0.0773 \quad |V_2^2| = 0.8 + (-0.0773) = 0.7227$$
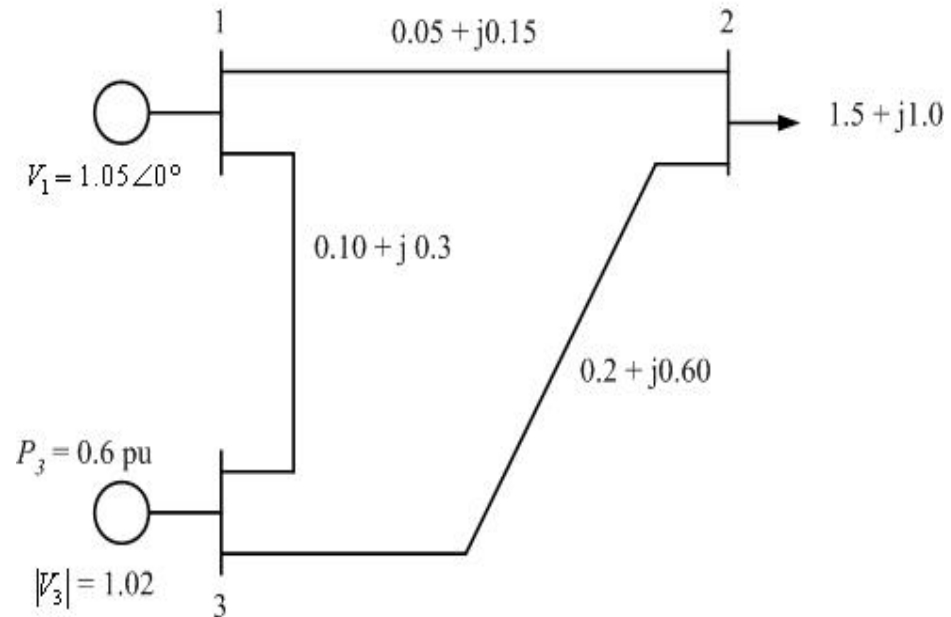
CONTINUE WITH THE 3RD ITERATION!
Good Luck......

Example of Exam Question

a)   In the Gauss Seidel method there is term called acceleration factor. Explain acceleration factor briefly.   (5 marks)

b)   For the power system shown, the bus admittance matrix Y_{bus} in per unit is given by:

$$\begin{bmatrix} 3-j9 & -2+j6 & -1+j3 \\ -2+j6 & 2.5-j7.5 & -0.5+j1.5 \\ -1+j3 & -0.5+j1.5 & 1.5-j4.5 \end{bmatrix}$$

$V_1 = 1.05\angle 0°$

$0.05 + j0.15$

$1.5 + j1.0$

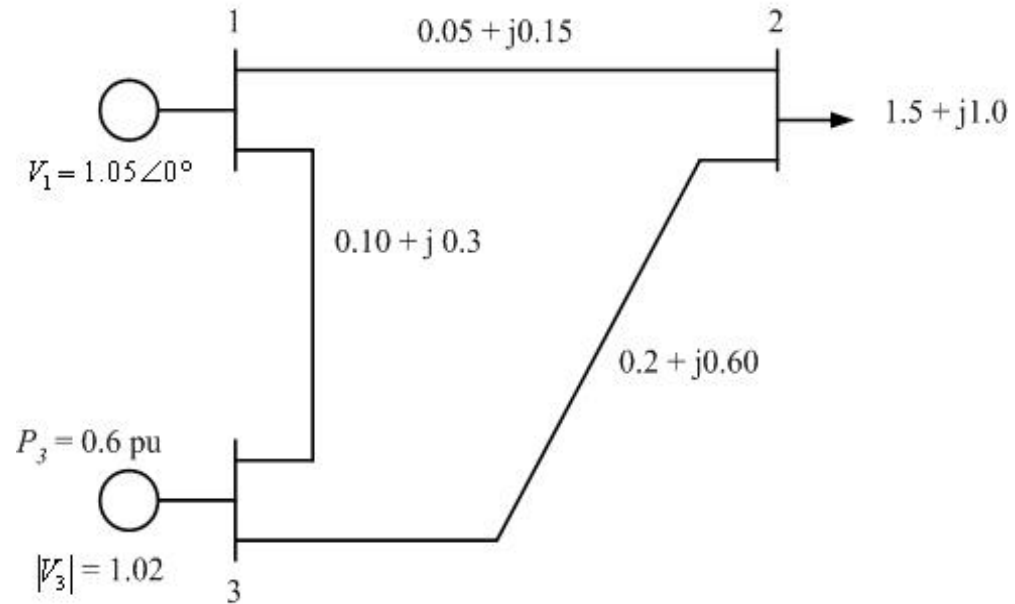$0.10 + j\,0.3$

$0.2 + j0.60$

$P_3 = 0.6$ pu

$|V_3| = 1.02$

For each bus k, specify the bus type, and determine which of the variables V_k, δ_k, P_k and Q_k are input data and which are unknowns. (2 marks)

Example of Exam Question

$$\begin{bmatrix} 3-j9 & -2+j6 & -1+j3 \\ -2+j6 & 2.5-j7.5 & -0.5+j1.5 \\ -1+j3 & -0.5+j1.5 & 1.5-j4.5 \end{bmatrix}$$

$V_1 = 1.05\angle 0°$

$0.05+j0.15$

$1.5+j1.0$

$0.10+j0.3$

$0.2+j0.60$

$P_3 = 0.6$ pu

$|V_3| = 1.02$

Set up the mismatch equation in vector-matrix form of the Newton-Raphson power flow method. **(3 marks)**

The mismatch equation in vector-matrix form of the Newton-Raphson power flow method. **(3 marks)**

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}$$
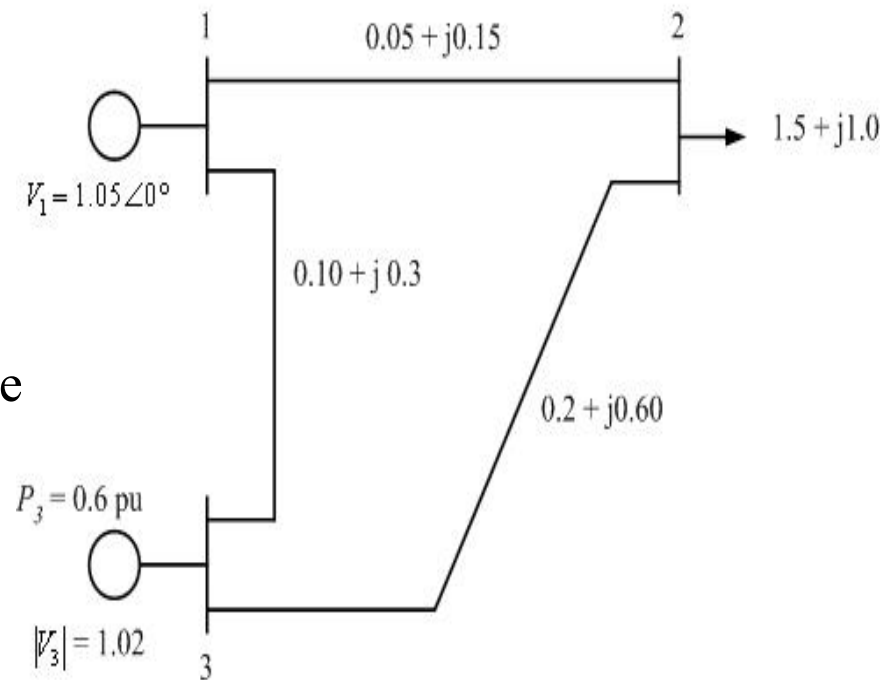
General equation

The mismatch equation in vector-matrix form of the Newton-Raphson power flow method. **(3 marks)**

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{vmatrix} \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial \delta_3} & \dfrac{\partial P_2}{\partial |V_2|} \\ \dfrac{\partial P_3}{\partial \delta_2} & \dfrac{\partial P_3}{\partial \delta_3} & \dfrac{\partial P_3}{\partial |V_2|} \\ \dfrac{\partial Q_2}{\partial \delta_2} & \dfrac{\partial Q_2}{\partial \delta_3} & \dfrac{\partial Q_2}{\partial |V_2|} \end{vmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

Example of Exam Question

$$\begin{bmatrix} 3 - j9 & -2 + j6 & -1 + j3 \\ -2 + j6 & 2.5 - j7.5 & -0.5 + j1.5 \\ -1 + j3 & -0.5 + j1.5 & 1.5 - j4.5 \end{bmatrix}$$

Assume an initial estimate of voltage $V_2 =$ 1.0$\underline{/0°}$ and the rotor angle $\delta_3 = 0°$, calculate the bus real and reactive power mismatches to be used in the first iteration of the Newton-Raphson power flow method. **(9 marks)**

1   0.05 + j0.15   2

1.5 + j1.0

$V_1 = 1.05\angle 0°$

0.10 + j0.3

0.2 + j0.60

$P_3 = 0.6$ pu

$|V_3| = 1.02$ 3

Solve the linear simultaneous equation

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \qquad \therefore \qquad \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Compute new voltage magnitudes and phase angles
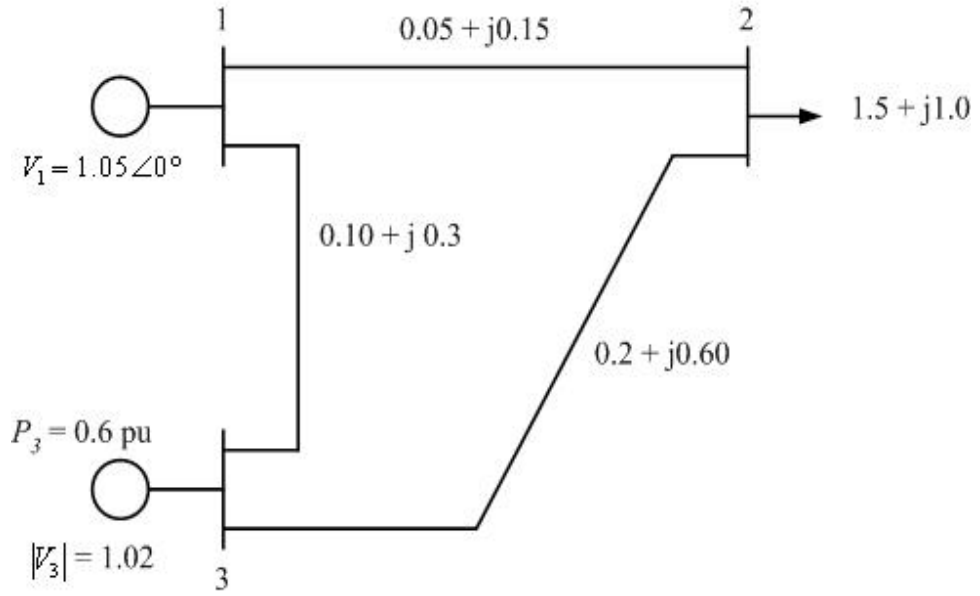
$$\delta_i^{(k+1)} = \delta_i^k + \Delta \delta_i^{(k)} \qquad \left| V_i^{(k+1)} \right| = \left| V_i^k \right| + \Delta \left| V_i^{(k)} \right|$$

Continue until the residuals $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are less than the specified accuracy.
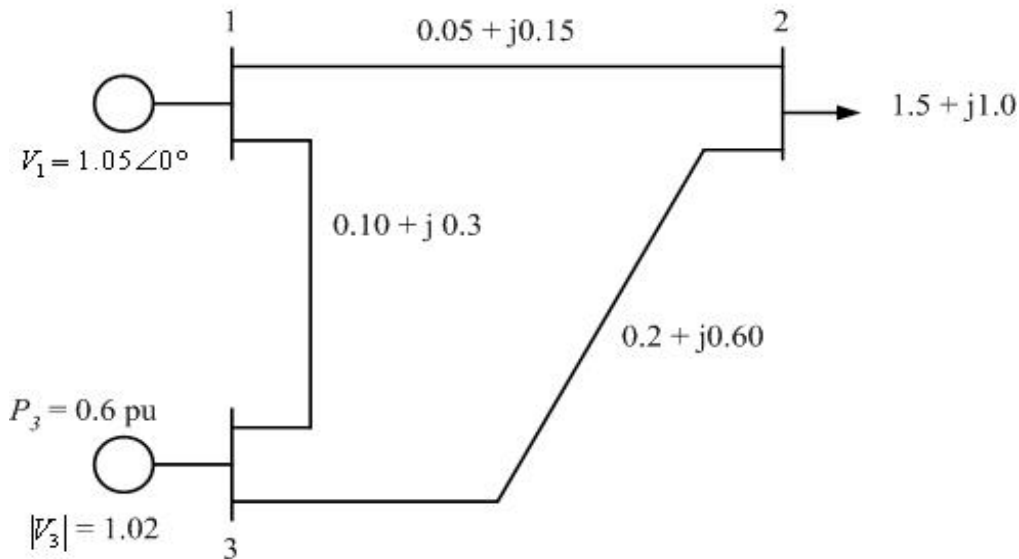
$$\left| \Delta P_i^{(k)} \right| = P_i^{sch} - P_i^{(k)}$$
$$\left| \Delta Q_i^{(k)} \right| = Q_i^{sch} - Q_i^{(k)}$$

$$\left| \Delta P_i^{(k)} \right| \le \varepsilon$$
$$\left| \Delta Q_i^{(k)} \right| \le \varepsilon$$

$$
\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix}
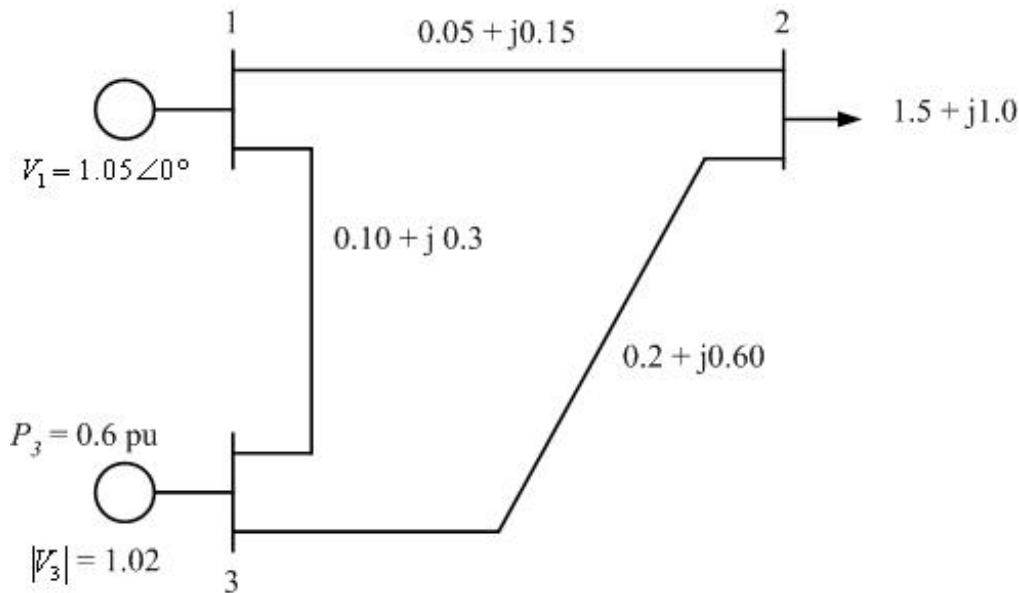$$

$V_1 = 1.05 \angle 0°$

$0.05 + j0.15$

$1.5 + j1.0$

$0.10 + j\,0.3$

$0.2 + j0.60$

$P_3 = 0.6$ pu

$|V_3| = 1.02$

$$
\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial \delta_3} & \dfrac{\partial P_2}{\partial |V_2|} \\[2mm] \dfrac{\partial P_3}{\partial \delta_2} & \dfrac{\partial P_3}{\partial \delta_3} & \dfrac{\partial P_3}{\partial |V_2|} \\[2mm] \dfrac{\partial Q_2}{\partial \delta_2} & \dfrac{\partial Q_2}{\partial \delta_3} & \dfrac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}
$$

$$Y_{bus} = \begin{bmatrix} 3 - j9 & -2 + j6 & -1 + j3 \\ -2 + j6 & 2.5 - j7.5 & -0.5 + j1.5 \\ -1 + j3 & -0.5 + j1.5 & 1.5 - j4.5 \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} 9.4868\angle-71.565 & 6.3246\angle108.435 & 3.1623\angle108.435 \\ 6.3246\angle108.435 & 7.9057\angle-71.565 & 1.5811\angle108.435 \\ 3.1623\angle108.435 & 1.5811\angle108.435 & 4.7134\angle-71.565 \end{bmatrix}$$

For bus 2

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j)$$

$$P_2 = \sum_{j=1}^{3} |V_2||V_j||Y_{2j}| \cos(\theta_{2j} - \delta_2 + \delta_j)$$

$$P_2 = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1)$$
$$+ |V_2||V_2||Y_{22}| \cos(\theta_{22} - \delta_2 + \delta_2) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$Y_{bus} = \begin{bmatrix} 9.4868\angle -71.565 & 6.3246\angle 108.435 & 3.1623\angle 108.435 \\ 6.3246\angle 108.435 & 7.9057\angle -71.565 & 1.5811\angle 108.435 \\ 3.1623\angle 108.435 & 1.5811\angle 108.435 & 4.7134\angle -71.565 \end{bmatrix}$$
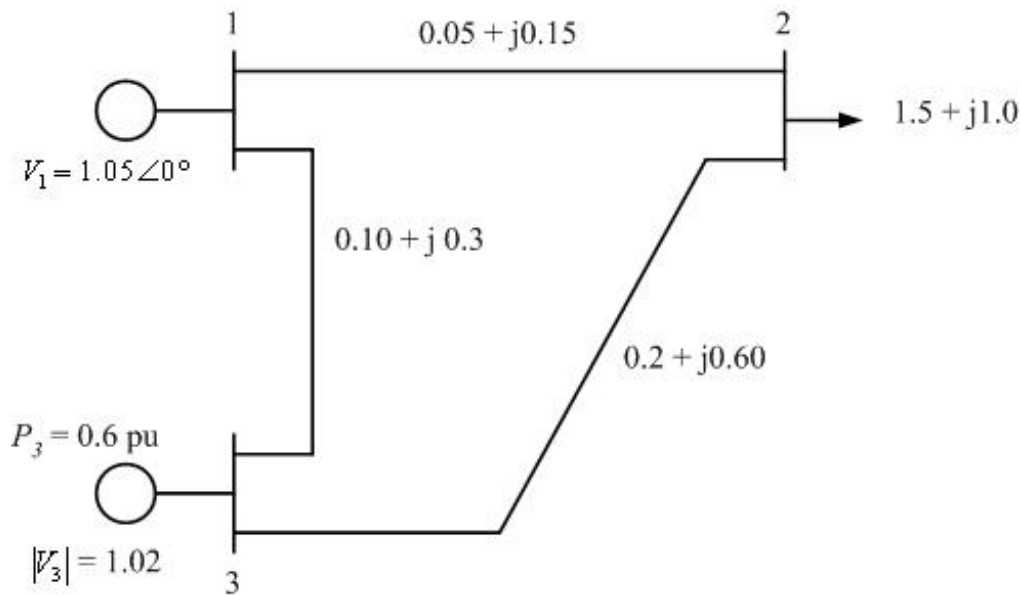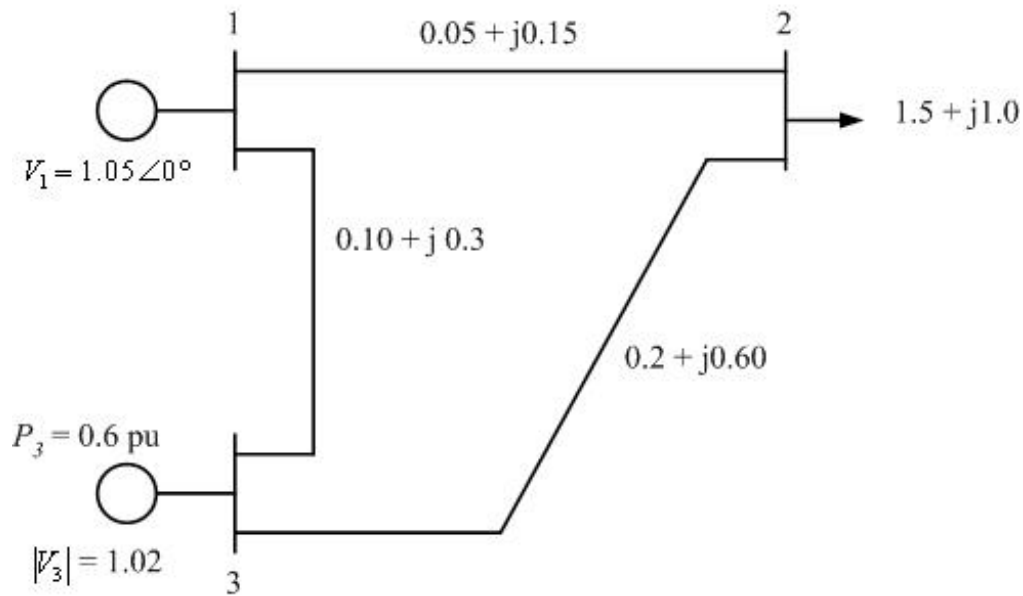
$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1)$$

$$+ |V_2||V_2||Y_{22}|\cos(\theta_{22} - \delta_2 + \delta_2) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$
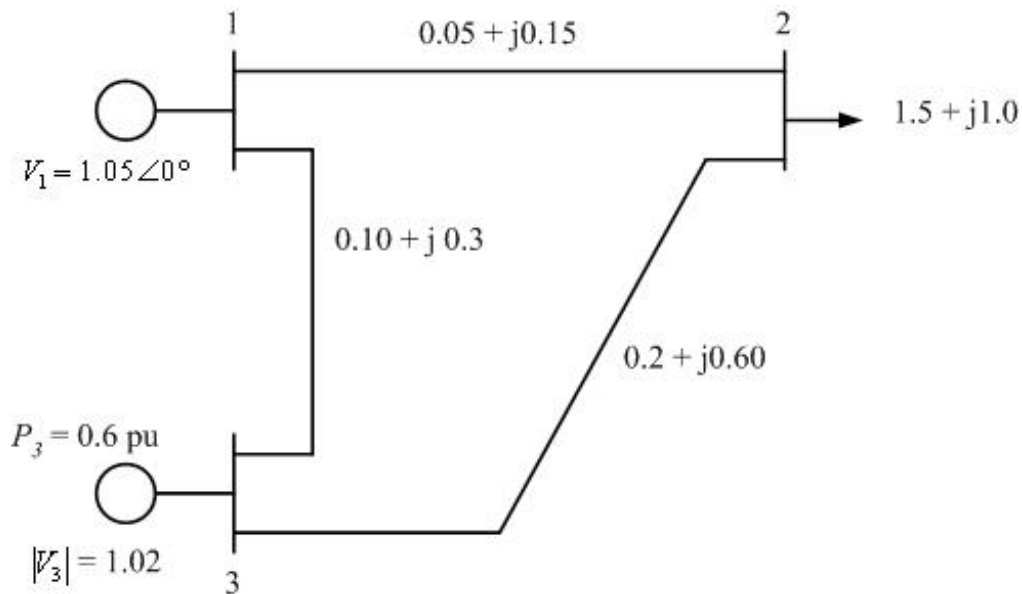
$$P_2 = |V_2||V_1|6.3246\cos(108.435 - \delta_2 + \delta_1) + |V_2|^2 7.9057\cos(-71.565)$$

$$+ |V_2||V_3|1.5811\cos(108.435 - \delta_2 + \delta_3)$$

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Circuit diagram with:
- Bus 1: $V_1 = 1.05\angle0°$
- Line 1-2: $0.05 + j0.15$
- Load at bus 2: $1.5 + j1.0$
- Line 1-3: $0.10 + j0.3$
- Line 3-2: $0.2 + j0.60$
- Bus 3: $P_3 = 0.6$ pu, $|V_3| = 1.02$

$$Y_{bus} = \begin{bmatrix} 9.4868\angle-71.565 & 6.3246\angle108.435 & 3.1623\angle108.435 \\ 6.3246\angle108.435 & 7.9057\angle-71.565 & 1.5811\angle108.435 \\ 3.1623\angle108.435 & 1.5811\angle108.435 & 4.7134\angle-71.565 \end{bmatrix}$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1)$$
$$+ |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3||V_3||Y_{33}|\cos(\theta_{33} - \delta_3 + \delta_3)$$

$$P_3 = |V_3||V_1|3.1623\cos(108.435 - \delta_3 + \delta_1) + |V_3||V_2|1.5811\cos(108.435)$$
$$+ |V_3|^2 4.7134\cos(-71.565)$$

$V_1 = 1.05\angle 0°$

$0.05 + j0.15$

$1.5 + j1.0$

$0.10 + j\,0.3$

$0.2 + j0.60$

$P_3 = 0.6$ pu

$|V_3| = 1.02$

$$Y_{bus} = \begin{bmatrix} 9.4868\angle -71.565 & 6.3246\angle 108.435 & 3.1623\angle 108.435 \\ 6.3246\angle 108.435 & 7.9057\angle -71.565 & 1.5811\angle 108.435 \\ 3.1623\angle 108.435 & 1.5811\angle 108.435 & 4.7134\angle -71.565 \end{bmatrix}$$
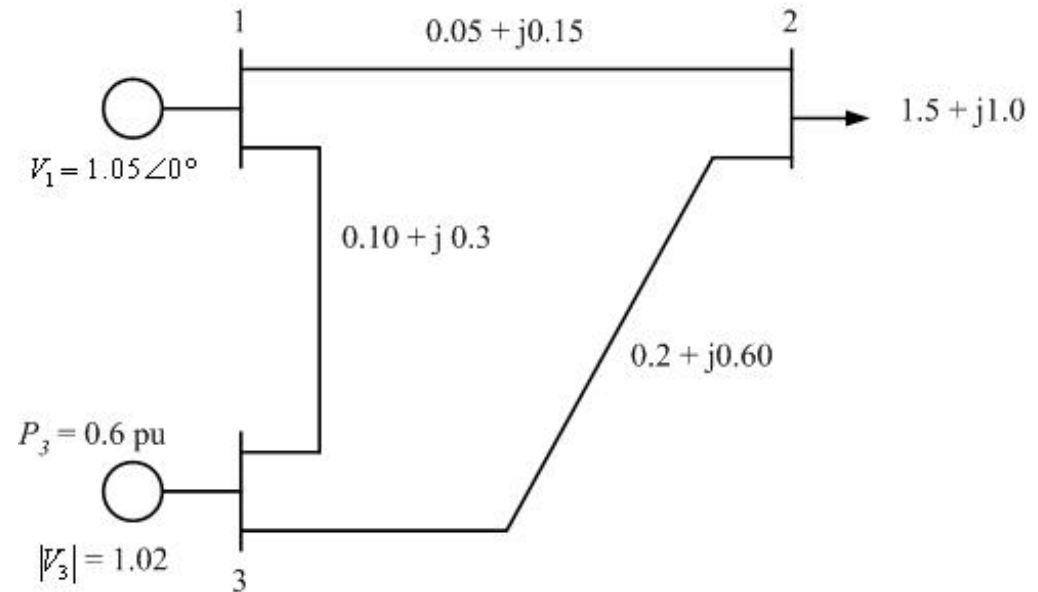
$$Q_2 = |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1)$$
$$+ |V_2||V_2||Y_{22}|\sin(\theta_{22} - \delta_2 + \delta_2) + |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$Q_2 = |V_2||V_1|6.3246\sin(108.435 - \delta_2 + \delta_1) + |V_2|^2 7.9057\sin(-71.565)$$
$$+ |V_2||V_3|1.5811\sin(108.435 - \delta_2 + \delta_3)$$

$$|V_1| = 1.05 \qquad \delta_1 = 0$$

$$|V_2| = 1.0 \qquad \delta_2 = 0$$

$$|V_3| = 1.02 \qquad \delta_2 = 0$$

$V_1 = 1.05\angle 0°$

$0.05 + j0.15$

$1.5 + j1.0$

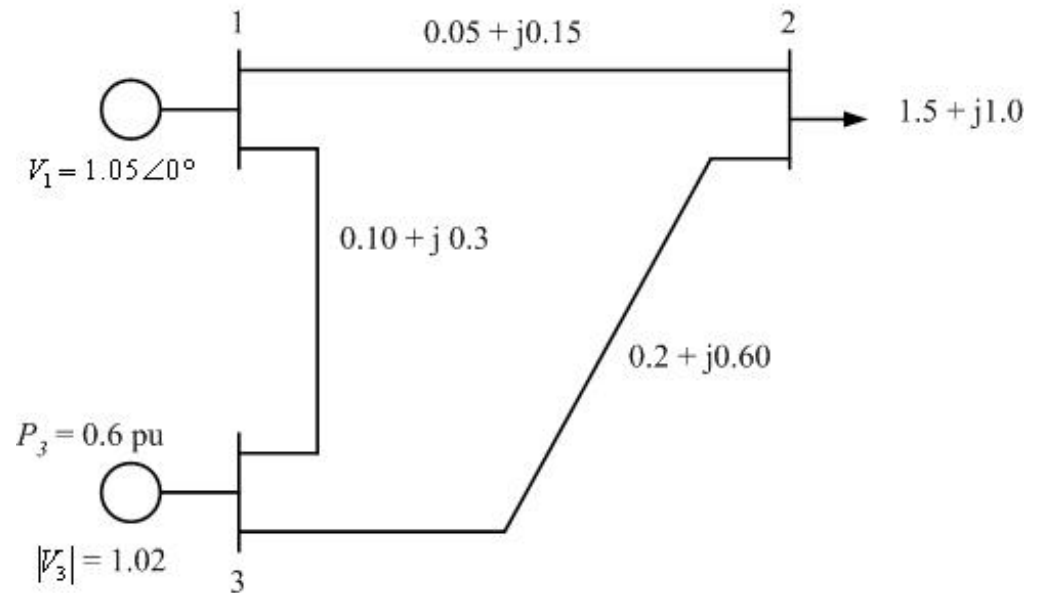$0.10 + j\,0.3$

$0.2 + j0.60$

$P_3 = 0.6$ pu

$|V_3| = 1.02$

$$P_2 = |V_2||V_1|6.3246\cos(108.435 - \delta_2 + \delta_1) + |V_2|^2 7.9057\cos(-71.565)$$
$$+ |V_2||V_3|1.5811\cos(108.435 - \delta_2 + \delta_3)$$

$$P_2^{\,0} = 1.0 \times 1.05 \times 6.3246\cos(108.435 - 0 + 0) + 1^2 \times 7.9057\cos(-71.565)$$
$$+ 1.0 \times 1.02 \times 1.5811\cos(108.435 - 0 + 0)$$
$$= -0.11$$

$$|V_1| = 1.05 \qquad \delta_1 = 0$$

$$|V_2| = 1.0 \qquad \delta_2 = 0$$

$$|V_3| = 1.02 \qquad \delta_2 = 0$$

$$P_3 = |V_3||V_1|3.1623\cos(108.435 - \delta_3 + \delta_1) + |V_3||V_2|1.5811\cos(108.435)$$
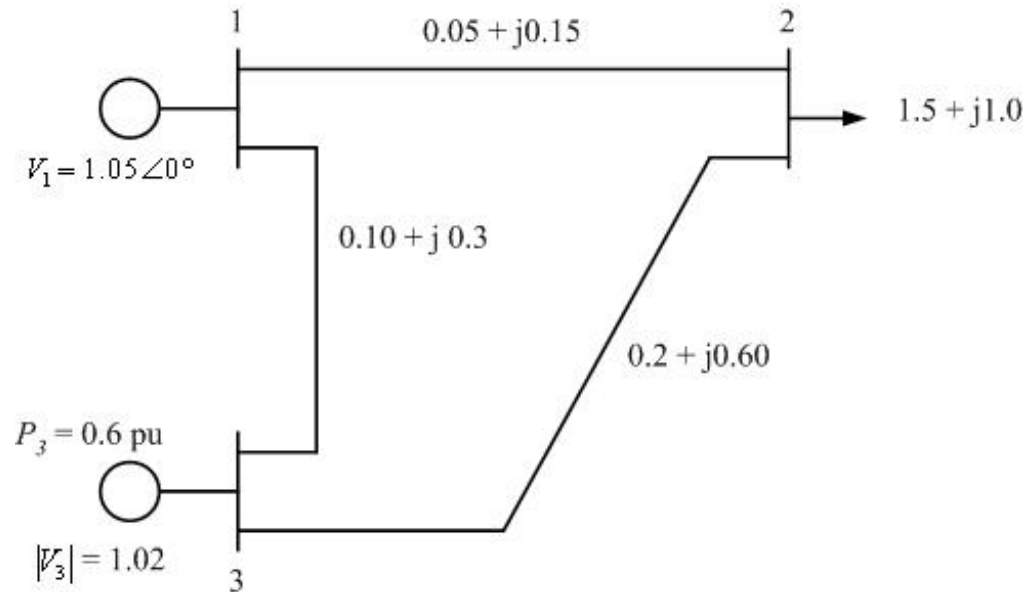
$$+ |V_3|^2 4.7134\cos(-71.565)$$

$$P_3^{\,0} = |V_3||V_1|3.1623\cos(108.435 - \delta_3 + \delta_1) + |V_3||V_2^{\,0}|1.5811\cos(108.435)$$

$$+ |V_3|^2 4.7134\cos(-71.565)$$

$$= 1.02 \times 1.05 \times 3.1623\cos(108.435) + 1.02 \times 1 \times 1.5811\cos(108.435)$$

$$+ 1.02^2 \times 4.7134\cos(-71.565)$$

$$= -0.02$$

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$$|V_1| = 1.05 \qquad \delta_1 = 0$$

$$|V_2| = 1.0 \qquad \delta_2 = 0$$

$$|V_3| = 1.02 \qquad \delta_2 = 0$$

$V_1 = 1.05\angle 0°$

$0.05 + j0.15$

$1.5 + j1.0$

$0.10 + j\,0.3$

$0.2 + j0.60$

$P_3 = 0.6$ pu

$|V_3| = 1.02$

$$Q_2 = |V_2||V_1|6.3246\sin(108.435 - \delta_2 + \delta_1) + |V_2|^2\,7.9057\sin(-71.565)$$
$$+\,|V_2||V_3|1.5811\sin(108.435 - \delta_2 + \delta_3)$$

$$Q_2 = 1.0 \times 1.05 \times 6.3246\sin(108.435) + 1^2 \times 7.9057\sin(-71.565)$$
$$+\,1.0 \times 1.02 \times 1.5811\sin(108.435)$$
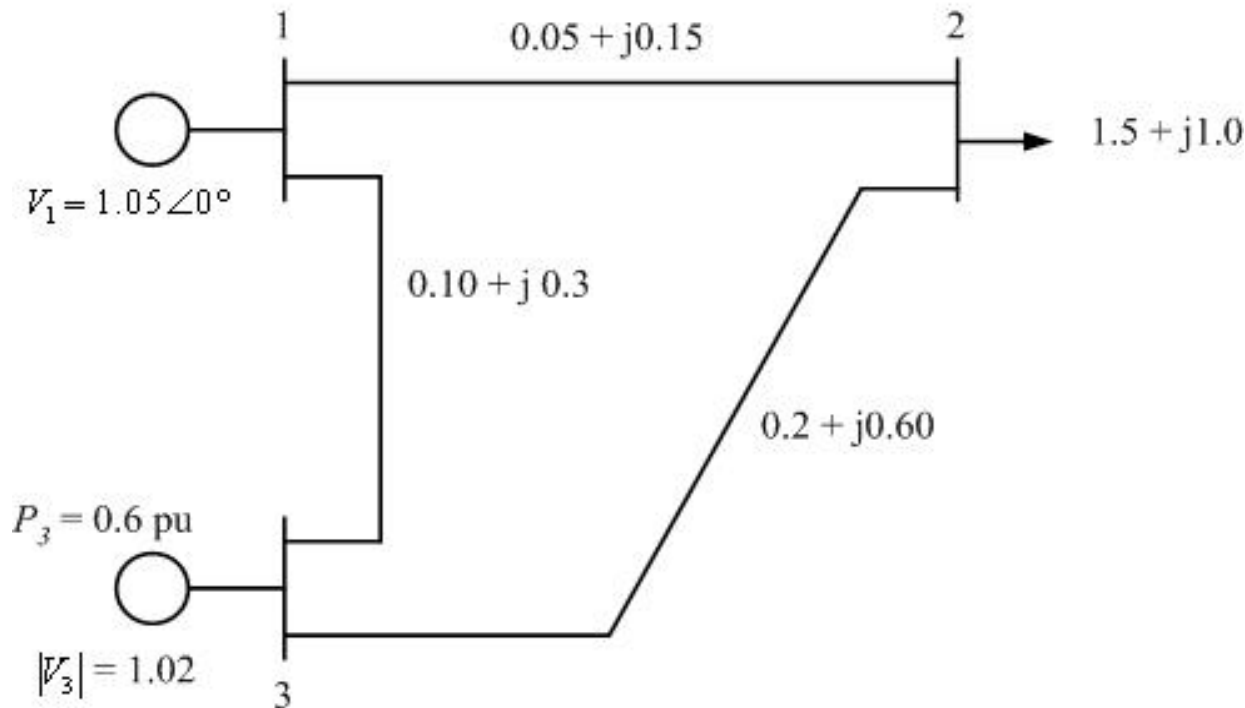$$= -0.33$$

$$\left| \Delta P_i^{(k)} \right| = P_i^{sch} - P_i^{(k)}$$

$$\left| \Delta Q_i^{(k)} \right| = Q_i^{sch} - Q_i^{(k)}$$

$$\left| \Delta P_2^{(0)} \right| = P_2^{sch} - P_2^{(0)} = -1.5 - (-0.11) = -1.39$$

$$\left| \Delta P_3^{(0)} \right| = P_3^{sch} - P_3^{(0)} = 0.6 - (-0.02) = 0.62$$

$$\left| \Delta Q_2^{(0)} \right| = Q_2^{sch} - Q_2^{(0)} = -1.0 - (-0.33) = -0.67$$

Assume an initial estimate of voltage $V_2 = 1.0\underline{/0°}$ and the rotor angle $\delta_3 = 0°$, calculate Jacobian Matrix to be used in the first iteration of the Newton-Raphson power flow method. (9 marks)

$$\begin{bmatrix} 3 - j9 & -2 + j6 & -1 + j3 \\ -2 + j6 & 2.5 - j7.5 & -0.5 + j1.5 \\ -1 + j3 & -0.5 + j1.5 & 1.5 - j4.5 \end{bmatrix}$$

$V_1 = 1.05\angle 0°$

$0.05 + j0.15$

1

2

$1.5 + j1.0$

$0.10 + j\,0.3$

$0.2 + j0.60$

$P_3 = 0.6$ pu

$|V_3| = 1.02$

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A four bus power system data is shown in Tables Q1a and Q1b. Form a bus admittance matrix, Y_{bus}, and determine the voltages at the end of first iteration using Gauss-Seidel method. The reactive power constraint on generator 2 is $0.2 \leq Q_2 \leq 1.0$. Use flat start estimates to start calculation. (16 marks)

Lines	Admittance (per unit)
1 – 2	$2 - j8.0$
1 – 3	$1 - j4.0$
2 – 3	$0.666 - j2.664$
2 – 4	$1 - j4.0$
3 - 4	$2 - j8.0$

Bus	Generation		Load		Bus Voltage		Remarks
	P	Q	P	Q	$\lvert V \rvert$	$\delta°$	
1	-	-	0.2	0.1	1.06	0	Slack
2	0.5	-	-	-	1.04	-	PV
3	-	-	0.4	0.3	-	-	PQ
4	-	-	0.3	0.1	-	-	PQ

Lines	Admittance (per unit)
1 – 2	2 – j8.0
1 – 3	1 – j4.0
2 – 3	0.666 – j2.664
2 – 4	1 – j4.0
3 - 4	2 – j8.0

The admittance bus matrix

$$Y_{bus} = \begin{bmatrix} 3 - j12 & -2 + j8 & -1 + j4 & 0 \\ -2 + j8 & 3.666 - j14.664 & -0.666 + j2.664 & -1 + j4 \\ -1 + j4 & -0.666 + j2.664 & 3.666 - j14.664 & -2 + j8 \\ 0 & -1 + j4 & -2 + j8 & 3 - j12 \end{bmatrix}$$

What is the type of Bus 2? What is known? What is unknown?

Bus 2 is PV bus. Known P and $|V_2|$. Unknown Q_2 and δ_2

Write the equation for Q_2

$$Q_2^{\,0} = -\Im\left\{V_2^{\,0*}\left(Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4\right)\right\}$$
$$= 0.1108$$

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What's next?

The value of Q_2 is less than the minimum specified

Reactive power is fixed at 0.2 (lower limit)

<u>Characteristics of the NR LF</u>

With sparse programming techniques and optimally ordered triangular factorisation, the Newton method for solving LF has become faster than other methods for large systems. The number of iterations is virtually independent of system size (from a flat voltage start and with no automatic adjustments) due to the quadratic characteristic of convergence. Most systems are solve in 2-5 iterations with no acceleration factor being necessary. With good programming the time per iteration rises nearly linearly with the number of system buses N, so that the overall solution time varies as N. One Newton iteration is equivalent to about seven Gauss Seidal iterations.

For a 500 bus system, the conventional Gauss Seidal method takes about 500 iterations and the speed advantage of the Newton method is then 15:1.

Storage requirements of the Newton method are greater, however, but increase linearly with system size. It is therefore attractive for large systems. The Newton method is very reliable in system solving, given good starting approximations. Heavily loaded systems with phase shifts up to 90° can be solved. The method did not troubled by ill-conditioned systems and the location of slack bus is not critical.

Due to the quadratic convergence of bus voltages, high accuracy (near exact solution) is obtained in only a few iterations. This is important for the use of LF is short circuit and stability studies.

The method is readily extended to include tap changing transformers, variable constrains on bus voltages, and reactive and optimal power scheduling. Network modifications are easily made. The success of the Newton method is critical on the formulation of the problem defining equation.

Comparison of LF methods

G-S and NR methods are compared when both use Y_{bus} as the network model.

G-S method works well when programmed using rectangular coordinates, whereas NR requires more memory when rectangular coordinates are used.

G-S method requires the fewest number of arithmetic operations to complete an iteration due to the sparsity of the network matrix and the simplicity of the solution techniques. Consequently, this method requires less time per iteration. With the NR method, the elements of the Jacobian are to be computed in each iteration, so the time is considerably longer. For typical large system, the time per iteration in the NR method is roughly equivalent to 7 times that of G-S method. The time per iteration in both methods increases directly as the number of buses of the network. The number of iterations for the G-S method increases directly as the number of buses of the network, whereas the number of iterations for the NR method remains practically constant (3-5 iterations). In the G-S method, convergence is affected by the choice of slack bus and the presence of series capacitor, but the

sensitivity of the NR method is minimal to these factors which cause poor convergence. Therefore, for large systems the NR method is faster, more accurate and more reliable than G-S method.