SKEE 4443
POWER SYSTEM ANALYSIS

CHAPTER 3

SYMMETRICAL FAULT

Introduction

- The cause of electric power system faults is insulation breakdown

- In a typical power system, the fault currents are several orders greater than normal load currents. Hence, the fault currents can cause extensive thermal and mechanical damage if they are allowed to persist for more than a couple of seconds.

- This breakdown can be due to a variety of different factors
  - lightning
  - wires blowing together in the wind
  - animals or plants coming in contact with the wires
  - salt spray or pollution on insulators



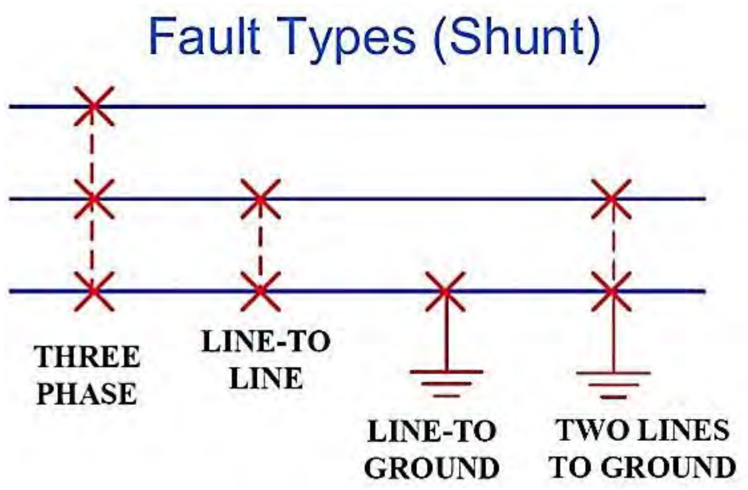
Introduction

- Goal of fault analysis is to determine the magnitudes of the currents present during the fault.

- The fault studies in a power system help to:
  - To establish CB ratings to interrupt the fault currents
  - To establish the short time (s/c) ratings of the power system devices
  - To establish relay settings
  - The solution of power systems under fault conditions essentially involves solving
  - an electric network with voltage sources and impedance.

Analysis types

- Power flow-evaluate normal operating conditions
- Fault analysis- evaluate abnormal operating conditions
- Fault types:
- Balanced faults
  - Three phase
- Unbalanced faults
- Single line to round faults
  - Double line faults
  - Double line to ground faults
- Magnitude of fault currents depends
  - The impedance of the network
  - The internal impedance of the generator
  - The resistance of the fault (arc resistance)
- Network impedances are governed by
  - Transmission line impedances
  - Grounding connections and resistances



Generator behavior during short circuit

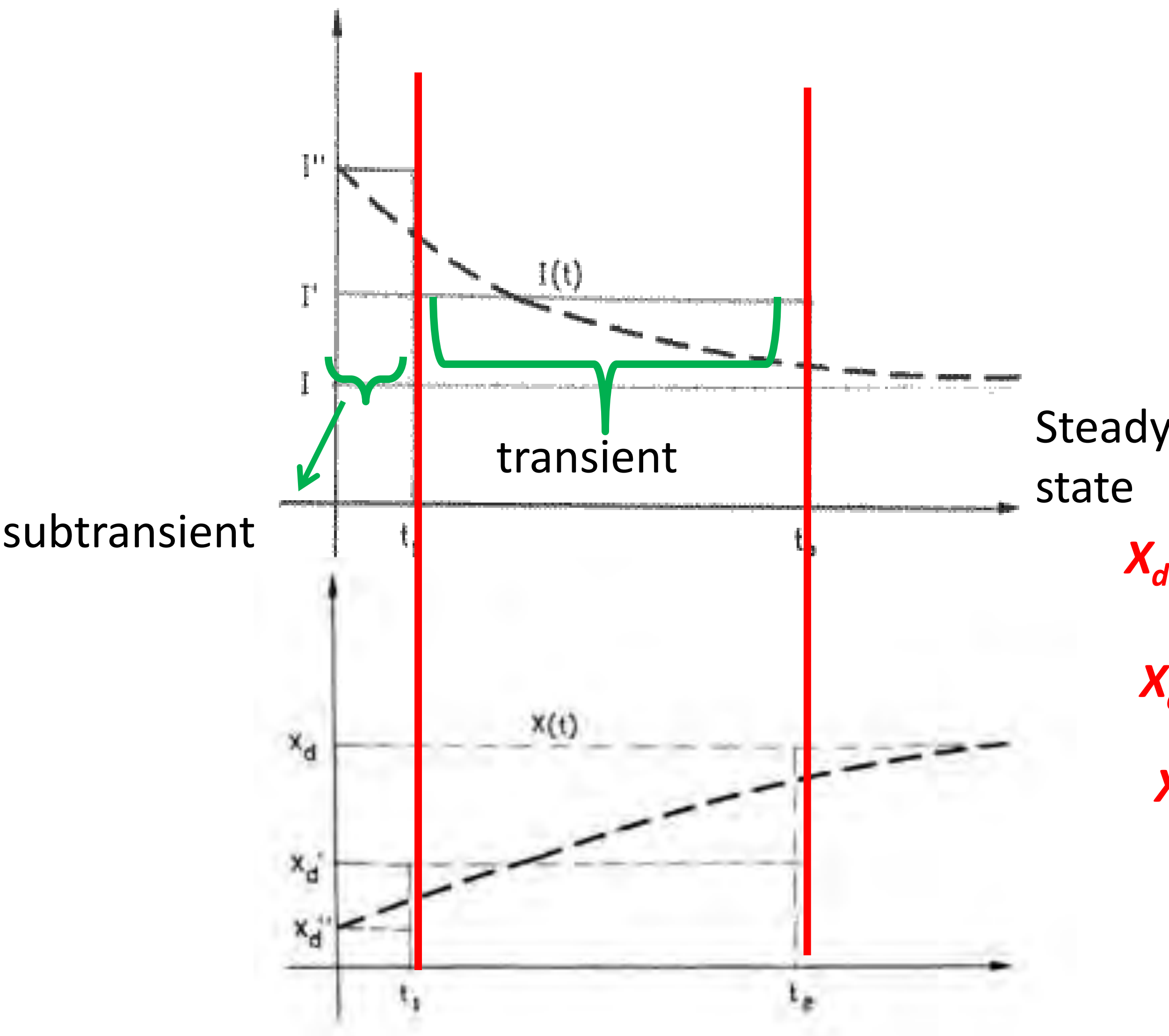
- The value of the generator inductance (reactance) varies substantially consequent to a disturbance in the power system.

- Immediately following a fault, the generator reactance not only varies with time, but is substantially less than the steady state value. Hence, it is essential to use appropriate reactance value depending on the study requirement.

- The regions and the corresponding reactances are termed as follows:
  - 0 to Approx. 2 cycles => Subtransient Region=> Subtransient Reactance (X”)
  - 2 to Approx. 25 cycles => Transient Region=> Transient Reactance (X’)
  - >Approx. 25 cycles => Steady state Region=> Synchronous Reactance (X)

Generator behavior during short circuit

Variation of current with time during a fault



I" = subtransient current

I' = transient current

I = steady state current

X_d" = subtransient reactance

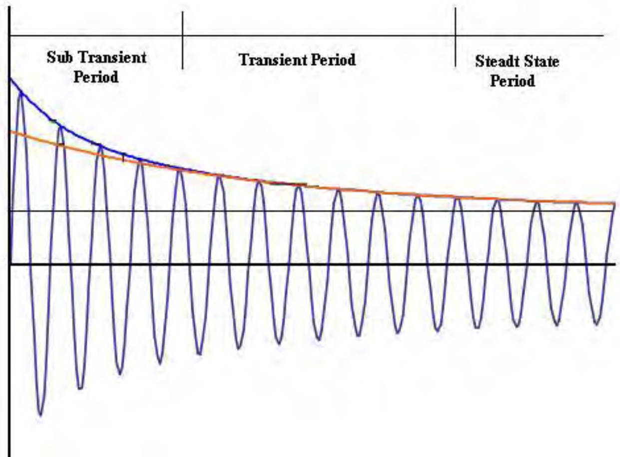
X_d' = transient reactance

X_d = steady state reactance

Generator behavior during short circuit

- Typical average reactance values for synchronous machines.

	Two pole turbine generator	Four pole turbine generator	Salient pole with dampers	Synchronous condensers
X_d	1.2	1.2	1.25	2.2
X_d'	0.15	0.23	0.30	0.48
X_d''	0.09	0.14	0.20	0.32
X_2	0.09	0.14	0.20	0.31
X_0	0.03	0.08	0.18	0.14

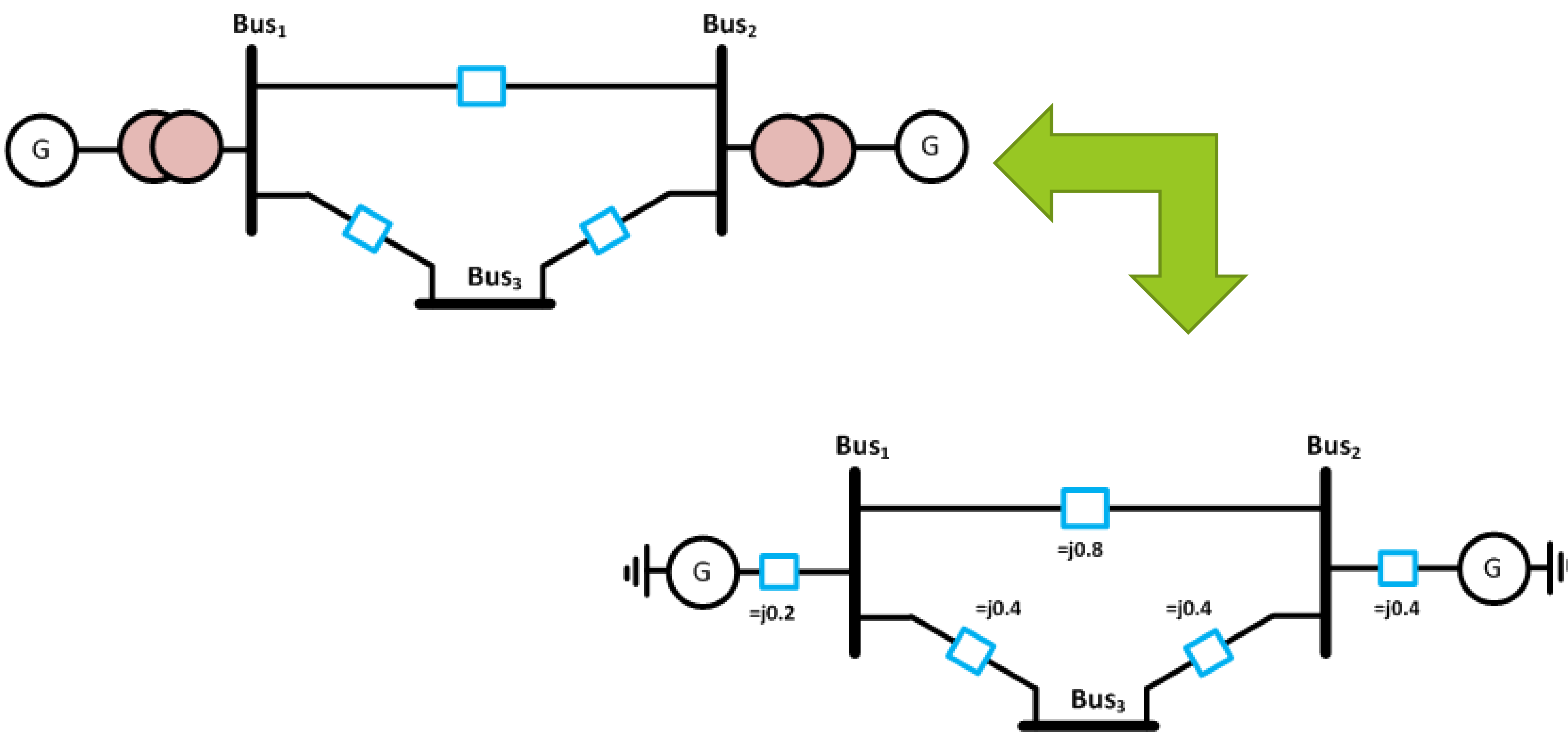


Fault representation

- A fault represents a structural network change
  - Equivalent to the addition of an impedance at the place of the fault
  - If the fault impedance is zero, the fault is referred to as a bolted fault or solid fault
- First order method
  - The faulted network can be solved conveniently by Thevenin's method
  - Network resistances are neglected
  - Generators are modeled as an emf behind the sub-transient or transient reactance
  - Shunt capacitances are neglected
  - System is considered as having no load

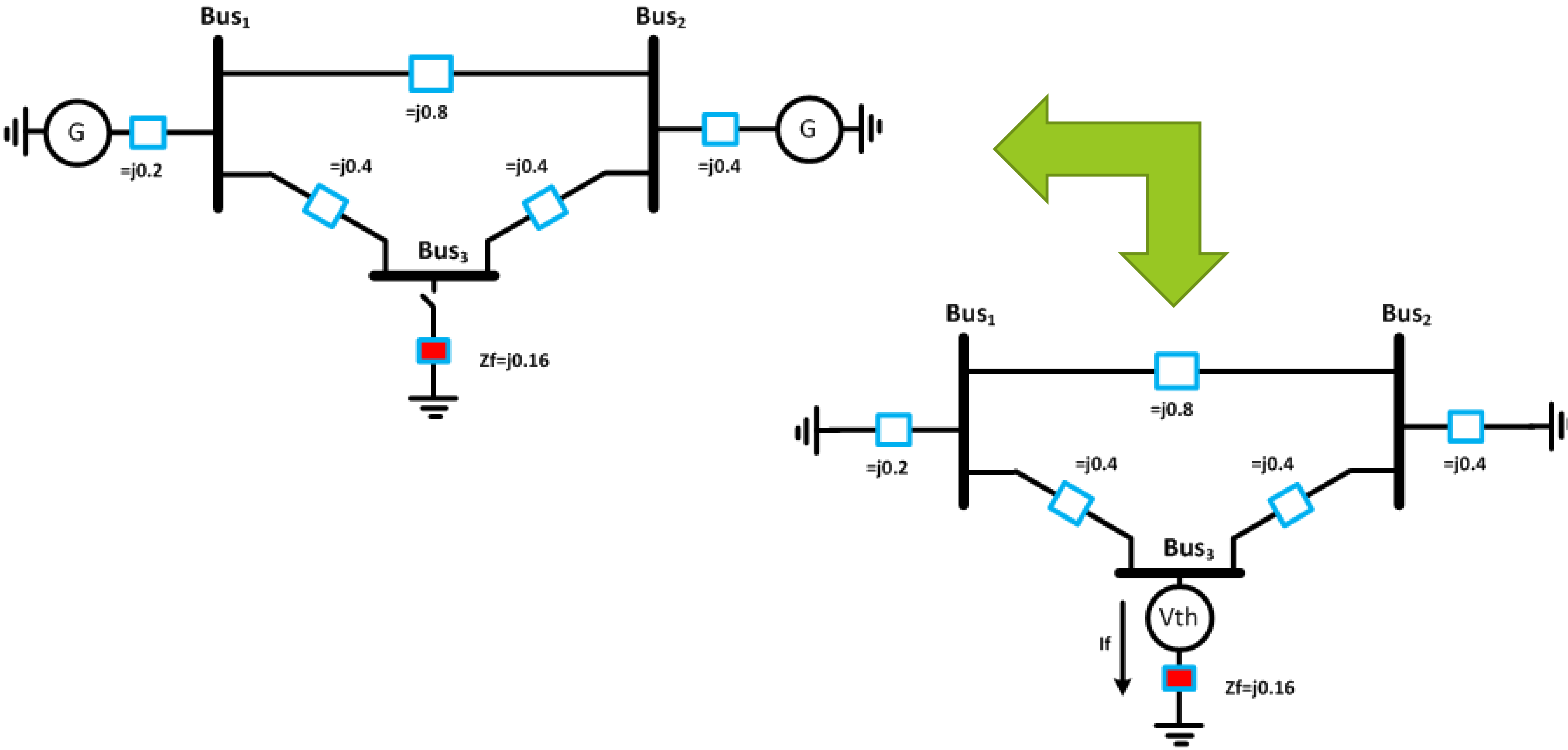
Thevenin's method

- The fault is simulated by switching a fault impedance at the fault bus
- The change in the network voltages is equivalent to adding a prefault bus voltage with all other sources short circuited



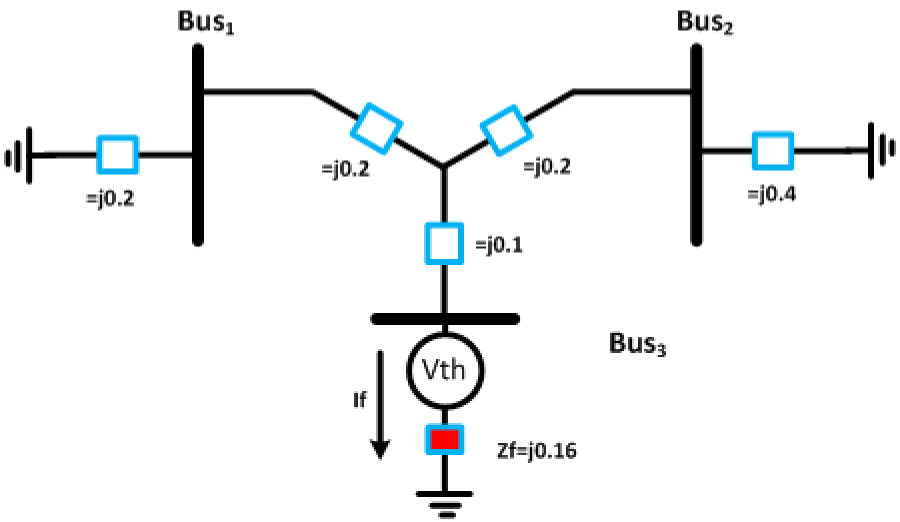
Thevenin's method

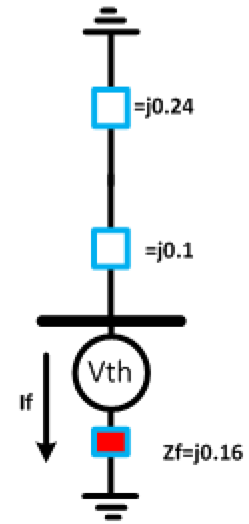
- 3 phase fault with Zf=j0.16 on bus 3



Thevenin's method

- 3 phase fault with Zf=j0.16 on bus 3





$$I_3^f = \frac{V_3^0}{Z_{33} + Z_f}$$

$$V_1^0 = V_2^0 = V_3^0 = 1.0\angle 0$$

$$Z_{1\_Y} = Z_{2\_Y} = \frac{j0.4 \times j0.8}{j0.16} = j0.2$$

$$Z_{3\_Y} = \frac{j0.4 \times j048}{j0.16} = j0.1$$

$$Z_{33} = \frac{j0.4 \times j06}{j0.4 + j0.6} + j0.1 = j0.34$$

$$I_3^f = \frac{V_3^0}{Z_{33} + Z_f}$$

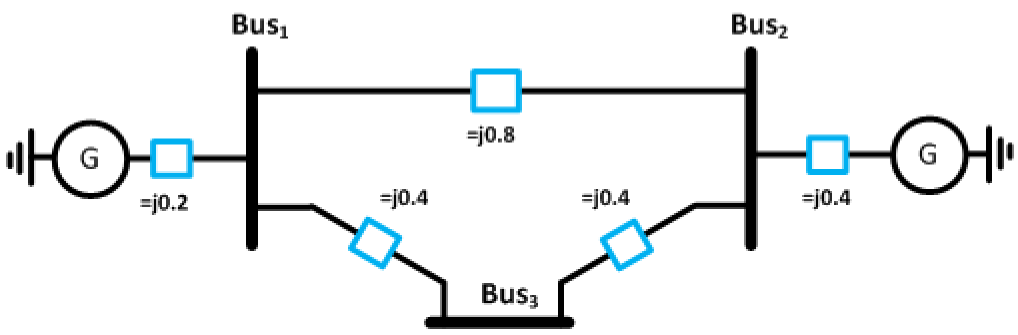
$$= \frac{1\angle 0}{j0.34 + j0.16} = -j2.0$$

Short circuit capacity (SCC)

- Measure the electrical strength of the bus
- Stated in MVA
- Determines the dimension of bus bars and the interruption capacity of circuit breaker

- Definition $$SCC = \sqrt{3} \times V_{LL\_k}^{pre-f} \times I_k^f$$

- In per unit $$I_k^f = \frac{V_k^{pre\_f}}{X_{kk}}$$ $$SCC = \frac{S_B}{|X_{kk}|}$$

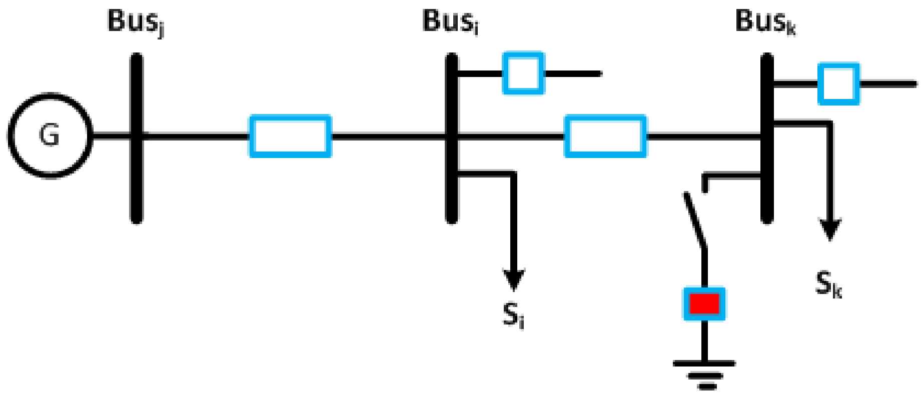


- For 3 bus example

$$Z_{33} = j0.34$$
$$S_{base} = 100 MVA$$
$$SCC = \frac{S_B}{|X_{kk}|} = \frac{100}{0.34} = 294 \text{ MVA}$$

Fault analysis using impedance matrix

- Network reduction by Thevenin's method is not efficient
- Difficult to apply to large systems/networks
- Matrix algebra where the diagonal elements represent the source impedance for those buses
- Consider the following system
- Operating under the balance conditions
- Each generator represented by a constant emf behind a proper reactance (sub transient, transient reactance)
- Lines represented by their equivalent π model

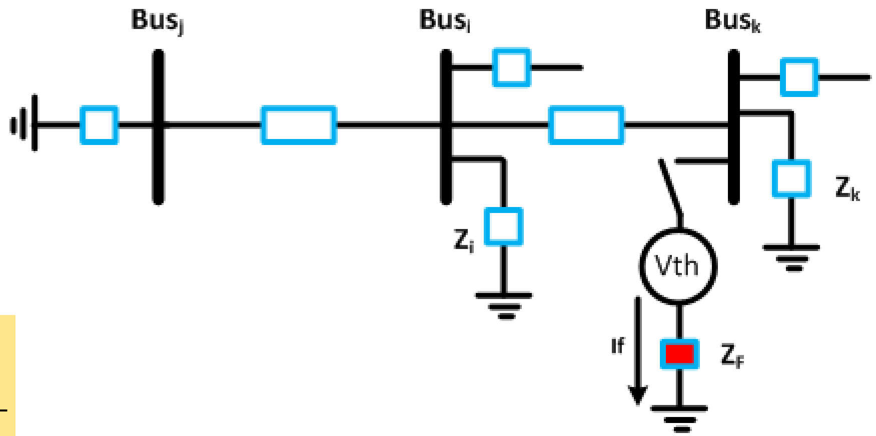


Fault analysis using impedance matrix

- Place the prefault voltages into a vector
- Replace the loads by a constant impedance model using the prefault bus voltages
- The change in the network voltages caused by the fault is equivalent to placing a fault voltage at the faulted bus with all the other sources short circuited

$$\left[V_{bus}^{pre\_f}\right] = \begin{bmatrix} V_1^{pre\_f} \\ \vdots \\ V_k^{pre\_f} \\ \vdots \\ V_n^{pre\_f} \end{bmatrix}$$

$$Z_k = \frac{\left|V_k^{pre\_f}\right|^2}{S_{k\_load}^*}$$



Fault analysis using impedance matrix

- Using superpositioning, the fault voltages are calculated from the prefault voltages by adding the change in the bus voltages due to the fault

$$\left[V_{bus}^{f}\right]=\left[V_{bus}^{pre\_f}\right]+\left[\Delta V_{bus}\right]$$

$$\left[\Delta V_{bus}\right]=\begin{bmatrix}\Delta V_1\\ \vdots\\ \Delta V_k\\ \vdots\\ \Delta V_n\end{bmatrix}$$

- The change in the bus voltages can be calculated from the network matrix

$$\left[I_{bus}^{f}\right]=\left[Y_{bus}\right]\left[\Delta V_{bus}\right]$$

$$\left[I_{bus}^{f}\right]=\begin{bmatrix}0_1\\ \vdots\\ -I_k^f\\ \vdots\\ 0\end{bmatrix}$$

In the Thevenin circuit, current entering every bus is zero except at the faulted bus. Since the current at faulted bus is leaving the bus, it is taken as a negative current entering bus k.

Fault analysis using impedance matrix

- . $\left[I_{bus}^{f}\right]=\left[Y_{bus}\right]\left[\Delta V_{bus}\right]$

$$\begin{bmatrix} 0_1 \\ \vdots \\ -I_k^f \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1k} & \cdots & Y_{1n} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ Y_{k1} & \cdots & Y_{kk} & \cdots & Y_{kn} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & \cdots & Y_{nk} & \cdots & Y_{nn} \end{bmatrix} \times \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}$$

- The change in the bus voltages can be calculated from the network matrix

$$\left[\Delta V_{bus}\right]=\left[Y_{bus}\right]^{-1}\times\left[I_{bus}^{f}\right]=\left[Z_{bus}\right]\times\left[I_{bus}^{f}\right]$$

$$\left[V_{bus}^{f}\right]=\left[V_{bus}^{pre\_f}\right]+\left[\Delta V_{bus}\right]$$

$$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \times \begin{bmatrix} 0_1 \\ \vdots \\ -I_k^f \\ \vdots \\ 0 \end{bmatrix}$$

$$\left[V_{bus}^{f}\right]=\left[V_{bus}^{pre\_f}\right]+\left[Z_{bus}\right]\times\left[I_{bus}^{f}\right]$$

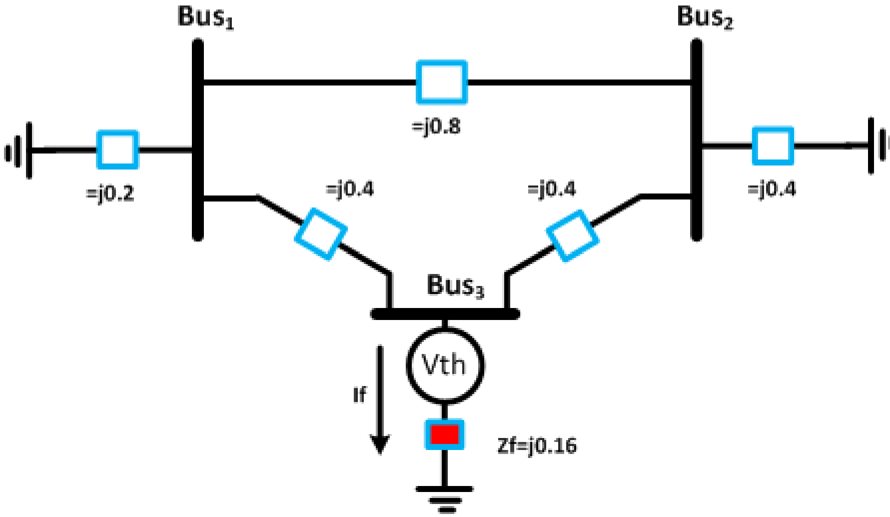
$$V_k^f = V_k^{pre\_f} + Z_{kk}\times I_k^f = Z_F \times I_k^f$$

$$I_k^f = \frac{V_k^{pre\_f}}{Z_{kk}+Z_F}$$

Fault analysis using impedance matrix

- 3 Bus example
- 3 phase fault with Zf=j0.16 on bus 3

$$[Y_{bus}] = \begin{bmatrix} -j8.75 & j1.25 & j2.50 \\ j1.25 & -j6.25 & j2.50 \\ j2.50 & j2.50 & -j5.00 \end{bmatrix}$$



$$[Z_{bus}] = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.24 \end{bmatrix}$$

- Therefore

$$I_k^f = \frac{V_k^{pre\_f}}{Z_{kk} + Z_F} = \frac{1\angle 0}{j0.34 + j0.16} = -j0.2$$

$$V_1^f = V_1^{pre\_f} + Z_{13} \times I_3^f = 1\angle 0 - (j0.12 \times -j2.0) = 0.76$$

$$V_2^f = V_2^{pre\_f} + Z_{23} \times I_3^f = 1\angle 0 - (j0.16 \times -j2.0) = 0.68$$

$$V_3^f = V_3^{pre\_f} + Z_{33} \times I_3^f = 1\angle 0 - (j0.34 \times -j2.0) = 0.32$$

Fault analysis general procedure using Z_{bus}

- Create a per-phase, per unit equivalent circuit for the power system.
- Calculate the bus admittance matrix Y_{bus}. Include the admittances of all transmission lines, transformers, between buses, including loads and generators.
- Calculate the bus impedance matrix Z_{bus}. It is the inverse of Y_{bus} .
- Assume that the power system is at no load conditions and determine the voltage at every bus. Voltage at every bus will be the same, and equal to internal voltage of generators. Given the symbol V_0.
- Calculate current at the faulted bus.
- Calculate voltage at each bus during fault.

$$I^f = \frac{V_r^0}{Z_{rr} + Z^f}$$

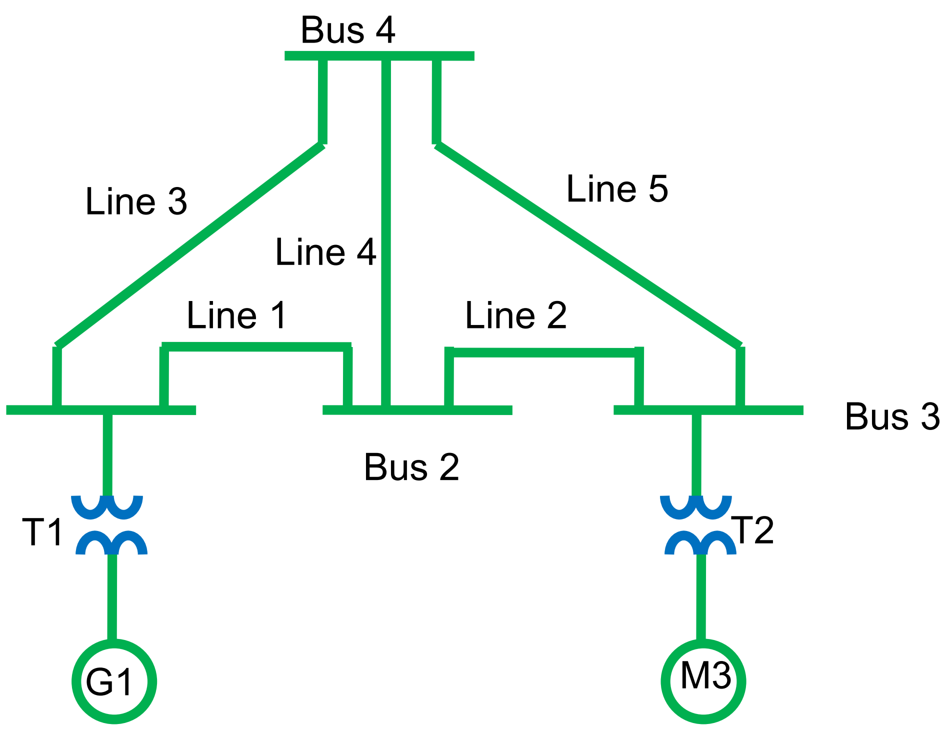
- Calculate currents in any desired transmission lines during fault.

$$V_i^f = V_i^0 - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^0, \Delta V_i = -Z_{ir} I^f$$

Fault analysis general procedure using Z_{bus}

4 bus system: Example

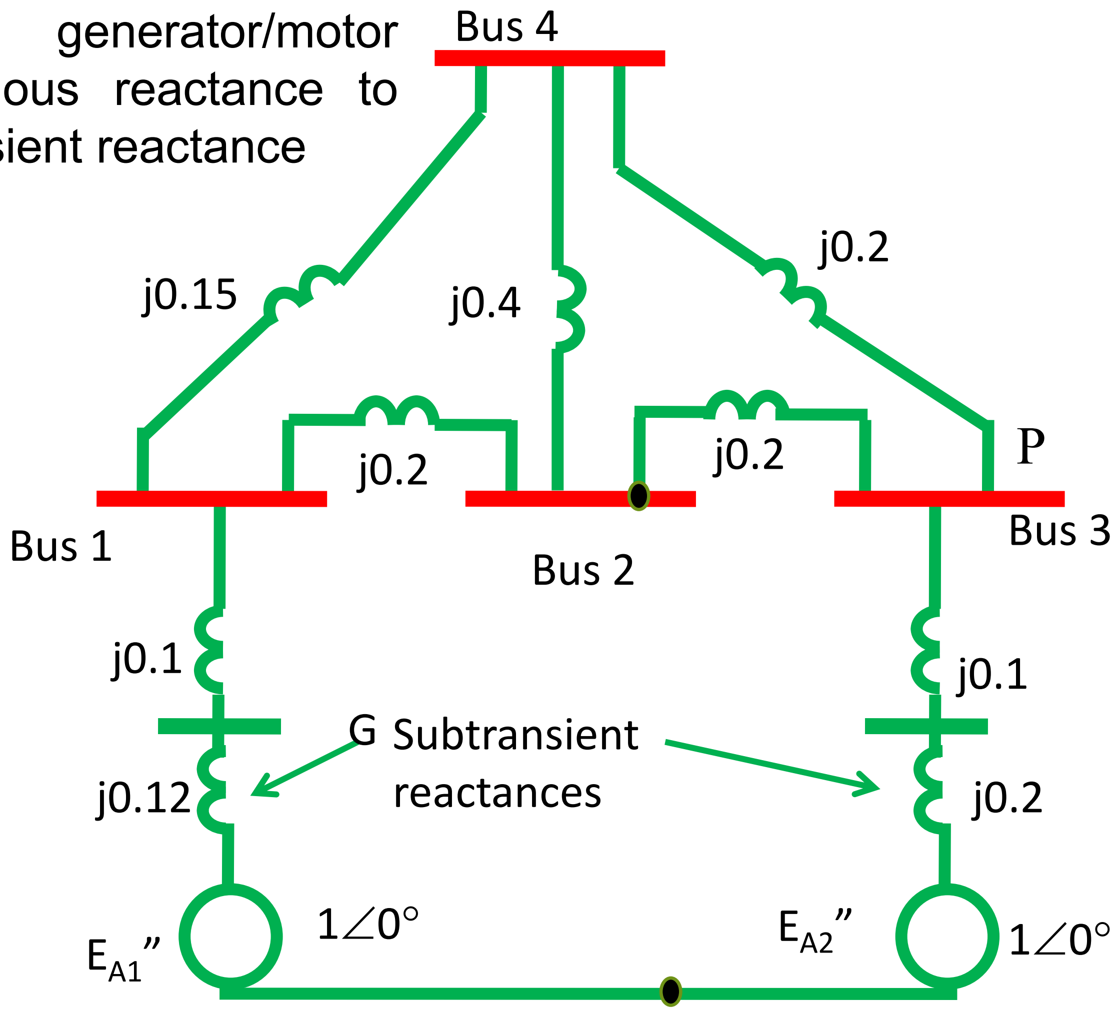
- Calculate the fault current and the system bus voltages during the fault what a symmetrical fault happens at bus 2



Fault analysis general procedure using Z_{bus}

4 Bus system: Solution

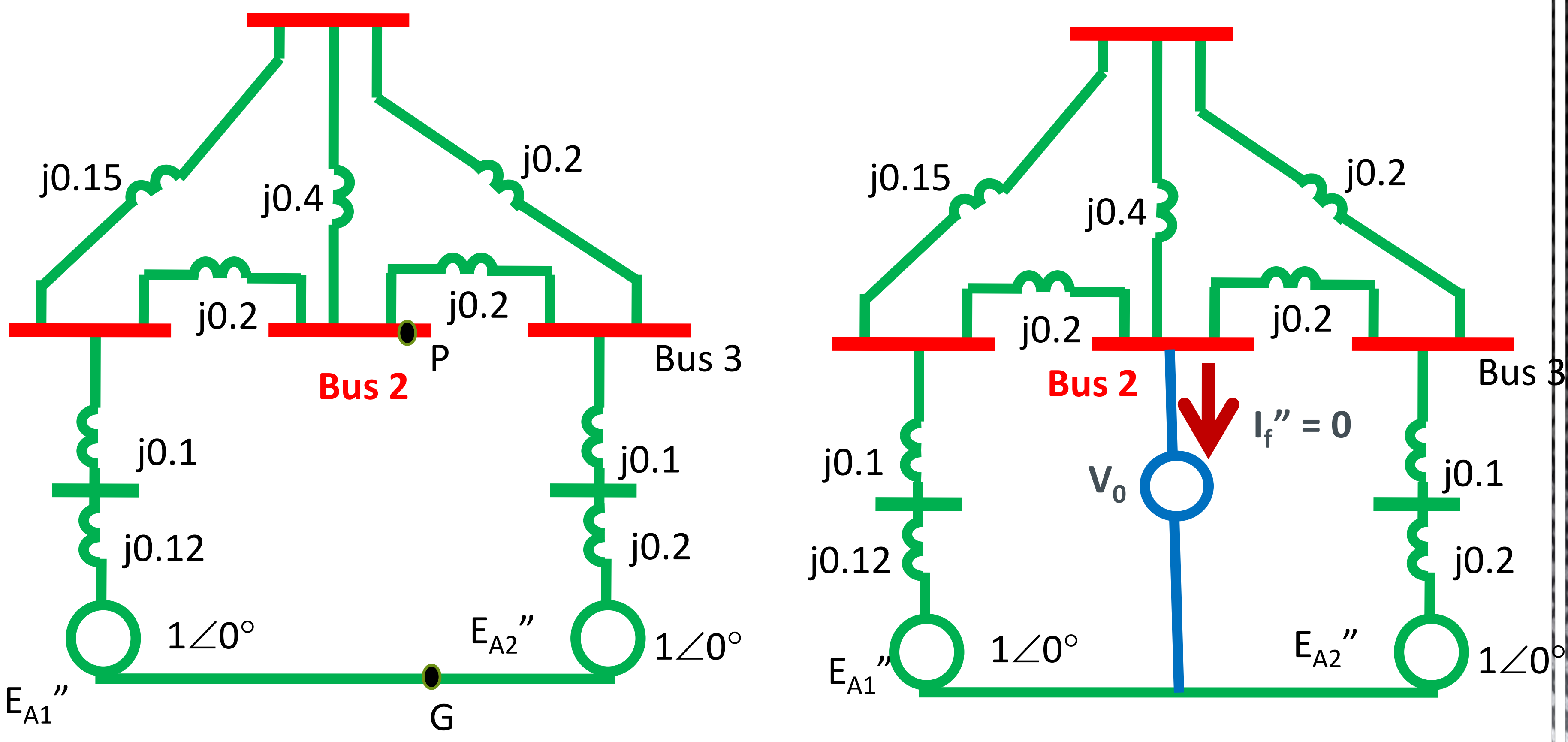
- Replace generator/motor synchronous reactance to sub transient reactance



Fault analysis general procedure using Z_{bus}

4 Bus system: Solution

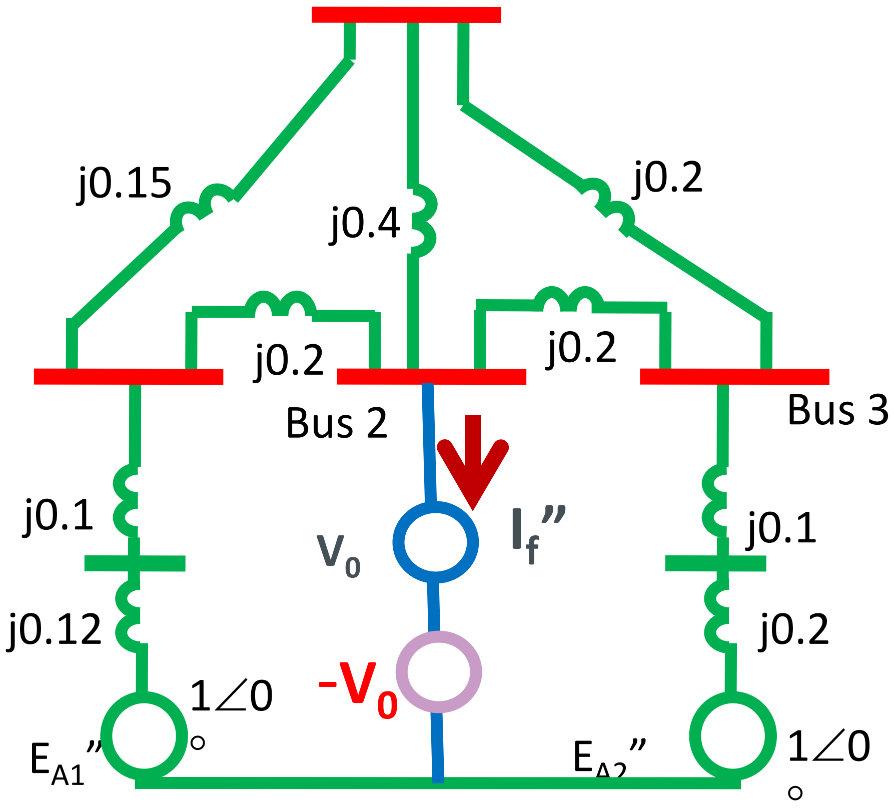
- Construct the equivalent circuit by adding pre fault voltage at bus 2



Fault analysis general procedure using Z_{bus}

4 Bus system: Solution

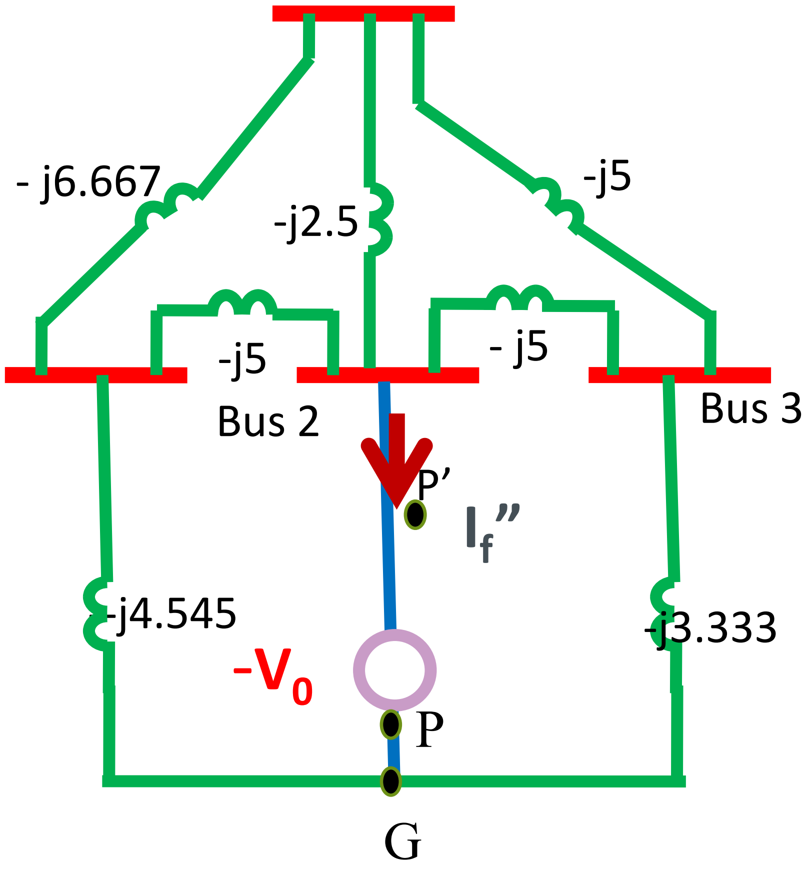
- Create fault at bus 2 by adding $-V_{02}$



Fault analysis general procedure using Z_{bus}

4 Bus system: Solution

- To apply Thevenin's theorem, Set all sources except for $-V_0$ to zero.
- Convert all impedances into admitances



Fault analysis general procedure using Z_{bus}

4 Bus system: Solution

- Construct admittance matrix and impedance matrix

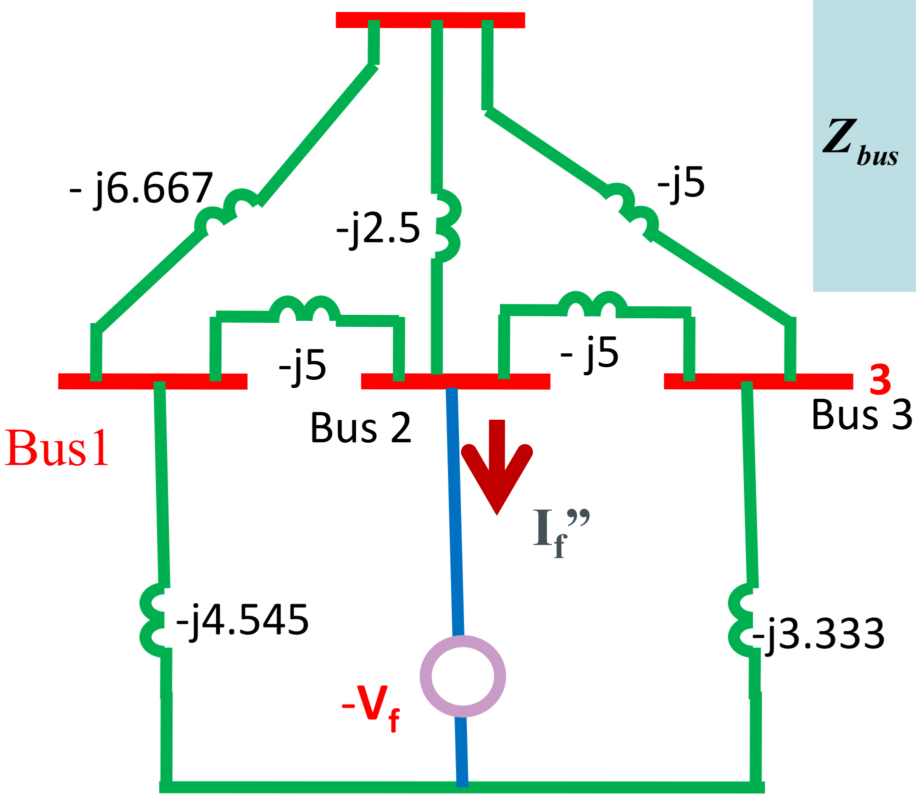
$$Y_{bus} = \begin{bmatrix} -j16.212 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -j13.333 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.1515 & j0.1232 & j0.0934 & j0.1260 \\ j0.1232 & j0.2104 & j0.1321 & j0.1417 \\ j0.0934 & j0.1321 & j0.1726 & j0.1282 \\ j0.1260 & j0.1417 & j0.1282 & j0.2001 \end{bmatrix}$$

Fault analysis general procedure using Z_{bus}

4 Bus system: Solution

- Assume V_0, = 1.0 and calculate faulted current



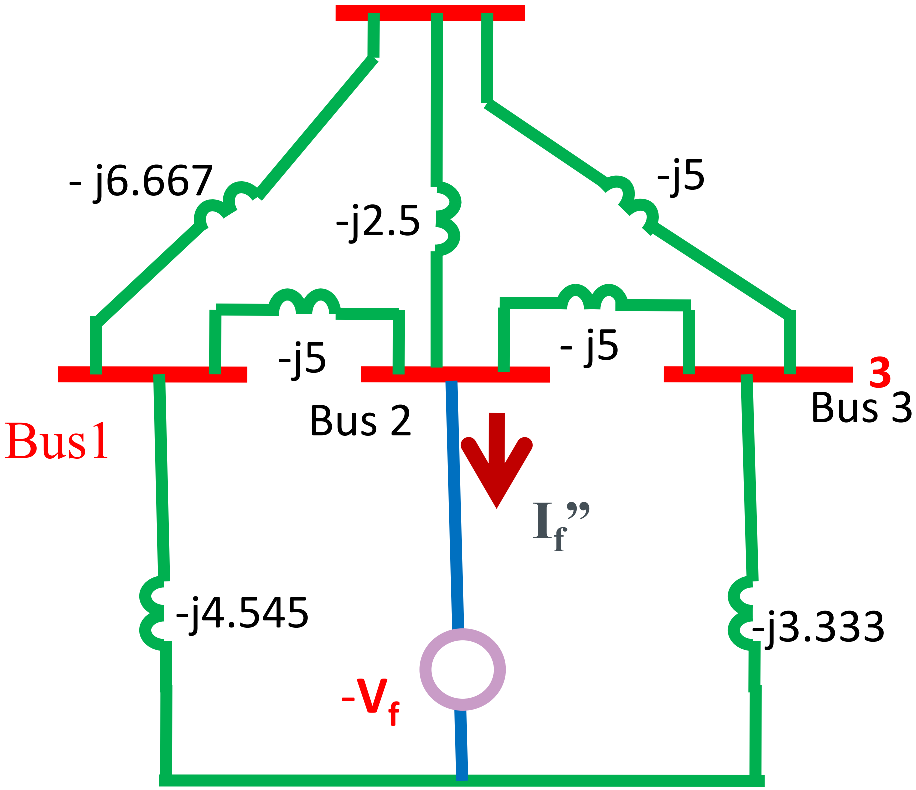
$$Z_{bus} = \begin{bmatrix} j0.1515 & j0.1232 & j0.0934 & j0.1260 \\ j0.1232 & j0.2104 & j0.1321 & j0.1417 \\ j0.0934 & j0.1321 & j0.1726 & j0.1282 \\ j0.1260 & j0.1417 & j0.1282 & j0.2001 \end{bmatrix}$$

$$I''_{f,2} = \frac{V_0}{Z_{22}} = \frac{1.00\angle 0}{j0.2104} = 4.753\angle -90$$

Fault analysis general procedure using Z_{bus}

- Calculate the voltages at buses during fault

4 Bus system: Solution



$$I''_{f,2} = \frac{V_0}{Z_{22}}$$

$$= \frac{1.00\angle 0}{j0.2104} = 4.753\angle -90$$

$$V_i^f\,(post\ \ fault) = V_i^0 - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^0, \quad V_i^0 = 1pu(prefault\,condition)$$

$$= \left(1 - \frac{Z_{ir}}{Z_{rr}}\right) V_0$$

Fault analysis general procedure using Z_{bus}

4 Bus system: Solution

- Calculate the voltages at buses during fault

$$V_1 = \left(1 - \frac{Z_{12}}{Z_{22}}\right)V_0 = \left(1 - \frac{j0.1232}{j0.2104}\right)1.00\angle 0 = 0.414\angle 0\ pu$$

$$V_2 = 0.0\angle 0$$

$$V_3 = \left(1 - \frac{Z_{32}}{Z_{22}}\right)V_0 = \left(1 - \frac{j0.1321}{j0.2104}\right)1.00\angle 0 = 0.372\angle 0\ pu$$

$$V_4 = \left(1 - \frac{Z_{42}}{Z_{22}}\right)V_0 = \left(1 - \frac{j0.1417}{j0.2104}\right)1.00\angle 0 = 0.327\angle 0\ pu$$

- Calculate the line current I_{12}

$$\begin{aligned} I_{12} &= -Y_{12}(V_1 - V_2) \\ &= (0.414\angle 0 - 0.00\angle 0)(-j5.0) \\ &= -j2.07\ pu \end{aligned}$$

SYMMETRICAL FAULT

In practical power system, most damage to equipment occurs due to short circuits occurring in the system.

Short circuit can occur due to various causes:

Birds dropping a conducting material across overhead lines

Lizards in switch boxes

Equipment failures due to aged or damaged insulation

In a typical power system, the fault currents are several orders greater than normal load currents. Hence, the fault currents can cause extensive thermal and mechanical damage if they are allowed to persist for more than a couple of seconds. If such conditions are allowed persist, they can result in extensive fire and explosion. Hence, it is important to recognise the fault conditions in the system and isolate the fault as quickly as possible.

In high voltage networks, this is normally achieved by means of relays to sense the fault current and the CB to isolate the fault. In addition, power system devices must be designed to withstand the fault currents for the short duration between the initiation and isolation of the fault. Hence, power system designers must have a very clear idea of the fault currents, which would flow in the system for various fault conditions. Consequently, modelling and analysis of power systems under fault conditions are an essential part of power system design and operation.

The objectives of the fault studies in a power system as below:

i) To establish CB ratings to interrupt the fault currents

ii) To establish the short time (s/c) ratings of the power system devices

iii) To establish relay settings

The solution of power systems under fault conditions essentially involves solving an electric network with voltage sources and impedance.

The generators and motors are treated as constant voltage sources, since the flux level, and hence the induced voltages in the machine coils, remain constant for a short period consequent to the fault.

Hence, for all practical purposes, the generators motors can be modelled as a constant voltage source behind machine impedance.

The fault itself can be modelled as a zero impedance branch or as an impedance branch, corresponding to the fault impedance.

It is also important to realise that the results of fault studies are concerned with the safety of the power system personnel and equipment.

Hence, fault system modelling and results must also meet statutory guidelines and obligations. Such guidelines are normally provided by the relevant standards.

It is important to be aware of relevant standards when conducting fault studies in practise.

During Fault

Resulting fault current is determined by the internal voltages of the synchronous machines and by the system impedances between the machine voltages and the fault.

Implication?

Short circuit current may be several orders of magnitude larger than normal operating Currents. If allowed to persist, may cause thermal damage to equipment

Winding and busbars may also suffer mechanical damage due to high magnetic forces during faults. It is necessary to remove faulted sections of power system from service as soon as possible.

Standard high voltage protective equipment is design to clear faults within 3 cycles.

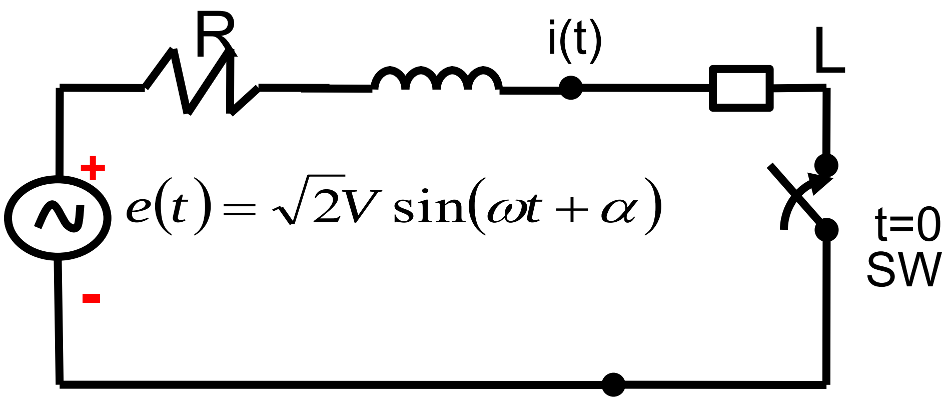
Lower voltage protective equipment operates more slowly (5 to 20 cycles)

- Delayed or inaction can be fatal!

50,000A Fault

Inadequate Interrupting Rating

Series R-L Circuit Transients



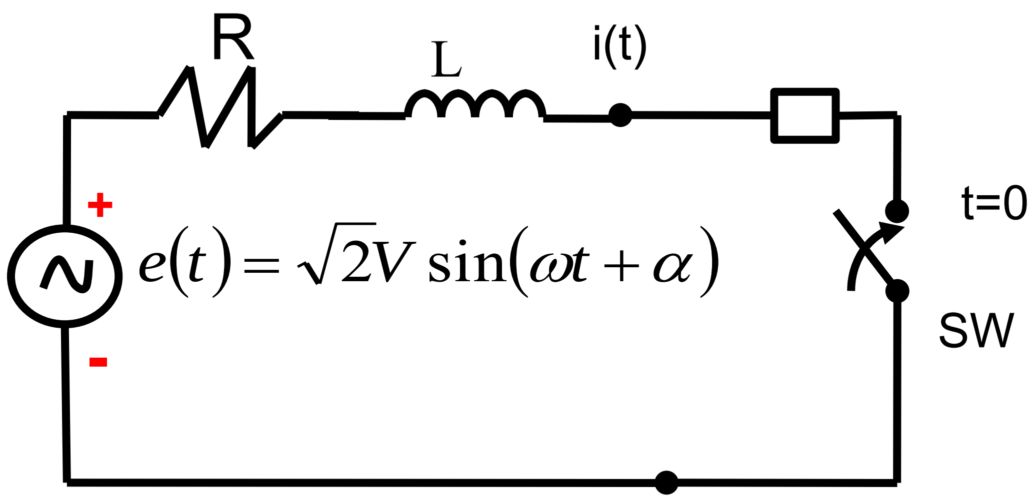
Consider the s/c transient on the transmission line. Certain simplifying assumptions are made at this stage.

i). The line is fed from a constant voltage source

ii). S/c takes place when the line is unloaded

iii). Line capacitance is negligible and the line can be represented by a lumped RL series circuit.

Series R-L Circuit Transients

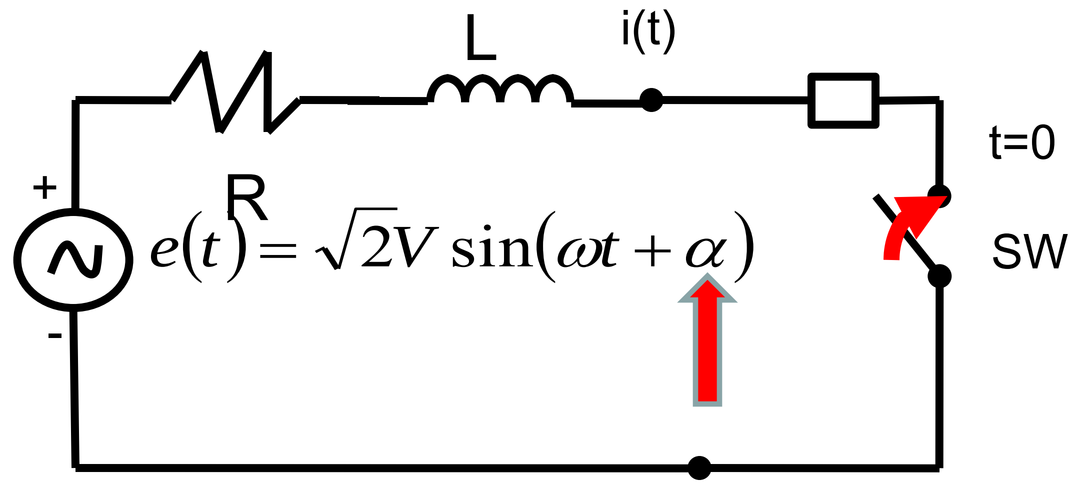


$$e(t) = \sqrt{2}V \sin(\omega t + \alpha)$$

The switch SW was closed at t = 0

(this is equivalent to three phase short circuit at the terminal of unloaded synchronous machine)

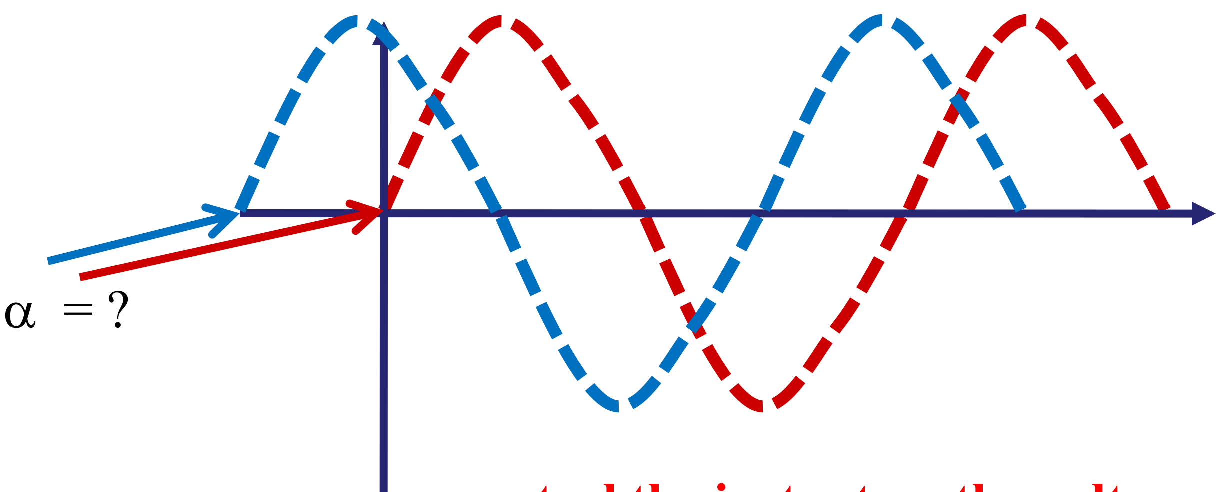
Series R-L Circuit Transients



Short circuit occurs at t=0

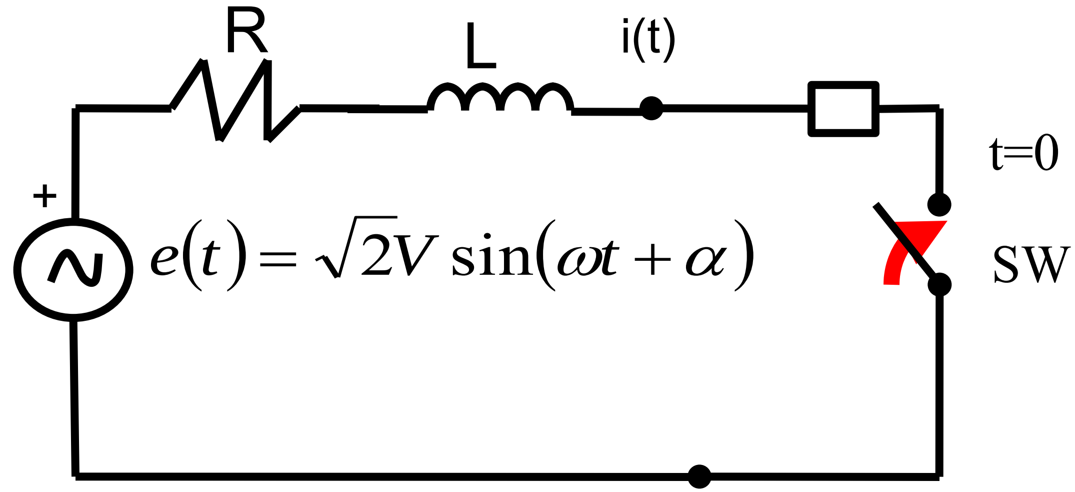
Source angle **α** determines the source voltage at **t = 0**

$$e(t) = \sqrt{2}V\sin(\omega t + \alpha)$$



α-control the instant on the voltage wave when s/c occurs

Series R-L Circuit Transients

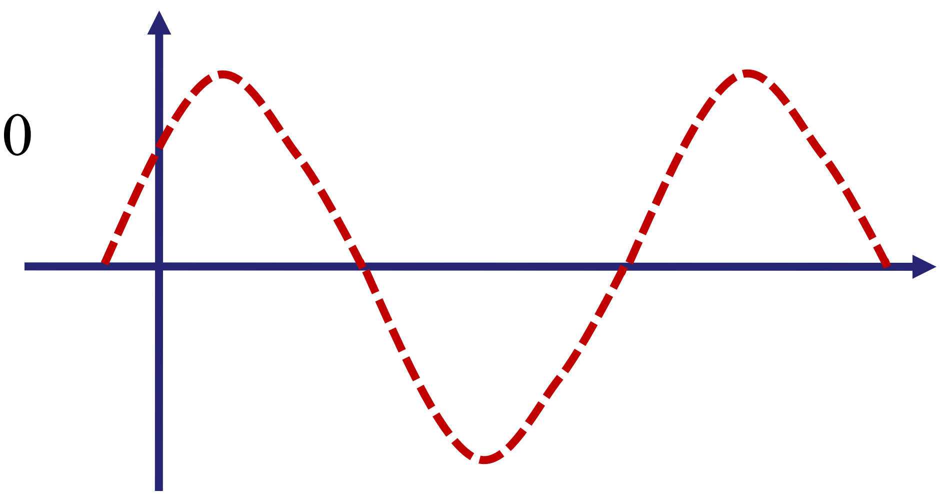


Assume **zero fault impedance** (s/c is a **solid or bolted** fault)

Current is zero before SW closes

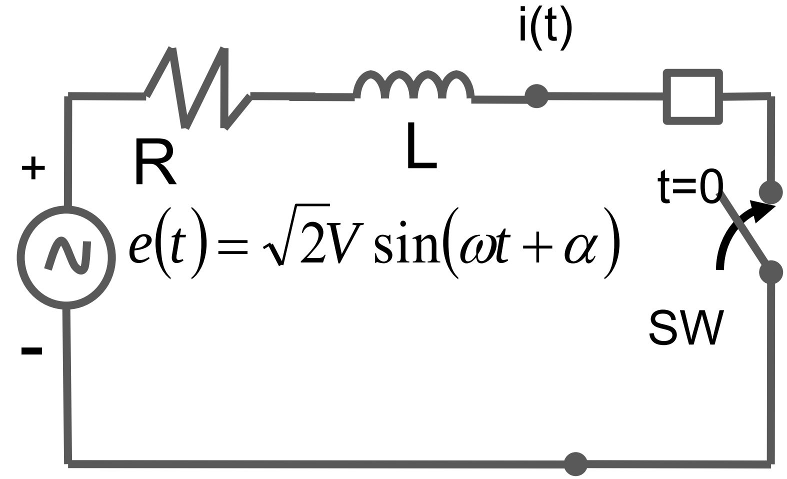
KVL equation for the circuit;

$$\frac{Ldi(t)}{dt} + Ri(t) = \sqrt{2}V\sin(\omega t + \alpha) \quad t \geq 0$$



Series R-L Circuit Transients

$$\frac{L\,di(t)}{dt} + Ri(t) = \sqrt{2}V\sin(\omega t + \alpha) \quad t \geq 0$$



The current after s/c is composed of two parts

Solution for the equation

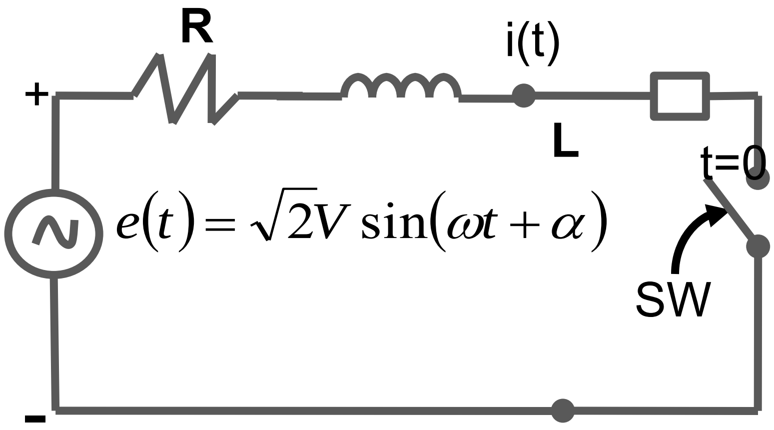
$$i(t) = i_{ac}(t) + i_{dc}(t)$$

$$i_{ac}(t) = \frac{\sqrt{2}\,V}{|Z|}\left[\sin(\omega t + \alpha - \theta)\right] \qquad |Z| = \sqrt{R^2 + (\omega L)^2}$$

$$i_{dc}(t) = -\frac{\sqrt{2}\,V}{|Z|}\left[\sin(\alpha - \theta)e^{-t/\tau}\right] \qquad \theta = \tan^{-1}\frac{\omega L}{R}$$

$$= \frac{\sqrt{2}\,V}{|Z|}\left[\sin(\theta - \alpha)e^{-t/\tau}\right]$$

Series R-L Circuit Transients

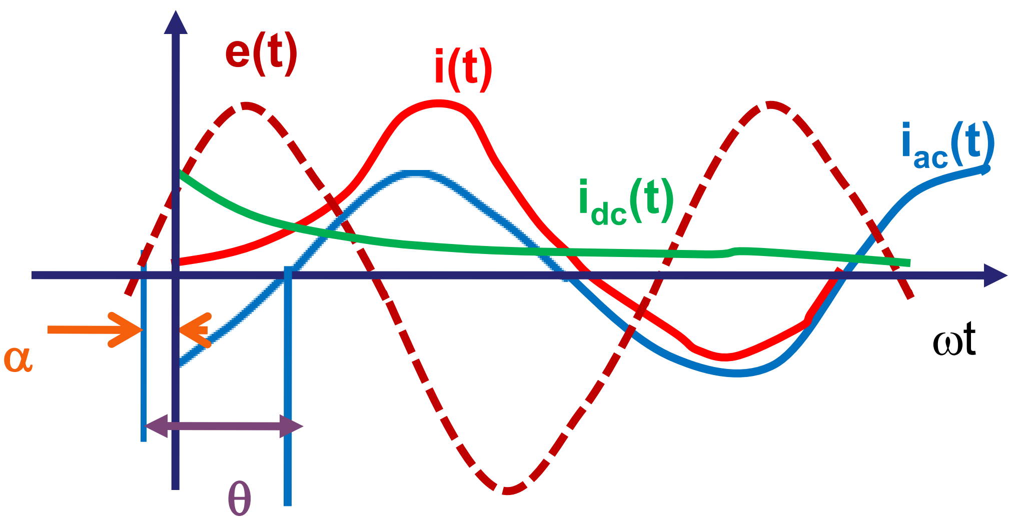


$$i(t) = i_{ac}(t) + i_{dc}(t)$$

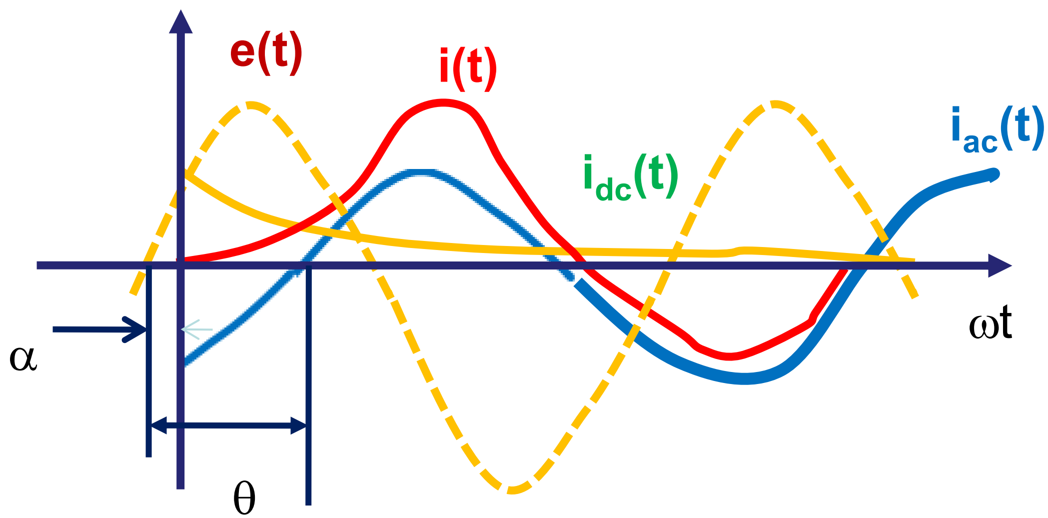
$$i_{ac}(t) = \underbrace{\frac{\sqrt{2}V}{Z}\left[\sin(\omega t + \alpha - \theta)\right]}_{\text{Symmetrical s/c current}}$$

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

$$i_{dc}(t) = \underbrace{\frac{\sqrt{2}V}{Z}\left[\sin(\theta - \alpha)e^{-t/\tau}\right]}_{\text{DC off-set current}}$$

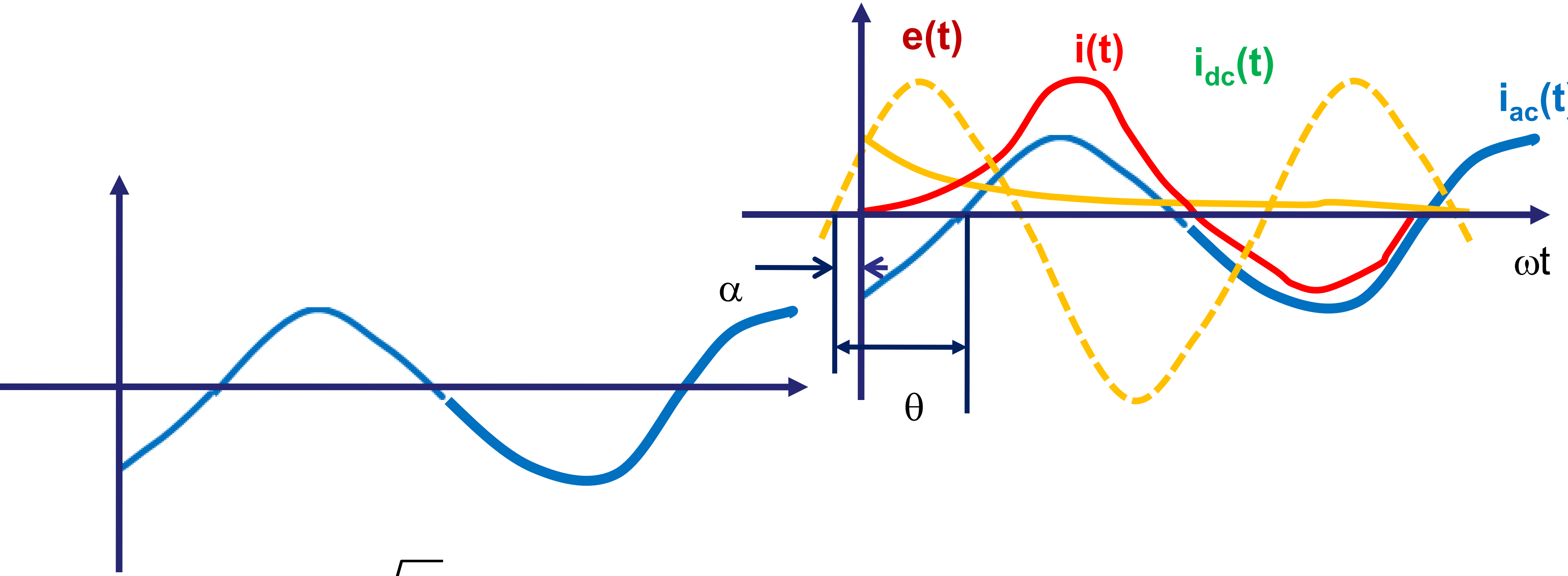


In power system terminology, the sinusoidal steady state current is called the symmetrical s/c current and the unidirectional transient component is called the DC off-set current, which causes the total s/c current to be unsymmetrical till the transient decays.



Total fault current is called asymmetrical fault current

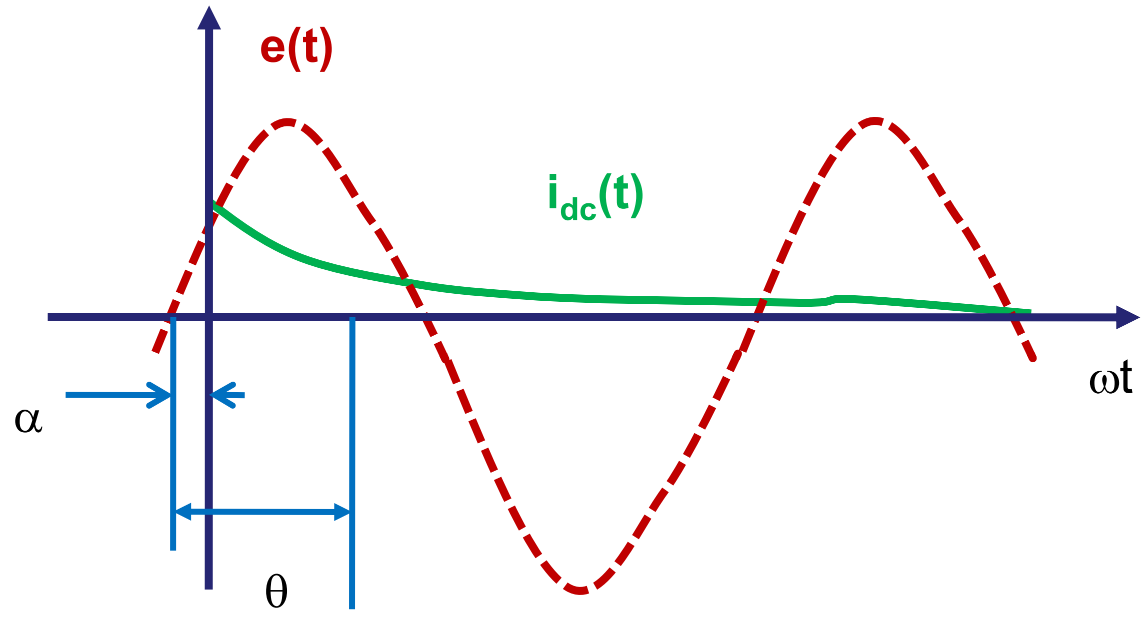
$$i(t) = i_{ac}(t) + i_{dc}(t)$$



$$i_{ac}(t) = \frac{\sqrt{2}V}{Z}\left[\sin(\omega t + \alpha - \theta)\right]$$

Ac fault current or symmetrical or steady-state fault current varies sinusoidally with time

The rms ac fault current $I_{ac} = \dfrac{V}{Z}$



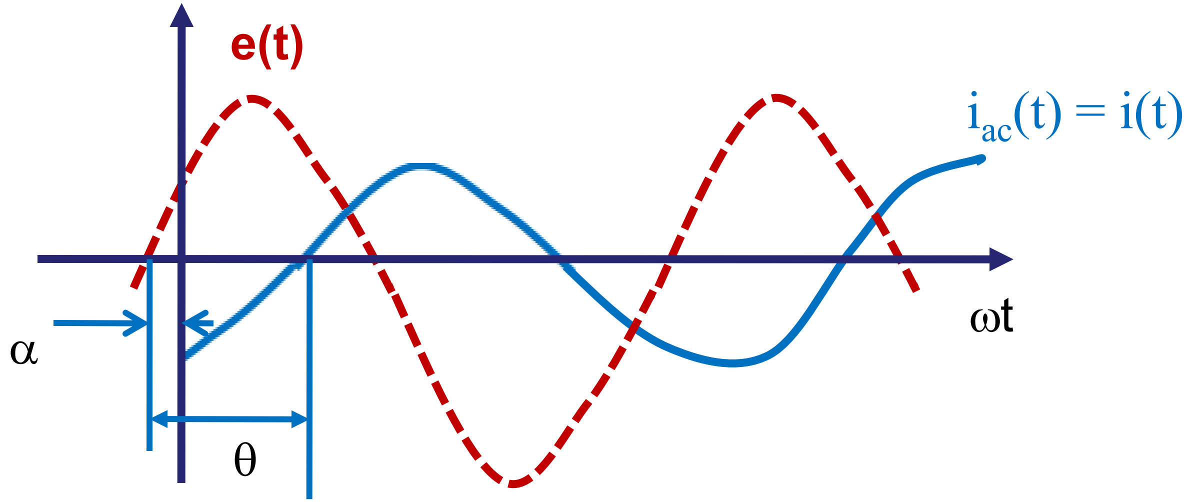
The dc fault current or dc offset current decays exponentially with time constant

$\tau = L/R$

$$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\left[\sin(\alpha-\theta)e^{-t/\tau}\right]$$

Depending on the instantaneous value of the voltage when the circuit is closed and on the power factor of the load, the dc fault current may have any value from 0 to

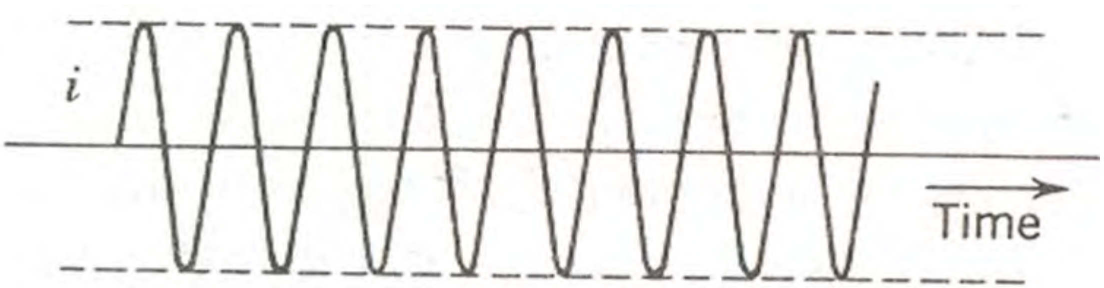
$$\frac{V_{\max}}{|Z|}$$



The dc fault current does not exist if the circuit is closed at a point on the voltage wave such that, $\alpha - \theta = 0$ or $\alpha - \theta = \pi$

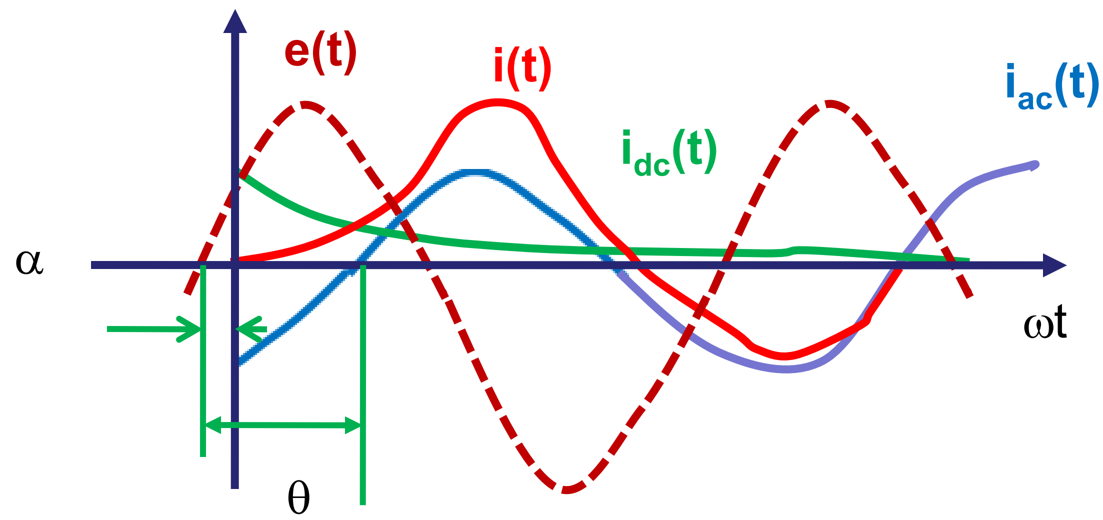
$$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\left[\sin(\alpha - \theta)e^{-t/T}\right]$$

Current as a function of time in an RL circuit



When $\alpha - \theta = 0$ no dc current

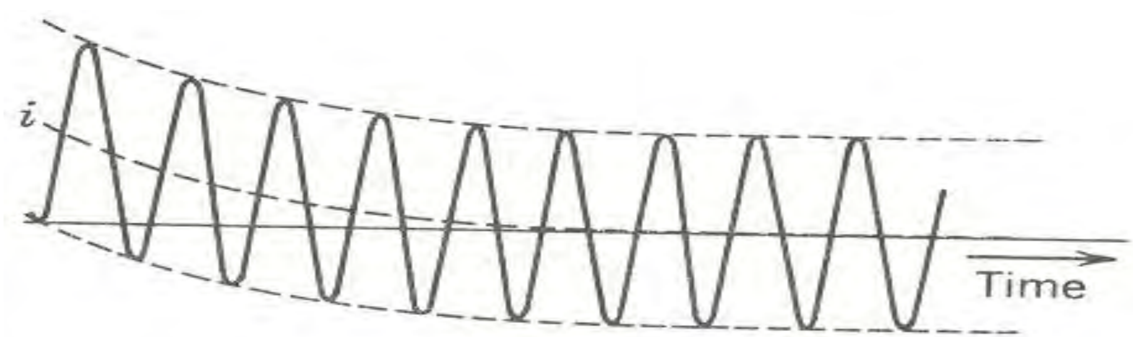
Total fault current is called asymmetrical fault current $i(t) = i_{ac}(t) + i_{dc}(t)$



$$i_{dc}(t) = -\frac{\sqrt{2}V}{Z}\left[\sin(\alpha-\theta)e^{-t/T}\right]$$

dc fault current has its maximum when $\alpha - \theta = \pm\frac{\pi}{2}$

Current as a function of time in an RL circuit



When $\alpha - \theta = -\pi/2$

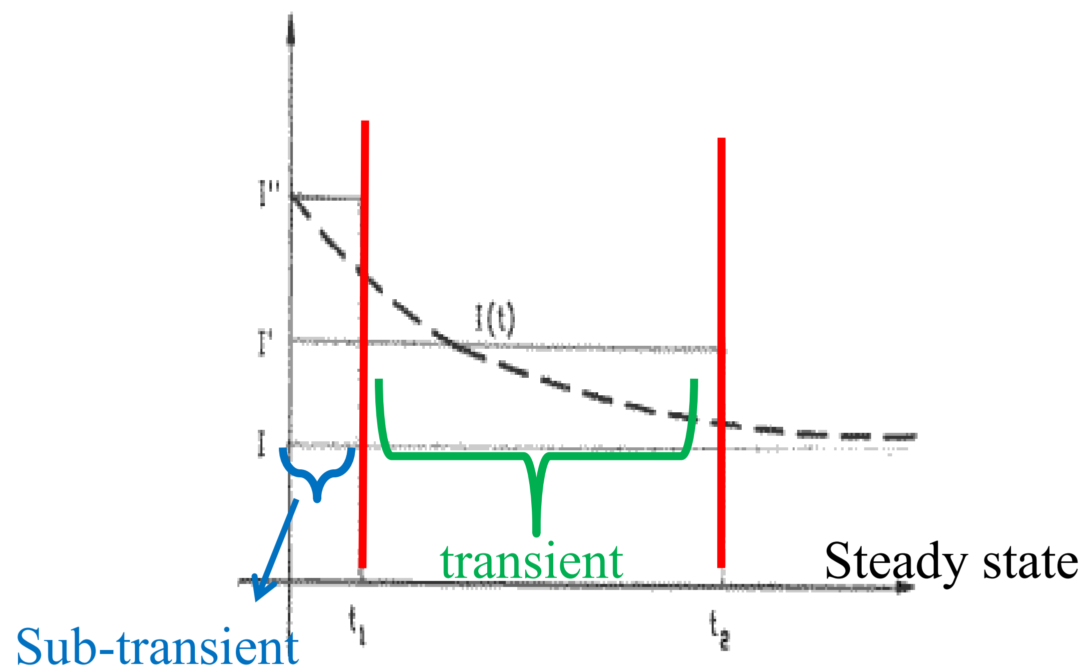
Variation of current with time during a fault

The envelope of the current wave is plotted. The s/c current can be divided into three periods-initial sub-transient period when the current is large as the machine offers sub-transient reactance, the middle transient period where the machine offers transient reactance, and finally the steady state period when the machine offers synchronous reactance.

I” = subtransient current

I’ = transient current

I = steady state current



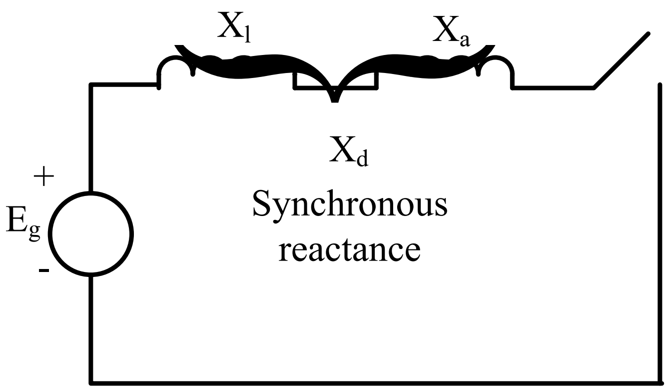
Generator Impedance

The value of the generator inductance (reactance) varies substantially consequent to a disturbance in the power system.

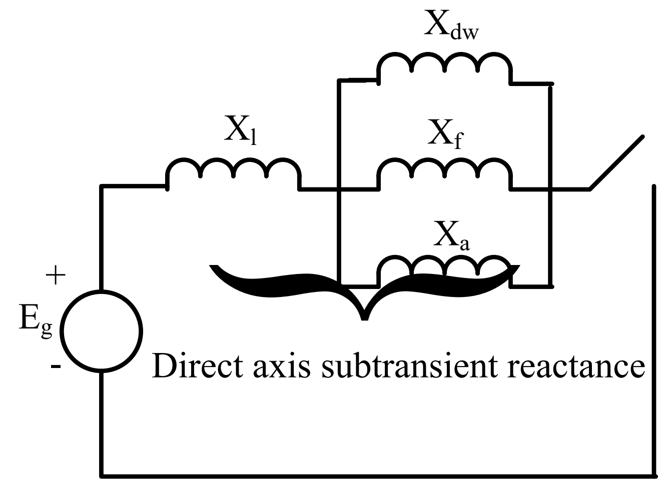
The most common disturbance is the s/c or fault condition.

Immediately following a fault, the generator reactance not only varies with time, but is substantially less than the steady state value. Hence, it is essential to use appropriate reactance value depending on the study requirement. The concept is illustrated by considering a balanced three-phase fault at the terminals of a generator. Since the system is balanced, we need to consider only the positive sequence network.

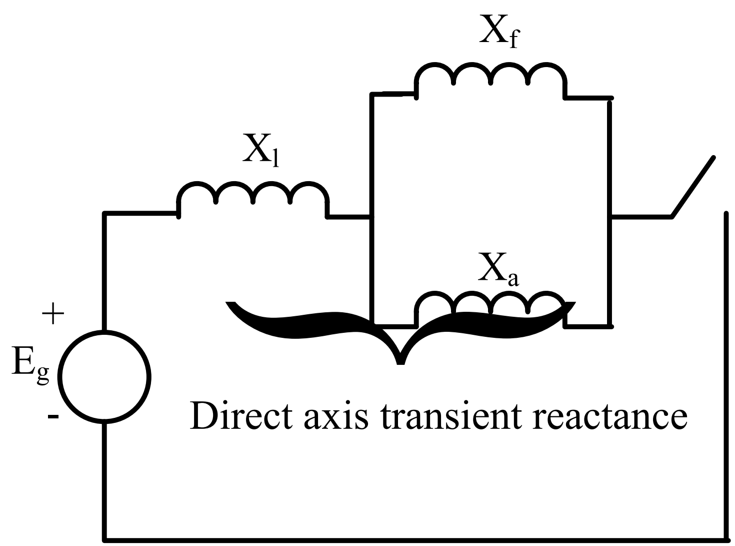
Under steady state s/c conditions, the armature reaction of a synchronous generator produces a demagnetizing flux. In terms of a circuit this effect is modeled as a reactance X_a in series with induced emf. This reactance when combined with the leakage reactance X_l of the machine is called synchronous reactance X_d (direct axis synchronous reactance in the case of salient pole).



(a) Steady state s/c model of a synchronous machine

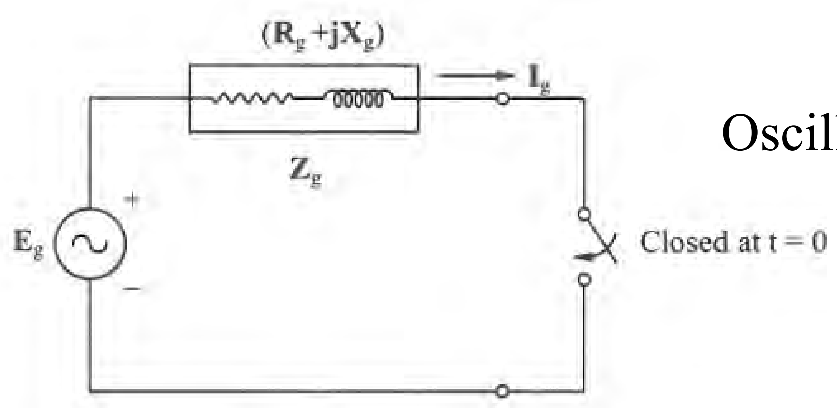


(b) Approximate circuit model during subtransient period of s/c



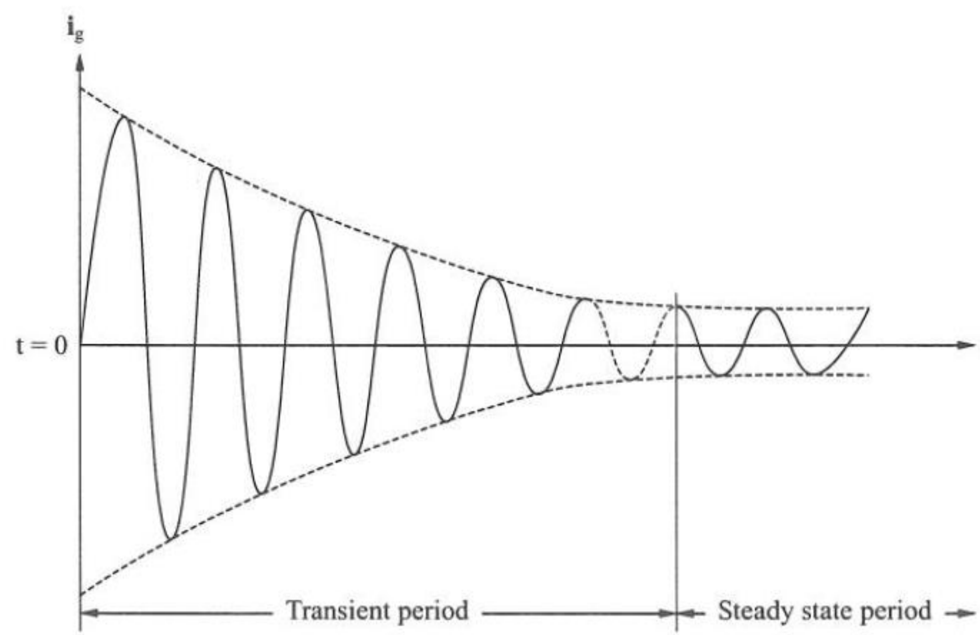
(c) Approximate circuit model during transient period of s/c

Armature resistance being small can be neglected. The steady state s/c model of a synchronous machine is shown below on per phase basis. The circuit diagram and the oscillogram of the generator current are as shown below.



Oscillogram of generator (fault) current

Balanced 3 phase fault at generator terminals



The physical explanation for the transient period is as follows:

At the instant of fault, the flux in a rotating machine is initially forced through the higher reluctance path.

Since the flux cannot change instantly to counter the demagnetization of the armature s/c current, the inductance (reactance) of the machine is reduced due to increased reluctance, thus resulting in a larger current. However, the flux distribution slowly realigns itself to the lower reluctance path over a period of time.

In practice, the inductance and consequently the fault current, normally settles down to the steady state after about 25 cycles (in accordance with the winding time constant). The time constant of the damper winding which has low leakage inductance is much less than that of the field winding, which has high leakage inductance. Thus during the initial part of the s/c, the damper and field windings have transformer currents induced in them so that in the circuit model their reactances, X_f of field winding and X_{dw} of damper winding appear in parallel with X_a. As the damper winding currents are first to die out, X_{dw} effectively becomes o/c and at a later stage X_f become o/c. The machine reactance thus changes from the parallel combination of X_a, X_f and X_{dw} during the initial period of the s/c to X_a and X_f in parallel in the middle period of the s/c, and finally to X_a in steady state. The machine thus offers a time-varying reactance which changes from X''_d to X'_d and finally to X_d. The larger transient current is not really important if we are only interested in I/V values after the steady state conditions are reached. That is, after about 0.5 sec consequent to fault initiation.

In fact, if the test is conducted using a traditional ammeter, instead of a high speed recorder you would only see the steady state fault current. Hence, the generator reactance corresponding to the steady state condition is adequate for voltage drop, power flow and power loss calculations. However, practical power systems must be designed to withstand transient currents under fault conditions and more importantly, the CB must be adequately rated to interrupt fault currents in the transient period. This is particularly relevant in the case of fast-acting CB, which are commonly used in modern power systems.

Therefore, the transient period has a definite significance for power system designers.

For design purposes, the current oscillogram is divided into three regions.

The maximum value of current in each region is assumed to be the steady state current for that region.

Consequently, the generator reactance values for each region are established.

Hence, they will enable the designer to calculate the fault currents in each region by the use of appropriate values of the machine reactance.

The regions and the corresponding reactances are termed as follows:

0 to Approx. 2 cycles => Subtransient Region=> Subtransient Reactance (X”)

2 to Approx. 25 cycles => Transient Region=> Transient Reactance (X’)

>Approx. 25 cycles => Steady state Region=> Synchronous Reactance (X)

In addition to the above, there is another factor while specifying reactances in the case of generators. The air gap in the case of high-speed generators (such as thermal units) is constant along the circumference.

Such machines are called cylindrical pole machines and typically have 2-4 poles.

However, such a design is not feasible in the case of slower-speed machines (such as hydro units), which require a larger number of poles.

Such machines are called salient pole machines.

In the case of cylindrical pole machines, only the machine reactance along the pole axis (direct axis) is specified, since the reactance is constant along the circumference.

However, in the case of salient pole machines, machine reactances along the pole axis and also along the axis between the poles (quadrature axis) are specified.

For most practical studies, it is common to ignore quadrature axis values.

In other words, the effect of saliency is neglected. In modern power systems, the subtransient reactance is invariably used for fault calculation and breaker ratings.

The synchronous reactance value is used for voltage drop and power flow calculations. Hence, in the case of positive sequence values, only X''_d and X_d are the most relevant quantities for power system calculations.

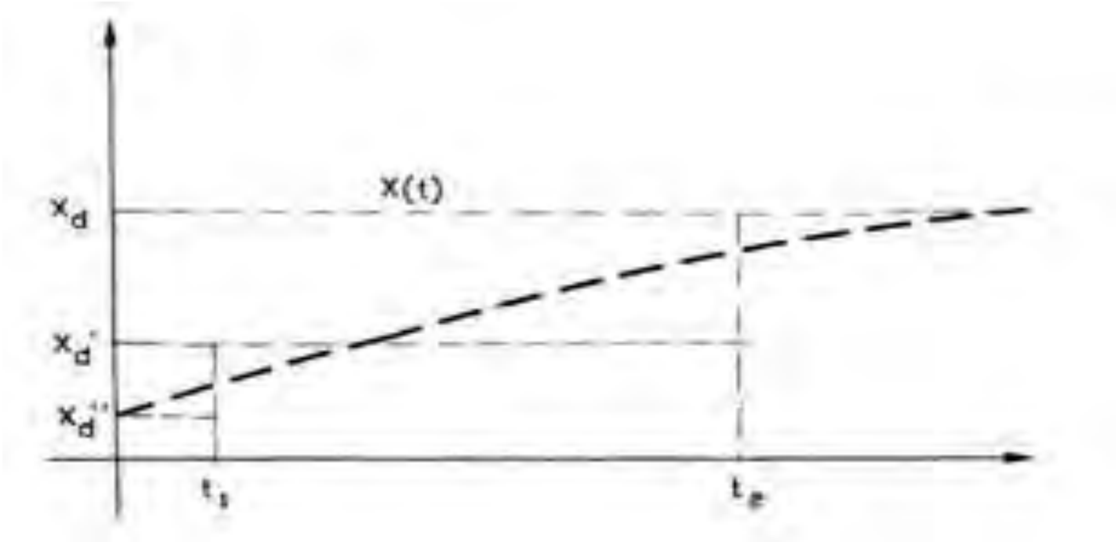
In the early days, the transient reactance value was commonly used for fault calculations, due to the usage of slow-operating CB.

Modern power systems employ fast-acting CB even at distribution voltage levels.

Hence, subtransient reactance is invariably used for fault calculations.

Note that usage of subtransient reactance (which is a smaller value) results in a higher value of fault current, and hence the system ratings will be more conservative.

Variation of generator reactance with time during a fault

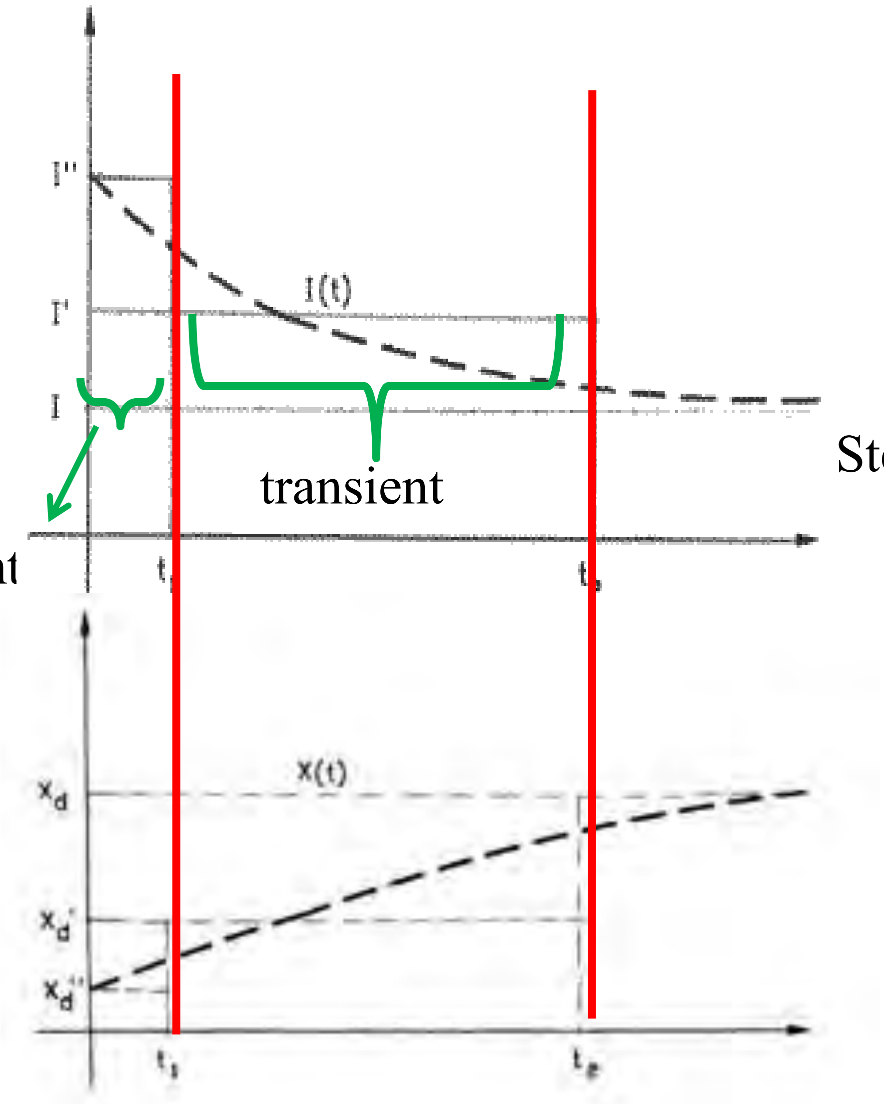


X_d" = subtransient reactance

X_d' = transient reactance

X_d = steady state reactance

Variation of current with time during a fault



Steady state

subtransient

I’’ = subtransient current

I’ = transient current

I = steady state current

X_d’’ = subtransient reactance

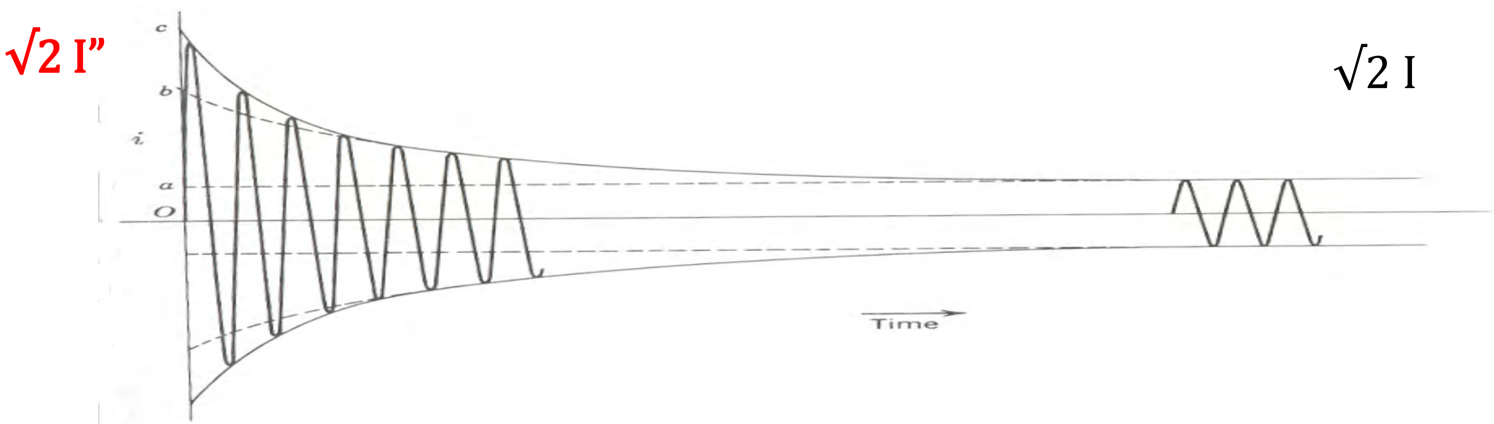
X_d’ = transient reactance

X_d = steady state reactance

Typical average reactance values for synchronous machines

	Two pole turbine generator	Four pole turbine generator	Salient pole with dampers	Synchronous condensers
X_d	1.2	1.2	1.25	2.2
X_d'	0.15	0.23	0.30	0.48
X_d''	0.09	0.14	0.20	0.32
X_2	0.09	0.14	0.20	0.31
X_0	0.03	0.08	0.18	0.14

Ac fault current in one phase of an unloaded synchronous machine during a three-phase short circuit.



Symmetrical fault calculation

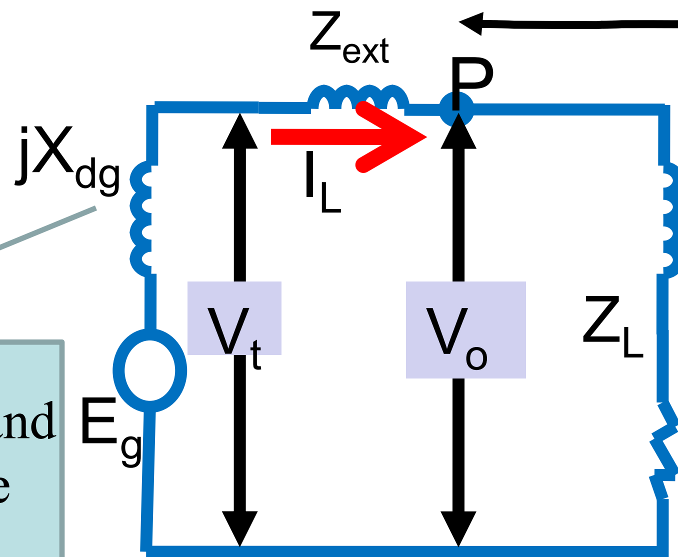
Three phase symmetrical faults on systems containing generators and motors may be analyzed using: 1. Subtransient internal voltages

2. Thevenin's theorem

3. Z_{bus}

Internal voltages of loaded machines under fault conditions

Consider the circuit model of a generator operating under steady conditions is loaded with a balanced 3 phase load when fault occurs.



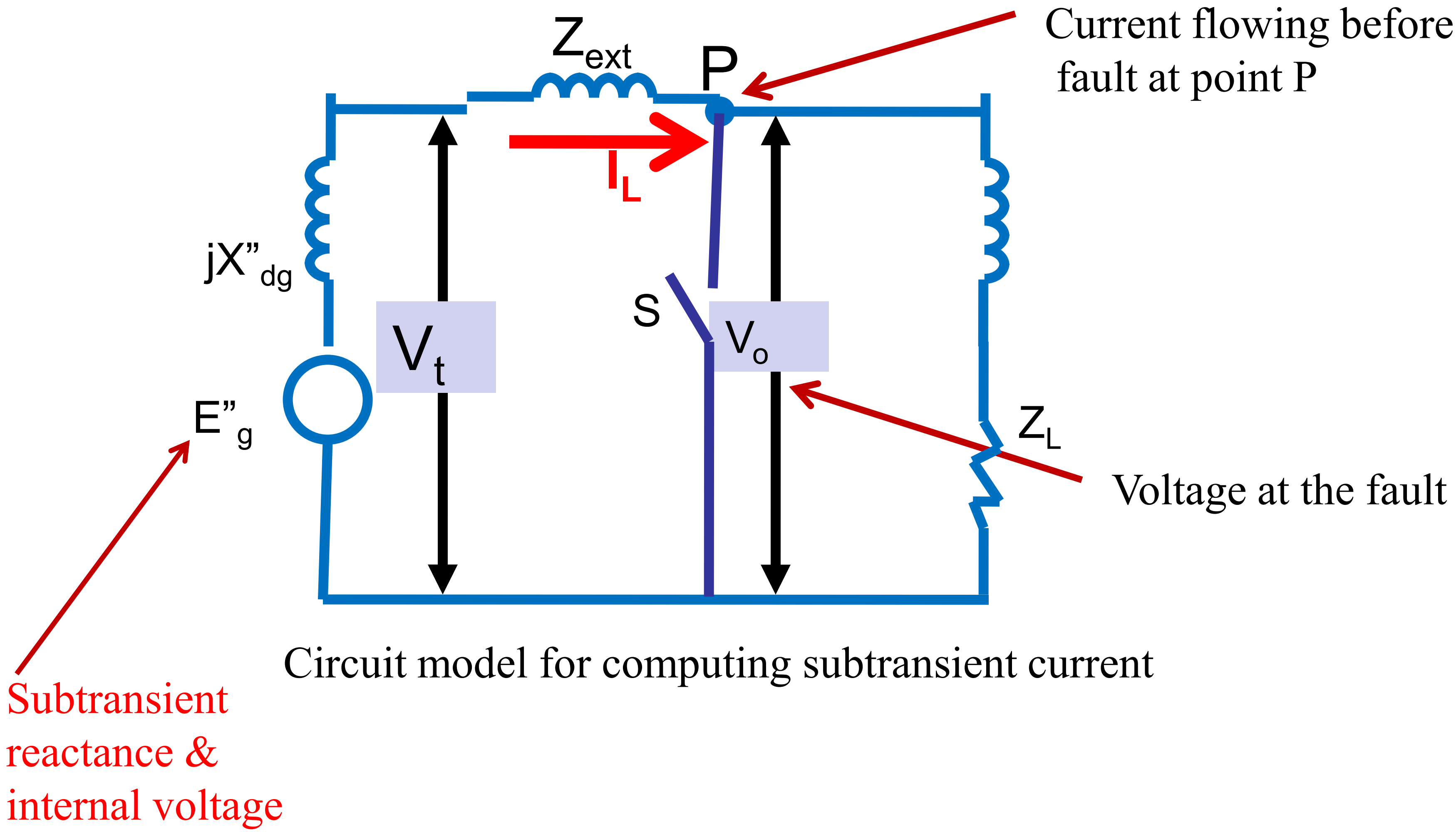
Internal voltage and reactance of generator

External impedance exist between generator terminals and point P where the fault occurs

E_g =induced emf under loaded condition

X_{dg} = direct axis synchronous reactance of the machine

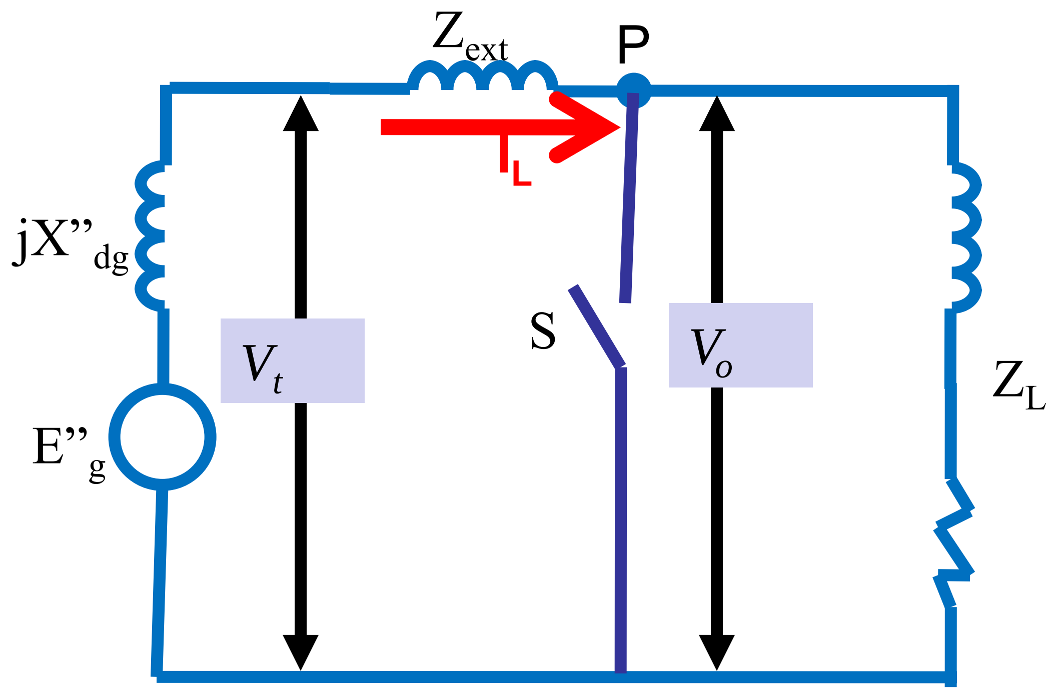
Application of a three-phase fault at P is simulated by closing switch S



Circuit model for computing subtransient current

To satisfy conditions of subtransient and transient steady state condition (parameters) need to be substituted with subtransient or transient .

Application of a three-phase fault at P is simulated by closing switch S



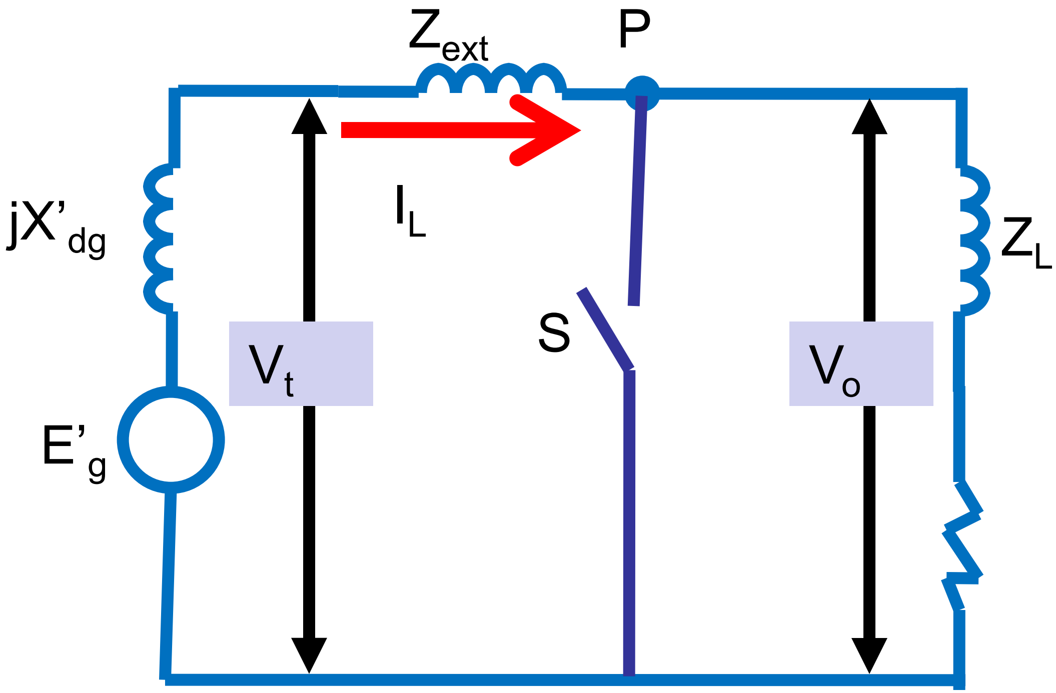
The induced emf to be used in this model is given by

With switch S open

$$\Rightarrow \quad E_g^{''} = V_t + jX_{dg}^{''} I_L = V_o + \left(Z_{ext} + jX_{dg}^{''}\right) I_L$$

E''_g =voltage behind the subtransient reactance

Application of a three-phase fault at P is simulated by closing switch S



Transient reactance & transient internal voltage for transient current

Circuit model for computing transient current

With switch S open

$$E'_g = V_t + jX'_{dg} I_L = V_o + \left(Z_{ext} + jX'_{dg}\right) I_L$$

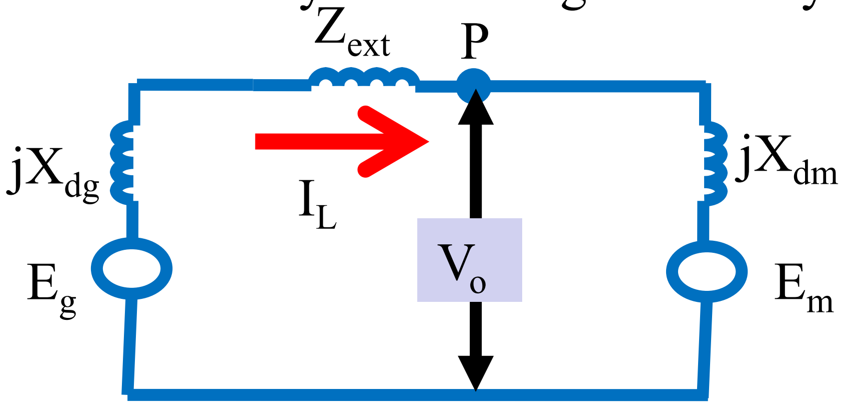
$$E''_g = V_t + jX''_{dg} I_L = V_o + \left(Z_{ext} + jX''_{dg}\right) I_L$$

$$E'_g = V_t + jX'_{dg} I_L = V_o + \left(Z_{ext} + jX'_{dg}\right) I_L$$

The value of load current determines E_g” and E_g’

Synchronous motor connected to synchronous generator by line impedance Z_{ext}

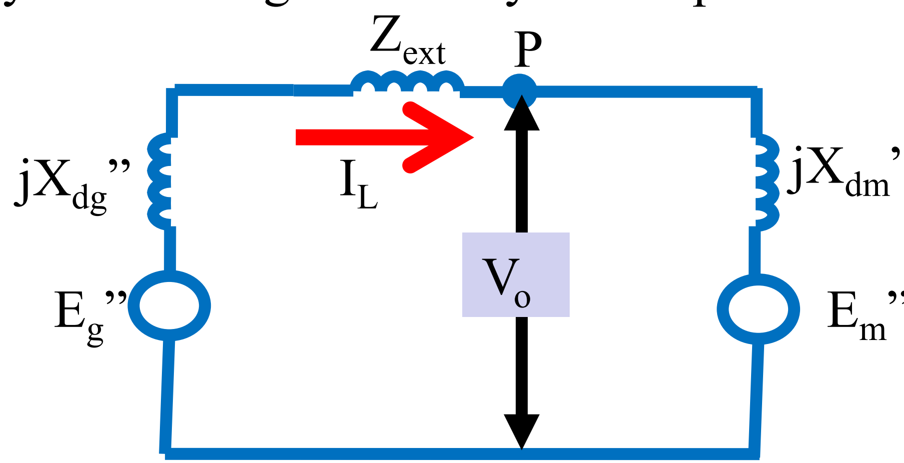
Before fault



Motor is drawing $\mathbf{I_L}$ when symmetrical three-phase fault occurs

Synchronous motor connected to synchronous generator by line impedance Z_{ext}

Before fault

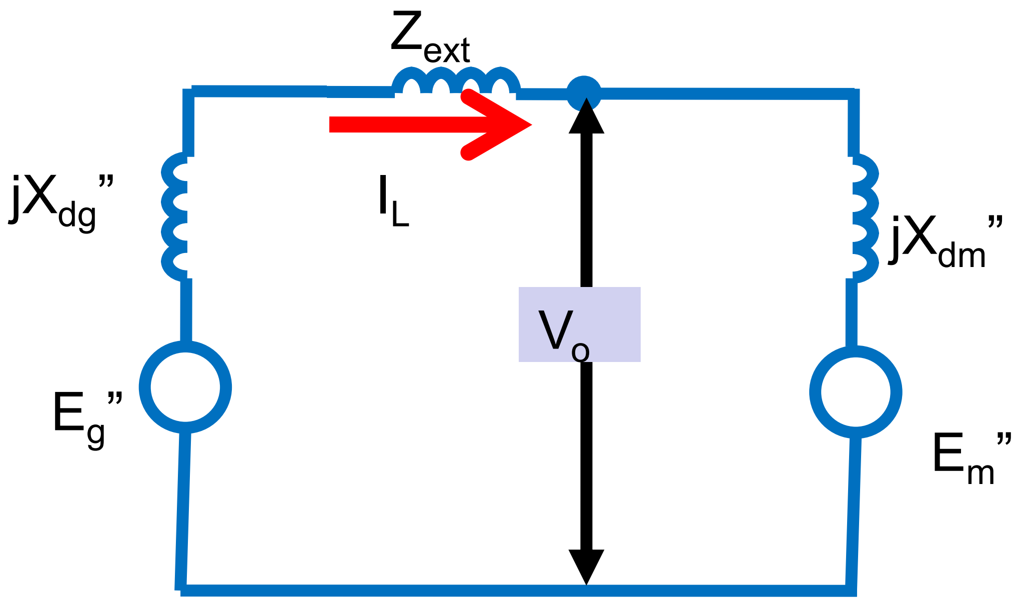


$$E''_m = V_o - jX'''_{dm} I_L$$

$$E''_g = V_t + jX''_{dg} I_L = V_o + (Z_{ext} + jX''_{dg}) I_L$$

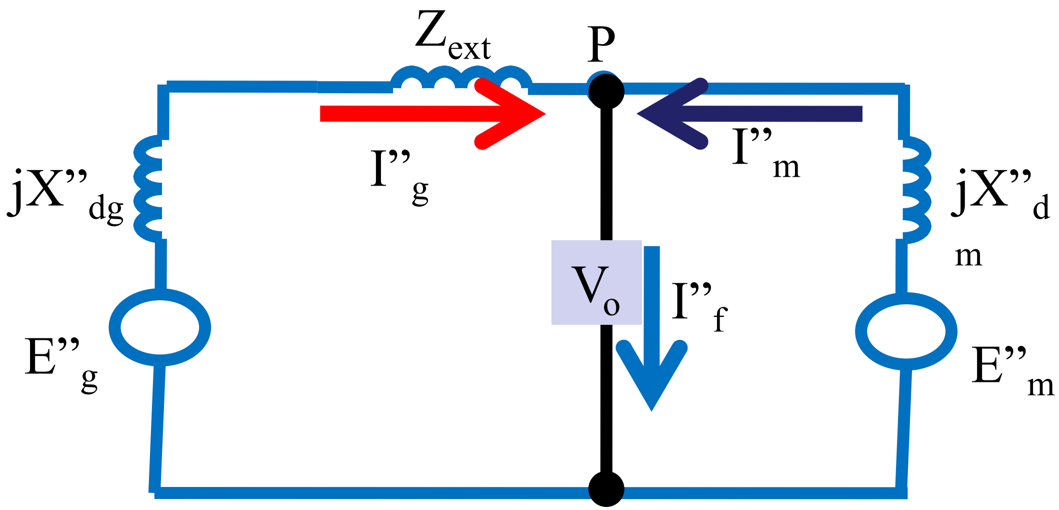
Synchronous motor connected to synchronous generator by line impedance Z_{ext}

Just after fault



When a motor is short circuited it no longer receives electric energy but its field remains energized and its rotor keeps rotating, hence it will behave like a generator

Motor subtransient internal voltage just before fault



$$E_m'' = V_o - jX_{dm}'' I_L$$

When fault

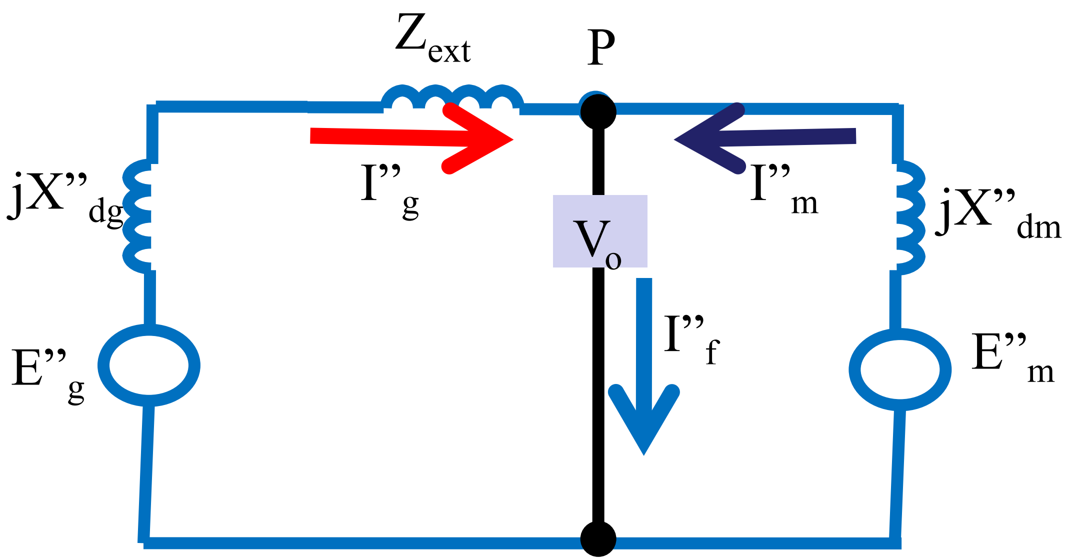
$$I_m'' = \frac{E_m''}{jX_{dm}''}$$

$$= \frac{V_o - jX_{dm}'' I_L}{jX_{dm}''}$$

$$= \frac{V_o}{jX_{dm}''} - I_L$$

$$\boxed{I_m'' = \frac{V_o}{jX_{dm}''} - I_L}$$

Generator subtransient internal voltage just before fault



$$E_g'' = V_0 + \left(Z_{ext} + jX_{dg}''\right) I_L$$

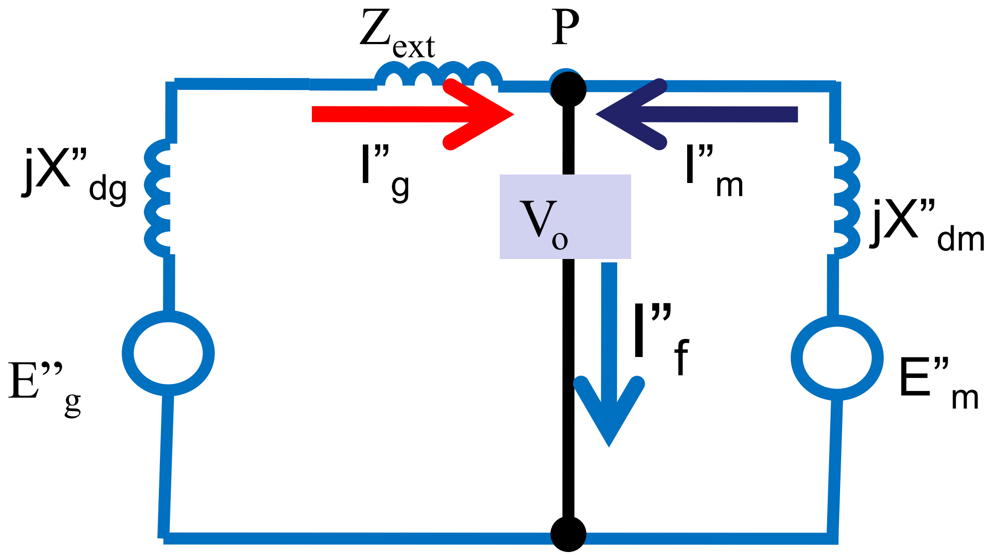
When fault

$$I_g'' = \frac{E_g''}{\left(Z_{ext} + jX_{dg}''\right)}$$

$$= \frac{V_o + \left(Z_{ext} + jX_{dg}''\right) I_L}{\left(Z_{ext} + jX_{dg}''\right)}$$

$$= \frac{V_o}{\left(Z_{ext} + jX_{dg}''\right)} + I_L$$

$$\boxed{I_g'' = \frac{V_o}{Z_{ext} + jX_{dg}''} + I_L}$$



P

Current from generator side:

$$\boxed{I_g'' = \frac{V_o}{Z_{ext} + jX_{dg}''} + I_L}$$

Current from motor side:

$$\boxed{I_m'' = \frac{V_o}{jX_{dm}''} - I_L}$$

$$I_f'' = I_g'' + I_m'' = \frac{V_o}{Z_{ext} + jX_{dg}''} + \frac{V_o}{jX_{dm}''}$$

Example

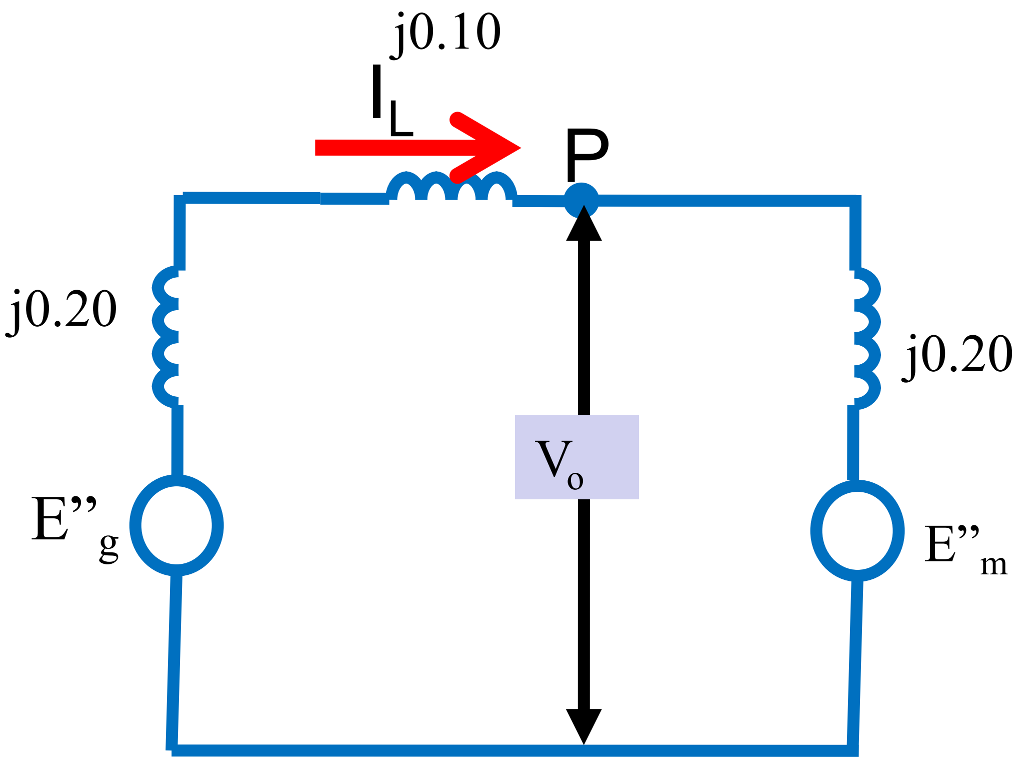
A synchronous generator and motor are rated 30,000 kVA, 13.2 kV, and both have subtransient reactances of 20%. The line connecting them has a reactance of 10% on the base of the machine ratings. The motor is drawing 20,000 kW at power factor 0.8 leading and terminal voltage of 12.8 kV when a symmetrical three-phase fault occurs at the motor terminals.

Find the subtransient currents in the generator, the motor, and the fault by using the internal voltages of the machines

Pre-fault equivalent circuit

Using ratings of gen and motor as base

Ratings of gen and motor: 30,000 kVA, 13.2 kV, X''=20% X_{line} = 10%



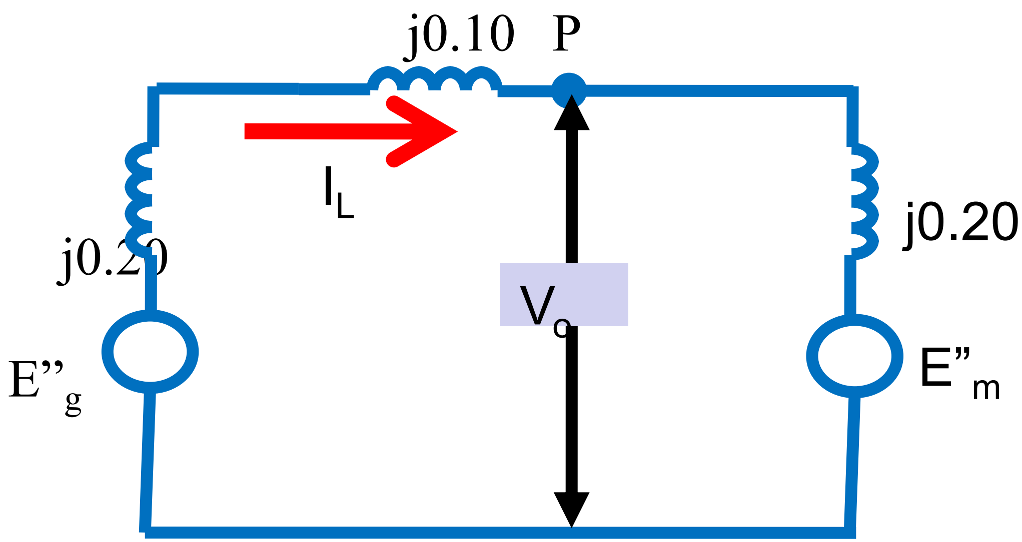
Terminal voltage of the Motor is 12.8 kV

V_o in p.u =?

$$V_o = \frac{12.8}{13.2} = 0.97\angle 0$$

Using ratings of gen and motor as base

Base kVA=30,000 kVA, V_{base} =13.2 kV, Base current ? Load current?



Motor takes 20,000 kW at 12.8 kV at p.f. 0.8 leading

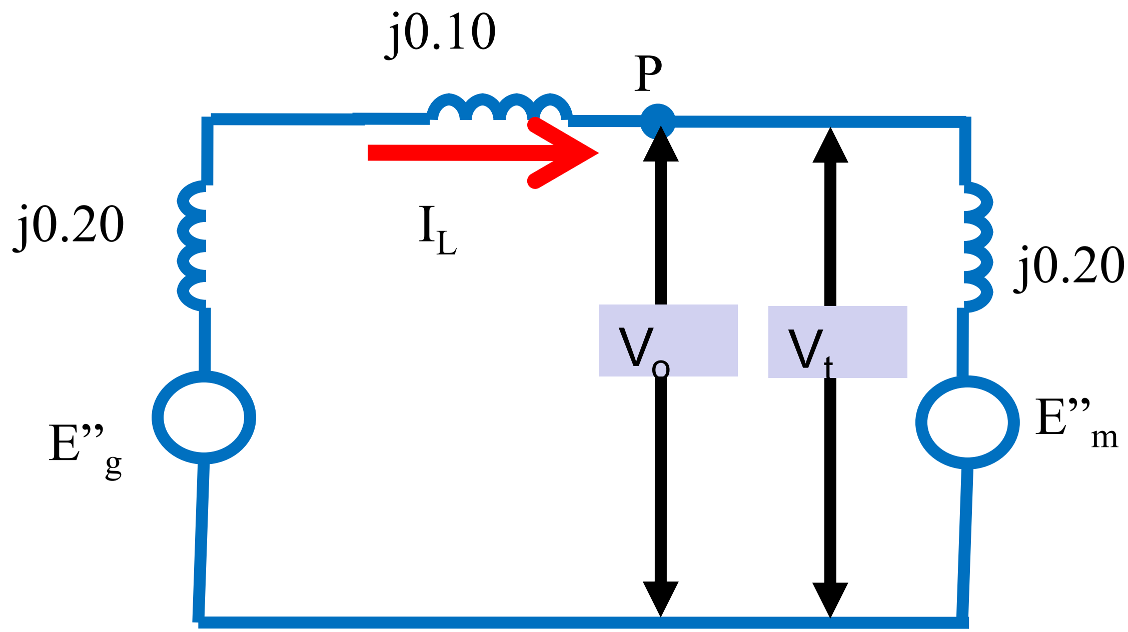
$$\text{Base current} = \frac{30,000}{\sqrt{3}\times 13.2} = 1312 \quad A$$

$$I_L = \frac{20,000}{0.8\times\sqrt{3}\times 12.8}\angle 36.9 = 1128\angle 36.9 \quad A$$

$$\text{Prefault current, } I^0 = I_L = \frac{1128\angle 36.9}{1312} = 0.86\angle 36.9 \quad p.u.$$

For generator

Base kVA=30,000 kVA, V_{base} =13.2 kV, $\text{Base current} = 1312 \quad A$



Write voltage equation V_t and E_g''

$$V_t = V_o(\text{Prefault voltage}) = 0.97\angle 0 \quad p.u.$$

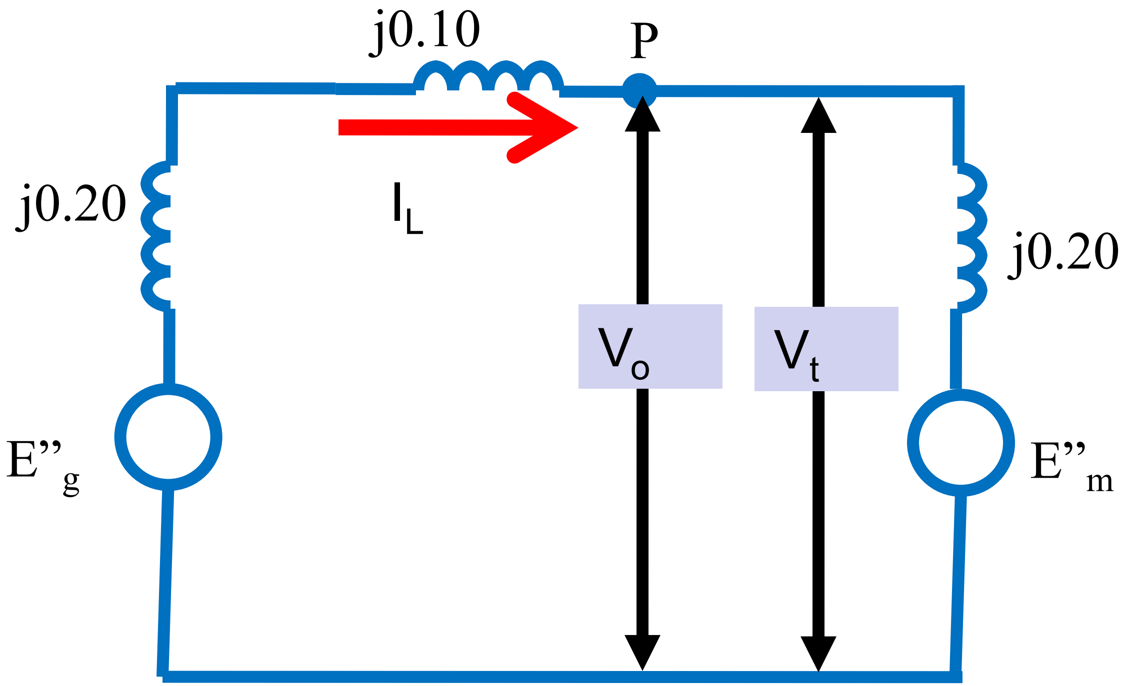
$$E_g'' = V_o + I_L(j0.10 + j0.20)$$

$$= (0.97 + j0) + 0.86\angle 36.9 \times 0.3\angle 90$$

$$= 0.8151 + j0.2063 = 0.8408\angle 14.2 \quad p.u.$$

For motor

Base kVA=30,000 kVA, Vbase =13.2 kV, $\text{Base current} = 1312 \quad A$



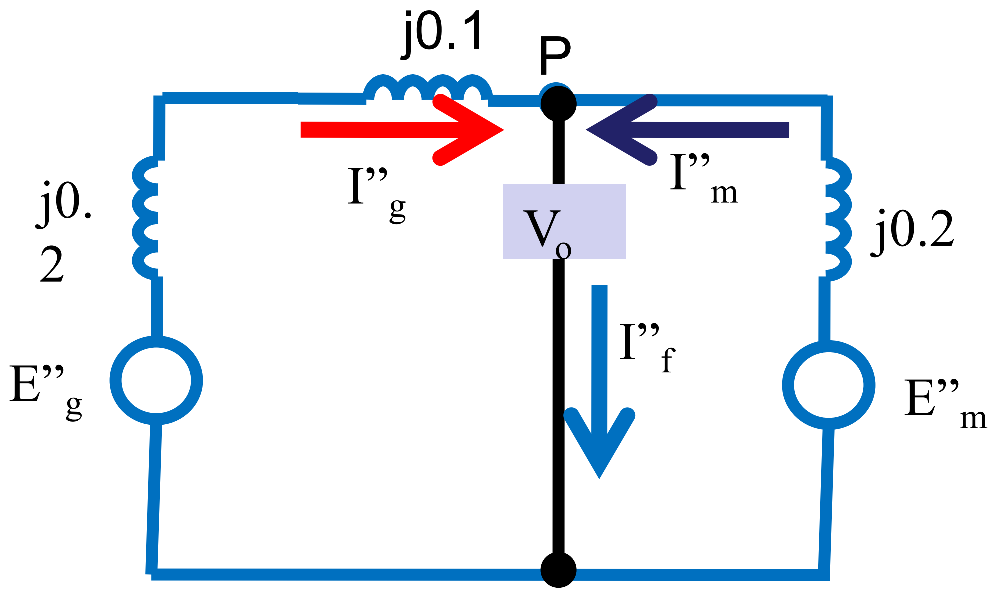
Write voltage equation V_t and E_m''

$$V_t = V_o = 0.97\angle 0 \quad p.u.$$

$$E_m'' = (0.97 + j0) - j0.2(0.69 + j0.52) = 1.074 - j0.138 \quad p.u.$$

Generator- After fault

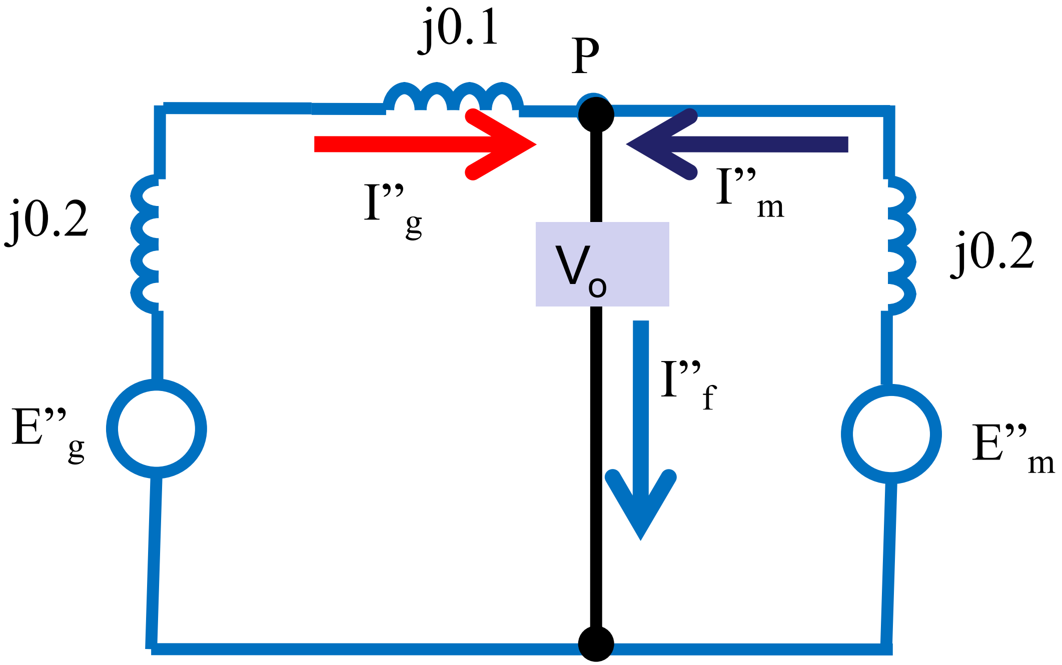
Calculate I_g''



$$I_g'' = \frac{E_g''}{j(0.3)} = \frac{0.8408\angle 14.2}{0.3\angle 90} = 2.8027\angle -75.8 \quad p.u$$

Motor- After fault

Calculate I_m''

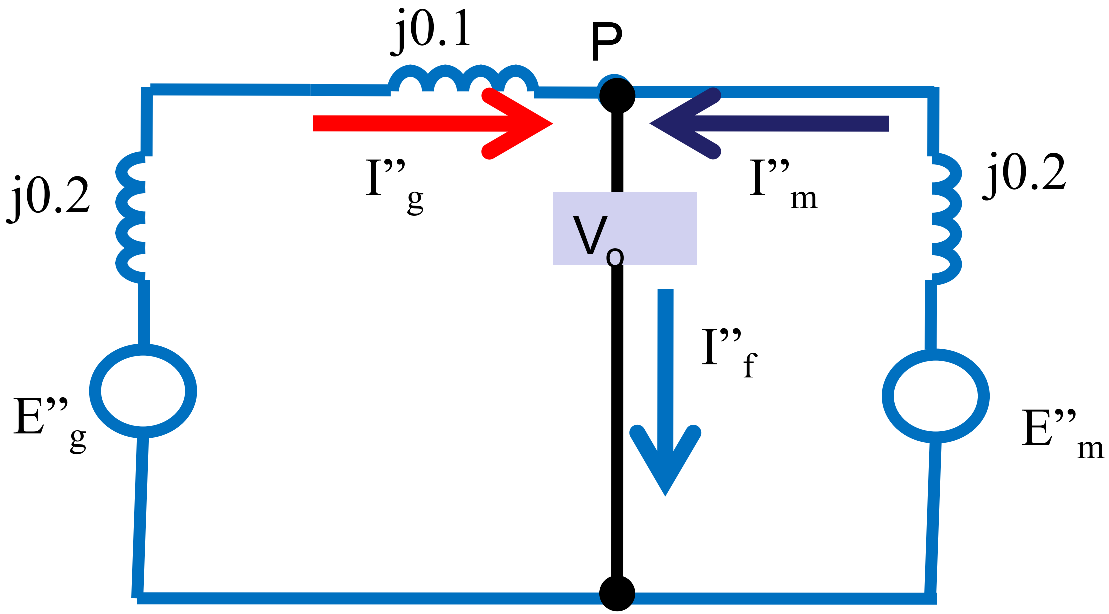


$$I_m'' = \frac{E_m''}{j(0.2)} = \frac{1.074 - j0.138}{0.2\angle 90} = -0.69 - j5.37 \quad p.u$$

$$I_m'' = -905 - j7050 \quad A$$

-In the fault

Calculate fault current



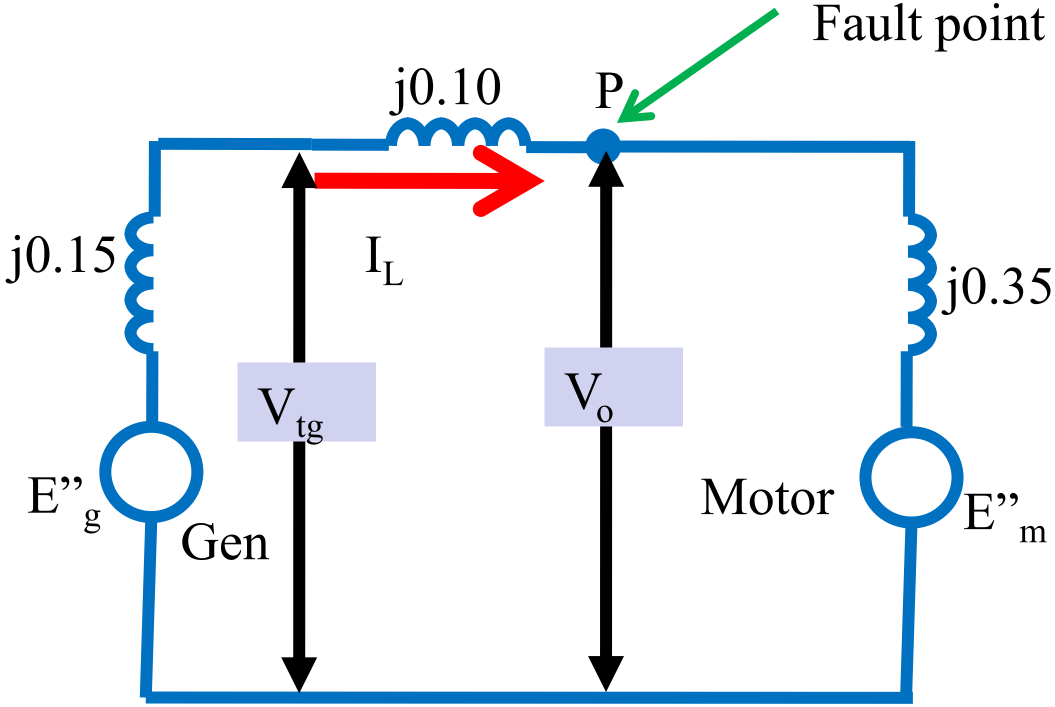
$$I_f'' = I_g'' + I_m'' = 0.69 - j2.71 - 0.69 - j5.37 = -j8.08 \quad p.u$$

$$I_f'' = -j10600 \quad A$$

Exercise

- A generator is connected through a transformer (j0.1Ω) to a synchronous motor. Reduced to the same base, the per-unit subtransient reactances of the generator and motor are 0.15 and 0.35, respectively.
- A three phase fault occurs at the terminals of the motor when the terminal voltage of the generator is 0.9 p.u and the output current of the generator is 1.0 p.u. at 0.8 power factor leading.
- Find the subtransient current in p.u. in the fault, in the generator and the motor.
- Using the terminal voltage of the generator as the reference phasor. Obtain the solution by computing the voltages behind subtransient reactance in the generator and motor.

Before fault



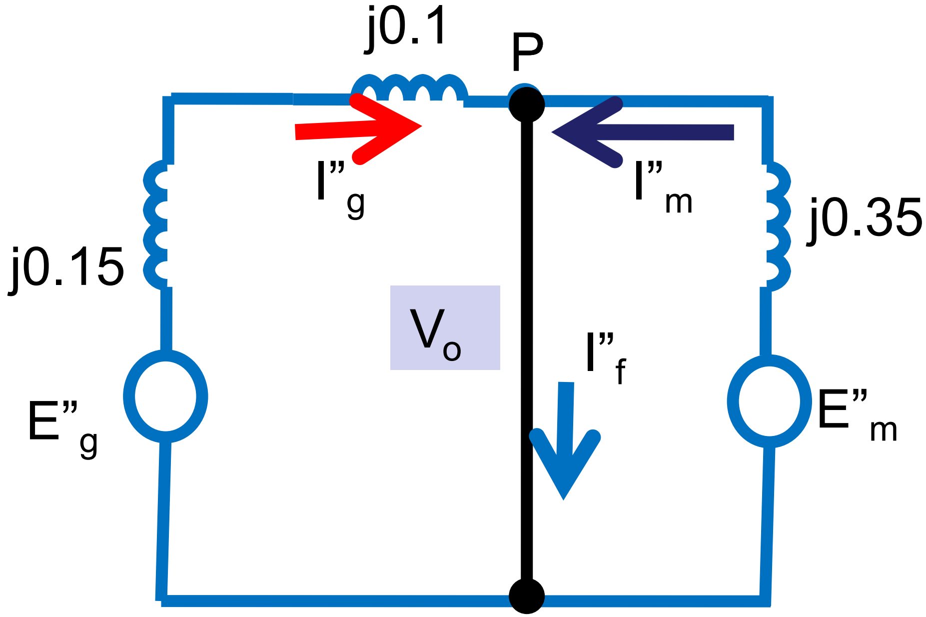
$$V_{tg} = 0.9\angle 0 \quad p.u.$$

A three phase fault occurs at the terminals of the motor when the terminal voltage of the generator is 0.9 p.u and the output current of the generator is 1.0 p.u. at 0.8 power factor leading.

$$\begin{aligned} E_g'' &= (0.9 + j0) + j0.15(0.8 + j0.6) \\ &= 0.81 + j0.12 \quad p.u. \end{aligned}$$

$$\begin{aligned} E_m'' &= (0.9 + j0) - j0.45(0.8 + j0.6) \\ &= 1.17 - j0.36 \quad p.u. \end{aligned}$$

- In the fault



$$I_g'' = \frac{0.81 + j0.12}{j0.25}$$

$$= 0.48 - j3.24 \quad p.u$$

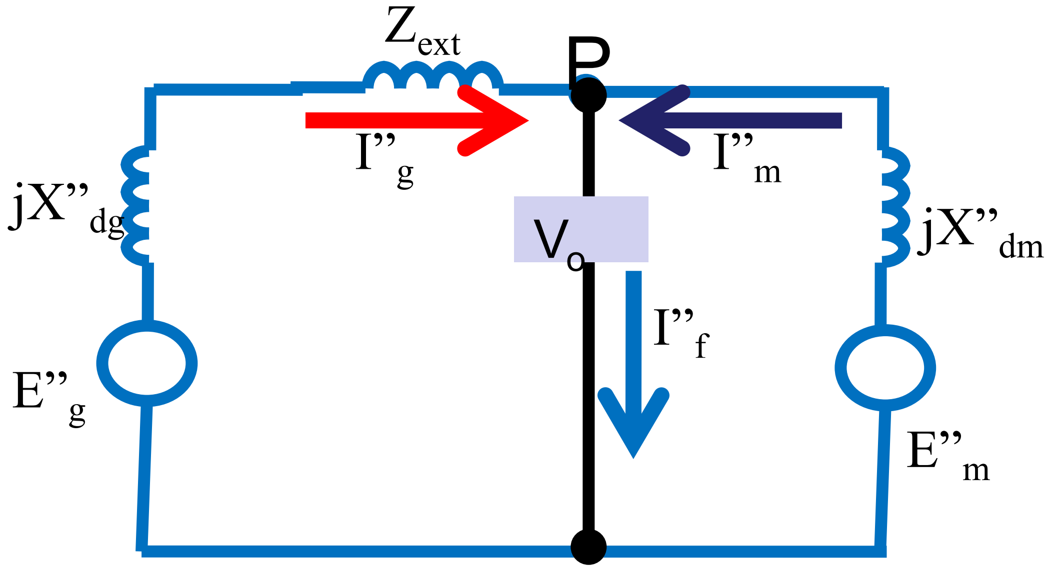
$$I_m'' = \frac{1.17 - j0.36}{j0.35}$$

$$= -1.03 - j3.34 \quad p.u$$

$$I_f'' = I_g'' + I_m'' = -0.55 - j6.58 \quad p.u$$

Using internal voltage approach we have shown that the fault current is determined by:

$$I''_f = I''_g + I''_m = \frac{V_o}{Z_{ext} + jX''_{dg}} + \frac{V_o}{jX''_{dm}}$$

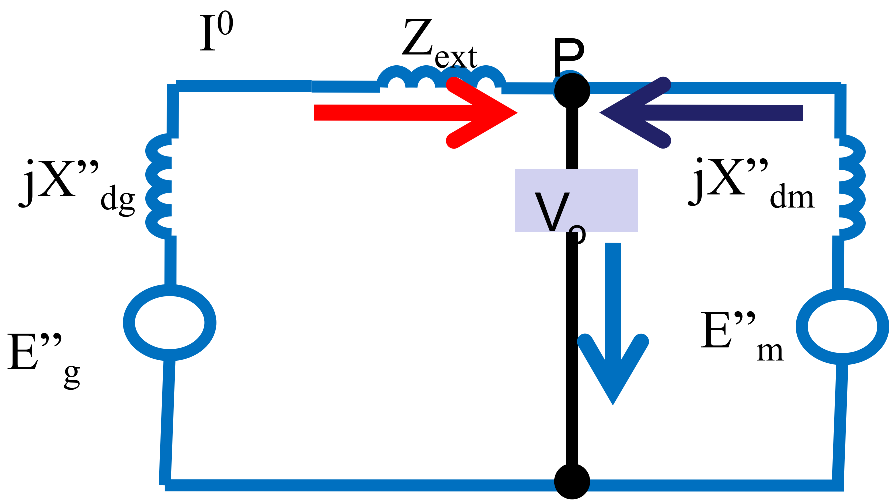


Observation

Only **prefault voltage** at the fault point and the **parameters** of the network with the subtransient **reactances** representing the machines are needed for the calculation of fault current (subtransient fault current)

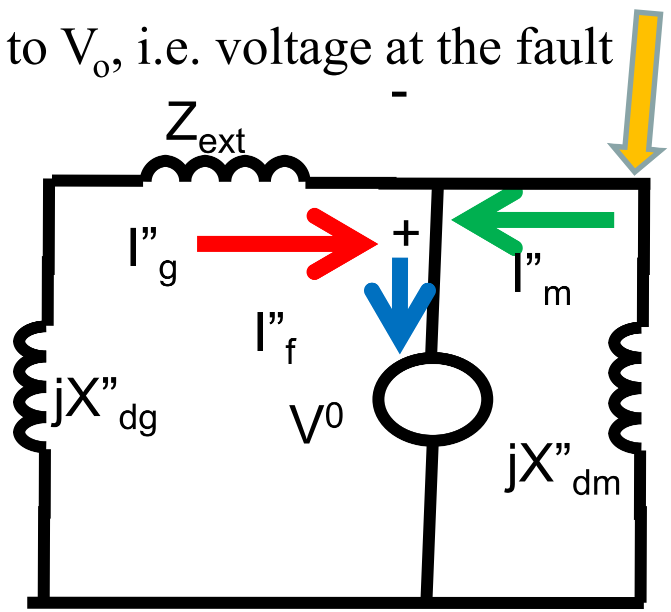
After fault

$$I''_f = I''_g + I''_m = \frac{V_o}{Z_{ext} + jX''_{dg}} + \frac{V_o}{jX''_{dm}}$$



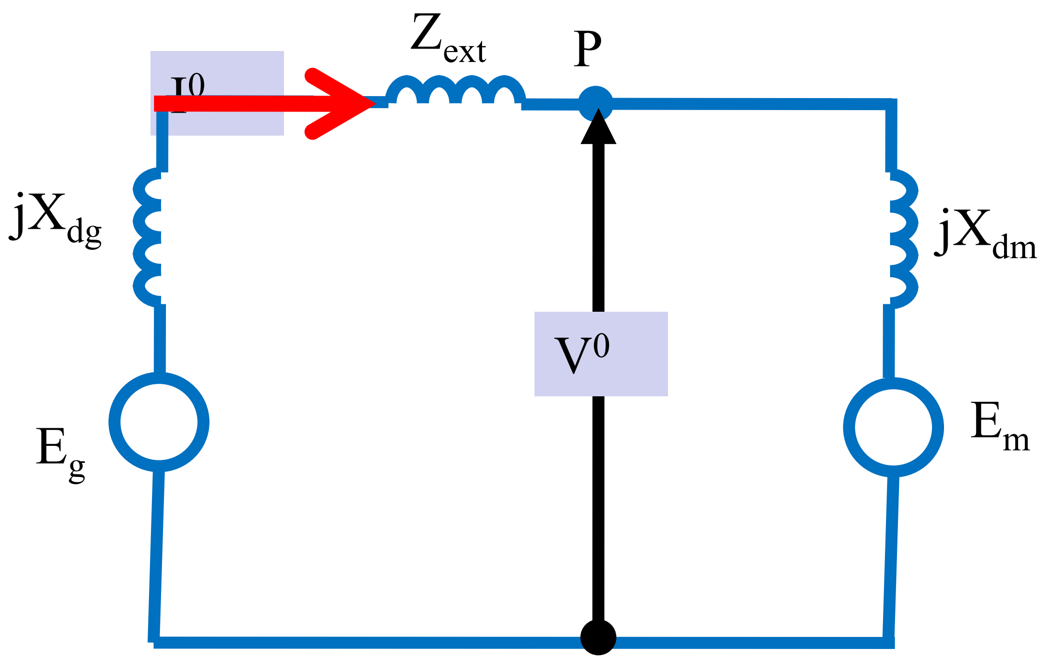
The equivalent generator has internal voltage equal to V_o, i.e. voltage at the fault point before the fault occur

We can represent the network as a single generator and a single impedance terminating at a point of application of fault :

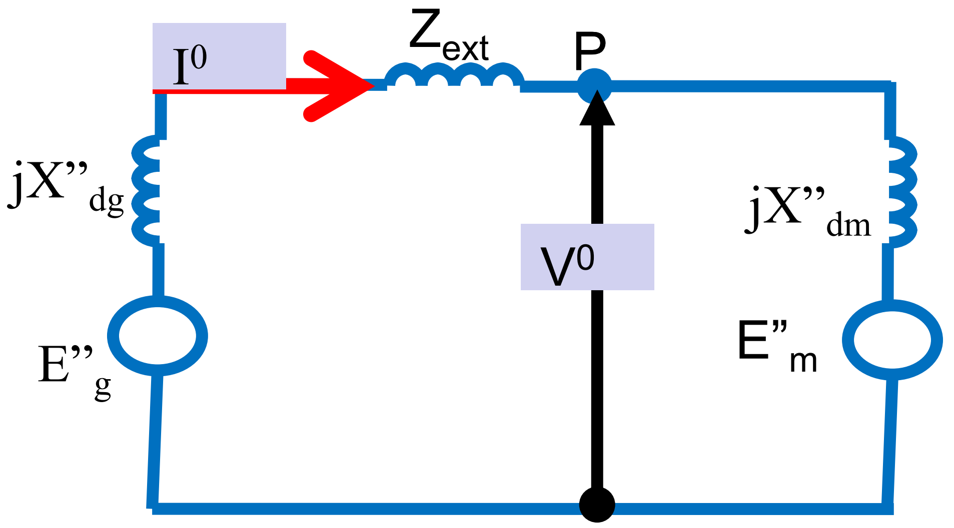


thevenin equivalent circuit

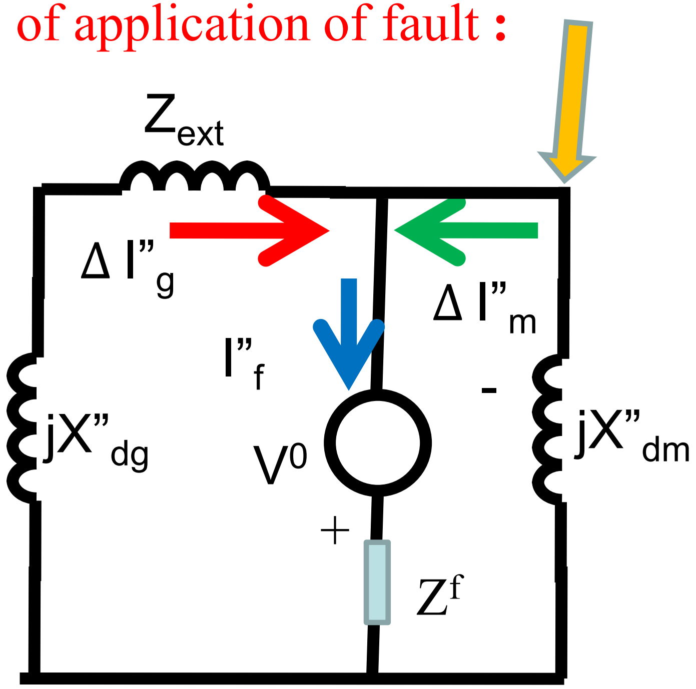
An alternate method of computing s/c currents is through the application of the Thevenin theorem. This method is faster and easily adopted to systematic computation for large networks. Consider a synchronous generator feeding a synchronous motor over a line. Fig. below shows the circuit model of the system under conditions of steady load. Fault computations are to be made for a fault at P, at the motor terminals.



As a first step the circuit model is replaced by the one shown in Fig below, wherein the synchronous machines are represented by their transient or subtransient reactances in series with voltages behind transient reactances.

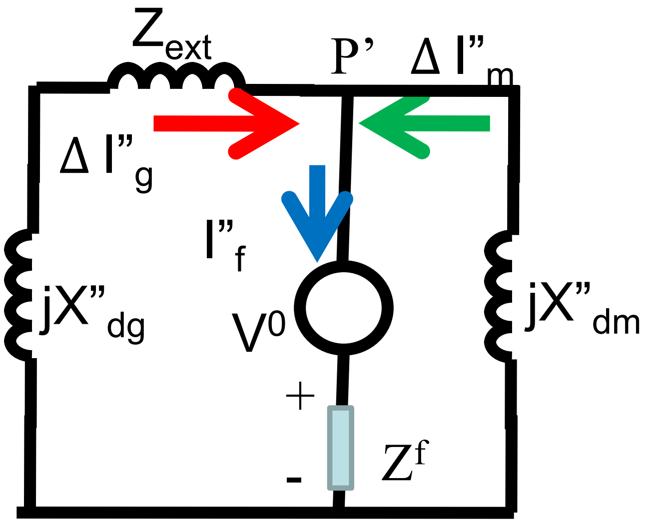


We can represent the network as a single generator and a single impedance terminating at a point of application of fault **:**



thevenin equivalent circuit

It comprises pre-fault voltage V^0 in series with the passive Thevenin impedance network. The pre-fault current I^0 does not appear in the passive Thevenin impedance network. This current must be accounted for by superposition after the SC solution is obtained through use of the Thevenin equivalent. Consider now a fault at P through an impedance Z^f.



thevenin equivalent circuit

Current caused by fault in motor circuit

$$I_f'' = \frac{V^0}{Z_{th} + Z^f}$$

$$\Delta I_m'' = \frac{X_{dg}'' + Z_{ext}}{\left(X_{dg}'' + Z_{ext} + X_{dm}''\right)} \times I_f''$$

Current caused by fault in generator circuit

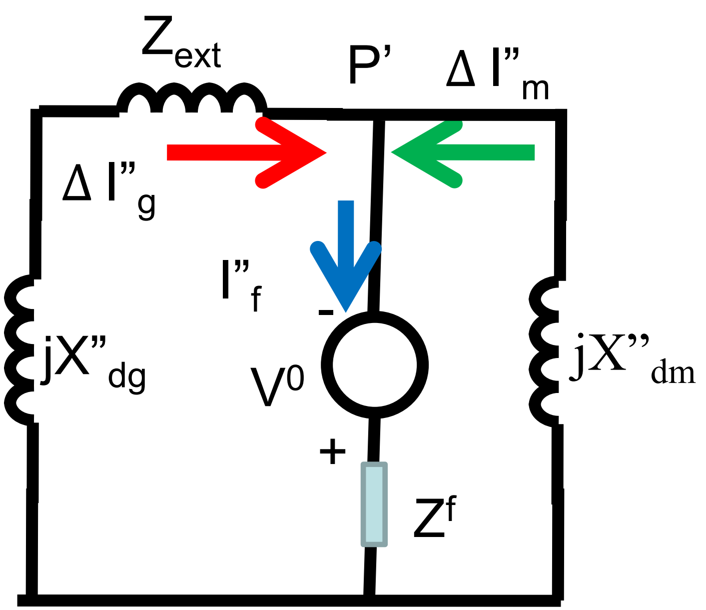
$$\Delta I_g'' = \frac{X_{dm}''}{\left(X_{dg}'' + Z_{ext} + X_{dm}''\right)} \times I_f''$$

Post fault currents and voltages are obtained as follows by superposition:

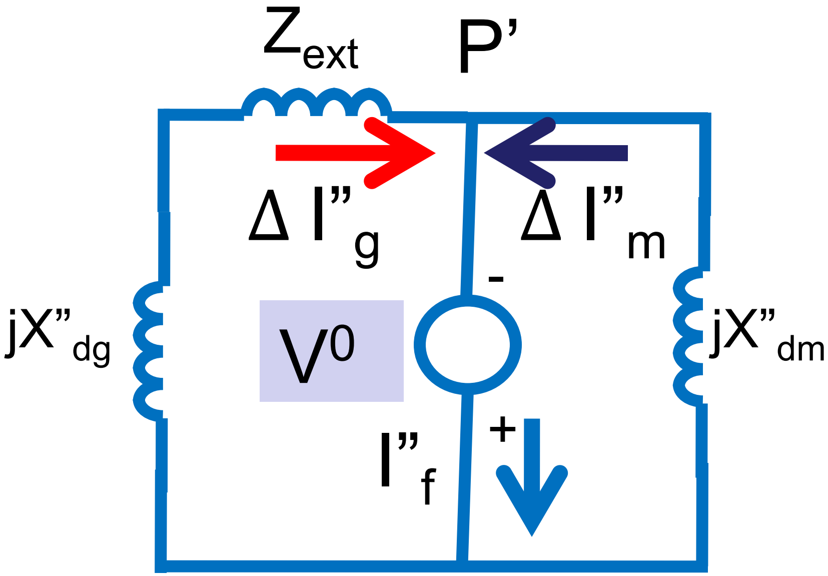
$$I_g'' = I^0 + \Delta I_g$$

$$I_m'' = -I^0 + \Delta I_m (in \;\; \text{the direction of} \;\; \Delta \mathrm{I}_m)$$

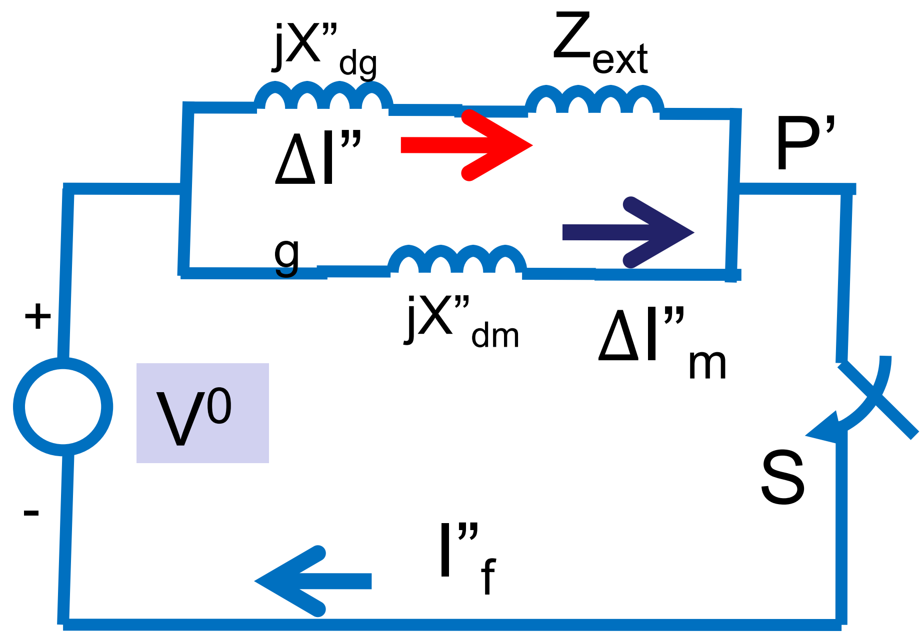
$$V^n = V^0 + \left(-Z_{Th} I^f\right) = V^0 + \Delta V$$

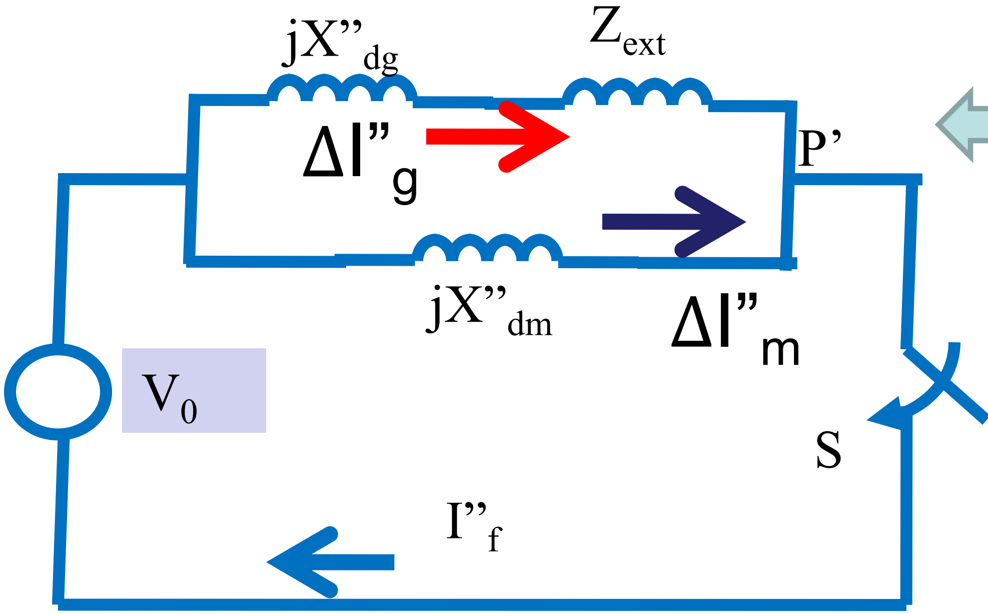


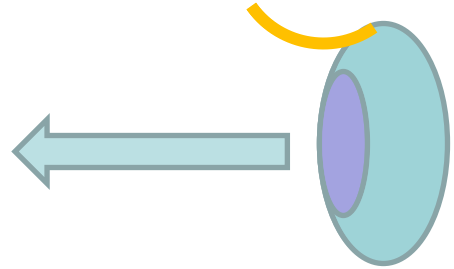
thevenin equivalent circuit



We can redraw this







Thevenin equivalent looking into the circuit at point P'

Thevenin impedance:

$$Z_{th} = \frac{jX''_{dm}\left(Z_{ext} + jX''_{dg}\right)}{Z_{ext} + j\left(X''_{dg} + X''_{dm}\right)}$$

$$V^{f} = V^{0} + \left(-jX_{Th} I^{f}\right) = V^{0} + \Delta V$$

$$I''_{m} = -I^{0} + \Delta I_{m} (in \ \text{the direction of } \Delta \mathrm{I}_{\mathrm{m}})$$

Subtransient current in the fault

$$I''_{f} = \frac{V^{0}}{Z_{th}} = \frac{V^{0}\left[Z_{ext} + j\left(X''_{dg} + X''_{dm}\right)\right]}{jX''_{dm}\left(Z_{ext} + jX''_{dg}\right)}$$

$$I''_{g} = I^{0} + \Delta I_{g}$$

The above approach to SC computation is summarized in the following four steps:

1. Obtain steady state solution of loaded system

2. Replace reactances of synchronous machines by their subtransient/transient values. SC all emf sources. The result is the passive Thevenin network.

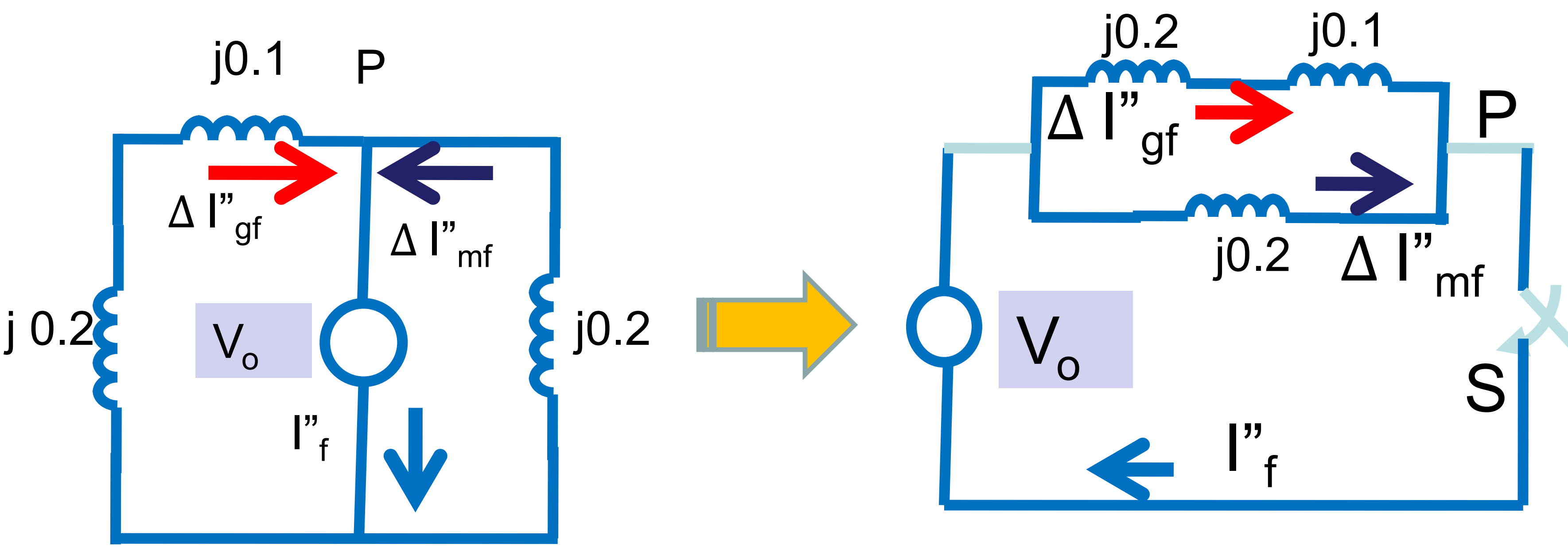
3. Excite the passive network of Step 2 at the fault point by negative of pre-fault voltage in series with the fault impedance. Compute voltages and currents at all points of interest.

4. Post fault currents and voltages are obtained by adding results of steps 1 and 3.

Example

A synchronous generator and motor are rated 30,000 kVA, 13.2 kV, and both have subtransient reactances of 20%. The line connecting them has a reactance of 10% on the base of the machine ratings. The motor is drawing 20,000 kW at power factor 0.8 leading and terminal voltage of 12.8 kV when a symmetrical three-phase fault occurs at the motor terminals.

Find the subtransient currents in the generator, the motor, and the fault by using the Thevenin approach.

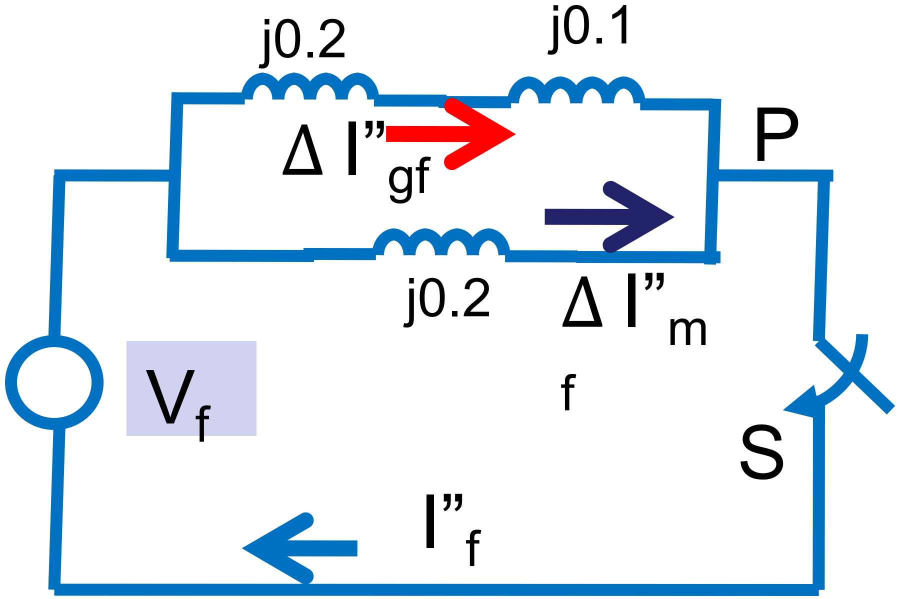


$V_o = ?$ $\quad\quad Z_{th} = ?$

$V_o = 0.97\angle 0°$ p.u.

$$Z_{th} = \frac{j0.2\left(j0.1 + j0.2\right)}{j0.1 + j\left(0.2 + 0.2\right)} = j0.12 \quad pu$$

$$I_f^{"} = \frac{V_o}{Z_{th}} = \frac{0.97\angle 0}{0.12\angle 90} = -j8.08 \quad p.u.$$



$$I_f^{"} = -j8.08 \quad p.u.$$

By simple current division

Change in generator current due to fault

$$\Delta I_{gf}^{"} = -j8.08 \times \frac{j0.2}{j0.5} = -j3.23 \quad p.u.$$

Change in motor current due to fault

$$\Delta I_{mf}^{"} = -j8.08 \times \frac{j0.3}{j0.5} = -j4.85 \quad p.u.$$

To these changes we add the pre-fault current to obtain the subtransient current in machines

$$I_g'' = I^0 + \Delta I_g = (0.687 + j0.516) + (-j3.23) = 0.687 - j2.717 \quad p.u.$$

$$I_m'' = -I^0 + \Delta I_m = (-0.687 - j0.516) + (-j4.85) = -0.687 - j5.366 \quad p.u.$$

Exercise

• A generator is connected through a transformer to a synchronous motor. Reduced to the same base, the per-unit subtransient reactances of the generator and motor are 0.15 and 0.35, respectively. Reactance of the transformer (on the same base) is 0.1 p.u. A three phase fault occurs at the terminals of the motor when the terminal voltage of the generator is 0.9 p.u and the output current of the generator is 1.0 p.u. at 0.8 power factor leading.

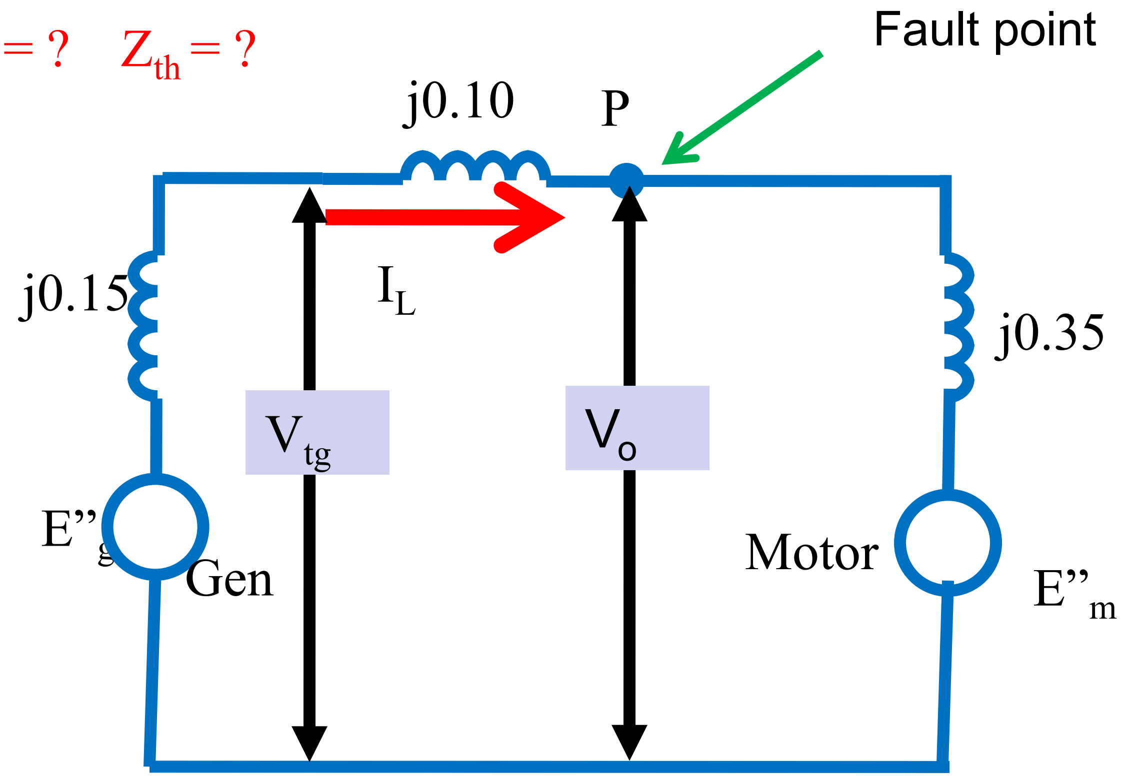
Find the subtransient current in p.u. in the fault, in the generator and the motor.

Use the terminal voltage of the generator as the reference phasor.

Obtain the solution by using Thevenin's theorem.

Solution

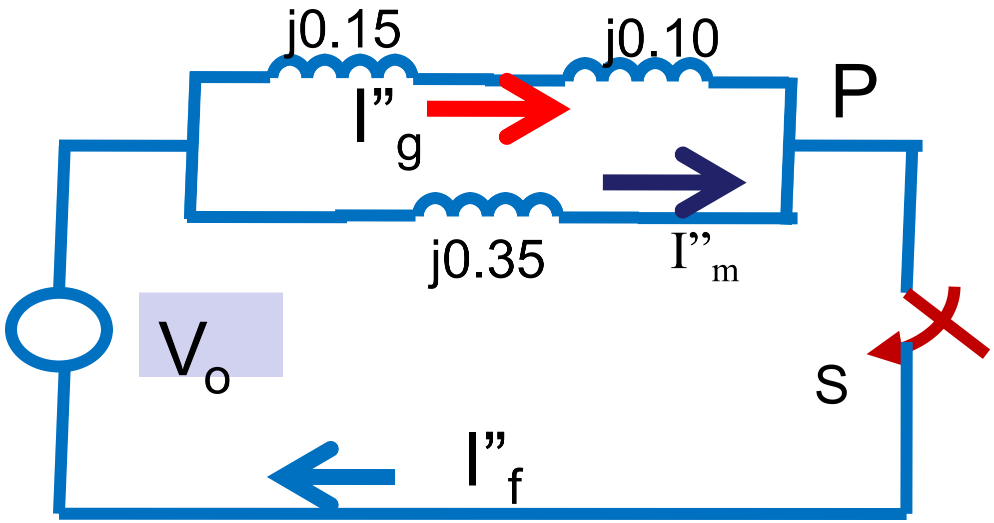
$V_o = ?$ $Z_{th} = ?$



$$V_{tg} = 0.9\angle 0 \quad p.u. \qquad I_L = 1.0\angle 36.87 \quad p.u.$$

$$V_o = 0.9 - (0.8 + j0.6)(j0.1) = 0.96 - j0.08 \quad p.u.$$

$Z_{th} = ?$



$$Z_{th} = \frac{jX''_{dm}\left(Z_{ext} + jX''_{dg}\right)}{Z_{ext} + j\left(X''_{dg} + X''_{dm}\right)}$$

$$= \frac{j0.25 \times j0.35}{j0.6} = j0.146$$

$$I''_f = \frac{V_o}{Z_{th}} = \frac{0.96 - j0.08}{j0.146} = -0.55 - j6.58 \quad p.u.$$

Calculate I_g'' and I_m''

CALCULATION OF FAULT CURRENT USING

BUS IMPEDANCE MATRIX

To handle really large power system, we need to have a systematic approach to solve for the voltages and currents in the power system.

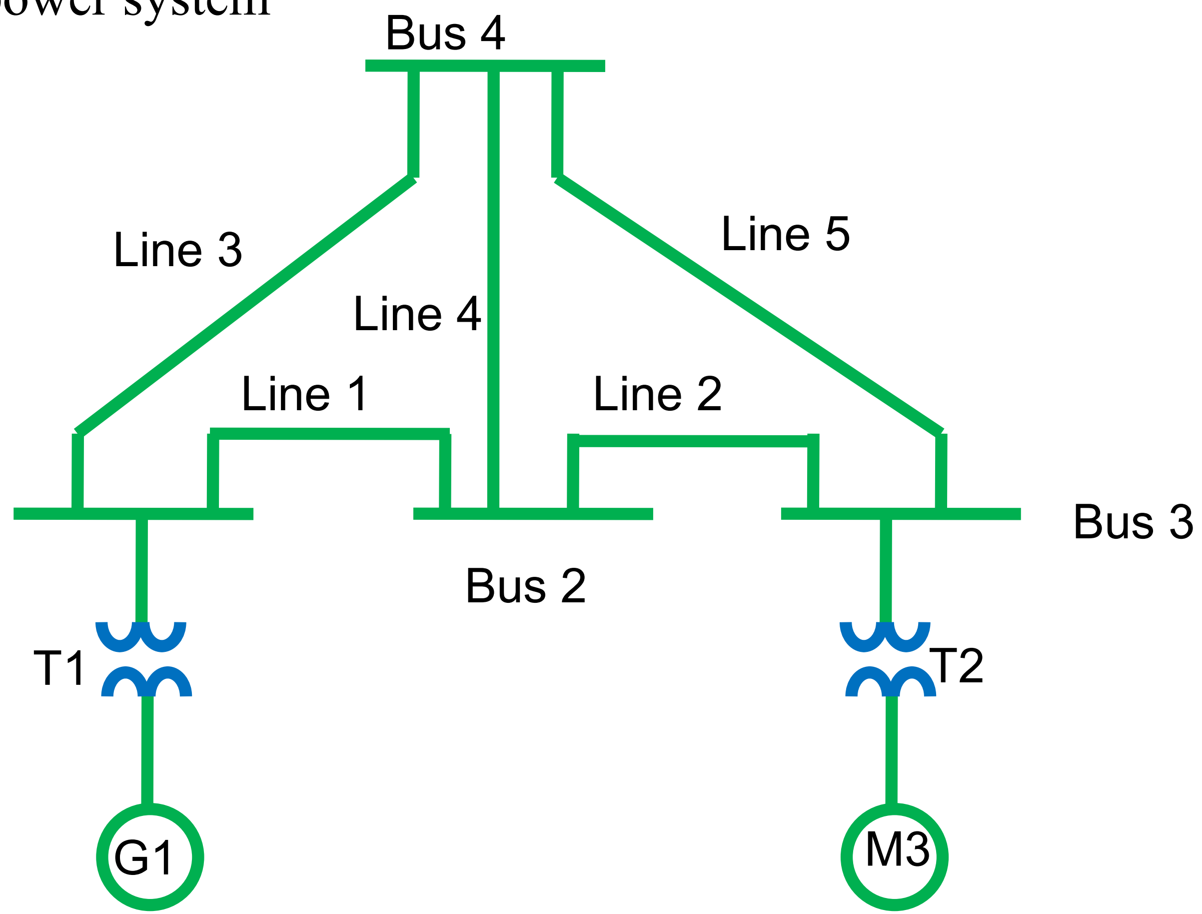
We do this by adapting the nodal analysis technique.

To solve for the fault currents in the power system, we introduce a new voltage source in the system to represent the effect of fault at a bus.

By solving for the currents introduced by this additional voltage source, we will automatically be solving for the fault currents that flow at the bus.

The first step towards S/C computation is to obtain pre-fault voltages (V_0) at all buses and currents in all lines through a LF study.

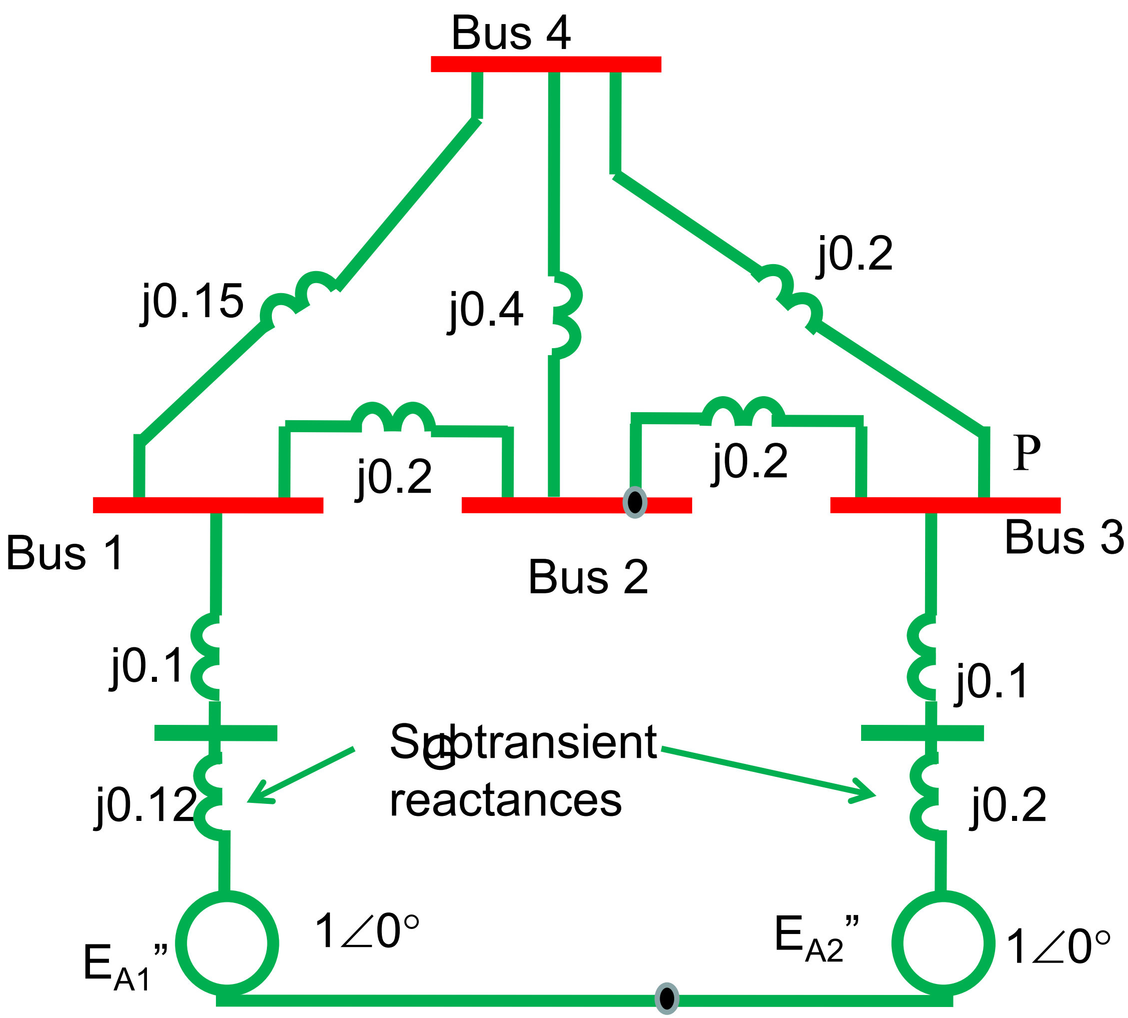
Consider a simple 3-phase power system



$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

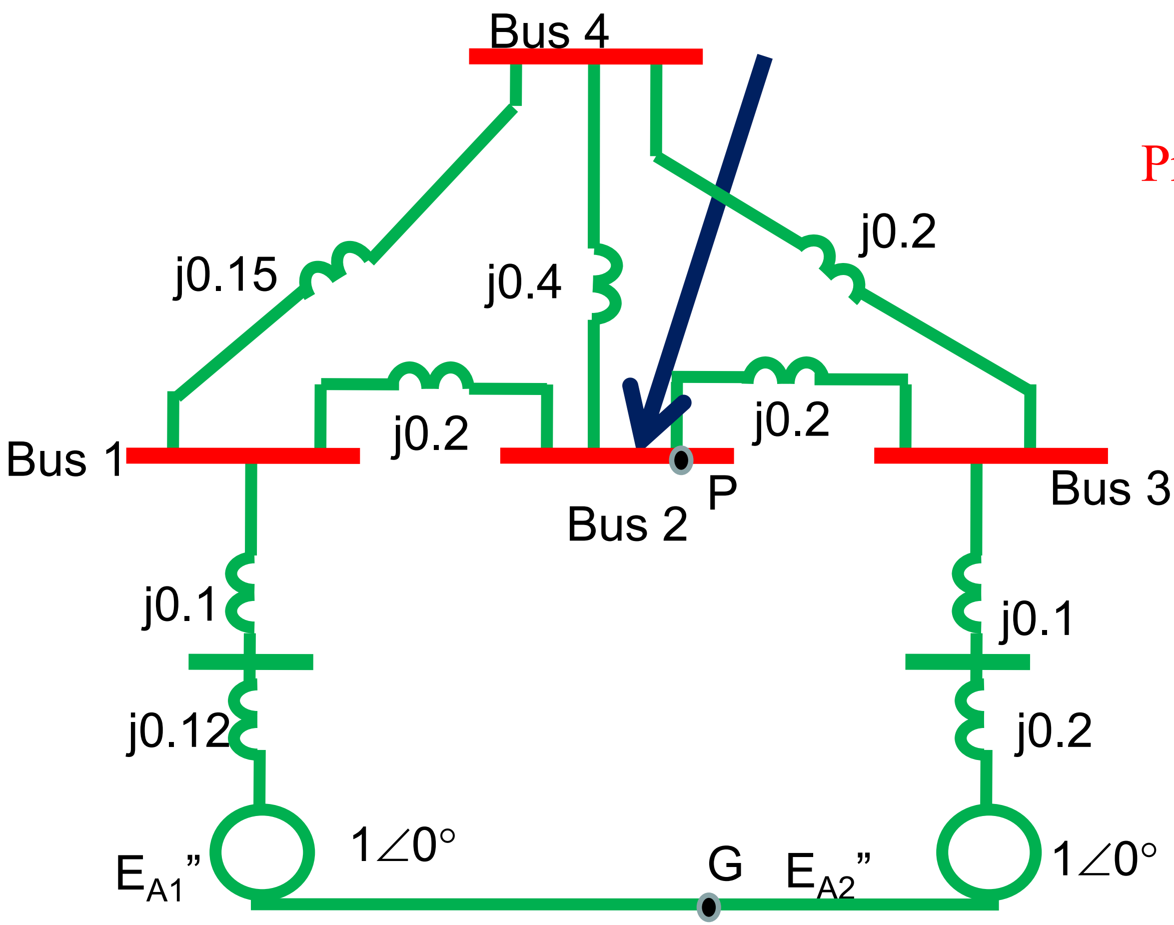
The per-phase, per-unit equivalent circuit of the power system

The values shown are per-unit impedances. Subtransient reactances are used instead of the synchronous reactances for each synchronous machines. We assume that the voltage behind synchronous reactances equal to $1\angle 0°$.



Before fault voltage at bus 2 is V_0

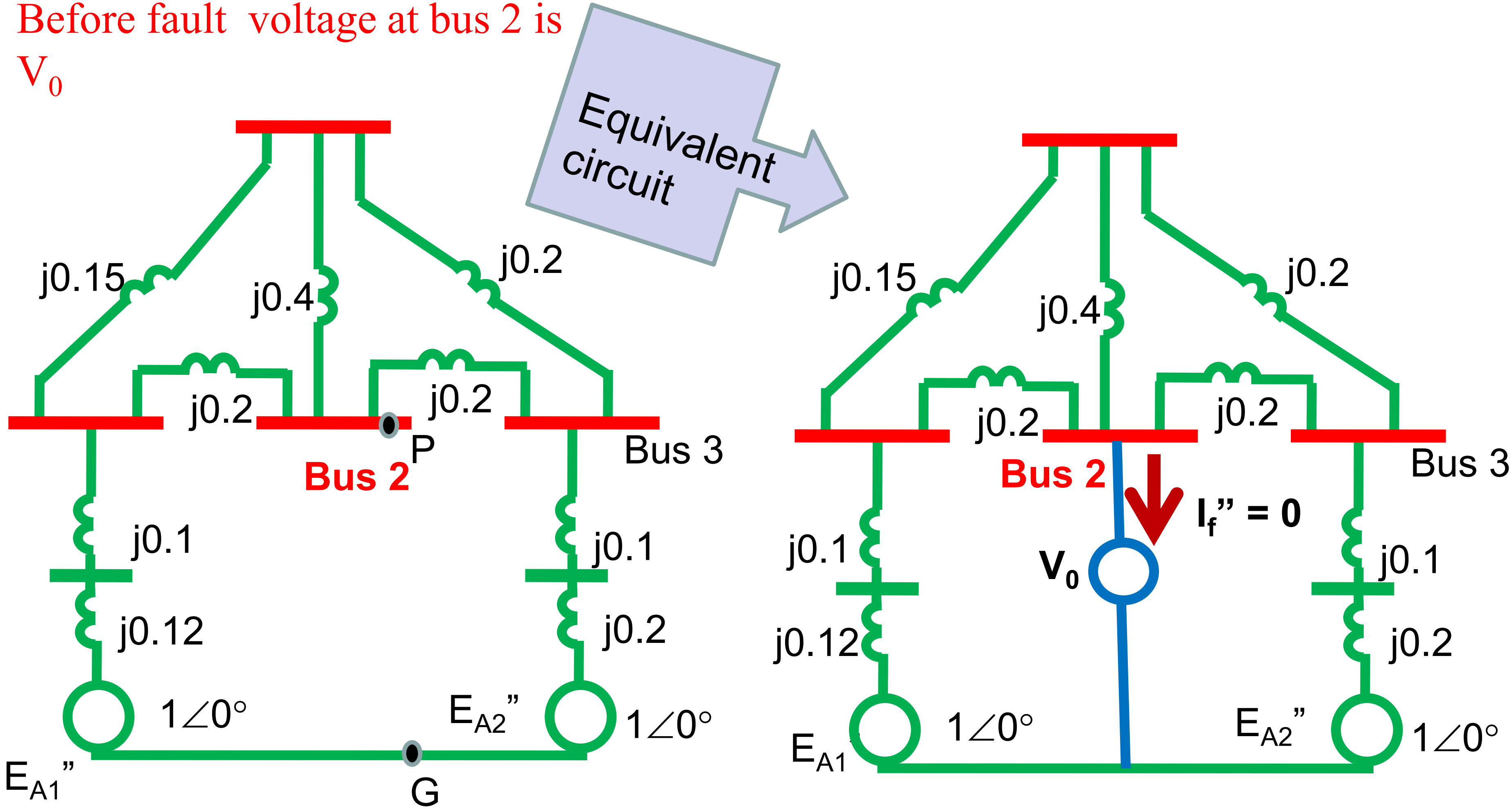
We want to determine the subtransient fault current at bus 2 when a symmetrical 3-phase fault occurs on the bus



Pre-fault bus voltage vector

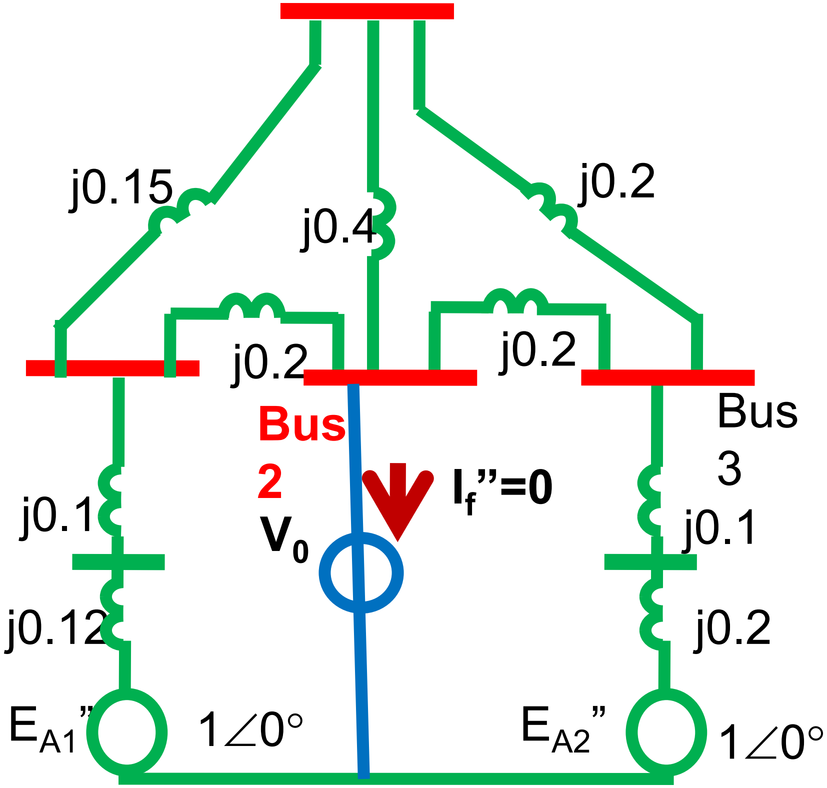
$$V_{BUS}^{0} = \begin{bmatrix} V_1^{0} \\ V_2^{0} \\ \vdots \\ V_n^{0} \end{bmatrix}$$

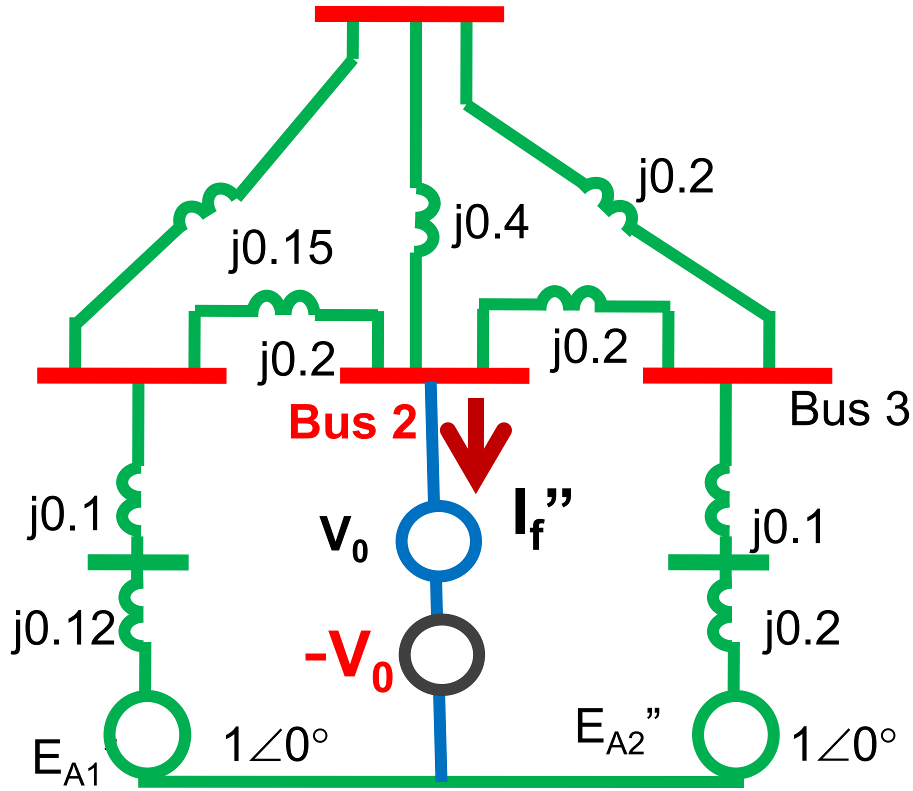
Before fault voltage at bus 2 is V_0



Voltage source V_0 is added at bus 2.

Since voltage at bus 2 is V_0 to begin with, nothing is changed and $I_f'' = 0$

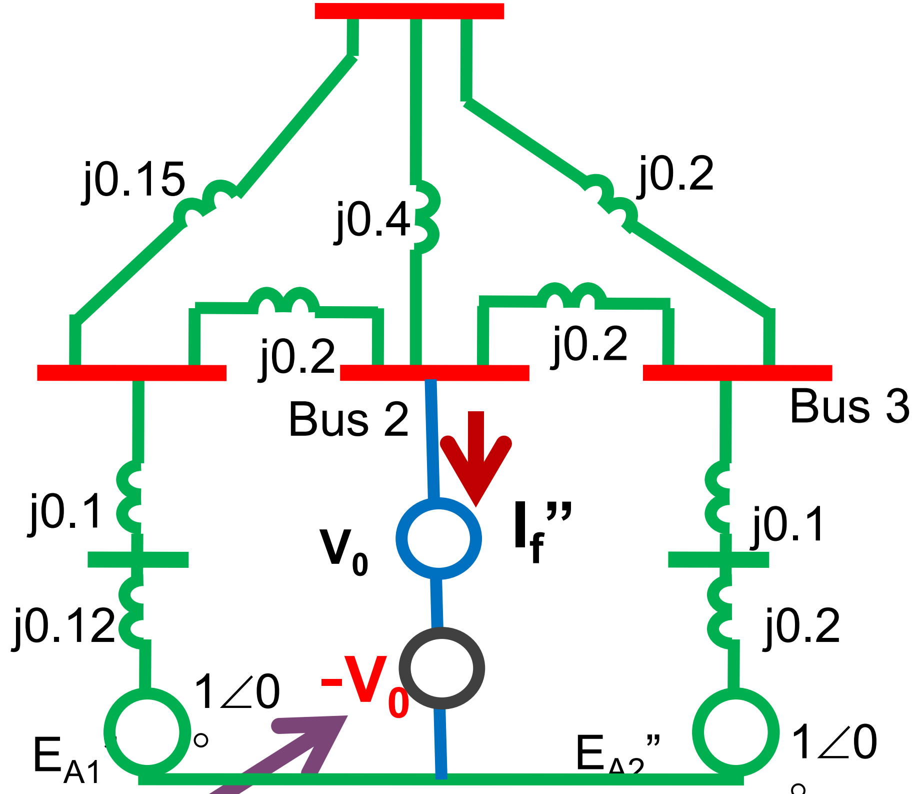




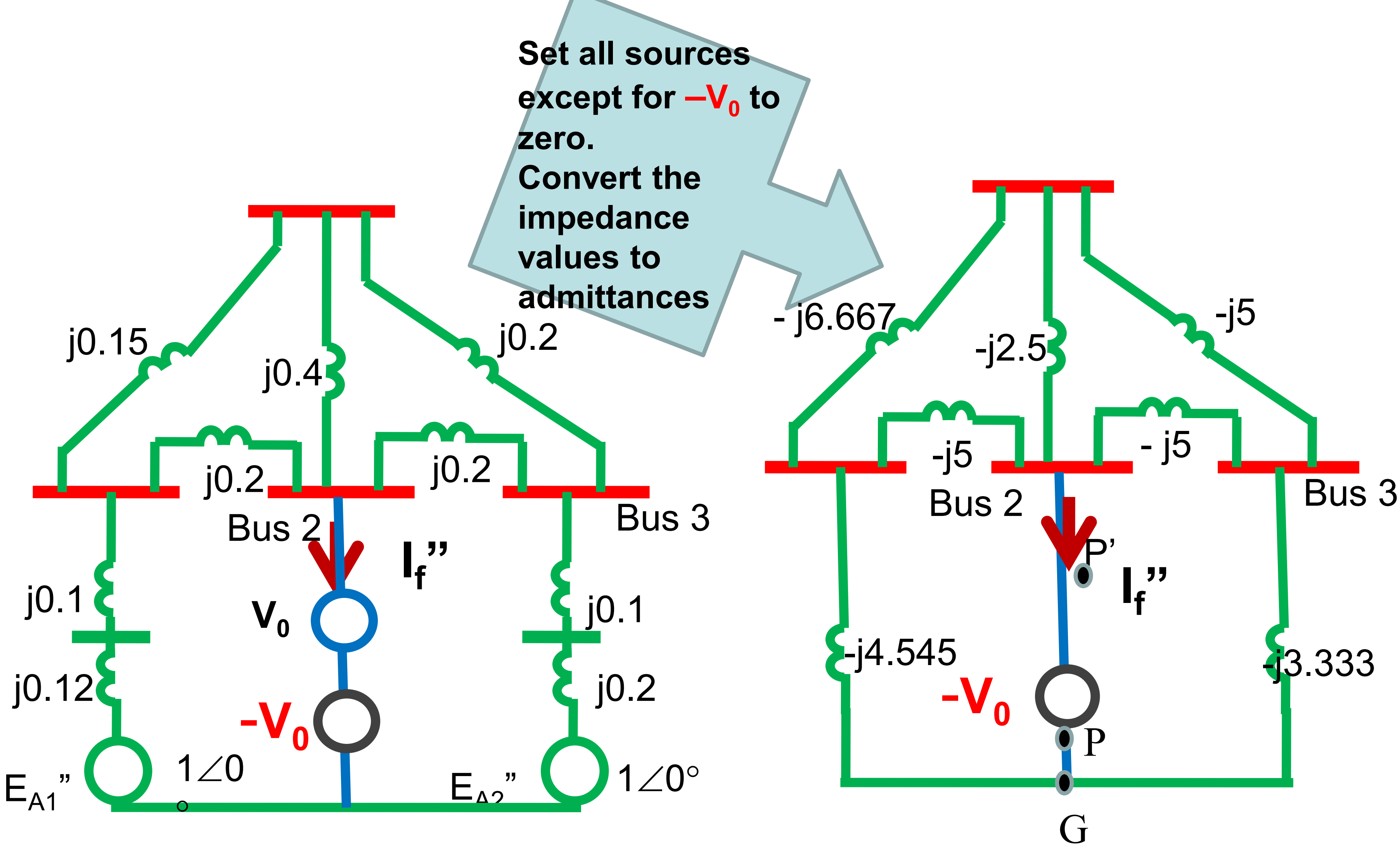
Short circuit at bus 2 brings voltage on bus 2 down to 0.

This is equivalent to inserting an additional voltage $-\mathbf{V_0}$ in series with V_0

Voltage at bus 2 drops to zero, fault current $\mathbf{I_f}$" flows



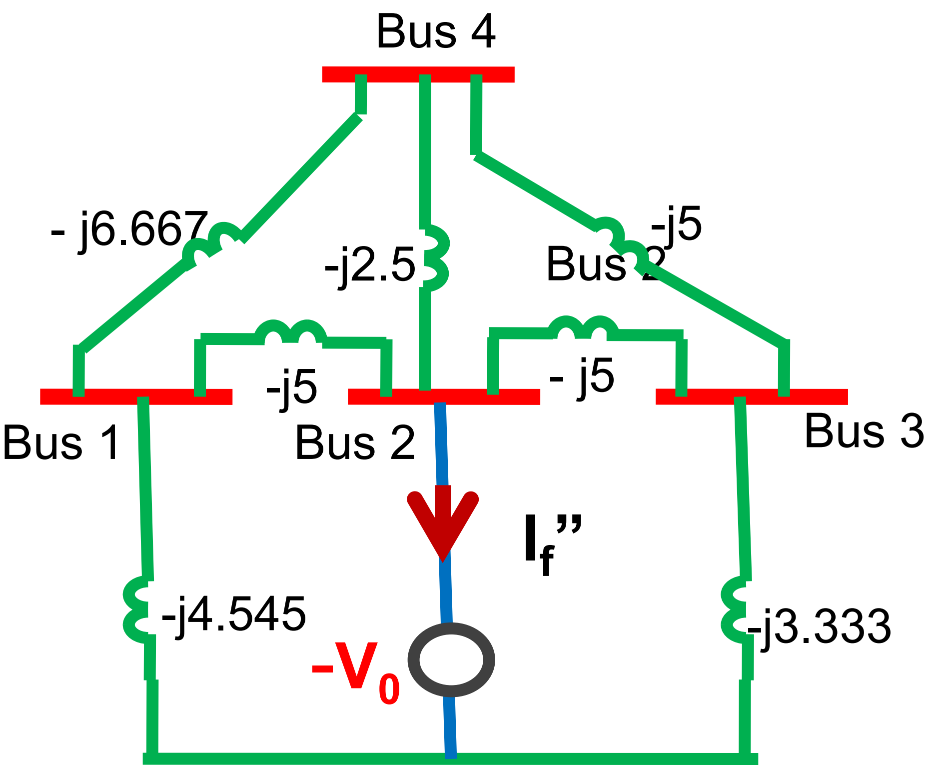
Since there was no fault current before, the current flowing is entirely due to the effect of adding this source (-V_0)



If E_{A1}'', E_{A2}'', and V_0 are s/c, the voltages and currents are those due only to $-V_0$ .Then the only current entering a node from the source is that from $-V_0$ and is $-I_f''$ into node 2 (I_f'' from node 2) since there is no current in this branch until the insertion of $-V_0$.

Bus admittance matrix Y_{bus} for the system:

$$Y_{bus} = \begin{bmatrix} -j16.212 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -j13.333 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$



$$I = Y_{bus} V$$

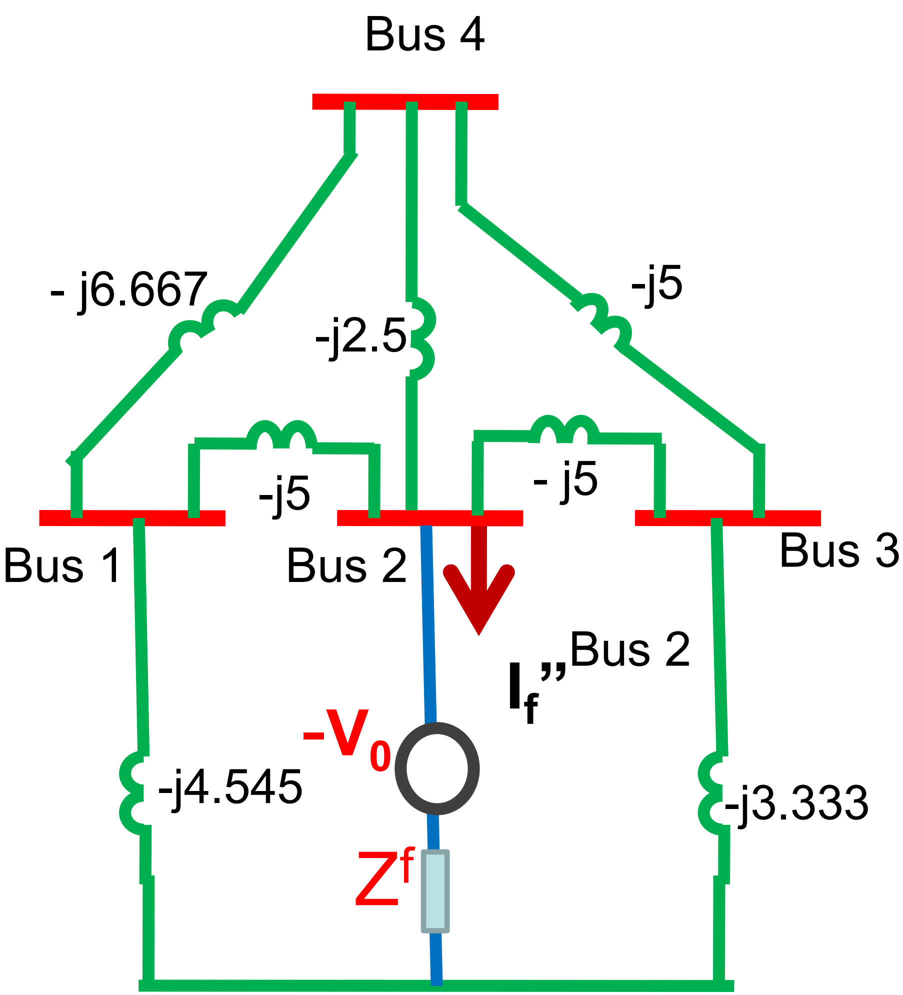
The node eqs. in matrix for the network with $-V_0$ the only source are:

$$\begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix}$$

$$\Delta V_2 = -V_0$$

Let us assume that the rth bus is faulted through a fault impedance Z^f.

The post fault bus voltage vector is $V_{BUS}^{f} = V_{BUS}^{0} + \Delta V$



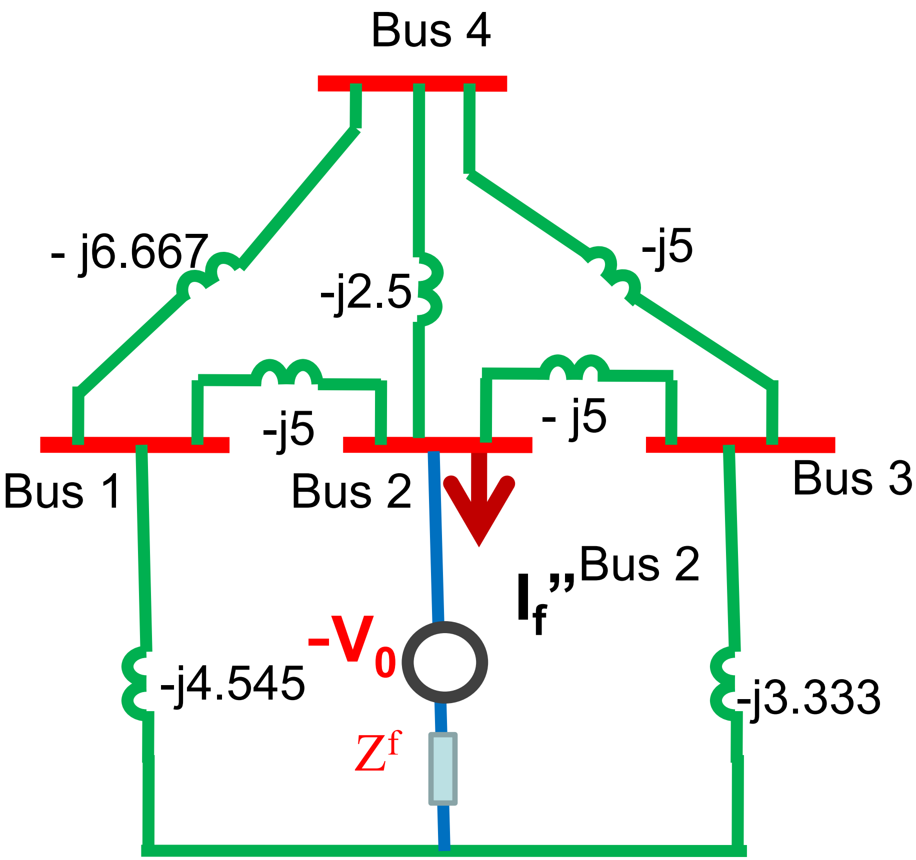
Vector of changes in bus voltages caused by the fault ($-V_0$)

Bus current injection vector

$$\Delta V = Z_{BUS} I^{f}$$

Bus impedance matrix of the passive Thevenin network

Since the network is injected with current-I^f only at the rth bus, we have



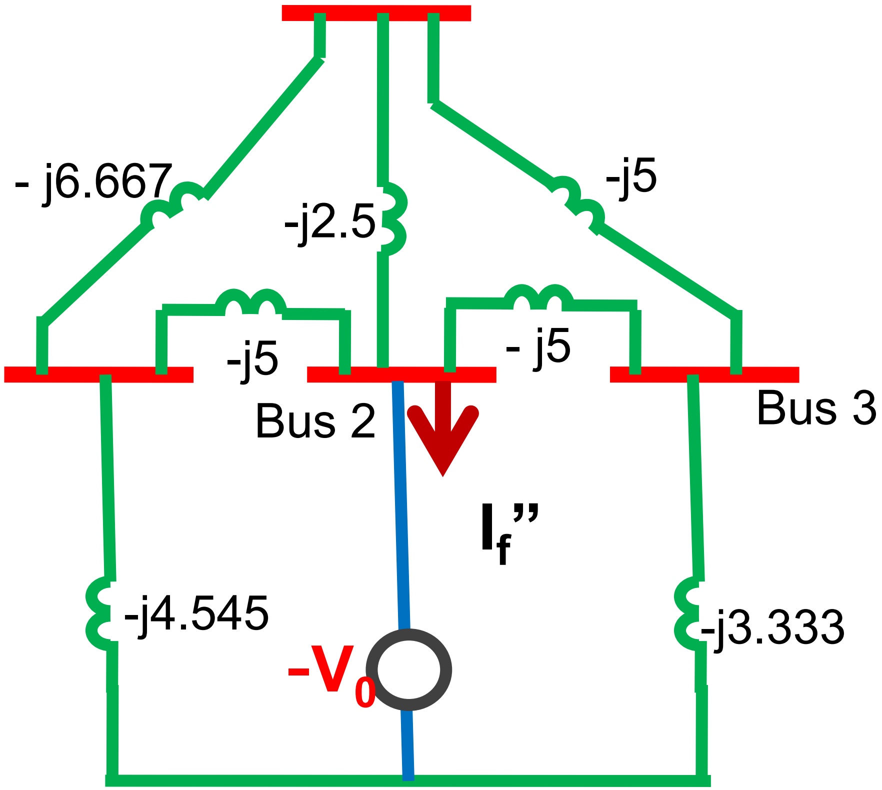
$$I^f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_r^f = -I^f \\ \vdots \\ 0 \end{bmatrix}$$

Substituting eq. in $\Delta V = Z_{BUS} I^f$

We have for the rth bus

$$\Delta V_r = -Z_{rr} I^f$$

Since the network is injected with current - $\mathbf{I_f''}$ only at the fault bus



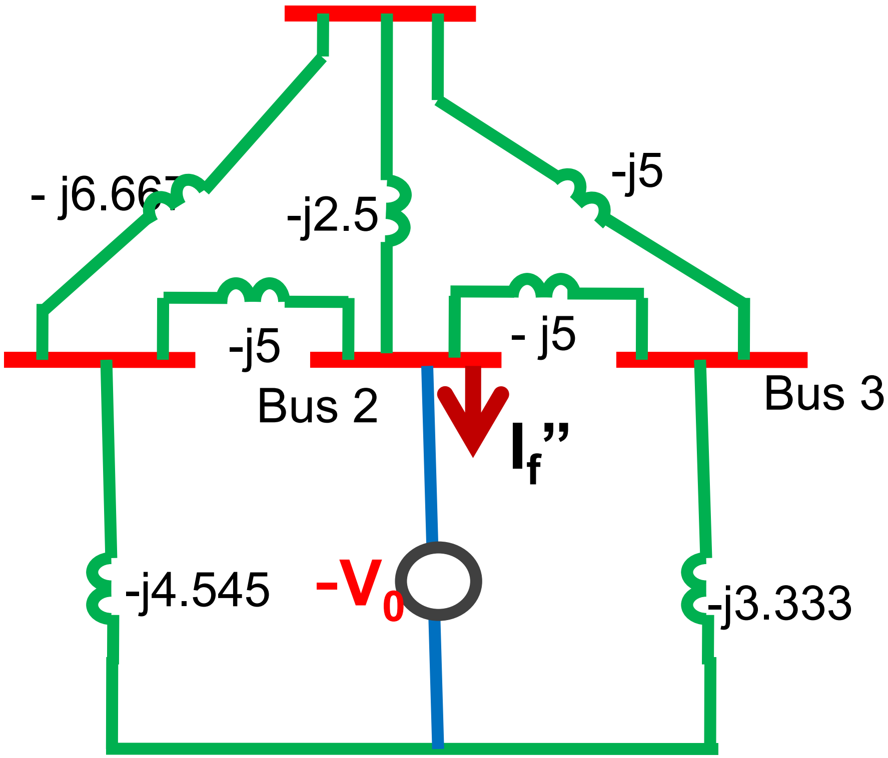
$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix}$$

$$Y_{bus} V = I$$

Changes in the voltages at buses due to $\mathbf{I_f''}$

$$\Delta V = Z_{BUS} I^f$$

$$\Delta V_i = -Z_{ir} I^f$$



$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix}$$

The solution of the equation:

$$\boldsymbol{V} = \boldsymbol{Y_{bus}}^{-1} \boldsymbol{I} = \boldsymbol{Z_{bus}} \boldsymbol{I}$$

$$\underbrace{\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix}}$$

The equation reduces to

$$\Delta V_1 = -Z_{12} I_f''$$

$$\Delta V_2 = -Z_{22} I_f''$$

$$\Delta V_3 = -Z_{32} I_f''$$

$$\Delta V_4 = -Z_{42} I_f''$$

When the generated voltage $-V_0$ is s/c and E_{A1}", E_{A2}", and V_0 are in the circuit, the currents and voltages everywhere in the network are those existing before the fault.

By the principle of superposition these pre-fault voltages added to the change in voltage due to the fault yield the voltages existing after the fault occurs.

Usually the faulted network is assumed to have been without loads before the fault.

In such a case no current is flowing before the fault, and all voltages throughout the network are the same and equal to V_0.

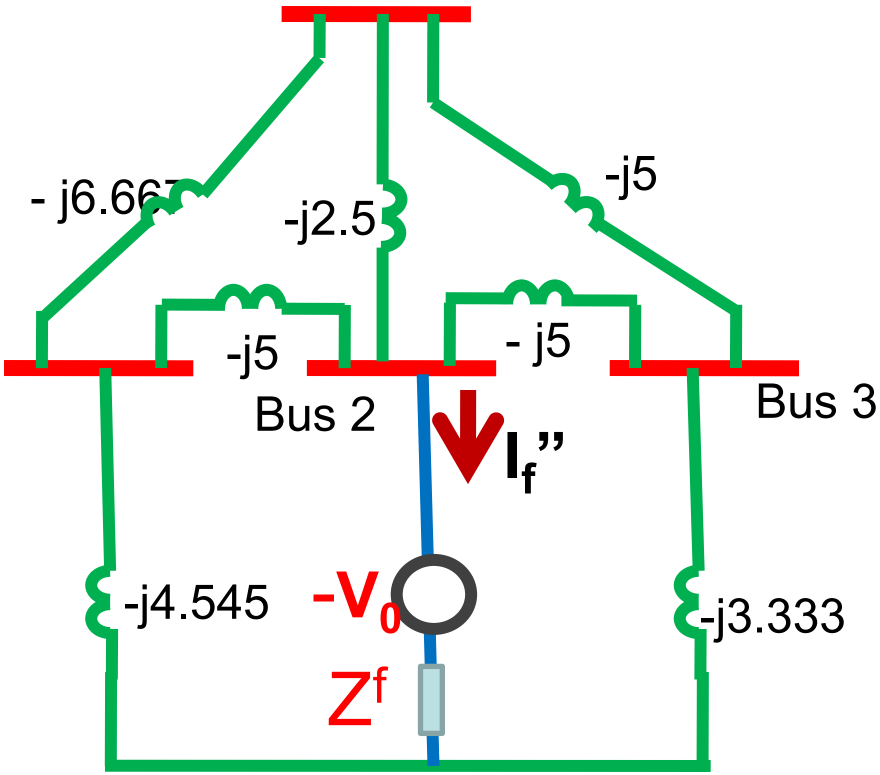
This assumption simplifies our work considerably, and applying the principle of superposition gives.

$$V_i^f = V_i^0 + \Delta V_i,$$

$$\Delta V_i = -Z_{ir} I^f$$

$$V_i^f = V_i^0 - Z_{ir} I^f, \quad \mathrm{i} = 1, 2, \ldots, \mathrm{n}$$

These voltages exist when subtransient current flows and Z_{bus} has been formed for a network having subtransient values for generator reactance



$$\begin{bmatrix}\Delta V_1\\ \Delta V_2\\ \Delta V_3\\ \Delta V_4\end{bmatrix}=\begin{bmatrix}Z_{11} & Z_{12} & Z_{13} & Z_{14}\\ Z_{21} & Z_{22} & Z_{23} & Z_{24}\\ Z_{31} & Z_{32} & Z_{33} & Z_{34}\\ Z_{41} & Z_{42} & Z_{43} & Z_{44}\end{bmatrix}\begin{bmatrix}0\\ -I_f^{''}\\ 0\\ 0\end{bmatrix}$$

$$\Delta V_1 = -Z_{12} I_f^{''}$$

$$\Delta V_2 = -Z_{22} I_f^{''}$$

$$\Delta V_3 = -Z_{32} I_f^{''}$$

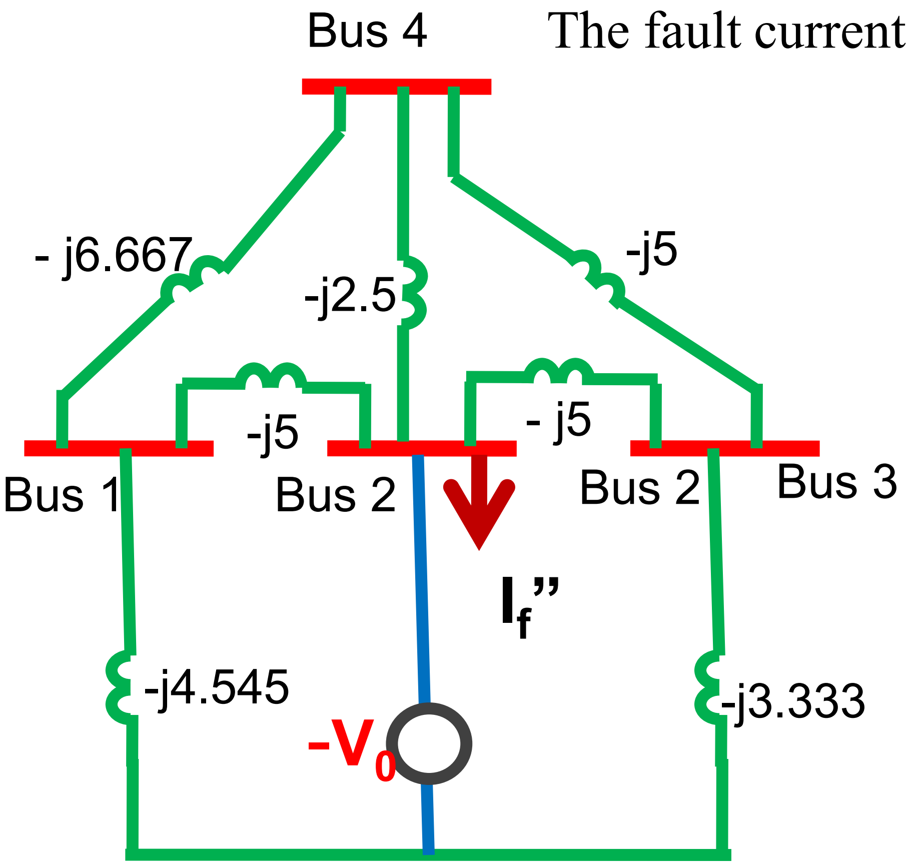
$$\Delta V_4 = -Z_{42} I_f^{''}$$

The voltage at the rth bus under fault is

$$V_r^f = V_r^0 + \Delta V_r^0 = V_r^0 - Z_{rr} I^f$$

However, this voltage must equal

$$V_r^f = Z^f I^f \quad \Longrightarrow \quad I^f = \frac{V_r^0}{Z_{rr} + Z^f}$$



The fault current at bus 2:

$$\Delta V_1 = -Z_{12} I_f^{''}$$

$$\Delta V_2 = -Z_{22} I_f^{''}$$

$$\Delta V_3 = -Z_{32} I_f^{''}$$

$$\Delta V_4 = -Z_{42} I_f^{''}$$

$$I_f^{''} = \frac{V_2^0}{Z_{22}} = \frac{V_0}{Z_{22}}$$

Voltage difference at each of the nodes due to fault current:

$$\Delta V_1 = -Z_{12} I_f^{''} = -\frac{Z_{12}}{Z_{22}} V_0$$

$$\Delta V_2 = -V_0$$

$$\Delta V_3 = -Z_{32} I_f^{''} = -\frac{Z_{32}}{Z_{22}} V_0$$

$$\Delta V_4 = -Z_{42} I_f^{''} = -\frac{Z_{42}}{Z_{22}} V_0$$

At the ith bus:

$$\Delta V_i = -Z_{ir} I^f$$

$$V_i^f = V_i^0 - Z_{ir} I^f, \quad \mathrm{i} = 1, 2, \ldots, \mathrm{n}$$

The total voltage during the fault is: $$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_1^0 \\ V_2^0 \\ V_3^0 \\ V_4^0 \end{bmatrix} + \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix}$$

Voltage at every bus in the power system during the fault can be determined from a knowledge of the pre-fault voltage at the faulted bus and the bus impedance matrix

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_1^0 \\ V_2^0 \\ V_3^0 \\ V_4^0 \end{bmatrix} + \begin{bmatrix} -\dfrac{Z_{12}}{Z_{22}} V_0 \\ -V_0 \\ -\dfrac{Z_{32}}{Z_{22}} V_0 \\ -\dfrac{Z_{42}}{Z_{22}} V_0 \end{bmatrix} = \begin{bmatrix} 1-\dfrac{Z_{12}}{Z_{22}} \\ 0 \\ 1-\dfrac{Z_{32}}{Z_{22}} \\ 1-\dfrac{Z_{42}}{Z_{22}} \end{bmatrix} V_0$$

Substituting for I^f,

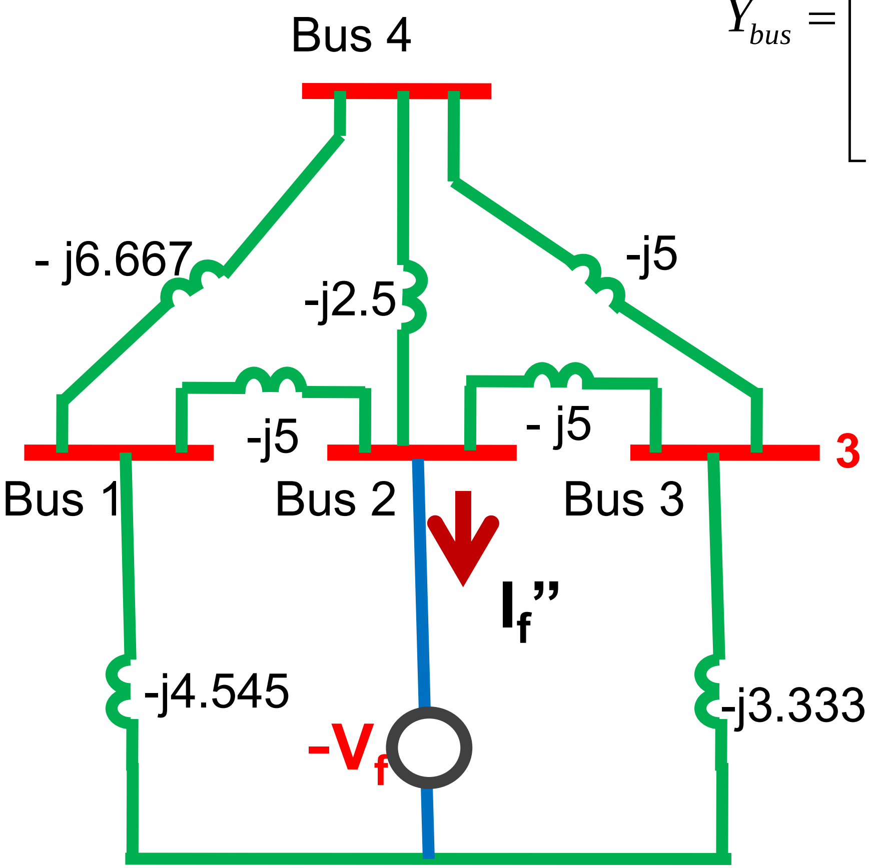
$$V_i^{\,f} = V_i^0 - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^0, \qquad V_r^{\,f} = \frac{Z^f}{Z_{rr} + Z^f} V_r^0 \; for\; i = r$$

The general procedure

1. Create a per-phase, per unit equivalent circuit for the power system.
2. Calculate the bus admittance matrix Y_{bus}. Include the admittances of all transmission lines, transformers, between buses, including loads and generators.
3. Calculate the bus impedance matrix Z_{bus}. It is the inverse of Y_{bus}. You may use MATLAB to perform this calculation.
4. Assume that the power system is at no load conditions and determine the voltage at every bus. Voltage at every bus will be the same, and equal to internal voltage of generators. Given the symbol $\mathbf{V_0}$.
5. Calculate current at the faulted bus.
6. Calculate voltage at each bus during fault. $I^f = \dfrac{V_r^0}{Z_{rr} + Z^f}$
7. Calculate currents in any desired transmission lines during fault.

$$V_i^f = V_i^0 - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^0, \Delta V_i = -Z_{ir} I^f$$

Bus admittance matrix Y_{bus} for the system:

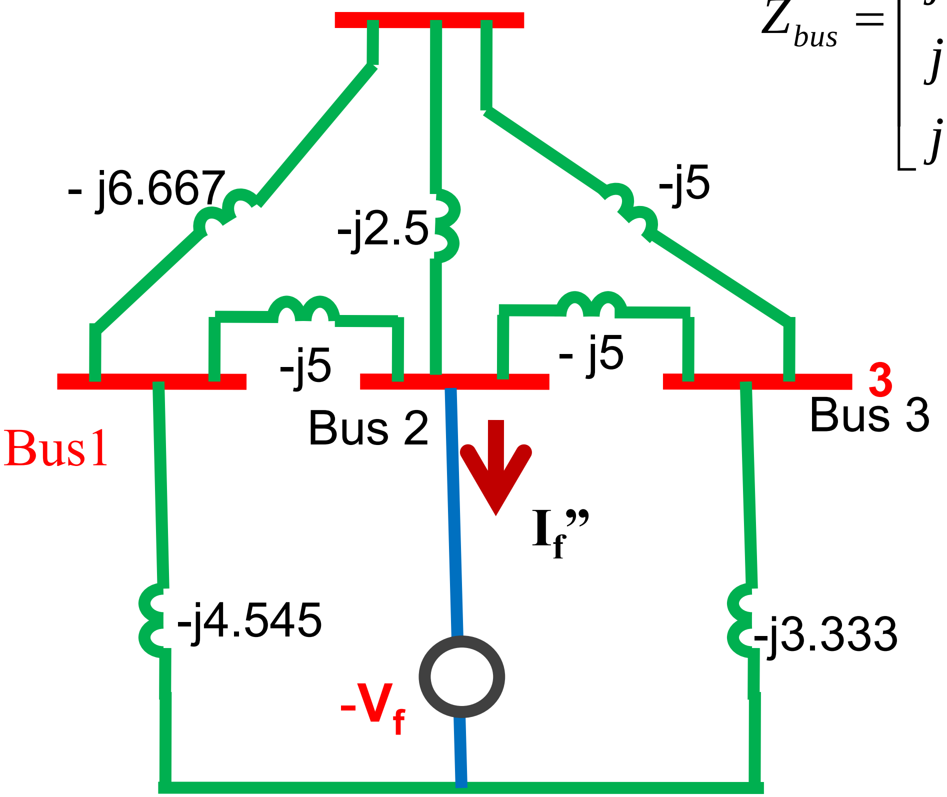


$$Y_{bus} = \begin{bmatrix} -j16.212 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -j13.333 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$

Calculate Z_{bus}

$$Z_{bus} = \begin{bmatrix} j0.1515 & j0.1232 & j0.0934 & j0.1260 \\ j0.1232 & j0.2104 & j0.1321 & j0.1417 \\ j0.0934 & j0.1321 & j0.1726 & j0.1282 \\ j0.1260 & j0.1417 & j0.1282 & j0.2001 \end{bmatrix}$$

$$\boxed{\textit{The}\text{ prefault condition being no load}, V_1^0 = V_2^0 = V_3^0 = V_4^0 = 1pu}$$



$$Z_{bus} = \begin{bmatrix} j0.1515 & j0.1232 & j0.0934 & j0.1260 \\ j0.1232 & j0.2104 & j0.1321 & j0.1417 \\ j0.0934 & j0.1321 & j0.1726 & j0.1282 \\ j0.1260 & j0.1417 & j0.1282 & j0.2001 \end{bmatrix}$$

Assume $V_0, = 1.00\angle 0°$ and calculate faulted current

$$I''_{f,2} = \frac{V_0}{Z_{22}} = \frac{1.00\angle 0}{j0.2104} = 4.753\angle -90$$

$$V_i^{\,f}\,(post\ \ fault) = V_i^{\,0} - \frac{Z_{ir}}{Z_{rr} + Z^{\,f}} V_r^{\,0}, \quad V_i^{\,0} = 1\,pu(\,prefault\,condition)$$

$$= \left(1 - \frac{Z_{ir}}{Z_{rr}}\right) V_0$$

Calculate the voltages at each bus during fault

$$V_1 = \left(1 - \frac{Z_{12}}{Z_{22}}\right)V_0 = \left(1 - \frac{j0.1232}{j0.2104}\right)1.00\angle 0 = 0.414\angle 0 \ pu$$

$$V_2 = 0.0\angle 0$$

$$V_3 = \left(1 - \frac{Z_{32}}{Z_{22}}\right)V_0 = \left(1 - \frac{j0.1321}{j0.2104}\right)1.00\angle 0 = 0.372\angle 0 \ pu$$

$$V_4 = \left(1 - \frac{Z_{42}}{Z_{22}}\right)V_0 = \left(1 - \frac{j0.1417}{j0.2104}\right)1.00\angle 0 = 0.327\angle 0 \ pu$$

Calculate the currents in any desired transmission lines during fault.

The current flowing in line 1:

$$\begin{aligned} I_{12} &= -Y_{12}(V_1 - V_2) \\ &= (0.414\angle 0 - 0.00\angle 0)(-j5.0) \\ &= -j2.07 \ pu \end{aligned}$$

Exercise

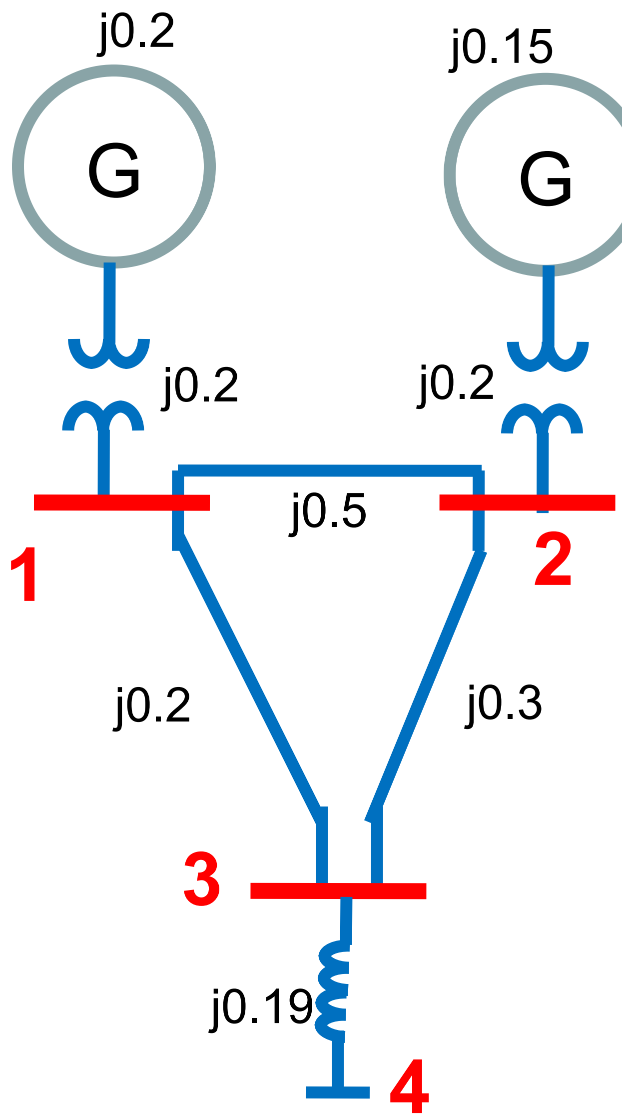
Per unit bus impedance matrix for a power system is given by

$$Z_{bus} = \begin{bmatrix} j0.24 & j0.14 & j0.2 & j0.2 \\ j0.14 & j0.2275 & j0.175 & j0.175 \\ j0.2 & j0.175 & j0.31 & j0.31 \\ j0.2 & j0.175 & j0.31 & j0.5 \end{bmatrix}$$

A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault. One line diagram of the system is given.

Repeat for a three-phase fault at bus 2 with a fault impedance of $Z_f = j0.0225$

One line diagram of the system

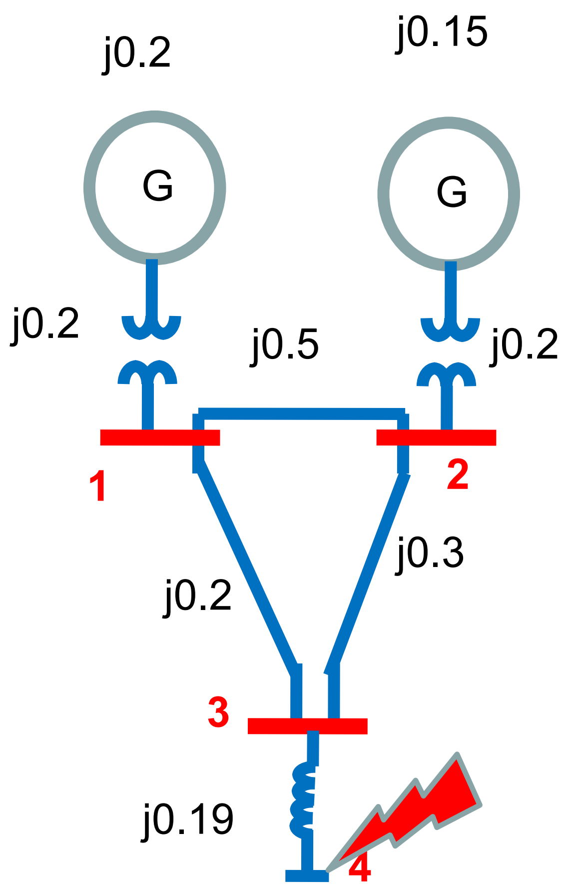


$$Z_{bus} = \begin{bmatrix} j0.24 & j0.14 & j0.2 & j0.2 \\ j0.14 & j0.2275 & j0.175 & j0.175 \\ j0.2 & j0.175 & j0.31 & j0.31 \\ j0.2 & j0.175 & j0.31 & j0.5 \end{bmatrix}$$

A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault.

Solution.

Assume $V_{0,} = 1.00\angle 0°$, faulted current at bus 4:



$$Z_{bus} = \begin{bmatrix} j0.24 & j0.14 & j0.2 & j0.2 \\ j0.14 & j0.2275 & j0.175 & j0.175 \\ j0.2 & j0.175 & j0.31 & j0.31 \\ j0.2 & j0.175 & j0.31 & j0.5 \end{bmatrix}$$

$$I''_{f,4} = \frac{V_0}{Z_{44}} = \frac{1.00\angle 0}{j0.5} = 2\angle -90$$

Calculate the voltages at each bus during fault

$$Z_{bus} = \begin{bmatrix} j0.24 & j0.14 & j0.2 & j0.2 \\ j0.14 & j0.2275 & j0.175 & j0.175 \\ j0.2 & j0.175 & j0.31 & j0.31 \\ j0.2 & j0.175 & j0.31 & j0.5 \end{bmatrix} \qquad V_j = \left(1 - \frac{Z_{ji}}{Z_{ii}}\right) V_0$$

$$V_1 = \left(1 - \frac{Z_{14}}{Z_{44}}\right) V_0 = \left(1 - \frac{j0.2}{j0.5}\right) 1.00\angle 0 = 0.6\angle 0 \ pu$$

$$V_2 = \left(1 - \frac{Z_{24}}{Z_{44}}\right) V_0 = \left(1 - \frac{j0.175}{j0.5}\right) 1.00\angle 0 = 0.65\angle 0 \ pu$$

$$V_3 = \left(1 - \frac{Z_{34}}{Z_{44}}\right) V_0 = \left(1 - \frac{j0.31}{j0.5}\right) 1.00\angle 0 = 0.38\angle 0 \ pu$$

$$V_4 = 0.0\angle 0$$

Calculate the currents in any desired transmission lines during fault.

The current flowing in line 12:

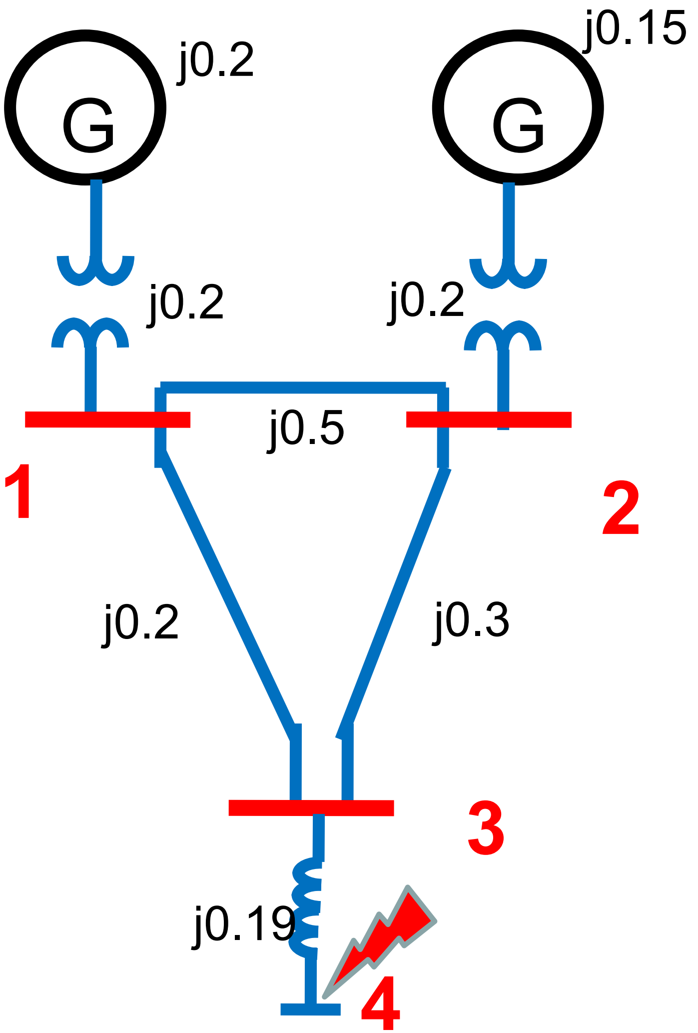
$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0.6\angle 0 \\ 0.65\angle 0 \\ 0.38\angle 0 \\ 0.0\angle 0 \end{bmatrix}$$

$$I_{21}(F) = \frac{(V_2(F) - V_1(F))}{z_{12}} = \frac{0.65 - 0.6}{j0.5} = -j0.1\ pu$$

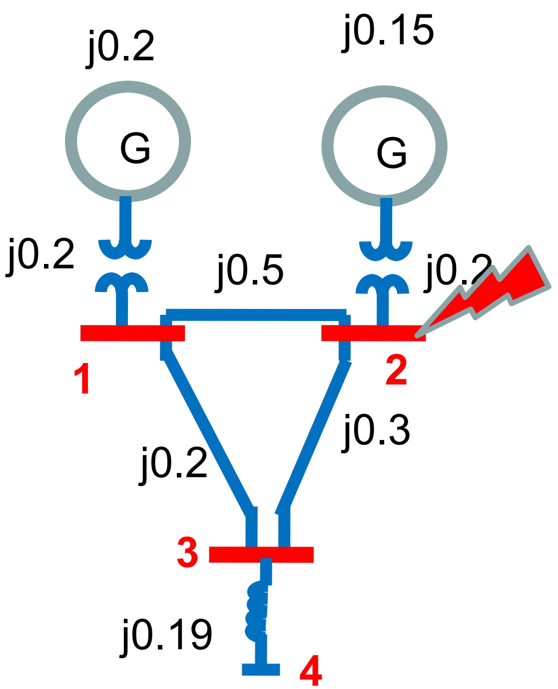
$I_{13}(F) = -j1.1\ pu$

$I_{23}(F) = -j0.9\ pu$

$I_{34}(F) = -j2\ pu$



A three-phase fault at **bus 2** with a fault impedance of **Z_f = j0.0225.** Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault. One line diagram of the system is given.



$$Z_{bus} = \begin{bmatrix} j0.24 & j0.14 & j0.2 & j0.2 \\ j0.14 & j0.2275 & j0.175 & j0.175 \\ j0.2 & j0.175 & j0.31 & j0.31 \\ j0.2 & j0.175 & j0.31 & j0.5 \end{bmatrix}$$

Assume $V_f = 1.00\angle 0°$, faulted current:

$$I''_{f,2} = \frac{V_f}{Z_{22} + Z_f} = \frac{1.00\angle 0}{j0.2275 + j0.0225} = 4\angle -90$$

Calculate the voltages at each bus during fault

$$V_i^f = V_i^0 - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^0, \quad Z_{bus} = \begin{bmatrix} j0.24 & j0.14 & j0.2 & j0.2 \\ j0.14 & j0.2275 & j0.175 & j0.175 \\ j0.2 & j0.175 & j0.31 & j0.31 \\ j0.2 & j0.175 & j0.31 & j0.5 \end{bmatrix}$$

$$V_1 = \left(1 - \frac{Z_{12}}{Z_{22} + Z_f}\right) V_f = \left(1 - \frac{j0.14}{j0.2275 + j0.0225}\right) 1.00\angle 0 = 0.44\angle 0 \ pu$$

$$V_2 = \left(1 - \frac{Z_{22}}{Z_{22} + Z_f}\right) V_f = \left(1 - \frac{j0.2275}{j0.25}\right) 1.00\angle 0 = 0.09\angle 0 \ pu$$

$$V_3 = \left(1 - \frac{Z_{32}}{Z_{22} + Z_f}\right) V_f = \left(1 - \frac{j0.175}{j0.25}\right) 1.00\angle 0 = 0.3\angle 0 \ pu$$

$$V_4 = \left(1 - \frac{Z_{42}}{Z_{22} + Z_f}\right) V_f = \left(1 - \frac{j0.175}{j0.25}\right) 1.00\angle 0 = 0.3\angle 0 \ pu$$

The current flowing in line 1:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0.44\angle 0 \\ 0.09\angle 0 \\ 0.3\angle 0 \\ 0.3\angle 0 \end{bmatrix}$$

$$I_{12}(F) = \frac{\left(V_1(F) - V_2(F)\right)}{z_{12}}$$

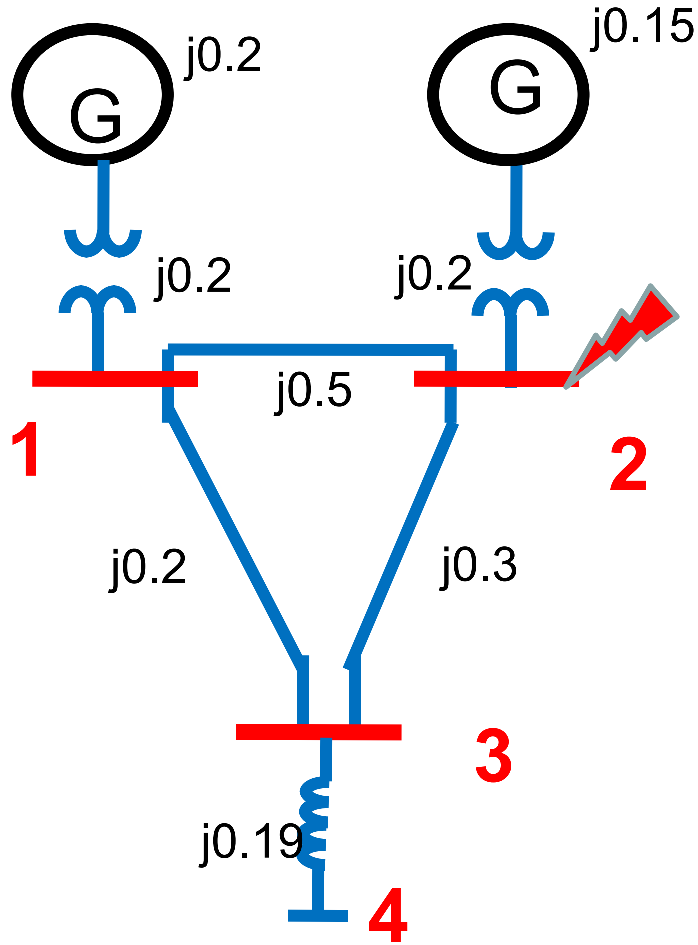
$$= \frac{0.44 - 0.09}{j0.5}$$

$$= -j0.7\ pu$$

$I_{13}(F)$ = -j0.7 pu

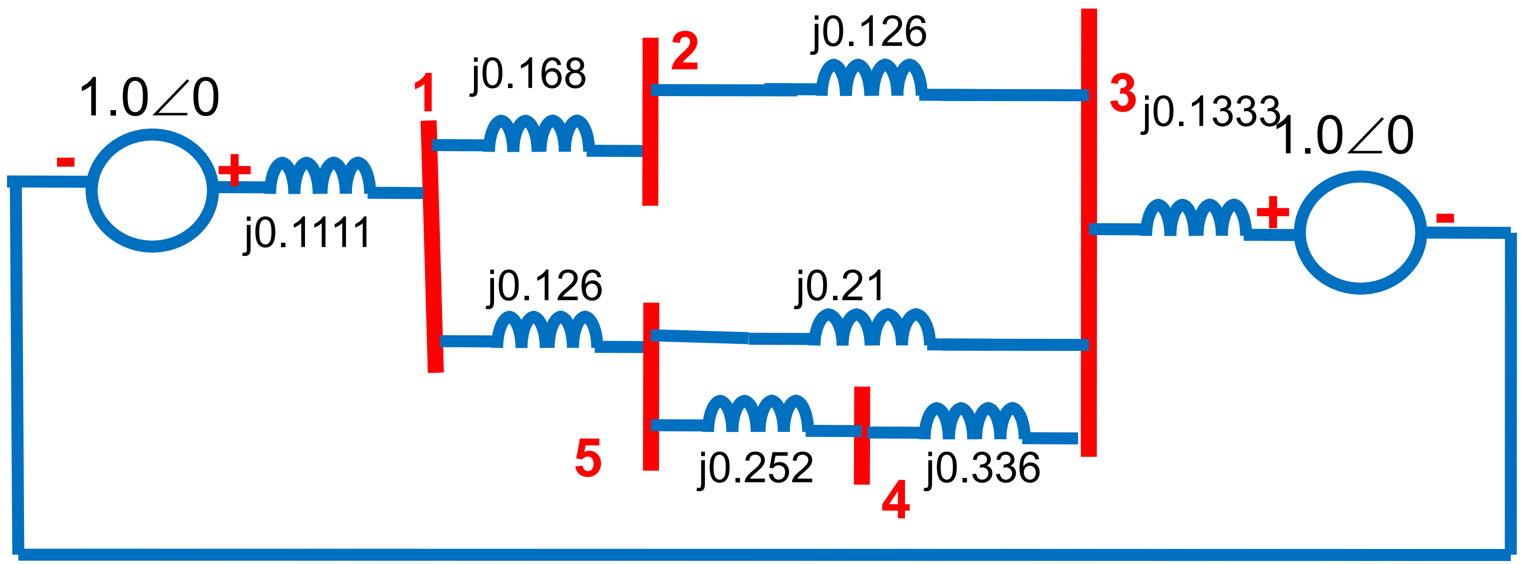
$I_{32}(F)$ = -j0.7 pu

$I_{34}(F)$ = 0 pu



Example

Using bus impedance matrix for the network, find subtransient current in a three-phase fault at bus 4 and the current coming to the faulted bus over each line. Neglect pre-fault current. All voltages are assumed to be 1.0 pu before fault



$$Z_{bus} = \begin{bmatrix} j0.0793 & j0.0558 & j0.0382 & j0.0511 & j0.0608 \\ j0.0558 & j0.1338 & j0.0664 & j0.0630 & j0.0605 \\ j0.0382 & j0.0664 & j0.0875 & j0.0720 & j0.0630 \\ j0.0511 & j0.0630 & j0.0720 & j0.2321 & j0.1002 \\ j0.0608 & j0.0605 & j0.0630 & j0.1002 & j0.1301 \end{bmatrix}$$

$$I_f'' = \frac{V_f}{Z_{44}}$$

$$= \frac{1.0}{j0.2321}$$

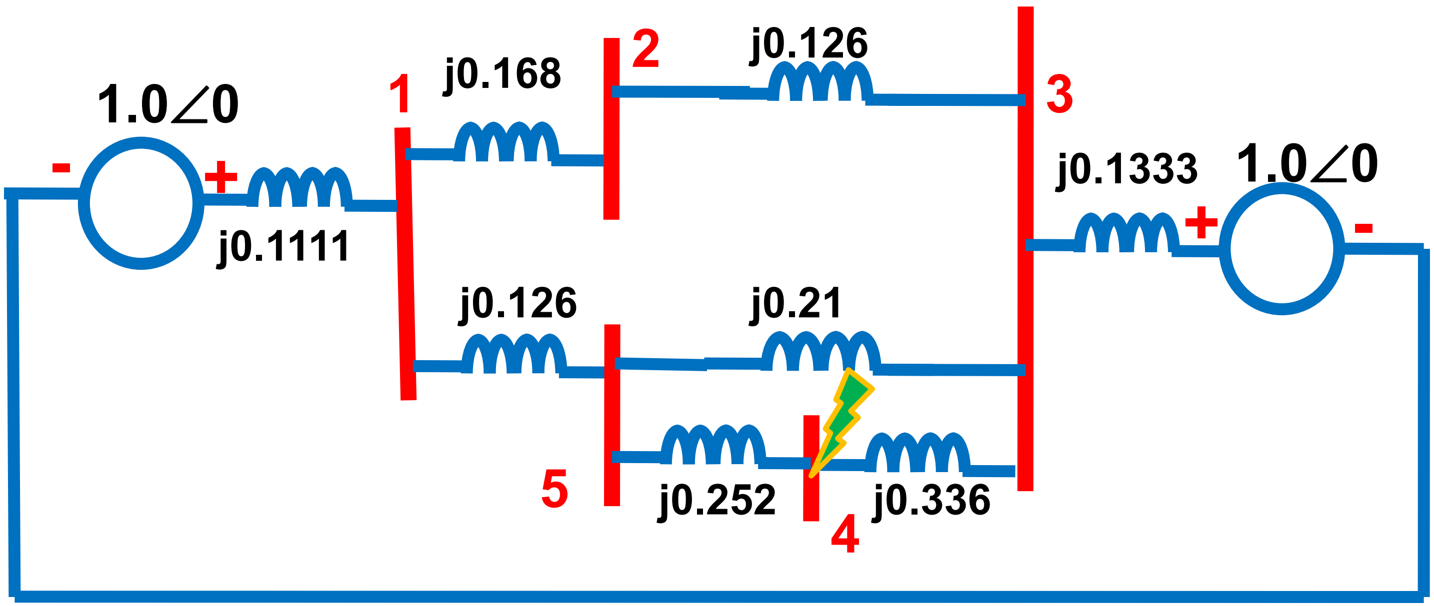
$$= -j4.308$$

$$V_3 = V_f - I_f'' Z_{34}$$

$$= 1.0 - (-j4.308)(j0.072) = 0.6898\ p.u.$$

$$V_5 = V_f - I_f'' Z_{54}$$

$$= 1.0 - (-j4.308)(j0.1002) = 0.5683\ p.u.$$



Currents into the fault at bus 4 over the line impedances Z_{line} are

From bus 3

$$I_{f,3}'' = \frac{V_3}{Z_{line34}} = \frac{0.6898}{j0.336} = -j2.053$$

From bus 5

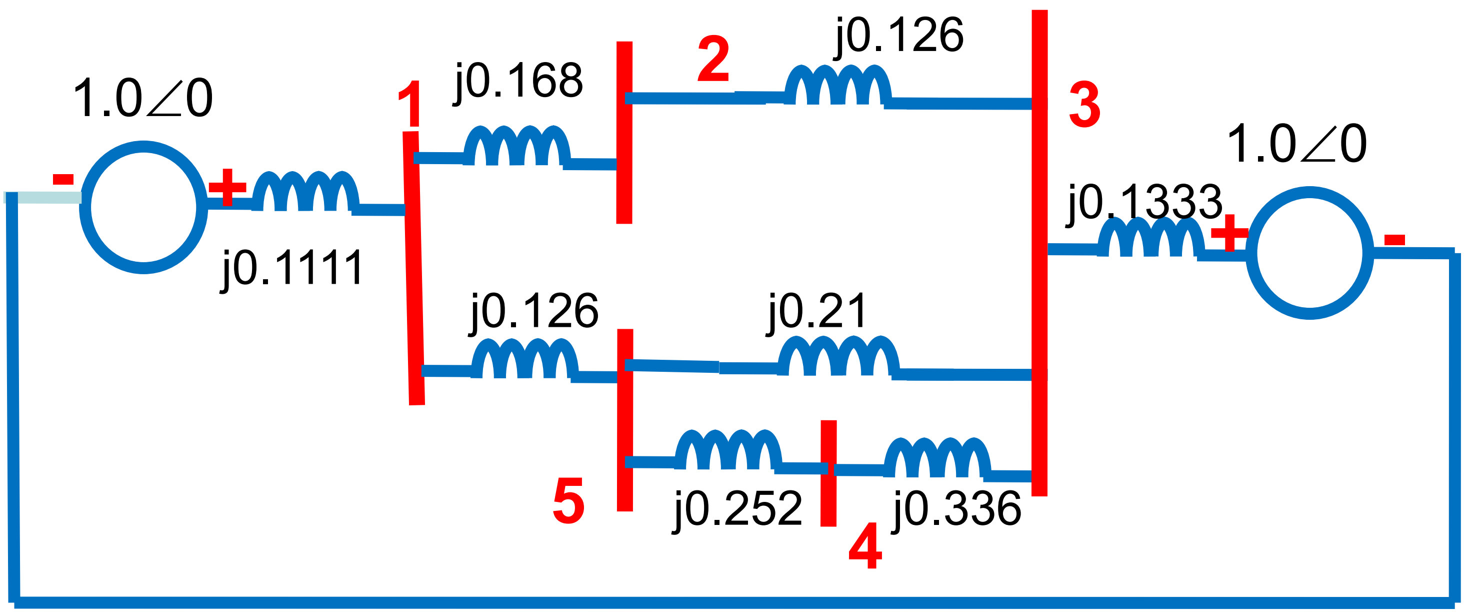
$$I_{f,5}'' = \frac{V_5}{Z_{line54}} = \frac{0.5683}{j0.252} = -j2.255$$

Total fault current at bus 4

$$I_f'' = -j4.308 \ pu$$

Exercise:

Find subtransient current in a three-phase fault at **bus 5** and the current coming to the faulted bus over each line. Pre-fault current is to be neglected and all voltages are assumed to be 1.0 pu before fault.



$$Z_{bus} = \begin{bmatrix} j0.0793 & j0.0558 & j0.0382 & j0.0511 & j0.0608 \\ j0.0558 & j0.1338 & j0.0664 & j0.0630 & j0.0605 \\ j0.0382 & j0.0664 & j0.0875 & j0.0720 & j0.0630 \\ j0.0511 & j0.0630 & j0.0720 & j0.2321 & j0.1002 \\ j0.0608 & j0.0605 & j0.0630 & j0.1002 & j0.1301 \end{bmatrix}$$

- **Short-Circuit Capacity (SCC)**
- Two of the CB ratings which require the computation of SC current are: rated momentary current and rated symmetrical interrupting current.
- Symmetrical SC current is obtained by using subtransient reactances for synchronous machines.
- Momentary current (rms) is then calculated by multiplying the symmetrical momentary current by a factor of 1.6 to account for the presence of DC off set current.
- Symmetrical current to be interrupted is computed by using subtransient reactances for synchronous generators and transient reactances for synchronous motors.
- The current that a CB can interrupt is inversely proportional to the operating voltage over a certain range i.e.

Ampere at operating voltage = ampere at rated voltage x rated voltage /operating voltage. Of course, operating voltage cannot exceed the maximum design values. Also, no matter how low the voltage is, the rated interrupting current cannot exceed the rated interrupting maximum current.

- Also called short-circuit MVA
- Is used for determining
  - the dimension of bus bar
  - Interrupting capacity of circuit breakers

Circuit breakers capacities

- Momentary duty – determine by asymetrical short circuit current
- Short-circuit MVA at bus k is the product of the magnitude of the rated bus voltage and the fault current.

$$SCC = \sqrt{3}\, V_{Lk} I_k(F) \times 10^{-3} \quad MVA$$

V_{Lk} is line to line voltage in kV

$I_k(F)$ is fault current in ampere

At bus k, short-circuit capacity (SCC) or short circuit MVA

$$SCC = \sqrt{3} V_{Lk} I_k(F) \times 10^{-3} \quad MVA$$

Line to line voltage in kV

Fault current expressed in amperes

$$I_k(F) = I_k(F)_{pu} I_B$$

$$= \frac{V_k(0)}{X_{kk}} \frac{S_B \times 10^3}{\sqrt{3} V_B}$$

Symmetrical current in p.u.

$$I_k(F)_{pu} = \frac{V_k(0)}{X_{kk}}$$

Base current

$$I_B = \frac{S_b \times 10^3}{\sqrt{3} V_B}$$

Per unit reactance at fault point

Per unit prefault bus voltage

$$I_k(F) = I_k(F)_{pu} I_B$$

$$SCC = \sqrt{3}\, V_{Lk} I_k(F) \times 10^{-3} \quad MVA$$

$$= \frac{V_k(0)}{X_{kk}} \frac{S_B \times 10^3}{\sqrt{3}\, V_B}$$

$$SCC = \sqrt{3}\, V_{Lk} I_k(F) \times 10^{-3}$$

$$= \sqrt{3}\, V_{Lk} \frac{V_k(0)}{X_{kk}} \frac{S_B \times 10^3}{\sqrt{3} V_B} \times 10^{-3}$$

$$SCC = \sqrt{3}\, V_{Lk} I_k(F) \times 10^{-3}$$

$$= \frac{V_k(0) S_B}{X_{kk}} \frac{V_{Lk}}{V_B}$$

Base voltage is equal to the rated voltage $V_L = V_B$

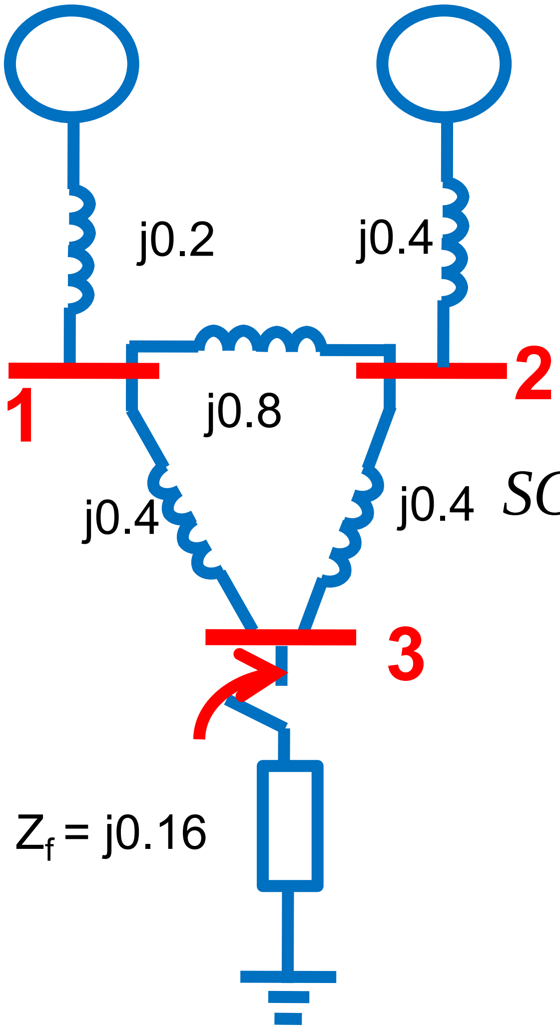
$$SCC = \frac{V_k(0) S_B}{X_{kk}}$$

Pre-fault bus voltage is usually assumed to be 1.0 p.u

$$\boxed{SCC = \frac{S_B}{X_{kk}} \quad MVA}$$

Example:

Determine short circuit capacity of the given network due to impedance fault at bus bar 3 $\quad S_B = 100 \quad MVA$



$$Z_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

$$Z_{33} = j0.34$$

$$SCC_3 = \frac{S_B}{|Z_{33}| + 0.16} = \frac{100}{0.34 + 0.16} = 200 \quad MVA$$

Selection of circuit breakers

- Industrial plants must specify their circuit breakers based on their fault current.
- This can be determined by the short-circuit MVA which can be expected at nominal voltage $\text{Short Circuit MVA} = \sqrt{3} \times (\text{nominal kV}) \times |I_{SC}| \times 10^{-3}$

In amperes the rms magnitude of the short circuit current in a three phase fault

$$\text{Short Circuit MVA} = \sqrt{3} \times (\text{nominal kV}) \times |I_{SC}| \times 10^{-3}$$

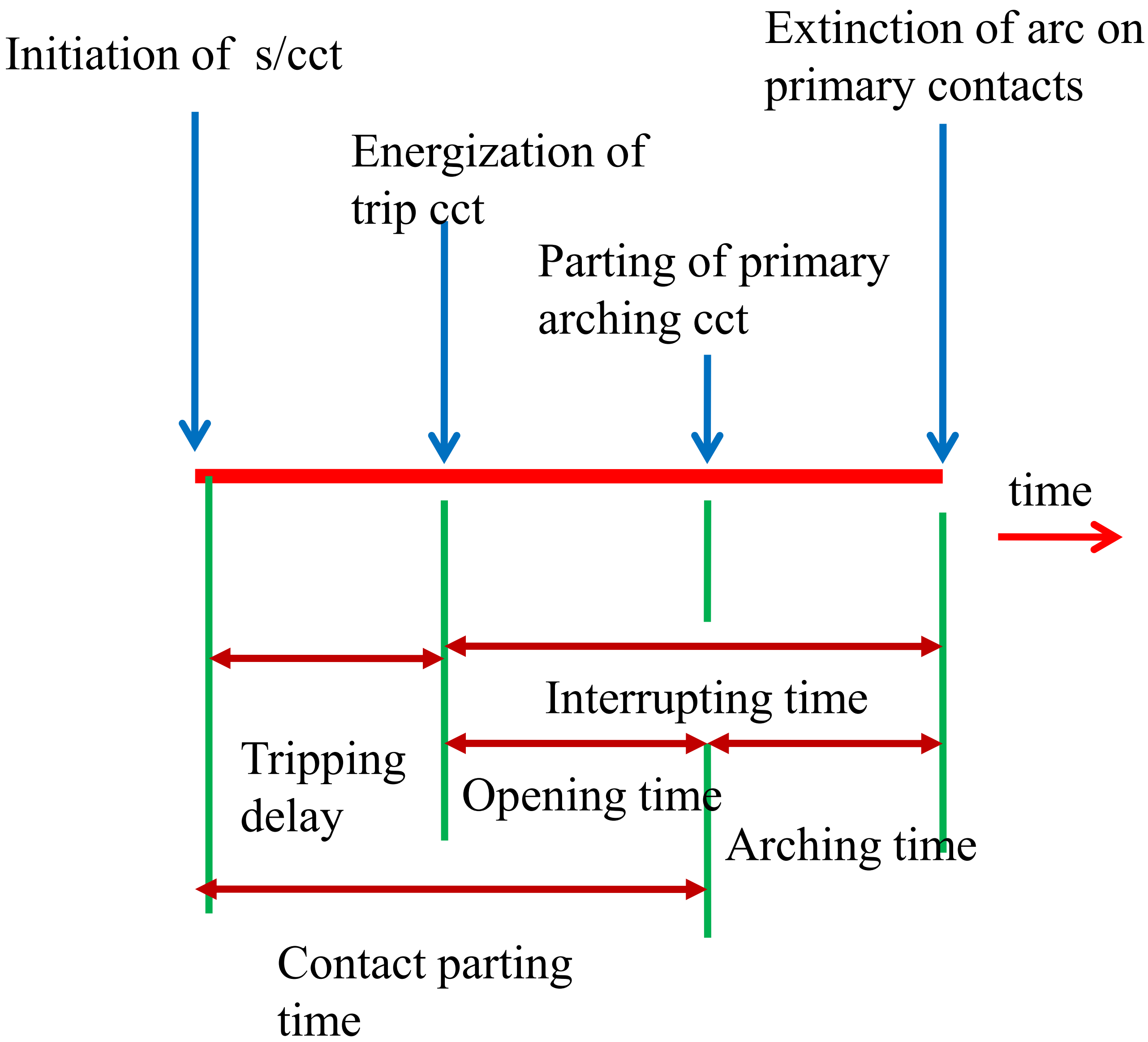
$$= V^0 I^f = V^0 \left(\frac{V^0}{Z} \right) = \frac{1}{Z}\ pu$$

$$\text{Base MVA} = \sqrt{3} \times (\text{base kV}) \times |I_{base}| \times 10^{-3}$$

$$\text{Short Circuit MVA in pu} = |I_{SC}| \text{ in pu}$$

$$|Z_{th}| = \frac{1.0}{|I_{SC}|} \text{ in pu}$$

$$= \frac{1.0}{\text{short-circuit MVA}} \text{ in pu}$$


Initiation of s/cct
Extinction of arc on primary contacts
Energization of trip cct
Parting of primary arching cct
time
Interrupting time
Tripping delay
Opening time
Arching time
Contact parting time

CURRENT LIMITING REACTOR

Fault current may be large enough to cause damage to the line and other equipment of a power system network. The interrupting capacities of circuit breakers to handle such currents would also be very large. Fault current is limited by the system reactance, which includes the impedance of the generators, transformers, lines, and other components of the system.

Modern generators have reactances large enough to limit fault currents. Old generators have low values of reactance. The fault levels increase with the growth of the interconnected system. Therefore, if the system is large or some of the generators are old, the fault current can be kept within safe limits by increasing the system reactance. This is done by connecting reactors at strategic points in the system.

Current-limiting reactors are coils used to limit current during fault conditions. Such reactors have large values of inductive reactances and low ohmic resistances.

Constructions of reactors

For current limiting reactors, it is important that magnetic saturation at high current does not reduce the coil reactance. Reactors can be either air-cored type or iron-cored type. Air-cored reactors do not have magnetic saturation, and therefore their reactances are independent of current. For this reason, air-cored reactors are most commonly used these days.

Location of reactors

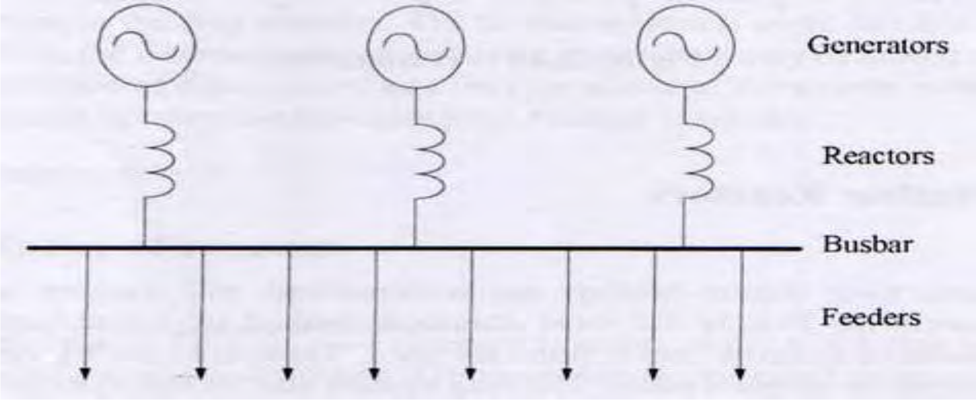
Current limiting reactors may be connected

(a) series with each generator,

(b) in series with each feeder, and

(c) between busbar sections.

Generators Reactors



- reactors connected in series with each generator.

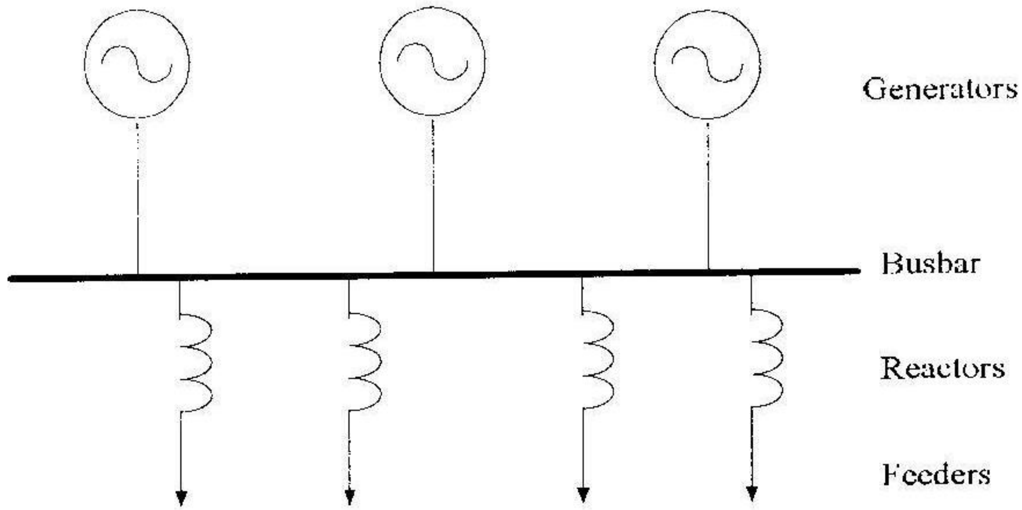
- feeders are connected directly to the bus-bars.

- modern generators are designed to have sufficiently large reactance to protect them even in dead short-circuits at their terminals. Thus, current limiting reactors suffer from the following drawbacks:-

a. the full load current flowing in the reactor under normal operation produces a constant voltage drop and power loss in each reactor.

b. if a bus-bar or feeder fault occurs close to the bus-bars, the voltage at the bus-bars drops to a low value with the result that generators may lose synchronism and the supply may interrupted.

Feeder Reactors



reactors are connected in series with each feeder advantages:-

a. if there is a fault on any feeder, the voltage drop in its reactor will not affect the bus-bar voltage, therefore there is a little chance for the generator to lose synchronism.

b. the fault on a feeder will not affect other feeders.

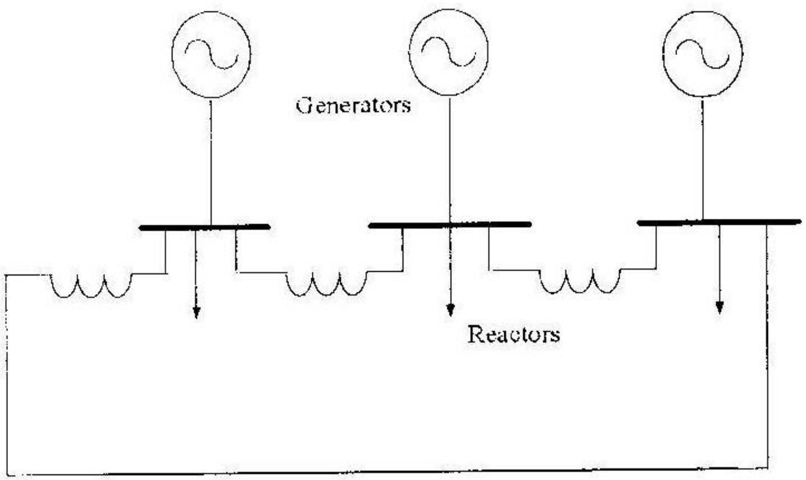
limitations:-

a. There is a constant voltage drop and power loss in each reactor during normal operating conditions.

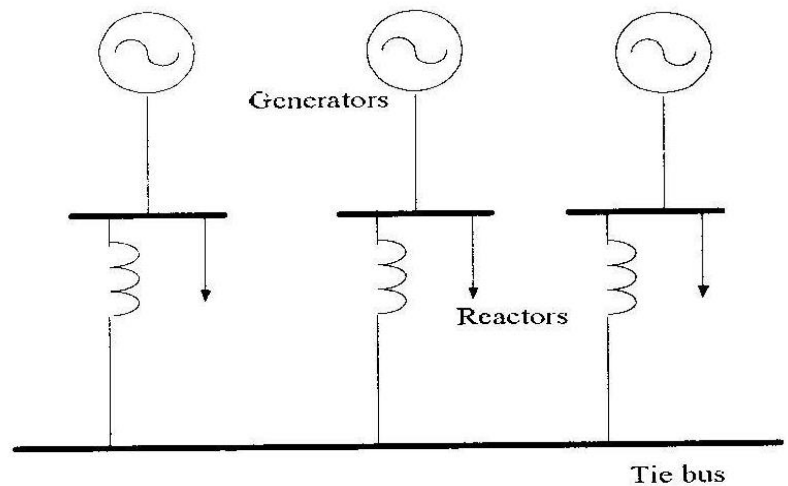
b. If a fault occurs at the busbars, no protection is provided to the generators.

c. If the number of generators is increased. The sizes of the feeder reactors should also be increased.

Bus-bar Reactors



(a) Ring System



(b) Tie bus system

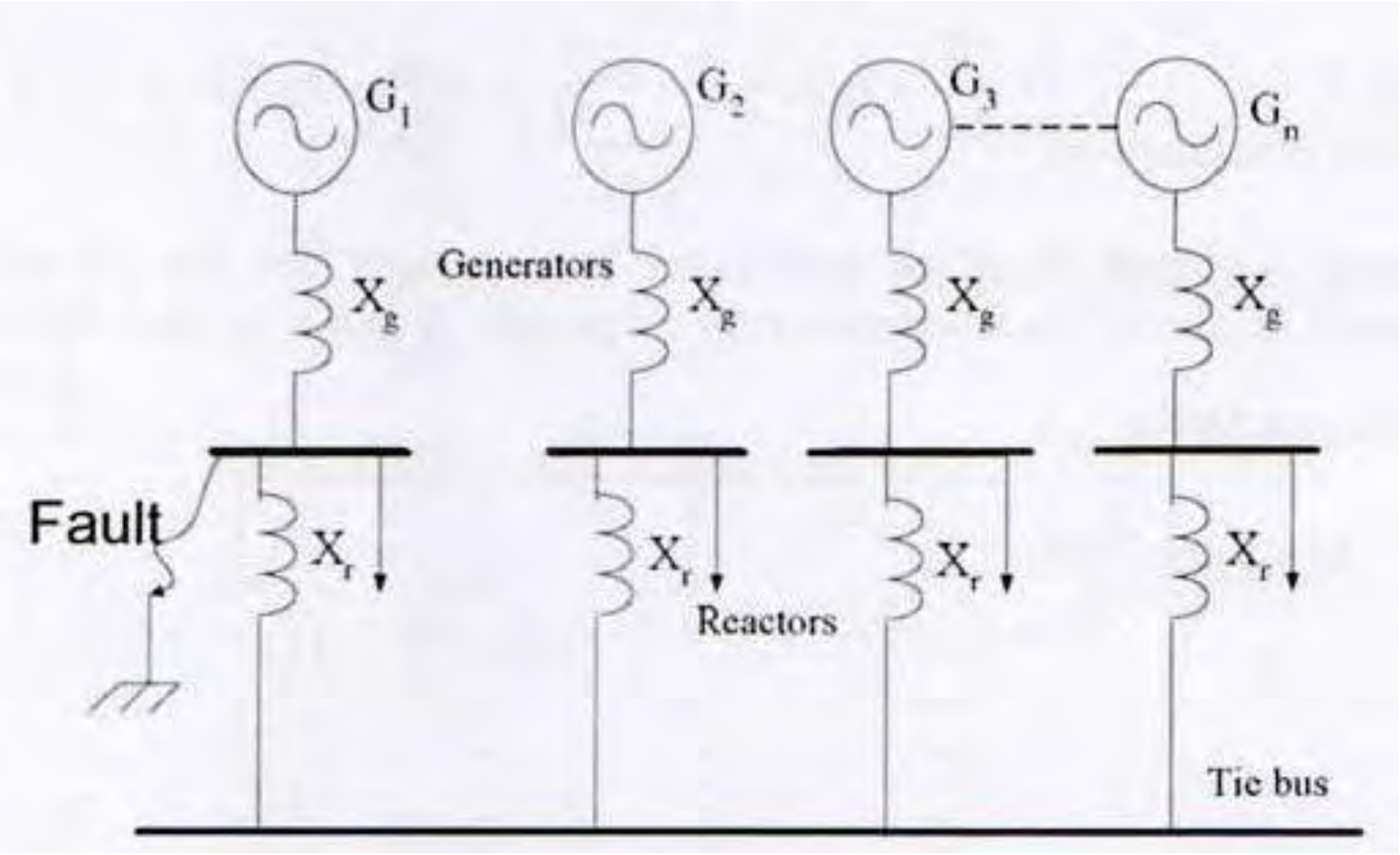
Generating station bus-bars are sectionalized and reactors are connected between sections. This is the most common method of connection of reactors. Under normal operation each generator supplies feeder connected to its own section, and there will be no current through the reactors. Thus, there is no voltage drop or power loss in the reactor during normal operation. The bus-bar reactors localize the faults. For example. if a fault occurs on a feeder, only that bus-bar section is affected to which it is connected. The other sections continue to operate normally. In the ring system, the current transferred between two sections flows through two paths in parallel, whereas in the tie-bus system the current flows through two reactors in series. Therefore, the reactors in the tie-bus system have only one-half of the reactance of ring reactors. However, they carry twice as much current as the ring reactors. The tie-bus system is more flexible than the ring system. Tie-bus - extra generators may be added to the system without addition of extra circuit breakers or increasing the existing reactance.

Rating of reactors

The rating of reactors is expressed in terms of the MVA that is designed to carry at rated current and voltage.

The reactance is expressed in per unit and is the ratio of voltage drop across the reactor at the rated current to the line-to-neutral voltage of the system.

Short Circuit MVA in a Tie Bus System



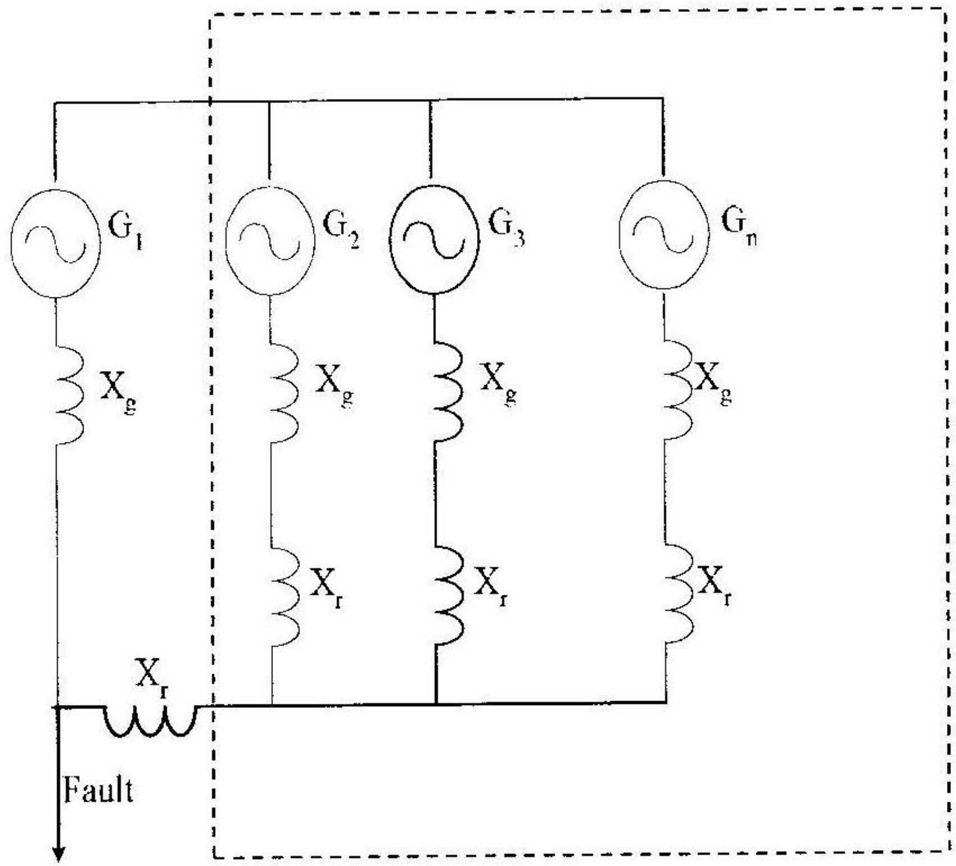


Figure 3.17

$$X_{n-1} = \frac{1}{n-1}(X_g - X_r)$$

X_{n-1} in series with reactance X_r of the faulty bus section

Thevenin reactance X_T

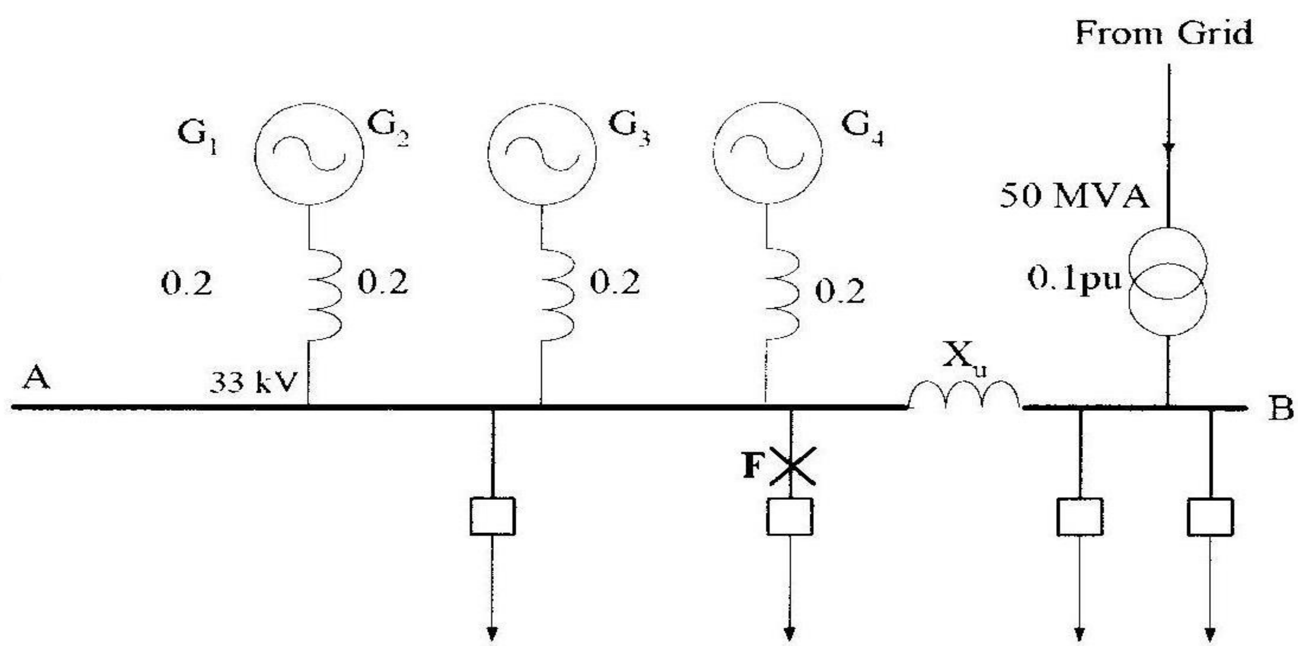
$$X_T = X_g // (X_{n-1} + X_r)$$

Short circuit MVA :

$$MVA_{sc} = \frac{S_{base}}{X_T}$$

Example

The 33 kV busbars of a station are in two sections A and B separated by a reactor. A is fed from four 10 MVA generators each having 0.2 per unit reactance and B is fed from the grid through a 50 MVA transformer of 0.10 per unit reactance. The CBs have each a rupturing capacity of 500 MVA. Find the reactance of the reactor to prevent the CBs being overloaded, if a symmetrical short circuit occurs on an outgoing feeder connected to it.



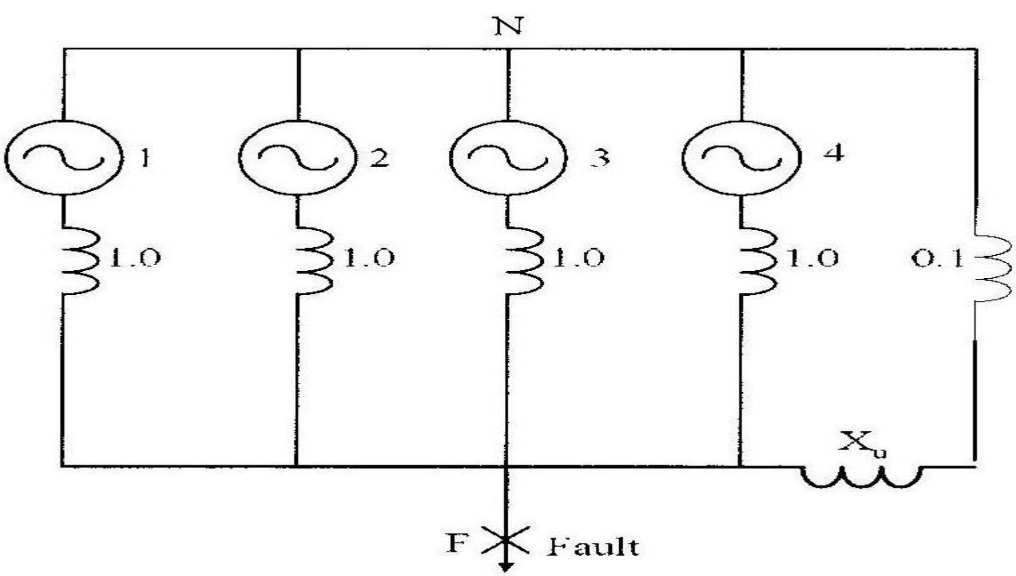
Solution

Taking 50 MVA as the base MVA.

Per unit reactance of each generator on 50 MVA

$$X_{new} = X_{old} \times \frac{S_{bnew}}{S_{bold}} \times \left(\frac{V_{bold}}{V_{bnew}}\right)^2 = 0.2 \times \frac{50}{10} \times \left(\frac{33}{33}\right)^2 = 1.0\, pu$$

Let the per unit reactance of the reactor be X_u on 50 MVA base. For the symmetrical fault at point F, the equivalent single-phase circuit is shown in the Figure 3.19.



The combined per unit reactance for 4 generators

$$= \frac{1.0}{4} = 0.25\ pu$$

Combined per unit reactance of transformer and reactor in series

$= X_u + 0.1$

Thevenin equivalent per unit reactance at the fault point F is

$$X_T = \frac{0.25(X_u + 0.1)}{0.25 + X_u + 0.1} = \frac{0.25(X_u + 0.1)}{X_u + 0.35}$$

Short circuit MVA fed into the fault at F

$$S_{sc} = \frac{S_b}{X_T} = \frac{50(X_u + 0.35)}{0.25(X_u + 0.1)}$$

If the short circuit MVA is not exceed 500 MVA, then

$$\frac{50(X_u + 0.35)}{0.25(X_u + 0.1)} = 500$$

$$X_u + 0.35 = \frac{500 \text{ x} 0.25}{50}(X_u + 0.35)$$

$$2.5X_u - X_u = 0.35 - 0.25$$

$$X_u = \frac{0.10}{1.5} = \frac{1}{15}\ pu$$

Full load current per phase corresponding to 500 MVA is

$$I_{fl} = \frac{S_b}{\sqrt{3}V_l} = \frac{50\text{x}10^6}{\sqrt{3}\text{x}33\text{x}10^3} = 874.8 \text{ A}$$

Voltage to neutral,

$$V_n = \frac{V_l}{\sqrt{3}} = \frac{33000}{\sqrt{3}} = 19052.6V$$

Per unit reactance

$$= \frac{I_{fl} X_{\Omega}}{V_n}$$

$$\frac{1}{15} = \frac{874.8 X_{\Omega}}{19052.6}$$

$$X_{\Omega} = \frac{19052.6}{15 \times 874.8} = 1.452 \ \Omega$$