# SKEE 4443 POWER SYSTEM ANALYSIS 

## CHAPTER 4

Symmetrical Components

## SYMMETRICAL COMPONENTS

- In the previous chapter, attention was confined to the analysis of symmetrical faults i.e. (L-L-L) or (L-L-L-G).

■ When such a fault occurs, it gives rise to symmetrical fault currents i.e. fault currents in the three lines are equal in magnitude and displaced $120^{\circ}$ electrical from one another.

- Although symmetrical faults are the most severe and impose heavy duty on the CB , yet the analysis of such faults can be made with a fair degree of ease.
- It is because the balanced nature of fault permits to consider only one phase in calculations; the conditions in the other two phases being similar.

■ The great majority of faults on the power system are of unsymmetrical nature; the most common type being a s/c from one line to ground.

- When such a fault occurs, it gives rise to unsymmetrical currents i.e. the magnitude of fault currents in the three lines are different having unequal
phase displacement.
- The calculation procedure known as method of symmetrical components is used to determine the currents and voltages on the occurrence of an unsymmetrical fault.
- Those faults on the power system which give rise to unsymmetrical fault currents (i.e. unequal fault currents in the lines with unequal phase displacement) are known as unsymmetrical faults.

■ On the occurrence of an unsymmetrical fault, the currents in the three lines become unequal and so is the phase displacement among them.

■ It may be noted that the term 'unsymmetry' applies only to the fault itself and the resulting line currents.

■ However, the system impedances and the source voltages are always symmetrical through its main elements.

- There are three ways in which unsymmetrical faults may occur in a power
system
i) Single line-to-ground fault (L-G)
ii) Line-to-line fault (L-L)
iii)Double line-to-ground fault (L-L-G)
- The solution of unsymmetrical fault problems can be obtained by either
a) Kirchhoff's laws
b) Symmetrical components method

The latter method is preferred because of the following reasons:
i) It is a simple method and gives more generality to be given to fault performance studies
ii) It provides a useful tool for the protection engineers, particularly in connection with tracing out of fault currents

- A clever way to treat an unbalanced three-phase power system as though it were three balanced power system

■ Invented by C.L Fortescue in 1918
■ Any unsymmetrical set of three-phase voltages or currents could be broken down into three symmetrical sets of balanced three-phased components.(three separate sets of balanced vectors)


Any unsymmetrical set of three-phase voltages or currents could be broken down into three symmetrical sets of balanced three-phased components.


Negative sequence components


Positive sequence components


Three sets of balanced phasors which are the symmetrical components of three unbalanced phasors

## Positive-sequence components

Consisting of three phasors


- $\mathrm{V}_{\mathrm{B} 1}$ lags $\mathrm{V}_{\mathrm{A} 1}$ by $120^{\circ}$ and $\mathrm{V}_{\mathrm{C} 1}$

$$
\begin{aligned}
& \text { lags } V_{\mathrm{B} 1} \text { by } 120^{\circ} \\
& \mathrm{V}_{\mathrm{A} 1}, \mathrm{~V}_{\mathrm{B} 1}=\mathrm{a}^{2} \mathrm{~V}_{\mathrm{A} 1}, \mathrm{~V}_{\mathrm{C} 1}=\mathrm{aV}_{\mathrm{A} 1} \\
& \mathrm{a}=\mathrm{e}^{\mathrm{j} 120^{\circ}} \\
& \mathrm{a}^{2}=\mathrm{e}^{\mathrm{j} 240^{\circ}}=\mathrm{e}^{-j 120^{\circ}}=\mathrm{a}^{*} \\
& \left(\mathrm{a}^{2}\right)^{*}=\mathrm{a} \\
& \mathrm{a}^{3}=1, \quad 1+\mathrm{a}+\mathrm{a}^{2}=0
\end{aligned}
$$

## Negative-sequence components:

- Three phasors


$$
\mathrm{V}_{\mathrm{A} 2}, \mathrm{~V}_{\mathrm{B} 2}=\mathrm{a} \mathrm{~V}_{\mathrm{A} 2}, \mathrm{~V}_{\mathrm{C} 2}=\mathrm{a}^{2} \mathrm{~V}_{\mathrm{A} 2}
$$



## Zero-sequence components:

- A set of three phasors equal in magnitude and phase.

$V_{B 0}$

$$
\mathrm{V}_{\mathrm{A} 0}, \mathrm{~V}_{\mathrm{B} 0}=\mathrm{V}_{\mathrm{A} 0}, \mathrm{~V}_{\mathrm{C} 0}=\mathrm{V}_{\mathrm{A} 0}
$$

$$
\mathrm{V}_{\mathrm{AO}} \longrightarrow
$$

Consider now a set of three voltages (phasors) $\mathrm{V}_{\mathrm{A}}, \mathrm{V}_{\mathrm{B}}$, and $\mathrm{V}_{\mathrm{C}}$ which in general may be unbalanced. The three phasors can be expressed as the sum of positive, negative and zero sequence phasors. Original phasors expressed in terms of their components:




$$
\mathrm{V}_{\mathrm{A}}=\mathrm{V}_{\mathrm{A} 1}+\mathrm{V}_{\mathrm{A} 2}+\mathrm{V}_{\mathrm{A} 0}
$$

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{B}}=\mathrm{V}_{\mathrm{B} 1}+\mathrm{V}_{\mathrm{B} 2}+\mathrm{V}_{\mathrm{B} 0} \\
& \quad=\mathrm{a}^{2} \mathrm{~V}_{\mathrm{A} 1}+\mathrm{aV}_{\mathrm{A} 2}+\mathrm{V}_{\mathrm{A} 0}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{V}_{\mathrm{C}} & =\mathrm{V}_{\mathrm{C} 1}+\mathrm{V}_{\mathrm{C} 2}+\mathrm{V}_{\mathrm{C} 0} \\
& =\mathrm{aV}_{\mathrm{A} 1}+\mathrm{a}^{2} \mathrm{~V}_{\mathrm{A} 2}+\mathrm{V}_{\mathrm{A} 0}
\end{aligned}
$$


Graphical addition of the components to obtain three unbalanced phasors
$V_{B}$

The synthesis of a set of three unbalanced phasors from the three sets of symmetrical ${ }_{10}$ components

## The a constant (operator a)

For simplicity we denote the sequence components $\left(\mathrm{V}_{\mathrm{A} 1}, \mathrm{~V}_{\mathrm{A} 2}\right.$ and $\left.\mathrm{V}_{\mathrm{A} 0}\right)$ as $\left(\mathrm{V}_{1}, \mathrm{~V}_{2}\right.$ and $\mathrm{V}_{0}$ )
$\mathrm{V}_{\mathrm{A}}, \mathrm{V}_{\mathrm{B}}$ and $\mathrm{V}_{\mathrm{C}}$ are defined by the following transformation:

$$
\left[\begin{array}{l}
V_{A} \\
V_{B} \\
V_{C}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a^{2} & a \\
1 & a & a^{2}
\end{array}\right]\left[\begin{array}{c}
V_{0} \\
V_{1} \\
V_{2}
\end{array}\right] \quad \text { With constant a represents }
$$

Phasor diagram showing relationships among the various powers of a

$$
\begin{aligned}
& a=1 \angle 120^{\circ} \\
& a^{2}=1 \angle 240^{\circ} \\
& a^{3}=1 \angle 360^{\circ}=1 \angle 0^{\circ}=1
\end{aligned}
$$

$$
\mathrm{V}_{\mathrm{B} 2}=\mathrm{a} \mathrm{~V}_{2}
$$

$$
\begin{aligned}
& V_{A 1} \\
& =V_{1}
\end{aligned}
$$



$$
\begin{aligned}
& v_{C 0} \pi / \\
& V_{B 0} V_{A 0}=V_{0}
\end{aligned}
$$

$$
\begin{aligned}
& V_{B 1} \\
& =a^{2} V_{1}
\end{aligned}
$$

$$
\begin{gathered}
V_{\mathrm{C}}=\mathrm{V}_{\mathrm{C} 1}+\mathrm{V}_{\mathrm{C} 2}+\mathrm{V}_{\mathrm{C} 0} \\
\mathrm{~V}_{\mathrm{C}}=\mathrm{aV}_{1}+\mathrm{a}^{2} \mathrm{~V}_{2}+\mathrm{V}_{0} \\
\mathrm{~V}_{\mathrm{B}}=\mathrm{V}_{\mathrm{B} 1}+\mathrm{V}_{\mathrm{B} 2}+\mathrm{V}_{\mathrm{B} 0} \\
\mathrm{~V}_{\mathrm{B}}=\mathrm{a}^{2} \mathrm{~V}_{1}+\mathrm{a} \mathrm{~V}_{2}+\mathrm{V}_{0}
\end{gathered}
$$

$$
\mathrm{V}_{\mathrm{A}}=\mathrm{V}_{\mathrm{A} 1}+\mathrm{V}_{\mathrm{A} 2}+\mathrm{V}_{\mathrm{A} 0}
$$

$$
V_{B}
$$

$$
\left.\left[\begin{array}{c}
V_{A} \\
V_{B} \\
V_{C}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a^{2} & a \\
1 & a & a^{2}
\end{array}\right] \underset{\uparrow}{[ }\right]\left[\begin{array}{c}
V_{0} \\
V_{1} \\
V_{2}
\end{array}\right]
$$

Vector of original phasors

$$
\left[\begin{array}{l}
V_{A} \\
V_{B} \\
V_{C}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a^{2} & a \\
1 & a & a^{2}
\end{array}\right]\left[\begin{array}{l}
V_{0} \\
V_{1} \\
V_{2}
\end{array}\right] \quad\left\lfloor\quad\left\lfloor\begin{array}{l}
V_{A} \\
V_{B} \\
V_{C}
\end{array}\right\rfloor=A\left\lfloor\begin{array}{l}
V_{0} \\
V_{1} \\
V_{2}
\end{array}\right\rfloor\right.
$$

The symmetrical components of the unbalanced three-phase voltage can be expressed as
$\left\lfloor\begin{array}{l}V_{0} \\ V_{1} \\ V_{2}\end{array}\right\rfloor=A^{-1}\left\lfloor\begin{array}{l}V_{A} \\ V_{B} \\ V_{C}\end{array}\right\rfloor$
$A^{-1}=\left[\begin{array}{ccc}1 & 1 & 1 \\ 1 & a^{2} & a \\ 1 & a & a^{2}\end{array}\right]^{-1}=\frac{1}{3}\left[\begin{array}{ccc}1 & 1 & 1 \\ 1 & a & a^{2} \\ 1 & a^{2} & a\end{array}\right] \quad\left[\begin{array}{l}V_{0} \\ V_{1} \\ V_{2}\end{array}\right]=\frac{1}{3}\left[\begin{array}{ccc}1 & 1 & 1 \\ 1 & a & a^{2} \\ 1 & a^{2} & a\end{array}\right]\left[\begin{array}{c}V_{A} \\ V_{B} \\ V_{C}\end{array}\right]$
Zero-sequence components are never present in the line voltages while voltages to neutral may contain zero-sequence components

Symmetrical components of an unbalanced three-phase current

$$
\begin{aligned}
& I_{C}=I_{C 1}+I_{C 2}+I_{C 0} \quad r_{C} \quad I_{A}=I_{A 1}+I_{A 2}+I_{A 0} \\
& I_{C}=\mathrm{al}_{1}+\mathrm{a}^{2} \mathrm{I}_{2}+\mathrm{I}_{0} \underset{\mathrm{I}_{\mathrm{B}}}{ } \quad \mathrm{I}_{\mathrm{A}}=\mathrm{I}_{1}+\mathrm{I}_{2}+\mathrm{I}_{0} \\
& \begin{array}{l}
\left\|_{\mathrm{B}}=\right\|_{\mathrm{B} 1}+\left\|_{\mathrm{B} 2}+\right\|_{\mathrm{B} 0} \\
\left\|_{\mathrm{B}}=\mathrm{a}^{2}\right\|_{1}+\mathrm{al}_{2}+\|_{0}
\end{array} \quad\left[\begin{array}{c}
I_{A} \\
I_{B} \\
I_{C}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a^{2} & a \\
1 & a & a^{2}
\end{array}\right]\left[\begin{array}{c}
I_{0} \\
I_{1} \\
I_{2}
\end{array}\right] \\
& {\left[\begin{array}{l}
I_{0} \\
I_{1} \\
I_{2}
\end{array}\right]=\frac{1}{3}\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a & a^{2} \\
1 & a^{2} & a
\end{array}\right]\left[\begin{array}{c}
I_{A} \\
I_{B} \\
I_{C}
\end{array}\right] \quad I_{\mathrm{n}}=3 I_{0}}
\end{aligned}
$$

## Example

Currents in an ungrounded Y-connected three-phase load are: $\mathrm{I}_{\mathrm{A}}=10 \angle 0^{\circ} \mathrm{A}$;
$\mathrm{I}_{\mathrm{B}}=10 \angle-180^{\circ} \mathrm{A} ; \mathrm{I}_{\mathrm{C}}=0 \angle 0^{\circ} \mathrm{A}$
Determine the symmetrical components of current in each phase in the load


$$
I_{C}=0 \angle 0^{\circ} \mathrm{A}
$$

$$
\left[\begin{array}{l}
I_{0} \\
I_{1} \\
I_{2}
\end{array}\right]=\frac{1}{3}\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a & a^{2} \\
1 & a^{2} & a
\end{array}\right]\left[\begin{array}{l}
I_{A} \\
I_{B} \\
I_{C}
\end{array}\right]
$$

The zero sequence current for phase a of the load:

$$
\begin{aligned}
I_{0} & =\frac{1}{3}\left(I_{A}+I_{B}+I_{C}\right) \\
& =\frac{1}{3}(10 \angle 0+10 \angle-180+0 \angle 0) \\
& =0
\end{aligned}
$$

The positive sequence current for phase a of the load:


$$
\begin{aligned}
I_{1} & =\frac{1}{3}\left(I_{A}+a I_{B}+a^{2} I_{C}\right) \\
& =\frac{1}{3}\left(10 \angle 0+a 10 \angle 180+a^{2} 0 \angle 0\right) \\
& =\frac{1}{3}(10 \angle 0+10 \angle-60+0) \\
& =5.77 \angle-30 A
\end{aligned}
$$

The negative sequence current for phase a of the load:

$$
\begin{aligned}
I_{2} & =\frac{1}{3}\left(I_{A}+a^{2} I_{B}+a I_{C}\right) \\
& =\frac{1}{3}\left(10 \angle 0+a^{2} 10 \angle-180+a 0 \angle 0\right) \\
& =\frac{1}{3}(10 \angle 0+10 \angle 60+0) \\
& =5.77 \angle 30 A
\end{aligned}
$$

The zero, positive and negative sequence currents for phase a of the load:


The zero, positive and negative sequence current for phase $\mathbf{c}$ of the load:

$$
\begin{aligned}
I_{c 0} & =I_{a o}=0 \\
I_{c 1} & =a I_{a 1}=1 \angle 120 \times 5.77 \angle-30=5.77 \angle 90 \\
I_{c 2} & =a^{2} I_{a 2}=1 \angle 240 \times 5.77 \angle 30=5.77 \angle-90
\end{aligned}
$$

## Exercise

■ The line-to-line voltages across a three-phase, wye-connected load consisting of $Z=100 \angle 30^{\circ} \Omega$ in each phase are:
$\mathrm{V}_{\mathrm{ab}}=205 \angle 0^{\circ}$
$\mathrm{V}_{\mathrm{bc}}=250 \angle-125^{\circ}$
$\mathrm{V}_{\mathrm{ca}}=214 \angle 107^{\circ}$
Determine the sequence components of the voltages.

- The sequence components of the current in phase a are:
$\mathrm{I}_{1}=6 \angle-15^{\circ} \mathrm{pu}$
$\mathrm{I}_{2}=8 \angle 225^{\circ} \mathrm{pu}$
$\mathrm{I}_{0}=5 \angle-165^{\circ}$
Determine the phase currents $\mathrm{I}_{\mathrm{a}}, \mathrm{I}_{\mathrm{b}}$ and $\mathrm{I}_{\mathrm{c}}$


## SEQUENCE IMPEDANCE

- The +ve and -ve sequence impedances of linear, symmetrical, static circuits are identical because the impedance is independent of phase order provided the applied voltages are balanced.
- The impedance of a transmission line to zero-sequence currents differs from the impedance to positive and negative sequence currents.

■ The impedances of rotating machines to currents of three sequences will generally be different for each sequence.

- The mmf produced by the -ve seq. armature current rotates in the direction opposite to that of the rotor on which is the dc field winding.
■ Unlike the flux produced by the +ve seq current, which is stationary with respect to the rotor, the flux produced by the -ve seq current is sweeping rapidly over the face of the rotor.
- The current induced in the field and the damper windings by the rotating armature flux keep the flux from penetrating the rotor.i.e.similar to the rapidly changing flux immediately upon the occurrence of a $\mathrm{s} / \mathrm{c}$ at the terminals of $a_{1}$ machine.


## SEQUENCE IMPEDANCE

■ Power system element (transmission line, transformers \& synchronous machines) have a three-phase symmetry because of which when currents of a particular sequence are passed through these elements, voltage drops of the same sequence appear, i.e. the elements posses only self impedances to sequence currents.

■ Each element can therefore be represented by three decoupled sequence networks (on single-phase basis) pertaining to +ve , -ve and zero sequences, respectively.

■ EMFs are involved only in a positive sequence network of synchronous machines.

■ For finding a particular sequence impedance, the element in question is subjected to currents and voltages of that sequence only.

- With the knowledge of sequence networks of elements, complete $+\mathrm{ve},-\mathrm{ve}$,
and zero sequence networks of any power system can be assembled.
- The voltage drop caused by the currents of a particular sequence depends on the impedance of the circuit to currents of that sequence
- Impedance of a circuit can differ for positive sequence, negative sequence and zero sequence currents
$Z_{\text {positive sequence }}\left(Z_{1}\right)$
$Z_{\text {negative sequence }}\left(Z_{2}\right)$
$Z_{\text {zero sequence }}\left(Z_{0}\right)$
- Impedance of a circuit when only positive-sequence current is flowing is known as the positive-sequence impedance of the circuit.
- Impedance of the circuit when only negative-sequence current is flowing is known as the negative-sequence impedance of the circuit.
- Impedance of the circuit when only zero sequence current is flowing is called the zero-sequence impedance

■ To analyzed an unsymmetrical fault we must construct three different perphase equivalent circuits, (one for each type of symmetrical component).

■ Positive-sequence network : per-phase equivalent circuit containing only positive sequence impedances and sources.

■ Negative-sequence network : per-phase equivalent circuit containing only negative sequence impedances.

■ Zero-sequence network: per-phase equivalent circuit containing only zerosequence impedances.

## Sequence impedances and networks of synchronous machine

Fig. below shows an unloaded (generator or motor) grounded through a reactor (impedance $\mathrm{Z}_{\mathrm{N}}$ ). $\mathrm{E}_{\mathrm{A}}, \mathrm{E}_{\mathrm{B}}$ and $\mathrm{E}_{\mathrm{C}}$ are the induced emfs of the three phases.

When a fault takes place at the machine terminals, currents $\mathrm{I}_{\mathrm{A}}, \mathrm{I}_{\mathrm{B}}$ and $\mathrm{I}_{\mathrm{C}}$ flow in the lines. Whenever the fault involves ground, current $I_{N}=I_{A}+I_{B}+I_{C}$ flows
to neutral from ground via $\mathrm{Z}_{\mathrm{N}}$. Unbalanced line currents can be resolved into their symmetrical components $\mathrm{I}_{0}, \mathrm{I}_{1}$ and $\mathrm{I}_{2}$.

Before proceed with fault analysis, we should know the equivalent circuits presented by the machine to the flow of +ve , -ve , and zero sequence currents, respectively.

Because of winding symmetry currents of a particular sequence produce voltage drops of that sequence only.

## Positive sequence impedance and network

Since a synchronous machine is designed with symmetrical windings, it induces emfs of + ve sequence only. i.e. no -ve or zero sequence voltages are induced in it.

When the machine carries +ve sequence currents only, this mode of operation is the balanced mode. The armature reaction field (interaction of armature mmf and the field mmf) caused by +ve sequence currents rotates at synchronous speed in the same direction as the rotor i.e. it is stationary with respect to field excitation.

The machine equivalently offers a direct axis reactance whose value reduces from $\mathrm{X}^{\prime \prime}{ }_{\mathrm{d}}$ to $\mathrm{X}^{\prime}{ }_{\mathrm{d}}$ and finally to $\mathrm{X}_{\mathrm{d}}$, as $\mathrm{s} / \mathrm{c}$ transient progresses in time. If armature resistance is assumed negligible, the +ve sequence impedance of the machine is $\mathrm{Z}_{1}=\mathrm{jX}{ }^{\prime \prime}{ }_{\mathrm{d}}$ (if 1 cycle transient is of interest), $\mathrm{jX}^{\prime}{ }_{\mathrm{d}}$ (if 3-4 cycle transient is of interest), $\mathrm{j}_{\mathrm{d}}$ (if steady state value is of interest).

If the machine $\mathrm{s} / \mathrm{c}$ takes place from unloaded conditions, the terminal voltage constitutes the +ve sequence voltage, on the other hand, if the $\mathrm{s} / \mathrm{c}$ occurs from loaded conditions, the voltage behind appropriate reactance constitutes the +ve sequence voltage.

## Negative sequence impedance and network

Synchronous machine has zero -ve seq. induced voltages. With the flow of negative sequence currents in the stator a rotating field is created which rotates in the opposite direction to that of the +ve sequence field and, therefore, at double syn. speed with respect to rotor. Currents at double the stator frequency are
therefore induced in rotor field and damper winding. In sweeping over the rotor surface, the -ve sequence mmf is alternately presented with reluctances of direct and quadrature axes. The -ve seq. impedance presented by the machine with consideration given to the damper windings, is defined as;
$\mathrm{Z}_{2}=\mathrm{j}\left(\mathrm{X}{ }_{\mathrm{q}}+\mathrm{X}{ }_{\mathrm{d}}{ }_{\mathrm{d}}\right) / 2 ;\left|\mathrm{Z}_{2}\right|<\left|\mathrm{Z}_{1}\right|$

## Zero sequence impedance and network

Again there is no zero sequence voltages are induced in syn. machine.
The flow of zero sequence currents creates three mmfs which are in time phase but are distributed in space phase by $120^{\circ}$.

The resultant air gap field caused by zero seq. currents is therefore zero.
Hence, the rotor windings present leakage reactance only to the flow of zero seq.
currents ( $\mathrm{Z}_{0 \mathrm{~g}}<\mathrm{Z}_{2}<\mathrm{Z}_{1}$ )
The current flowing in the impedance $\mathrm{Z}_{\mathrm{N}}$ between neutral and ground is $\mathrm{I}_{\mathrm{N}}=3 \mathrm{I}_{0}$

## Sequence impedances of transmission lines

A fully transposed 3phase line is completely symmetrical and therefore the per phase impedance offered by it is independent of the phase sequence of a balanced set of currents. In other words, the impedance offered by it to +ve and -ve seq. currents are identical. When only zero sequence currents flow in a transmission line, the currents in each phase are identical in both magnitude and phase angle.

Part of these currents return via the ground, while the rest return through the OHL ground wires. The ground wires being grounded at several towers, the return currents in the ground wires are not necessarily uniform along the entire length.

The flow of zero seq. currents through transmission lines, ground wires and ground creates a magnetic field pattern which is very different from that caused by the flow of +ve or - ve seq. currents where the currents have a phase difference of $120^{\circ}$ and the return current is zero. The zero sequence impedance of a transmission line also accounts for the ground impedance. Since the ground
impedance heavily depends on soil conditions, it is essential to make some simplifying assumptions to obtain analytical results. The zero sequence impedance of transmission lines usually ranges from 2 to 3.5 times the + ve seq. impedance.

## Sequence impedances and networks of transformers

The positive sequence series impedance of a transformer equals its leakage impedance. The leakage impedance does not change with alteration of phase sequence of balanced applied voltages. The transformer negative seq. impedance is also therefore equal to its leakage reactance. $\mathrm{Z}_{1}=\mathrm{Z}_{2}=\mathrm{Z}_{\text {leakage }}$

Assuming such transformer connections that zero sequence currents can flow on both sides, a transformer offers a zero sequence impedance which may differ slightly from the corresponding positive and negative sequence values.

It is normal practice to assume that the series impedances of all sequences are equal regardless of the type of transformer.

The object of obtaining the seq. impedances is to construct the seq. networks for the complete system.

The network of a particular seq. shows all the paths for the flow of current of that seq. in the system.

The transition from a + ve seq. network to -ve seq. network is simple.
Three-phase syn. Gens and motors have internal voltages of +ve seq. only, since they are designed to generate balances voltages.

Since the + ve and -ve seq. impedances are the same in a static symmetrical system, conversion of +ve to -ve seq. network is done by changing the impedances that represent the rotating machinery and by omitting the emf.

Emf are omitted on the assumption of balanced generated voltages and the absence of -ve-seq. voltages induced from the outside sources.

Since all the neutral points of a symmetrical three-phase system are at the same potential when balanced three-phase currents are flowing, all the neutral points must be at the same potential for either +ve or -ve seq. currents.

Therefore the neutral of a symmetrical three-phase system is the logical reference potential for specifying + ve and $-v e$ seq. voltage drops and is the reference bus of the + ve and - ve seq. networks.

Impedance connected between the neutral of a machine and ground is not a part of either the + ve or - ve seq. network neither + ve nor - ve seq. current can flow in an impedance so connected.

## Zero sequence networks of transformers

Before considering the zero sequence networks of various types of transformer connections, three observations are made.
i. When magnetizing current is neglected, transformer primary would carry current only if there is current flow on the secondary side.
ii. Zero sequence currents can flow in the legs of a star connection only if the star point is grounded which provides the necessary return path for zero sequence currents.
iii. No zero sequence currents can flow in the lines connected to a delta connection as no return path is available for these currents.

Zero sequence currents can, however, flow in the legs of a delta -such currents are caused by the presence of zero sequence voltages in the delta connection.

Case 1: Y-Y transformer bank with any one neutral grounded
If any one of the two neutrals of a Y-Y transformer is ungrounded, zero sequence currents cannot flow in the ungrounded star and consequently, these cannot flow in the grounded star. Hence, an open circuit exists in the zero sequence network between P and Q , i.e. between the two parts of the system connected by the transformer.

Case 2: Y-Y transformer bank both neutrals grounded
When both the neutrals of a Y-Y transformer are grounded, a path through the transformer exists for zero seq. currents in both windings via the two grounded neutrals. Hence, in the zero sequence network P and Q are connected by the zero sequence impedance of the transformer.

Case 3: $\mathrm{Y}-\Delta$ transformer bank with grounded Y neutral
If the neutral of star side is grounded, zero sequence currents can flow in star because a path is available to ground and the balancing zero sequence currents
can flow in delta. Of course no zero sequence currents can flow in the line on the delta side.

## Zero sequence networks of transformers

The zero sequence network must therefore have a path from the line P on the star side through the zero sequence impedance of the transformer to the reference bus, while an open circuit must exist on the line Q side of delta. If the star neutral is grounded through $\mathrm{Z}_{\mathrm{N}}$, an impedance $3 \mathrm{Z}_{\mathrm{N}}$ appears in series with $\mathrm{Z}_{0}$ in the sequence network.

Case 4: Y- $\Delta$ transformer bank with ungrounded star. This is the special case of Case 3 where the neutral is grounded through $Z_{N}=\infty$. Therefore no zero sequence current can flow in the transformer windings.

Case 5: $\Delta-\Delta$ transformer bank. Since a delta circuit provides no return path, the zero sequence currents cannot flow in or out of $\Delta-\Delta$ transformer; however, it can circulate in the delta windings.

Therefore, there is an open circuit between P and Q and $\mathrm{Z}_{0}$ is connected to the reference bus on both ends to account for any circulating zero sequence current in the two deltas.

Positive, negative and zero-sequence equivalent circuit for generators


Y - connected three-phase generator grounded through an inductive reactance $\mathrm{Z}_{\mathrm{N}}$


(b) Positive sequence network
(a) A synchronous generator as seen by positive-sequence currents

Fig. shows the $3 \emptyset+$ ve seq. net model of syn. machine. $Z_{N}$ does not appear in the model as $\mathrm{I}_{\mathrm{N}}=0$ for + ve sequence currents.

Since it is a balanced net. it can be represented by the single phase network model (b). The ref. bus is at neutral potential (no current flows from ground to neutral, (the neutral is at ground potential). The + ve seq. voltage:

$$
V_{1}=E_{A}-I_{1} Z_{1}
$$




Negative sequence network

A synchronous generator as seen by negative-sequence currents
Fig. shows the $3 \varnothing$ and $1 \varnothing$-ve seq. net model of syn. machine. The reference bus is of course at neutral potential which is the same as ground potential.
The negative sequence voltage is $\quad V_{2}=-I_{2} Z_{2}$


A synchronous generator as seen by zerosequence currents. The zero seq. voltage

Zero sequence network

$$
\begin{aligned}
& V_{0}=-3 Z_{N} I_{0}-Z_{g o} I_{0} \\
&=-\left(3 Z_{N}+Z_{g o}\right) I_{0} \\
& Z_{0}=3 Z_{N}+Z_{g o} \quad V_{0}
\end{aligned}=-I_{0} Z_{0}
$$

zero-sequence equivalent circuit for grounded generator



$$
\begin{aligned}
Z_{0} & =Z_{g 0} \\
V_{0} & =-I_{0} Z_{0}
\end{aligned}
$$

zero-sequence equivalent circuit for ungrounded generator



Grounded Y connected load


Grounded Y connected load


Delta connected load


Case 2




Case 5

$$
P-3 \varepsilon Q
$$



| Device | Capacity (MVA) | Voltage (kV) | $\begin{aligned} & X^{\prime \prime} \\ & \text { (pu) } \end{aligned}$ | $X^{\prime}$ <br> (pu) | $\mathrm{X}_{1}$ (pu) | $X_{2}$ (pu) | $X_{0}$ (pu) | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Generator $\mathbf{G}_{1}$ | 250 | 13.8 | 0.18 | 0.4 |  | 0.15 | 0.08 | Grounded through an impedance $Z_{n 1}=j 0.2 \Omega$ |
| Generator $\mathbf{G}_{2}$ | 500 | 20.0 | 0.15 | 0.35 |  | 0.15 | 0.05 | Grounded through an impedance $Z_{n 1}=j 0.2 \Omega$ |
| Generator $\mathbf{G}_{3}$ | 250 | 13.8 | 0.20 | 0.40 |  | 0.20 | 0.15 | Grounded through an impedance $Z_{n 1}=j 0.25 \Omega$ |
| Transform er $\mathrm{T}_{1}$ | 250 | 13.8 $/ 240 Y$ |  |  | 0.1 | 0.1 | 0.1 |  |
| Transform er $\mathrm{T}_{2}$ | 500 | 20.04/240Y |  |  | 0.08 | 0.08 | 0.08 |  |
| Transform er T3 | 250 | 13.8 $/ 240 Y$ |  |  | 0.1 | 0.1 | 0.1 |  |
| Each line |  |  |  |  | $40 \Omega$ | $40 \Omega$ | $80 \Omega$ |  |

Assume that the power system is initially unloaded, that the voltage at bus 4 is 250 kV , and all resistances may be neglected.
Convert the power system to per unit on a base of 500 MVA at 20 kV at generator $\mathrm{G}_{2}$.
Create the positive, negative and zero-phase sequence diagrams.


Line 3

Draw positive, negative and zero sequence network


|  | MVA | kV | Xs | X1 | X' | X" | X2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G1 | 100 | 13.8 | 0.9 |  | 0.2 | 0.1 | 0.1 |
| T1 | 100 | $13.8 / 120$ |  | 0.05 |  |  | 0.05 |
| T2 | 100 | $120 / 13.8$ |  | 0.05 |  |  | 0.05 |
| G2 | 100 | 13.8 |  | 1.1 | 0.3 | 0.18 | 0.05 |
| Line |  |  |  | $75 \Omega$ |  |  | $75 \Omega$ |

Draw positive sequence network


Draw negative sequence network


Draw zero sequence network


Develop positive, negative and zero sequence network for the system

|  | X1 | X2 | X0 |
| :--- | :--- | :--- | :--- |
| G1 | $\mathrm{X}_{\mathrm{g} 11}$ | $\mathrm{X}_{\mathrm{g} 12}$ | $\mathrm{X}_{\mathrm{g} 10}$ |
| T1 | $\mathrm{X}_{\mathrm{t} 11}$ | $\mathrm{X}_{\mathrm{t} 12}$ | $\mathrm{X}_{\mathrm{t} 10}$ |
| T2 | $\mathrm{X}_{\mathrm{t} 21}$ | $\mathrm{X}_{\mathrm{t} 22}$ | $\mathrm{X}_{\mathrm{t} 20}$ |
| G2 | $\mathrm{X}_{\mathrm{g} 21}$ | $\mathrm{X}_{\mathrm{g} 22}$ | $\mathrm{X}_{\mathrm{g} 20}$ |
| Line1 | $\mathrm{X}_{\mathrm{L} 11}$ | $\mathrm{X}_{\mathrm{L} 12}$ | $\mathrm{X}_{\mathrm{L} 10}$ |
| Line 2 | $\mathrm{X}_{\mathrm{L} 21}$ | $\mathrm{X}_{\mathrm{L} 22}$ | $\mathrm{X}_{\mathrm{L} 20}$ |
| Line 3 | $\mathrm{X}_{\mathrm{L} 31}$ | $\mathrm{X}_{\mathrm{L} 32}$ | $\mathrm{X}_{\mathrm{L} 30}$ |
| T3 | $\mathrm{X}_{\mathrm{t} 11}$ | $\mathrm{X}_{\mathrm{t} 32}$ | $\mathrm{X}_{\mathrm{t} 30}$ |
| T4 | $\mathrm{X}_{\mathrm{t} 41}$ | $\mathrm{X}_{\mathrm{t} 42}$ | $\mathrm{X}_{\mathrm{t} 40}$ |
| T5 | $\mathrm{X}_{\mathrm{t} 51}$ | $\mathrm{X}_{\mathrm{t} 52}$ | $\mathrm{X}_{\mathrm{t} 50}$ |
| T6 | $\mathrm{X}_{\mathrm{t} 61}$ | $\mathrm{X}_{\mathrm{t} 62}$ | $\mathrm{X}_{\mathrm{t} 60}$ |

Positive sequence network


Negative sequence network


Zero sequence network


Sketch sequence networks for the given system


Find the subtransient fault current at the fault for a symmetrical three-phase fault at the location indicated by $\mathbf{F}$


|  | MVA | kV | Xs/X1 | X' | X" | X2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| X0 |  |  |  |  |  |  |
| G1 | 100 | 13.8 | 0.9 | 0.2 | 0.1 | 0.1 |
| T1 | 100 | $13.8 / 120$ | 0.05 |  |  | 0.05 |
| T2 | 100 | $120 / 13.8$ | 0.05 |  |  | 0.05 |
| G2 | 100 | 13.8 | 1.1 | 0.3 | 0.18 | 0.15 |
| Line |  |  | $75 \Omega$ |  |  | 0.05 |

Positive sequence network during subtransient



Positive sequence network


$$
\begin{aligned}
Z_{\text {thevenin }} \text { at point of fault } & =? \\
& =\mathrm{j} 0.1713
\end{aligned}
$$

subtransient fault current at the fault for a symmetrical three-phase fault at the location indicated by $\mathbf{F}$


$$
\begin{aligned}
I_{s / c} & =\frac{1 \angle 0}{Z_{1}} \\
& =\frac{1 \angle 0}{j 0.1713} \\
& =5.8381 \quad \text { p.u }
\end{aligned}
$$

$$
V_{1}=1 \angle 0-I_{1} Z_{1}
$$

I fault in ampere?

Develop positive, negative and zero sequence network for the system when fault is located at the middle of transmission line (F).

Find the subtransient fault current at the fault for a symmetrical three-phase fault at the location indicated by $\mathbf{F}$ (middle of line).


|  | MVA | kV | Xs | X1 | X' | X" | X2 | X0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| G1 | 100 | 13.8 | 0.9 |  | 0.2 | 0.1 | 0.1 | 0.05 |
| T1 | 100 | $13.8 / 120$ |  | 0.05 |  |  | 0.05 | 0.05 |
| T2 | 100 | $120 / 13.8$ |  | 0.05 |  |  | 0.05 | 0.05 |
| G2 | 100 | 13.8 |  | 1.1 | 0.3 | 0.18 | 0.15 | 0.10 |
| Line |  |  |  | $75 \Omega$ |  |  | $75 \Omega$ | $125 \Omega$ |

Positive sequence network


Positive sequence network during


Negative sequence network


Zero sequence network


Fault current at the fault for a symmetrical three-phase fault at the location indicated by $\mathbf{F}$


$$
V_{1}=1 \angle 0-I_{1} Z_{1}
$$

$$
\begin{aligned}
\mathrm{E}_{1} & =1 \angle 0 \\
I_{s / c} & =\frac{1 \angle 0}{Z_{1}} \\
& =\frac{1 \angle 0}{j 0.2235} \\
& =4.4743 \quad \text { p.u }
\end{aligned}
$$

I fault in ampere?

$$
2
$$

