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Course Text:

1. Abd Wahid Md Raji *et al*. (2013), *The First Course of Calculus for Science and Engineering Students*. Penerbit UTM. (ISBN: 978–983–52–0862–1).

Supplementary Texts:

1. Fallow, Stanley J, Hagg, Gary M. (1990). *Calculus and its applications*. New York: McGraw–Hill (QA 303 F38 1990).
2. Mizrahi, Abe, Sullivan, Michael. (1986). *Calculus and analytical geometry*. California: Wadsworth Pub. (QA 303 M59 1986).
3. Varberg, Dale E, Fleming Walter. (1991). *Calculus with applications*. Eaglewood Cliffs, N.J.: Prentice Hall, 1991 (QA 303 V37 1991).

Assessment

NO.	TYPE OF ASSESSMENT	NO OF ASSESSMENT	% EACH	% TOTAL	DATE/ TIME / VENUE
1.	Quizes	2 (15 min each)	5	10	
1.	Test 1	1 (1 hr 15 min)	15	15	Date: **tbc** Day: **tbc** Time: **During class** Venue: Will inform later
2.	Test 2	1 (1 hr 15 min)	20	20	Date: **tbc** Day: **tbc** Time: **During class** Venue: Will inform later
3.	Assignments	1	5	5	
5.	Final	1 (3 h)	50	50	

Course Synopsis

This course provides a solid foundation of basic calculus prior to pursuance of any mathematics at university level. It comprises of various topic such as Limits and continuity of functions, Differentiations, Integrations, Differential Equations and Numerical Methods. The intention is to equip students with the necessary tools required for further mathematics and engineering courses.

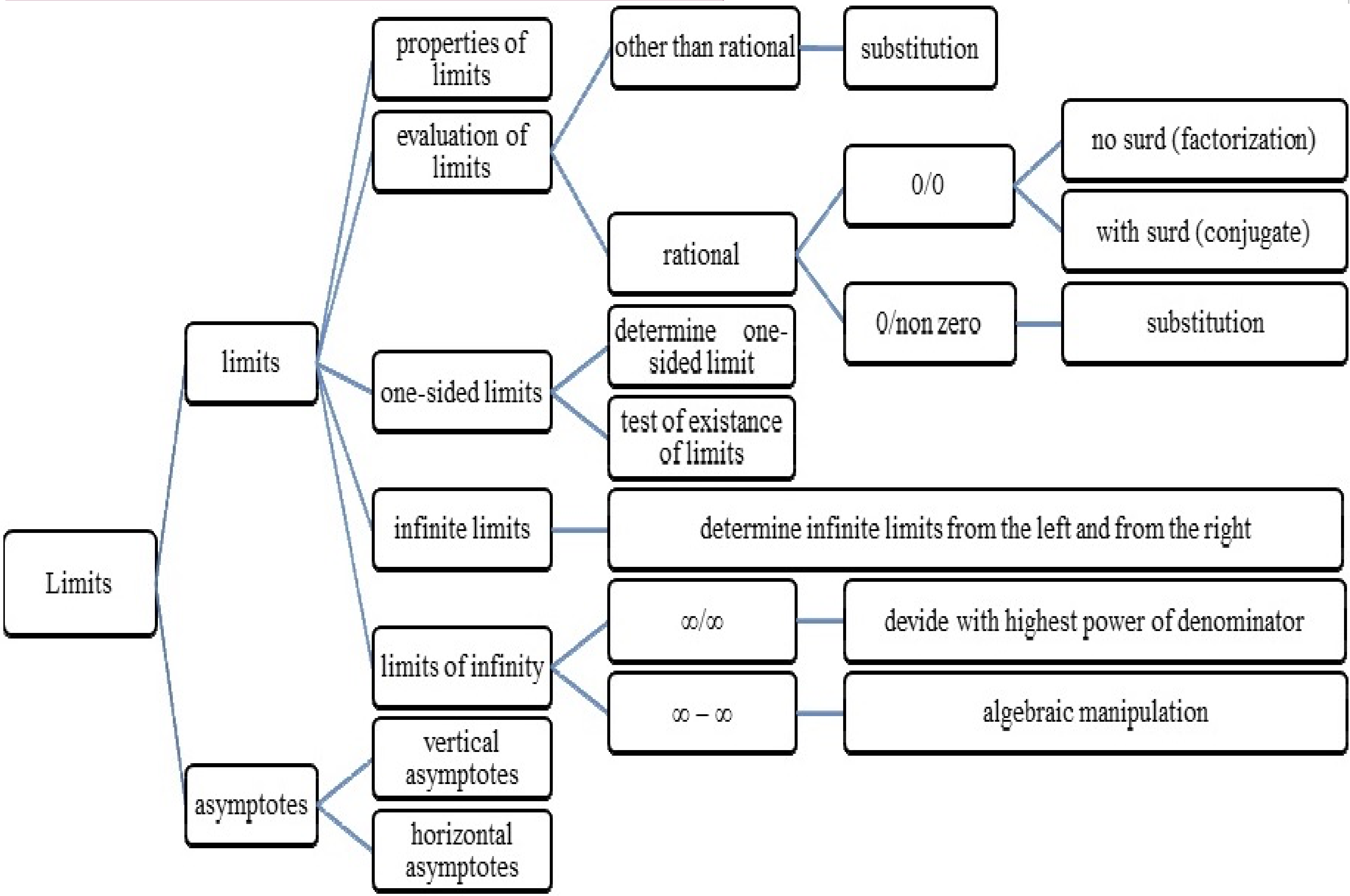
Weekly Schedule

Week	Contents	Notes
Week 1	***Limits***: *Left and right limits. Limits at infinity (polynomial and rational functions). Computational of limits by using factorisation, by rationalization (not including trigonometry).*	
Week 2	*Continuity of functions at a point and on an interval.*	
Week 3	***Differentiation***: *using first principle. Derivatives of algebraic functions & exponential functions.*	
Week 4	*Derivatives of trigonometric functions, and reciprocal of trigonometric functions such as sec x, cot x, cosec x.*	
Week 5	*Rules of differentiation: Product Rule, Quotient Rule, Chain Rule, Implicit differentiation and Parametric differentiations.*	
Week 6	*Higher order differentiations. Errors and approximations. Rate of change.*	
Week 7	*Decreasing and increasing functions. Relative maximum and minimum points by using first derivative tests.*	
Week 8	*Points of inflection. Graph sketching using the critical points, vertical and horizontal asymptotes.*	
	Mid Semester Break	

Weekly Schedule

Week	Contents	Notes
Week 9	***Integrations:*** *Anti-derivatives, the indefinite integral. Standard integrals. Integration techniques using substitutions.*	
Week 10	*Integration techniques using substitutions including trigonometric substitution. Integration by parts including tabular form.*	
Week 11	*Integration by partial fractions. Integration of exponential, logarithmic and trigonometric functions.*	
Week 12	*Area bounded by two curves. Volume of revolution about x-axis and about y-axis. Volume bounded by two curves.*	
Week 13	***Ordinary Differential Equation:*** *Formations of differential equations. Solving first order differential equation: Separable equations.*	
Week 14	*Solving first order differential equation: Linear Equations.*	
Week 15	***Non-linear equations:*** *Fixed point method and Newton Raphson Method.*	
Week 16	*Numerical integration: trapezium rule and Simpson rule.*	

LIMITS


Limits
limits
asymptotes
properties of limits
evaluation of limits
one-sided limits
infinite limits
limits of infinity
vertical asymptotes
horizontal asymptotes
other than rational
substitution
rational
0/0
no surd (factorization)
with surd (conjugate)
0/non zero
substitution
determine one-sided limit
test of existance of limits
determine infinite limits from the left and from the right
∞/∞
devide with highest power of denominator
∞ – ∞
algebraic manipulation

Limits are used to explain changes that arise for a particular function when the value of an independent variable approaches a certain value.

$$\lim_{x \to x_0} f(x) = f(x_0) = L$$

LIMITS

Right Limit

If the value of f(x) approaches to a number L_1 as x approaches x_0 from the right, then we right

$$\lim_{x \to x_0^+} f(x) = L_1$$

and we say the "limit of f(x) as x approaches x_0 from the right equals L_1"

Definition 1

LIMITS

Left Limit

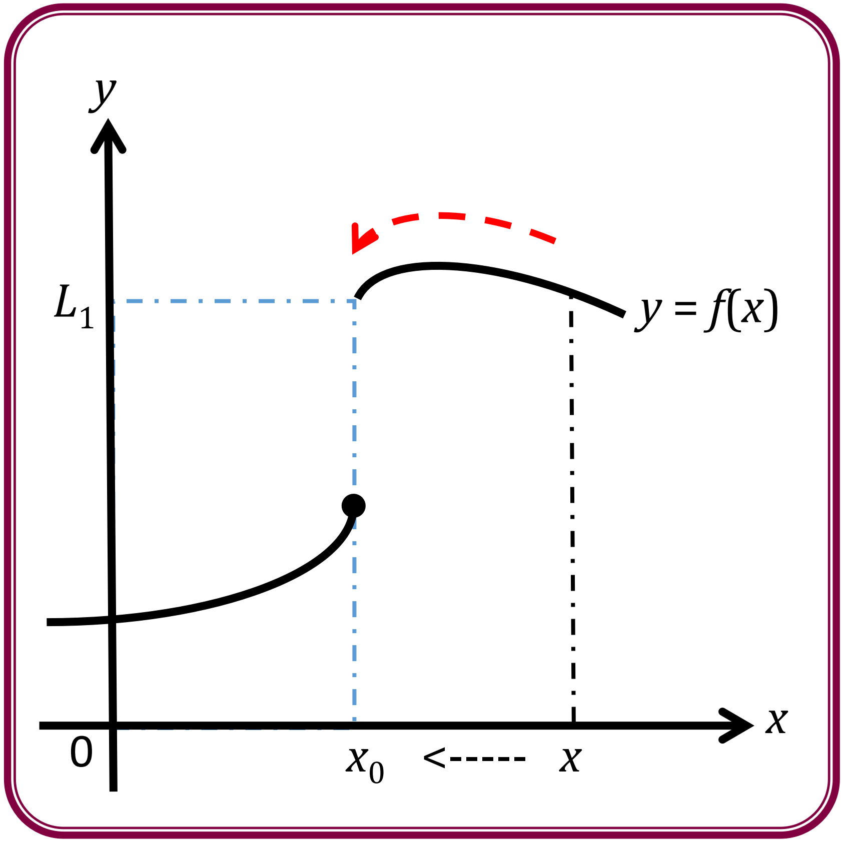
If the value of $f(x)$ approaches to a number L_2 as x approaches x_0 from the left, then we right

$$\lim_{x \to x_0^-} f(x) = L_2$$

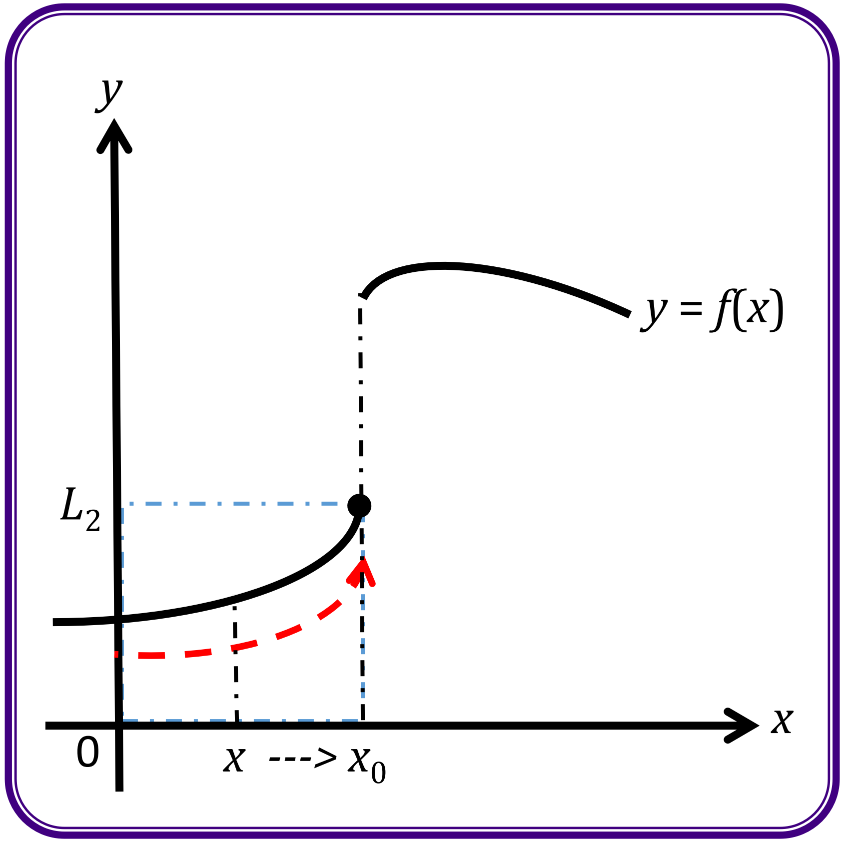
and we say the "limit of $f(x)$ as x approaches x_0 from the left equals L_2"

Definition 2

LIMITS



(a)



(b)

Figure (a) and (b) illustrate the Definition 1 and Definition 2, in cases when the two limits are different.

LIMITS

Limit of a Function

If the limits from the left and the right sides of f(x) have the same value, L that is

$$\lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^+} f(x) = L$$

Then $\lim_{x \to x_0} f(x)$ ***exist*** *and it is written as*

$$\lim_{x \to x_0} f(x) = L$$

and we say the "limit of f(x) as x approaches x_0 *equals L"*

Definition 3

The limit of $f(x)$ at x_0 exists only when the limits from the left and the right exist and both are equal.

However if either the limit from the left or from the right does not exist, or if

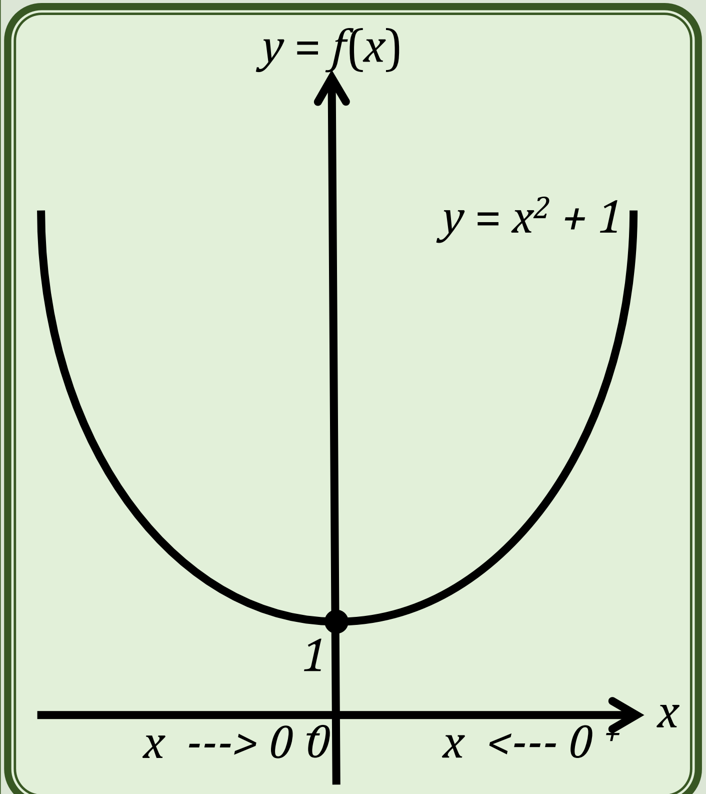
$$\lim_{x \to x_0^-} f(x) \neq \lim_{x \to x_0^+} f(x)$$

Then $\lim_{x \to x_0} f(x)$ ***does not exist.***

LIMITS

Example 1

Find the $\lim_{x \to 0} (x^2 + 1)$



x	**– 1**	**– 0.5**	**– 0.25**
f(x)	2	1.25	1.0625

x	**– 0.001**	**0**	**0.001**
f(x)	1.000001	1	1.000001

x	**0.25**	**0.5**	**1**
f(x)	1.0625	1.25	2

LIMITS

The closer x gets to the value 0, the closer the value of f(x) comes to 1. This limit could have been determined by simply substituting x = 0 into f(x). Since

$$\lim_{x \to 0^-}(x^2+1) = \lim_{x \to 0^+}(x^2+1) = 1$$

thus $\lim_{x \to 0}(x^2+1) = 1$

LIMITS

Example 2

Find $\lim_{x \to 4} f(x)$ *if* $f(x) = \begin{cases} 2x & \text{if } x \leq 4 \\ 2x + 3 & \text{if } x > 4 \end{cases}$

Solution:

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} 2x = 2(4) = 8$$

$$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} 2x + 3 = 2(4) + 3 = 11$$

Since $\lim_{x \to 4^-} f(x) \neq \lim_{x \to 4^+} f(x)$, *thus* $\lim_{x \to 4} f(x)$

does not exist.

LIMITS

Limits At Infinity

The independent variable "moves far away" from the origin along the x-axis. If x is allowed to increase indefinitely, we may write it as "$x \rightarrow +\infty$" and we say "x approaches positive infinity". Whereas, if x is allowed to decrease indefinitely, we may write it as "$x \rightarrow -\infty$" and we say "x approaches negative infinity".

Let say the limit of f(x) as x approaches positive infinity is L, where L is a real number. This statement can be written as

$$\lim_{x \to +\infty} f(x) = L$$

The line y = L is a horizontal asymptote for y = f(x).

LIMITS

Example 3

Find the $\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)$

x	1	10	100	1000
f(x)	2	1.1	1.01	1.001

x	10,000	100,000	1,000,000
f(x)	1.0001	1.00001	1.000001

x	10,000,000	100,000,000	1,000,000,000
f(x)	1.0000001	1.00000001	1.000000001

LIMITS

Example 4

Sketch the graph $y = \dfrac{2}{x}$.

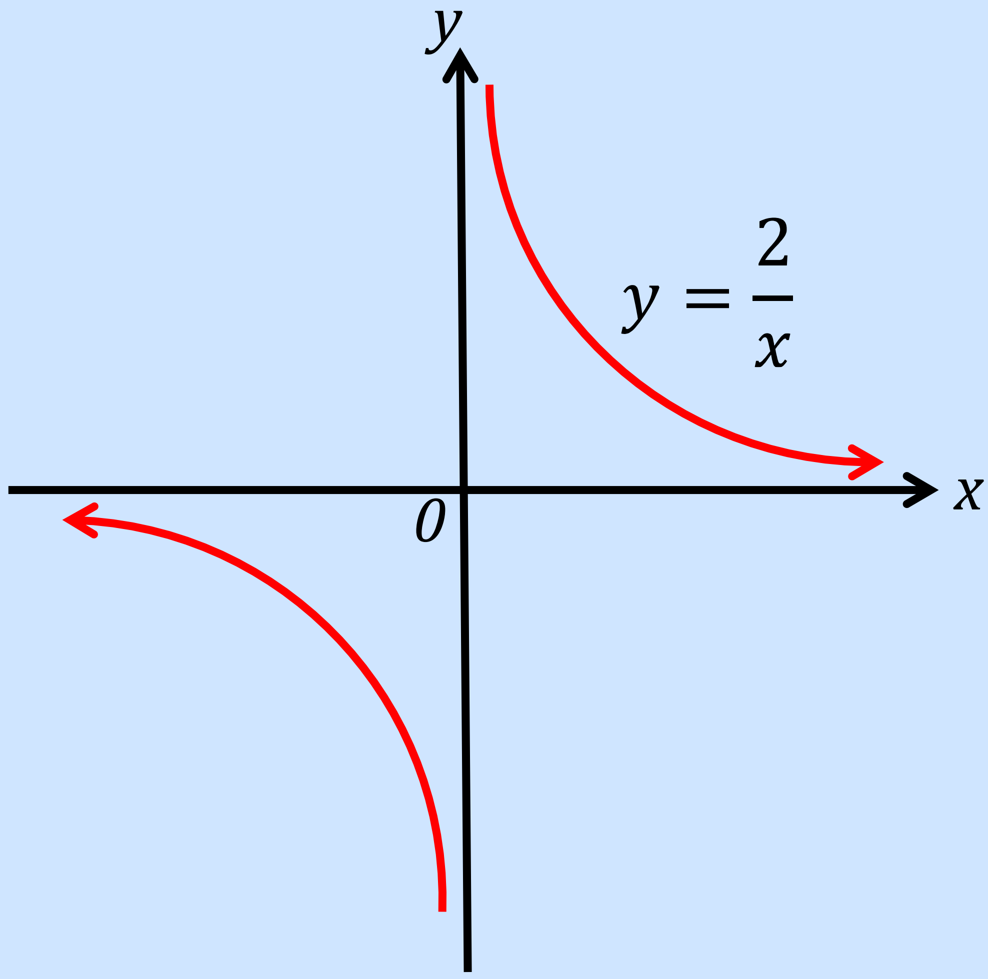
x	−1,000,000	−100,000	−10,000	−1,000	−100
f(x)	−0.000002	−0.00002	−0.0002	−0.002	−0.02

x	−10	−1	−0.1	−0.01	−0.001	−0.0001	−0.00001
f(x)	−0.2	−2	−20	−200	−2,000	−20,000	−200,000

x	0.00001	0.0001	0.001	0.01	0.1	1	10
f(x)	200,000	20,000	2,000	200	20	2	0.2

x	100	1,000	10,000	100,000	1,000,000
f(x)	0.02	0.002	0.0002	0.00002	0.000002

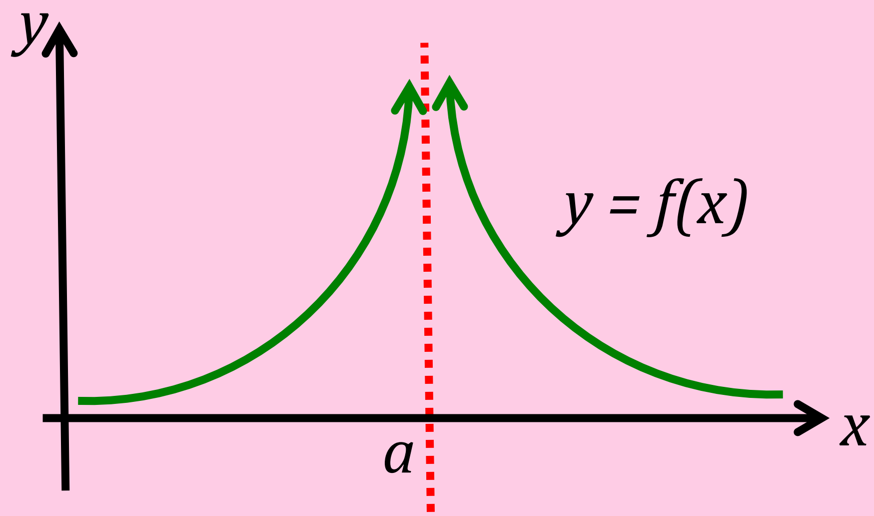
LIMITS



LIMITS

INFINITE LIMITS

- *If the function f(x) increases without bound as x approaches a from the left and right side,*



$$\lim_{x \to a^-} f(x) = +\infty \text{ and } \lim_{x \to a^+} f(x) = +\infty$$

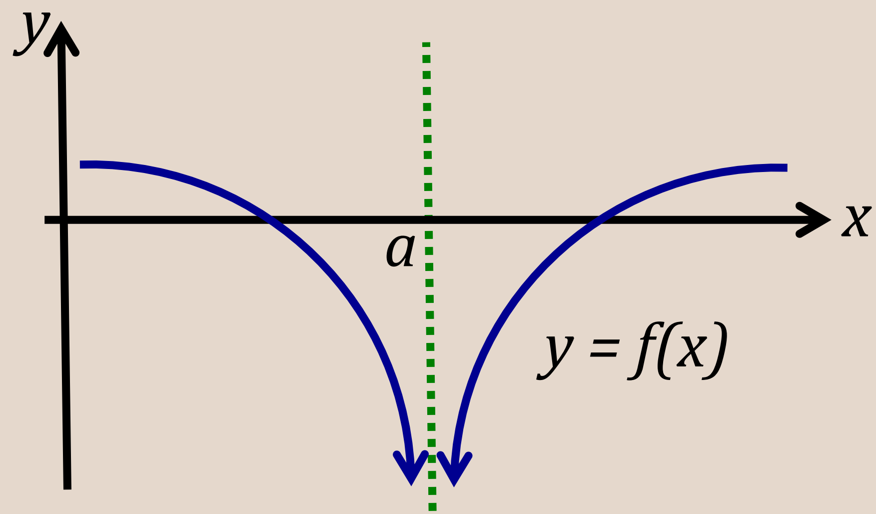
then

$$\lim_{x \to a} f(x) = +\infty$$

LIMITS

INFINITE LIMITS

- *If the function f(x) decreases without bound as x approaches a from the left and right side,*

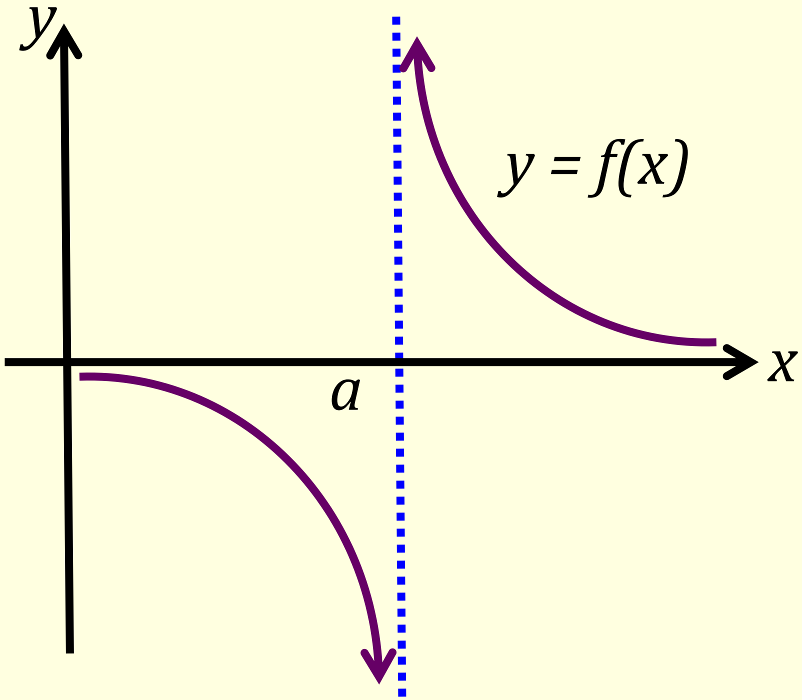


$$\lim_{x \to a^-} f(x) = -\infty \text{ and } \lim_{x \to a^+} f(x) = -\infty$$

then

$$\lim_{x \to a} f(x) = -\infty$$

- *If the function f (x) increases without bound as x approaches a from the right side, and decreases without bound as x approaches a from the left side,*



$$\lim_{x \to a^-} f(x) = -\infty \text{ and } \lim_{x \to a^+} f(x) = +\infty$$

then

$$\lim_{x \to a} f(x) \text{ does not exist}$$

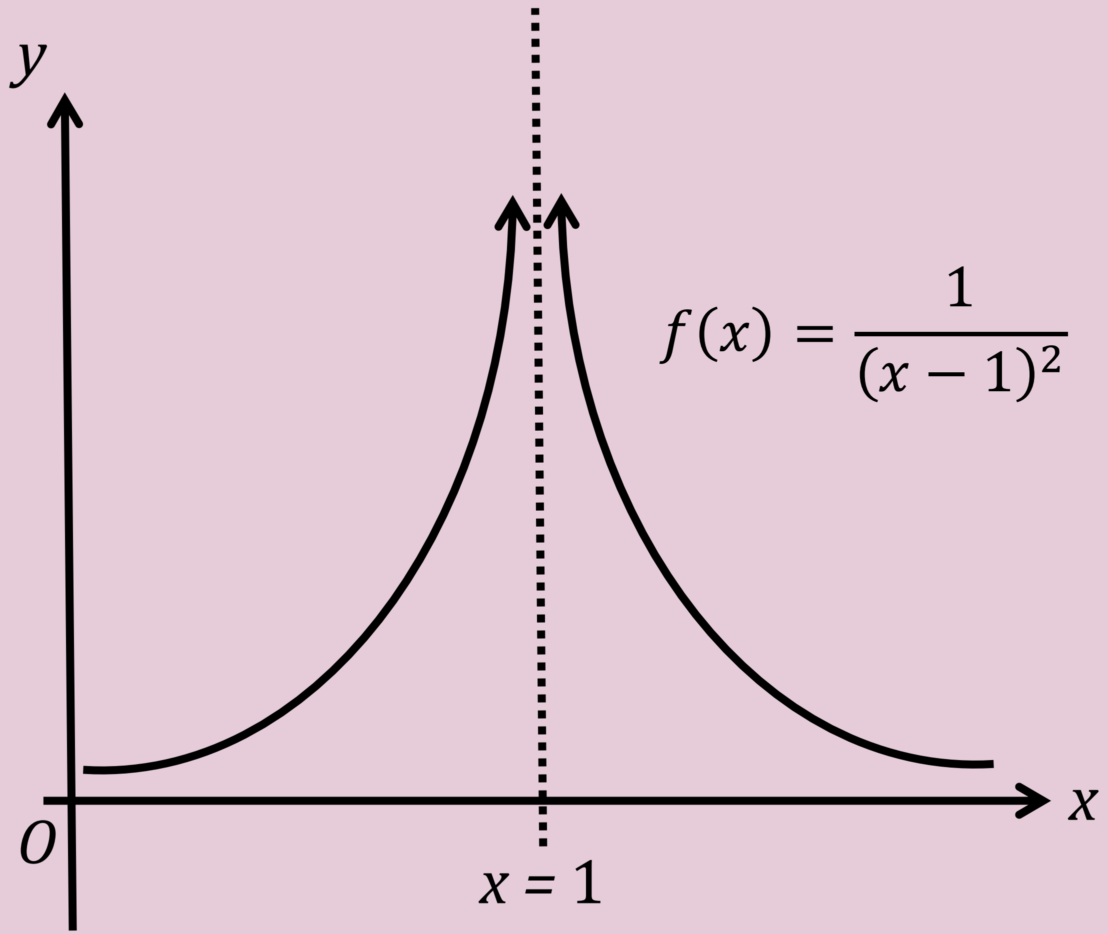
LIMITS

Example 5

To evaluate $\lim_{x \to 1} \frac{1}{(x-1)^2}$, *we must take left and right hand limits as x approaches 1.*

$x \to 1^-$	$x \to 1^+$
$f(0.9) = 100$	$f(1.1) = 100$
$f(0.99) = 10000$	$f(1.01) = 10000$
$f(0.9999) = 100000000$	$f(1.0001) = 100000000$

We should conclude that $\lim_{x \to 1} \frac{1}{(x-1)^2} = +\infty$



$$f(x) = \frac{1}{(x-1)^2}$$

LIMITS

Example 6

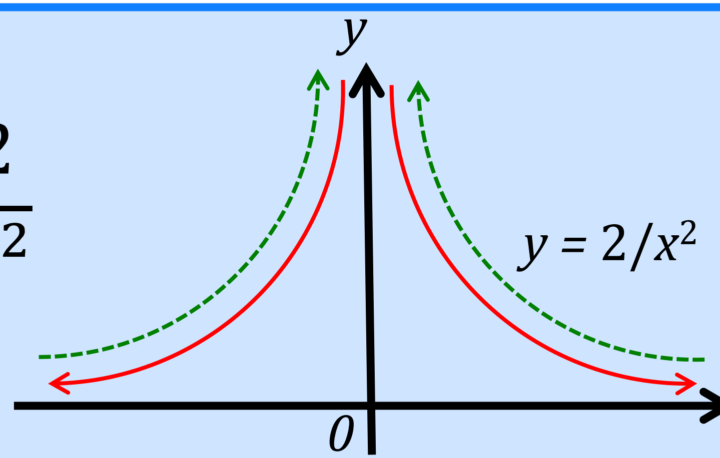
Evaluate

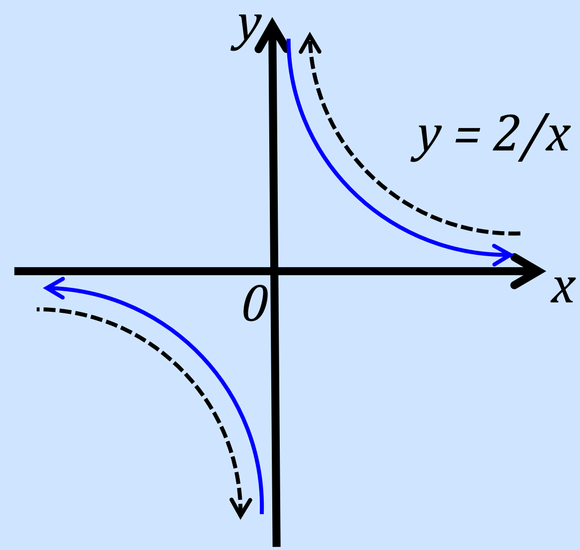
a) $\lim_{x \to -\infty} \frac{2}{x^2}$ *and* $\lim_{x \to +\infty} \frac{2}{x^2}$

b) $\lim_{x \to 0^-} \frac{2}{x^2}$ *and* $\lim_{x \to 0^+} \frac{2}{x^2}$

c) $\lim_{x \to -\infty} \frac{2}{x}$ *and* $\lim_{x \to +\infty} \frac{2}{x}$

d) $\lim_{x \to 0^-} \frac{2}{x}$ *and* $\lim_{x \to 0^+} \frac{2}{x}$





Exercises

$$Find \lim_{x\to a^-} f(x),\ \lim_{x\to a^+} f(x)\ and \lim_{x\to a} f(x)$$

a) $f(x) = \left(3 - \sqrt{x}\right); a = 0$

b) $f(x) = \sqrt{4 - x}; a = 4$

c) $f(x) = \sqrt{x^2 - 4}; a = 2$

LIMITS

Find $\lim_{x \to a^-} f(x)$, $\lim_{x \to a^+} f(x)$ *and* $\lim_{x \to a} f(x)$

a) $f(x) = \dfrac{1}{x^2}; a = 0$

b) $f(x) = \dfrac{1}{x-1}; a = 1$

c) $f(x) = \dfrac{2x}{x-3}; a = 3$

d) $f(x) = \dfrac{\sqrt{1+3x}}{1-x}; a = 1$

Exercises

LIMITS

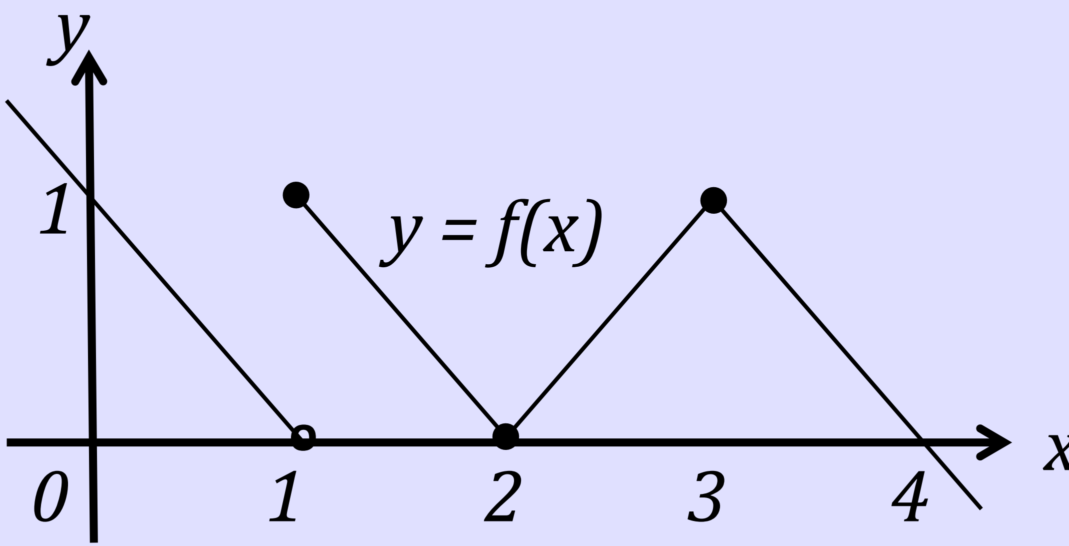
By referring to the following figure which shows the graph of y = f(x), find

a) $\lim_{x \to 1^-} f(x)$

b) $\lim_{x \to 1^+} f(x)$

c) $\lim_{x \to 2} f(x)$

d) $\lim_{x \to 3} f(x)$



Exercises

LIMITS

By referring to the following figure which shows the graph of y = g(x), find

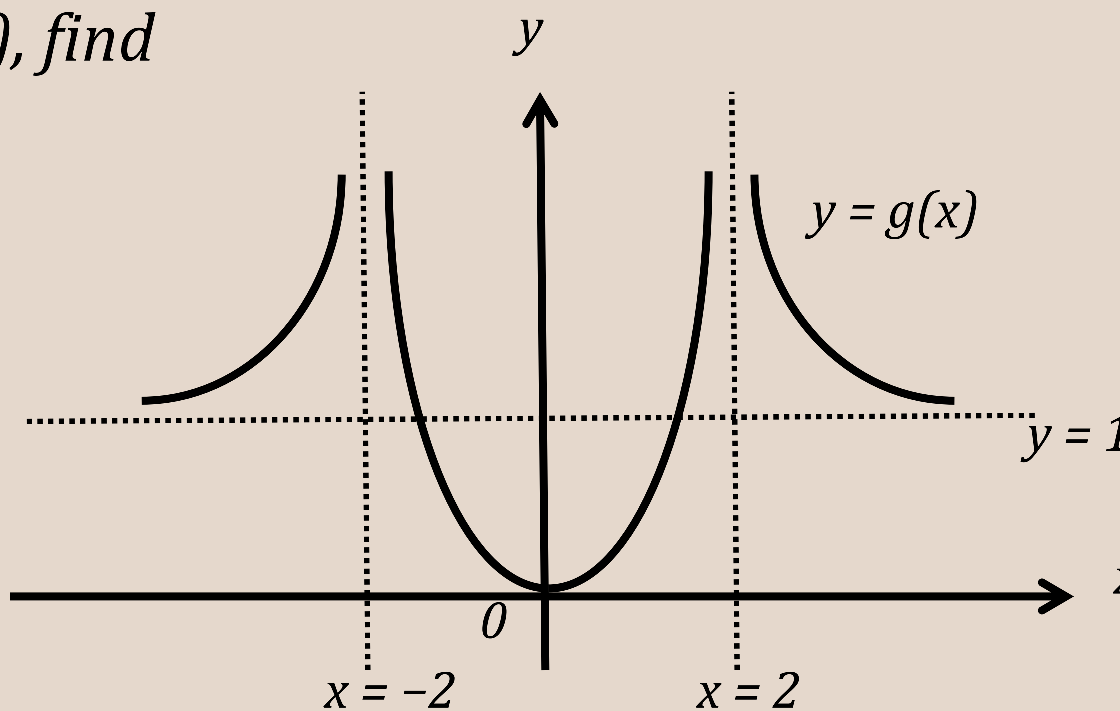
a) $\lim_{x \to -2^+} g(x)$

b) $\lim_{x \to -2^-} g(x)$

c) $\lim_{x \to -2} g(x)$

d) $\lim_{x \to 2^-} g(x)$

e) $\lim_{x \to 2^+} g(x)$

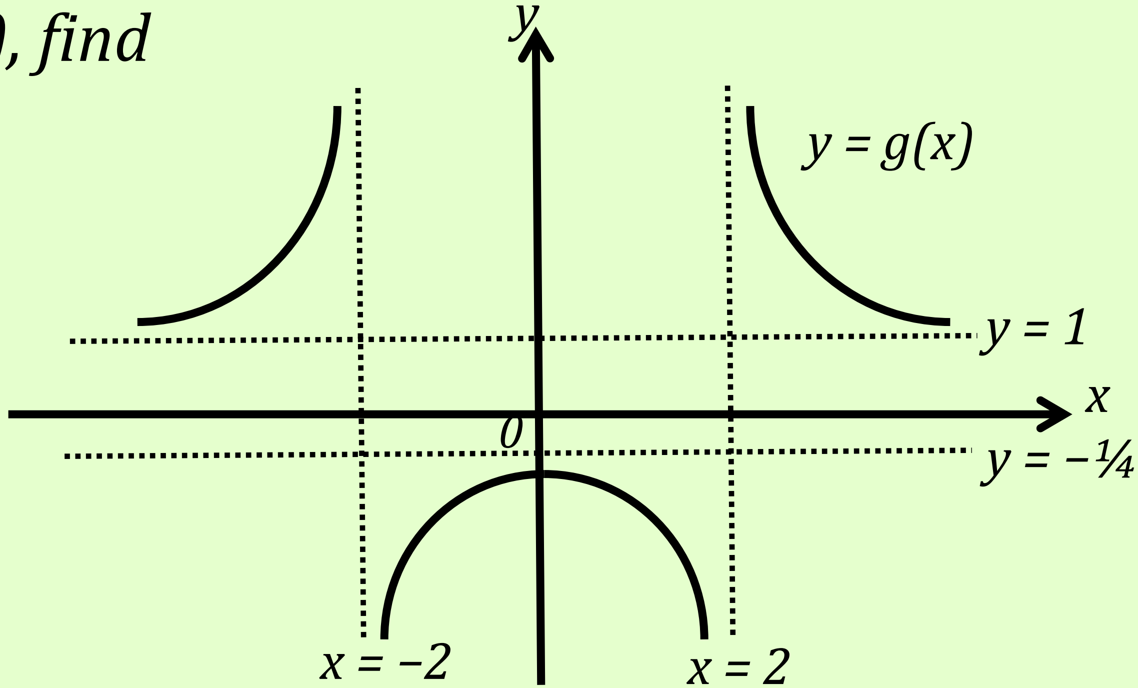


f) $\lim_{x \to 2} g(x)$

Exercises

LIMITS

By referring to the following figure which shows the graph of y = g(x), find



a) $\lim\limits_{x \to -2^{+}} g(x)$

b) $\lim\limits_{x \to -2^{-}} g(x)$

c) $\lim\limits_{x \to -2} g(x)$

d) $\lim\limits_{x \to 2^{-}} g(x)$

e) $\lim\limits_{x \to 2^{+}} g(x)$

f) $\lim\limits_{x \to 2} g(x)$

Exercises

LIMITS

1) Evaluate $\lim_{x \to 1} f(x)$ for the function below if exist.

$$f(x) = \begin{cases} x+2 & ; \quad x < -1 \\ x^2 & ; \quad -1 \le x \le 1 \\ 4 - x^2 & ; \quad x > 1 \end{cases}$$

1) Given $f(x) = \begin{cases} x-1 & ; \quad x \le 3 \\ 3x-7 & ; \quad x > 3 \end{cases}$, find

a) $f(3)$

b) $\lim_{x \to 3^-} f(x)$

c) $\lim_{x \to 3^+} f(x)$

d) $\lim_{x \to 3} f(x)$

Exercises

LIMITS

3) *Evaluate* $\lim_{x \to 1} f(x)$ *for the function below if exist.*

a) $f(x) = \begin{cases} 2 & ; \quad x \neq 1 \\ 5 & ; \quad x = 1 \end{cases}$

b) $f(x) = \begin{cases} 4 - x^2 & ; \quad x \leq 1 \\ 2 + x^2 & ; \quad x > 1 \end{cases}$

c) $f(x) = \begin{cases} 2x & ; \quad x \leq 1 \\ 2x + 3 & ; \quad x > 1 \end{cases}$

Exercises

LIMITS

Properties of Limits

1) If $f(x) = c$, where c is a constant, then

$$\lim_{x \to a} f(x) = \lim_{x \to a} c = c$$

2) If $f(x) = x$, then

$$\lim_{x \to a} f(x) = \lim_{x \to a} x = a$$

3) If $f(x) = x^n$, where n is a positive integer $(n > 0)$ then

$$\lim_{x \to a} f(x) = \lim_{x \to a} x^n = a^n$$

4) If c is a constant, then

$$\lim_{x \to a} c\left(f(x)\right) = c \lim_{x \to a} f(x)$$

LIMITS

Properties of Limits (cont)

5) If $f(x) = \sqrt[n]{x}$, where n is a positive integer (n>0) then

$$\lim_{x \to a} f(x) = \lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$$

6) If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

$$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

7) If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist,

$$\lim_{x \to a} [f(x) \times g(x)] = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x)$$

Properties of Limits (cont)

8) *If* $\lim_{x \to a} f(x)$ *and* $\lim_{x \to a} g(x)$ *exist, then*

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

where $\lim_{x \to a} g(x) \neq 0$

LIMITS

Evaluate the following limits.

a) $\lim_{x \to 2} 8$

b) $\lim_{x \to 2} x$

c) $\lim_{x \to -2} x^3$

d) $\lim_{x \to 10} 9x^2$

e) $\lim_{x \to -2} (x^5 + 2x - 5)$

f) $\lim_{x \to -1} (x^5 - 5)(x + 1)$

g) $\lim_{x \to -2} \sqrt{5x^2 - 4}$

h) $\lim_{x \to -3} \frac{x - 1}{x^2}$

Exercises

LIMITS

Direct Substitution Method

The limits of certain functions may be evaluated by substituting x = a in f(x). Thus we get

$$\lim_{x \to a} f(x) = f(a)$$

Exercises

a) $\lim_{x \to 5} x$

b) $\lim_{x \to 2} 3x^2 - 4$

c) $\lim_{x \to 4} \sqrt{x - 2}$

d) $\lim_{x \to -3} \dfrac{2x + 1}{x^2 - 3x + 1}$

e) $\lim_{x \to -1} 2e^{x+1}$

f) $\lim_{x \to 2} [\ln(x - 1) + 5]$

g) $\lim_{x \to 3} |x - 2|$

h) $\lim_{x \to 0} (x^3 + 1)\sqrt{x + 4}$

Limit of a Rational Function

The limit of a rational function can be found by substitution when the denominator is different from zero. If f(x) and g(x) are polynomials, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)} \; ; \; where\ g(c) \neq 0$$

Factorization Method

If f(x) and g(x) are polynomials, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}$$

will be $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0}$ *(indeterminate form)*

if f (c) = 0 and g(c) = 0. We use factorization method to solve the limit.

LIMITS

Exercises

Evaluate the following

$a)\ \lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x - 2}$

$b)\ \lim_{x \to 4} \dfrac{x - 4}{\sqrt{x} - 2}$

$c)\ \lim_{x \to 1} \dfrac{x^3 - 1}{x - 1}$

$d)\ \lim_{x \to 0} \dfrac{x^2 + x}{x^2 - x}$

$e)\ \lim_{x \to 2} \dfrac{x^3 - 8}{x^2 - 4}$

$f)\ \lim_{x \to 1} \dfrac{x^2 - 1}{x^2 - x}$

Multiplication of Conjugates Method

If by direct substitution, the limit of a rational function involving surd is 0/0 (indeterminate form), then the method of multiplication of conjugate can be used.

LIMITS

Exercises

Evaluate the following limits.

a) $\lim_{x\to 2} \dfrac{x-2}{\sqrt{x+2}-2}$

b) $\lim_{x\to 2} \dfrac{x^2-3x+2}{2-\sqrt{x+2}}$

c) $\lim_{x\to 0} \dfrac{\sqrt{x+9}-3}{x}$

d) $\lim_{x\to 1} \dfrac{3(x-1)+4(x-1)^2}{2(x-1)+5(x-1)^2}$

e) $\lim_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

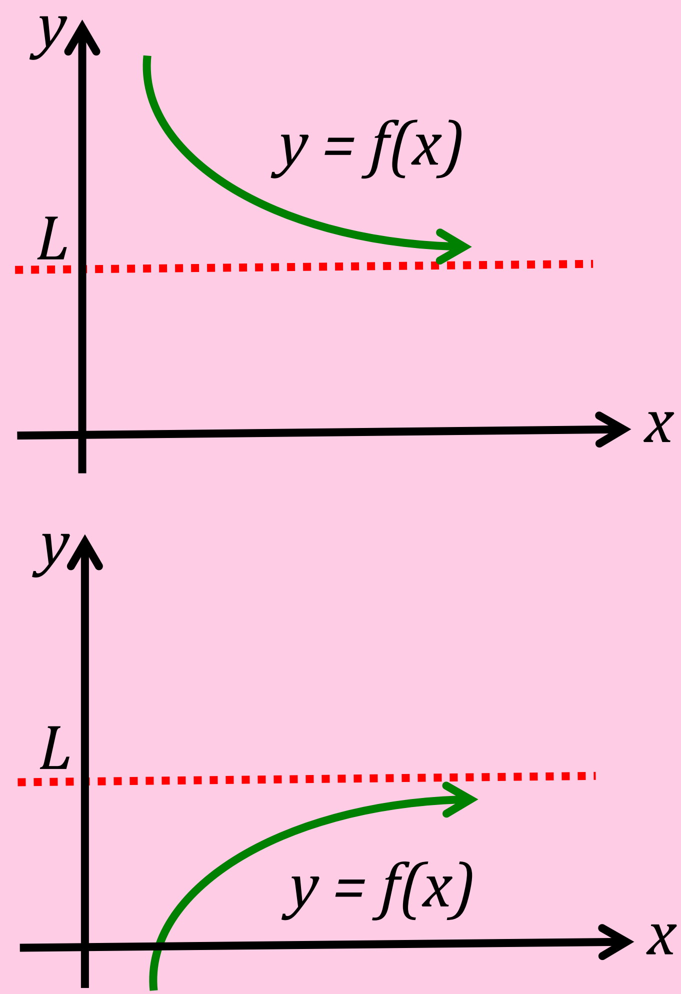
f) $\lim_{x\to 3} \dfrac{\sqrt{4-x}-1}{3-\sqrt{2x+3}}$

LIMITS

LIMITS AT INFINITY

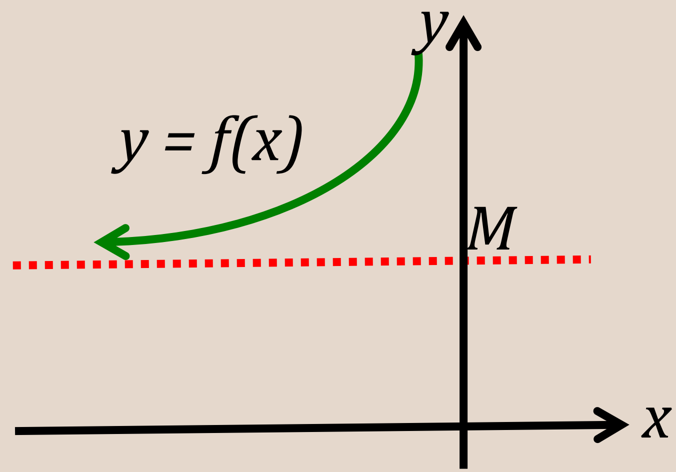
- *If x approaches positive infinity, f(x) approaches but never quite reaches a value of L, using limit notation, we state*

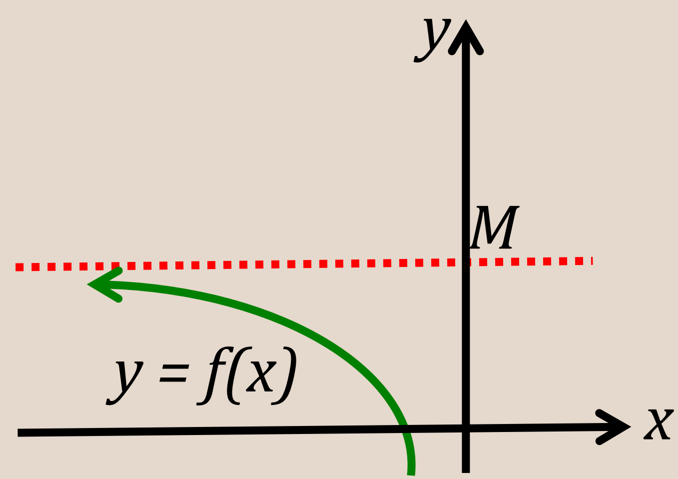
$$\lim_{x\to+\infty} f(x) = L$$



- *If x approaches negative infinity, f(x) approaches but never quite reaches a value of M, using limit notation, we state*

$$\lim_{x \to -\infty} f(x) = M$$





LIMITS

If f(x) is a rational function, to calculate

$$\lim_{x \to -\infty} f(x) \text{ or } \lim_{x \to +\infty} f(x)$$

we carry out the following steps:

STEP 1:

Divide the numerator and denominator of f(x) with x^n, where n is the highest power of x in the denominator.

STEP 2:

Use the limits theorem.

LIMITS

Evaluate the following limits.

a) $\lim_{x \to -\infty} (7x^5 - 9)$

b) $\lim_{x \to +\infty} (3x - 1)$

c) $\lim_{x \to +\infty} \sqrt{x^2 - 4}$

d) $\lim_{x \to -\infty} \left(2 + \frac{1}{x}\right)$

Exercises

LIMITS

Evaluate the following limits.

$a)\ \lim\limits_{x\to+\infty}\frac{3x-5}{6x+8}$

$b)\ \lim\limits_{x\to-\infty}\frac{x-4x^2}{5-6x^3}$

$c)\ \lim\limits_{y\to+\infty}\sqrt[3]{\frac{3y-5}{6y+8}}$

$d)\ \lim\limits_{t\to-\infty}\frac{\sqrt{3t^4+t}}{t^2-8}$

$e)\ \lim\limits_{x\to-\infty}\frac{x+2}{x^2-2x+1}$

$f)\ \lim\limits_{y\to-\infty}\frac{2+y}{\sqrt{6y^2-7}}$

$g)\ \lim\limits_{y\to\infty}\frac{y^2-y}{y^2+y}$

$h)\ \lim\limits_{t\to\infty}\frac{t^3-1}{t^2+1}$

Exercises

LIMITS

CONTINUITY

In this section, we will discuss another concept called continuity, which is closely related to the concept of limits

Definition 4

A function f is said to be continuous at a point x = a if the following conditions are satisfied.

(a) *The function f is defined at x = a, that is f (a) exist.*

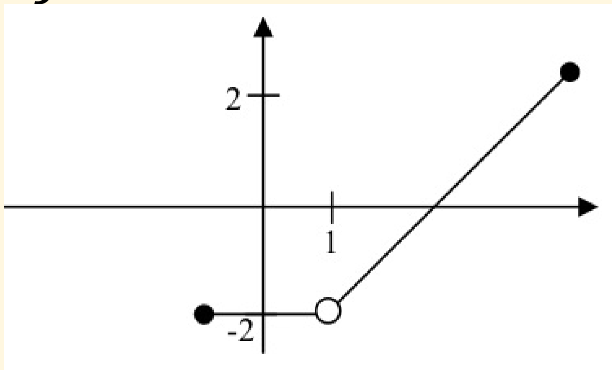
(b) $\lim_{x \to a} f(x)$ *exist.*

(c) $\lim_{x \to a} f(x) = f(a)$

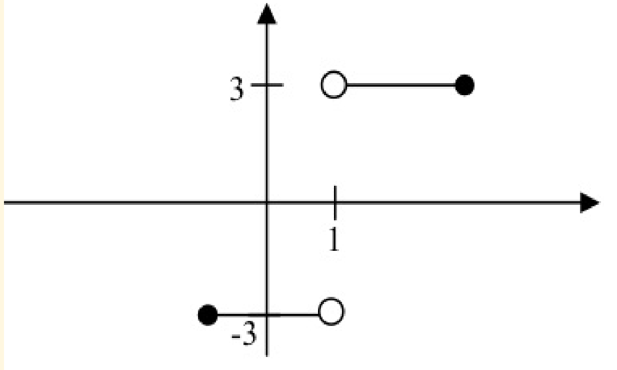
LIMITS

Example 7

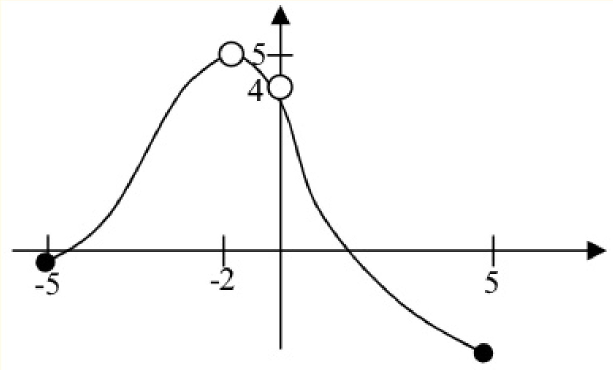
From the following graphs, determine whether each of the limits exist or not.



a) $\lim_{x \to 1} f(x)$



b) $\lim_{x \to 1} g(x)$

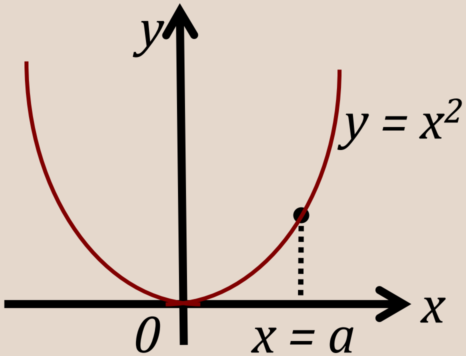


c) $\lim_{x \to 0} h(x)$

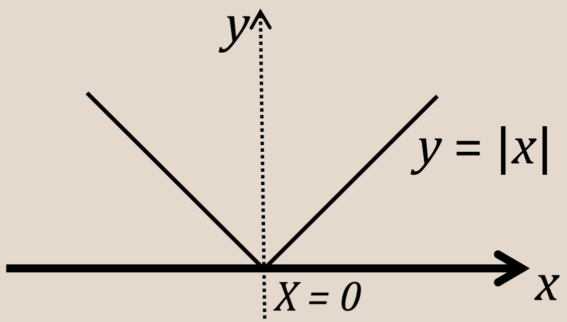
LIMITS

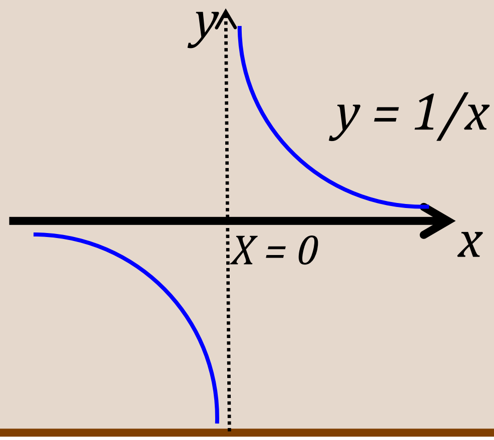
Find the point of discontinuity, if they exist, for the following functions.

$a)\ f(x) = x^2$



$b)\ f(x) = |x|$





$c)\ f(x) = \dfrac{1}{x}$

Exercises

LIMITS

Sketch the graph of the following functions and find the points of discontinuity.

$$a)\ f(x) = \begin{cases} -x & , \quad x < 0 \\ x^2 & , \quad x > 0 \end{cases}$$

$$b)\ g(x) = \begin{cases} 1 & , \quad x \neq 2 \\ 2 & , \quad x = 2 \end{cases}$$

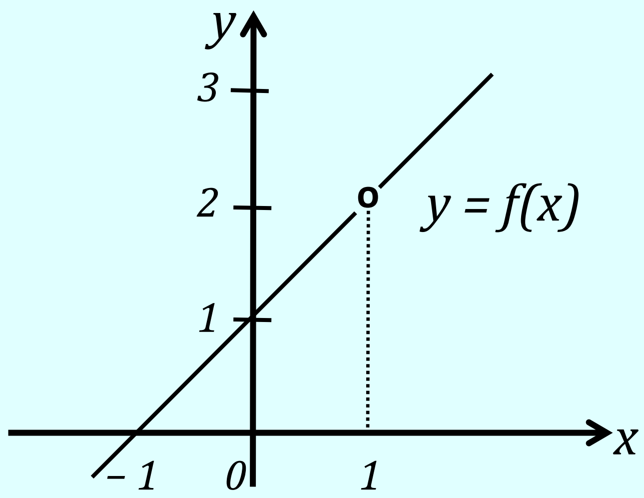
$$c)\ f(x) = \begin{cases} 2x & ; \quad x \leq 1 \\ 2x + 3 & ; \quad x > 1 \end{cases}$$

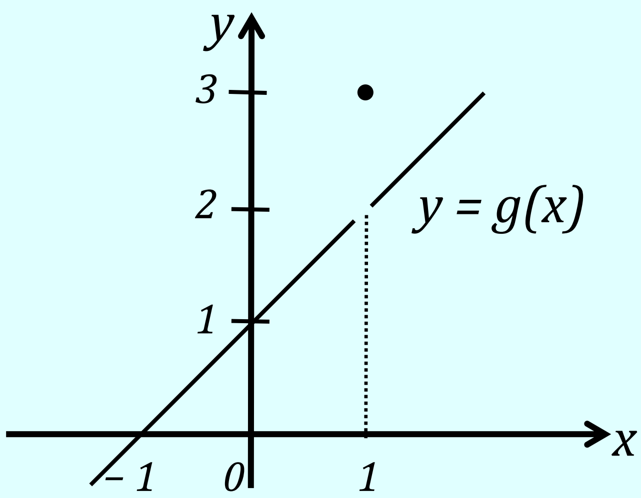
Exercises

LIMITS

Identify whether the following functions f(x) and g(x) are continuous or discontinuous at x = 1.

$$f(x) = \frac{x^2 - 1}{x - 1}; x \neq 1 \qquad g(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & ; \quad x \neq 1 \\ 3 & ; \quad x = 1 \end{cases}$$





Exercises

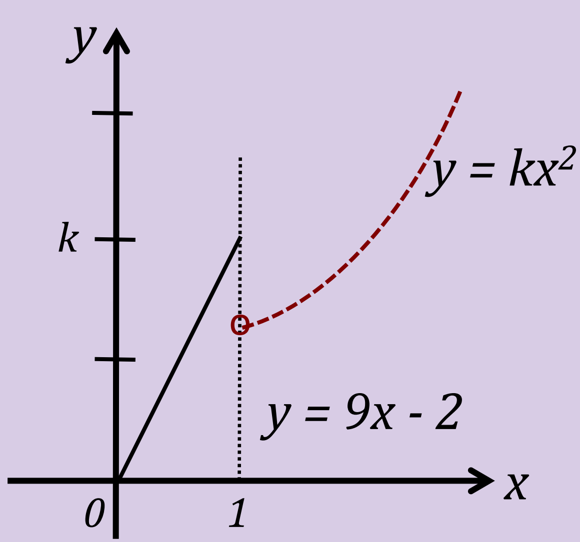
LIMITS

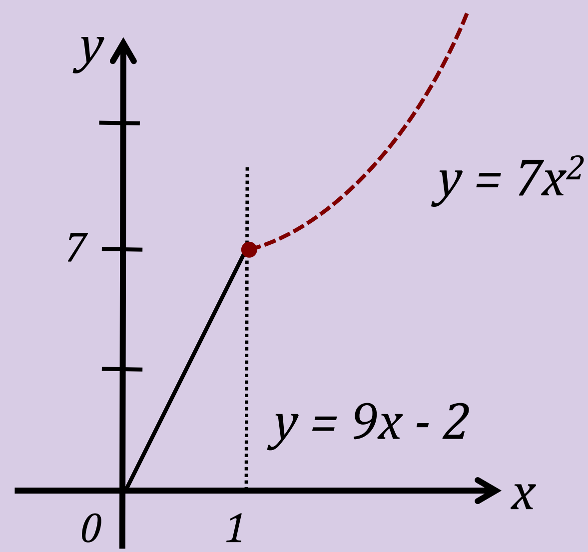
Exercises

Given

$$f(x) = \begin{cases} 9x - 2 & , \quad x \le 1 \\ kx^2 & , \quad x > 1 \end{cases}$$

Sketch the graph of y = f(x) and find the value of the constant k so that f(x) is continuous at x = 1.





CONTINUITY RULLES

If f and g are continuous at x = a, then the following functions are also continuous at x = a.

a) $f \pm g$

b) gf

c) fg

d) f^n

e) $\dfrac{f}{g}$, provided $g(a) \neq 0$

CONTINUITY IN AN INTERVAL

Let f be a function defined in the interval [a, b]. Then the function f is said to be continuous in the interval [a, b] if

a) f is continuous in the interval (a,b),

b) $\lim_{x \to a^+} f(x) = f(a)$ *and* $\lim_{x \to b^-} f(x) = f(b)$

LIMITS

If $f(x) = \sqrt{4 - x^2}$*, sketch the graph of y = f(x) and prove that f is continuous in the interval [−2, 2].*

a) For −2 < c < 2

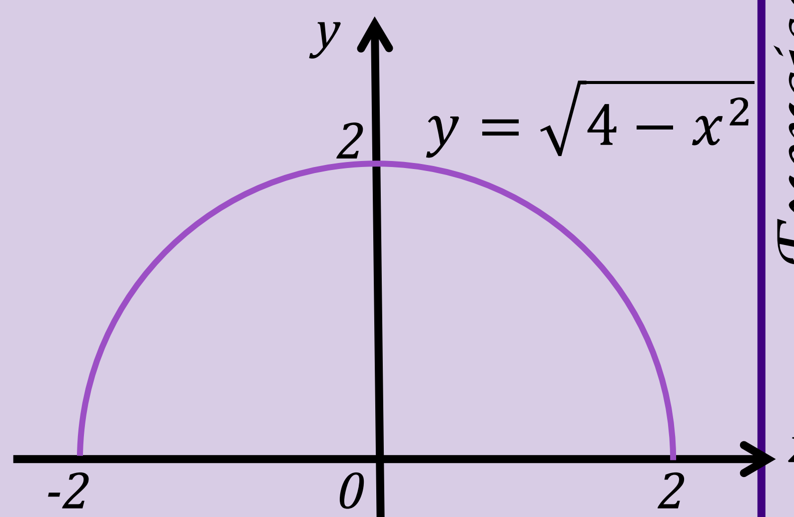
$$\lim_{x \to c} f(x) = \lim_{x \to c} \sqrt{4 - x^2} = \sqrt{4 - c^2} = f(c)$$

f is continuous at x = c.

b) $\lim_{x \to -2^+} f(x) = f(-2)?$

$\lim_{x \to 2^-} f(x) = f(2)?$

If yes, f is continuous in the interval [−2, 2].



Exercises

PAST YEAR QUESTIONS

Test 1 Sem 2 20182019

Find the limit for each of the following.

a) $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x}$.

(3 marks)

b) $\lim_{x\to \infty} \frac{3x^2+2x-1}{7-2x^2}$.

(3 marks)

PAST YEAR QUESTIONS

Test 1 Sem 1 20192020

1 Evaluate the limit for each of the following.

i. $\lim_{x \to -1} 3x(2x+1)$.

(1 mark)

ii. $\lim_{x \to \infty} \dfrac{2x^3+x^2+x+1}{x^3+1}$.

(3 marks)

2 Show that the $\lim_{x \to 9} \dfrac{x-9}{\sqrt{x}-3} = 6$.

(3 marks)

PAST YEAR QUESTIONS

Final Sem 2 20182019

QUESTION 1)

a) A function $f(x)$ is defined by

$$f(x)=\begin{cases} ax^2+\frac{1}{2}, & x<-1, \\ 1-bx, & -1\leq x<2, \\ \sin\left(\frac{\pi}{2}x\right), & x\geq 2, \end{cases}$$

where a and b are constants. Find the values of a and b if $f(x)$ is continuous at $x=-1$ and $x=2$.

(5 marks)

PAST YEAR QUESTIONS

Final Sem 2 20182019

QUESTION 1

b) Find the limit of each of the following

i. $\lim_{x \to 4} \frac{x^2 - 3x - 4}{x^2 - 16}$.

(5 marks)

ii. $\lim_{x \to \infty} \frac{4x - 1}{\sqrt{9x^2 + 1} + 2x}$.

(5 marks)

Final Sem 1 20192020

QUESTION 1)

a) A function $f(x)$ is defined by

$$f(x) = \begin{cases} x^2 - \alpha^2, & x < 4, \\ \alpha x + 20, & x \geq 4, \end{cases}$$

where α is a constant. Determine the value of α so that $f(x)$ is continuous for any value of x. Hence sketch the graph of $f(x)$.

(6 marks)

PAST YEAR QUESTIONS

Final Sem 1 20192020

QUESTION 1

b) Show for each of the following

i. $\lim_{x \to 4} \frac{x^2 - 16}{\sqrt{x} - 2} = 32.$

(4 marks)

ii. $\lim_{x \to 0} \frac{4 - \sqrt{16 + x}}{x} = -\frac{1}{8}.$

(5 marks)