DISCRETE-TIME SIGNAL

SKEL 4223 - DIGITAL SIGNAL PROCESSING

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Introduction

- Digital Signal Processing (DSP) has developed rapidly over the past 40 years due to significant advances in digital computer technology and integrated-circuit fabrication (MSI→LSI→VLSI).
- In telecommunication:
  - ADC - samples an analog signal and encodes the signal into pulse code modulation (PCM) format.
  - Filter noise and interference from signal, transcode, compress, and encrypt the signal prior to placing it on a carrier of an electrically based digital circuit for transmission.
  - Sending the signal to DAC for decoding back into real world analog form, perhaps with improved clarity, a shifted frequency, or demodulated for transmission.

What is a signal?

Description of the evolution of the physical phenomenon

Example:

- Pressure (sound)
- Temperature (weather)
- Magnetic deviation (recorded sound)
- Gray level on paper (photograph)

Introduction (cont.)

- A discrete-time signal is a series consisting of a sequence of quantities. In other words, it is a series that is a function over a domain of discrete integers. Each value in the sequence is called a sample.

- As most of the signals encountered in science and engineering are analog in nature, a process on converting the analog signal to discrete signal is needed. This process is called digitization, which consists of Sampling and Quantization process.

Digitization

Referring to figure below:

- converting continuous value to discrete value on horizontal-axis is called sampling
- converting the continuous value to discrete value on vertical- axis is called quantization.

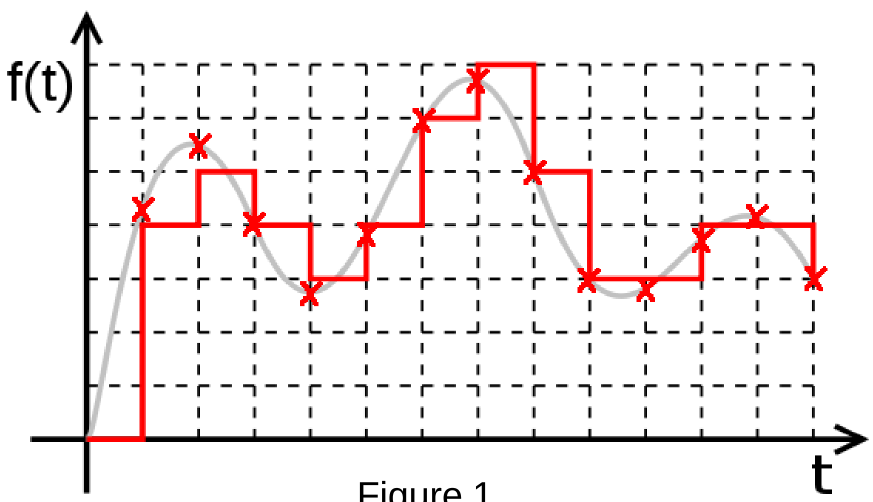


Figure 1

Digitization (cont.)

- In Figure 1, the continuous signal is first sampled where values of at only selected t are kept. This is shown by the red cross. So far only values on the horizontal have been digitized.

- To digitize the vertical axis, values on the red cross are quantized where in this example these values are round toward nearest values of 3 bit integer. For example the first red cross value is quantized from 4.2 to 4 (or 100 in binary).

Classification of Signal

- Continuous-time and discrete time – Continuous time signal take on values in the continuous interval while discrete time signal take samples only at certain values of time.
- Periodic and aperiodic – Periodic signal is consistently repeated at certain time period from $-\infty$ to ∞ .
- Deterministic and random – Deterministic signal can be represented with mathematical description while random signal is not where the signal evolves in time in an unpredictable manner.

Sampling

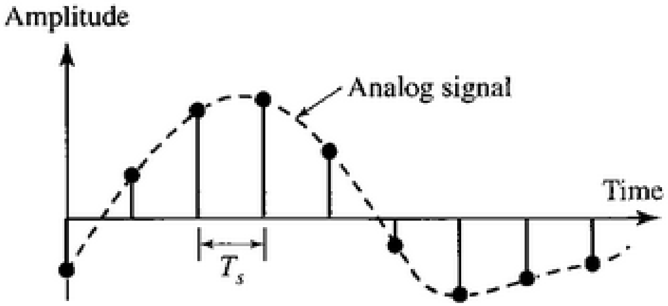
The sampling process can be viewed as:

$$t = nT_s$$

t – continuous time (s)

n – discrete time (integer). Also called sample

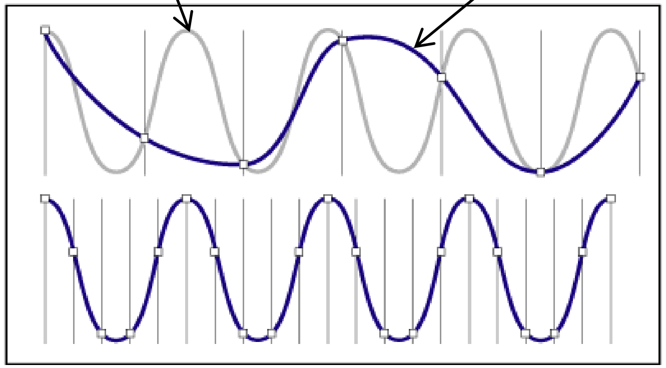
T_S – sampling time (s)



Sampling Theorem

To ensure the digitized discrete-time signal represents its original continuous-time signal sufficiently and correctly, the sampling process need to follow nyquist theorem as below:

$$F_S \geq 2F_N$$



Where:

F_S – Sampling Frequency

F_N – Highest frequency component in the signal

Signal Representation

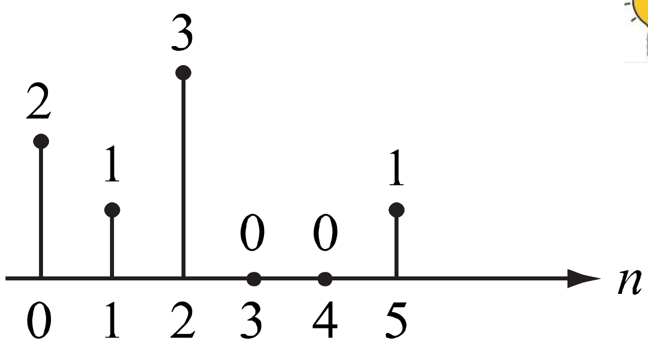
– analog/continuous $x(t)$

– digital/discrete $x[n]$

Discrete Signal Representation

- Numerical Method
  - $x[n]=[\underset{\uparrow}{2},1,3,0,0,1]$ where the arrow shows the position at *n*=0
  - if no arrow showed, the first sample holds the position at *n*=0

- Graphical Method

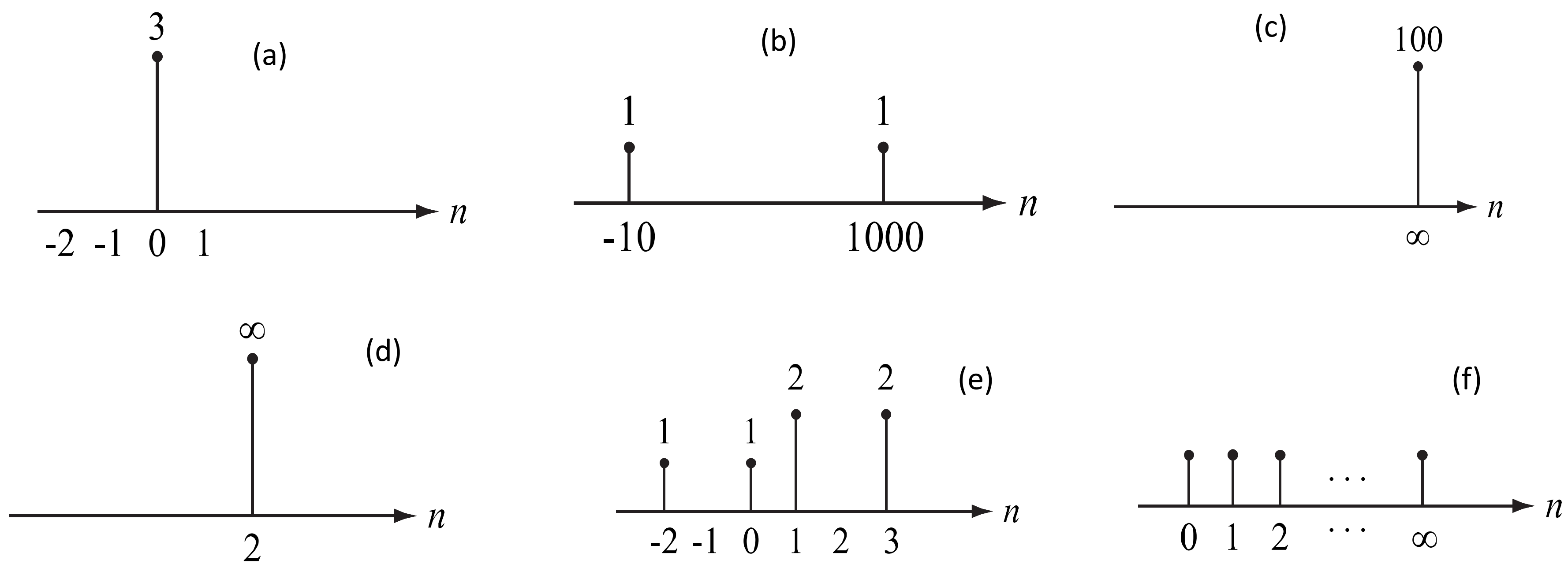


Finite length sequences are defined only for a range of indices, say N_1 to N_2. The length of this finite sequence is given by $|N_2 - N_1 + 1|$

– Discrete signal length (also called size): Number of samples in the signal. Count from the lowest to the highest that has a nonzero value

Quiz 1

What is the length of these signals?

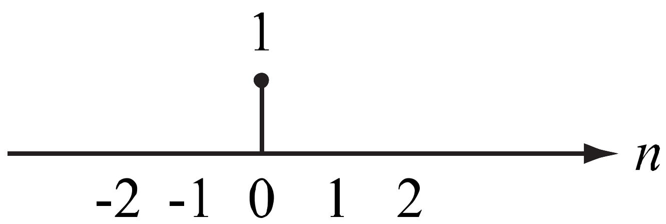


Type of Discrete Signal

Basic signal

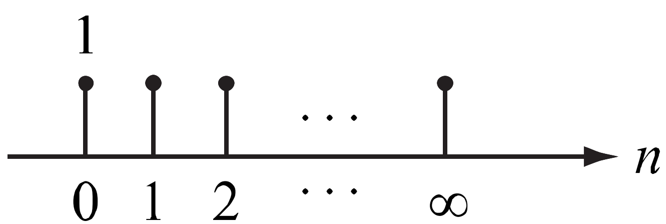
- **Impulse**

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



- **Unit Step**

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

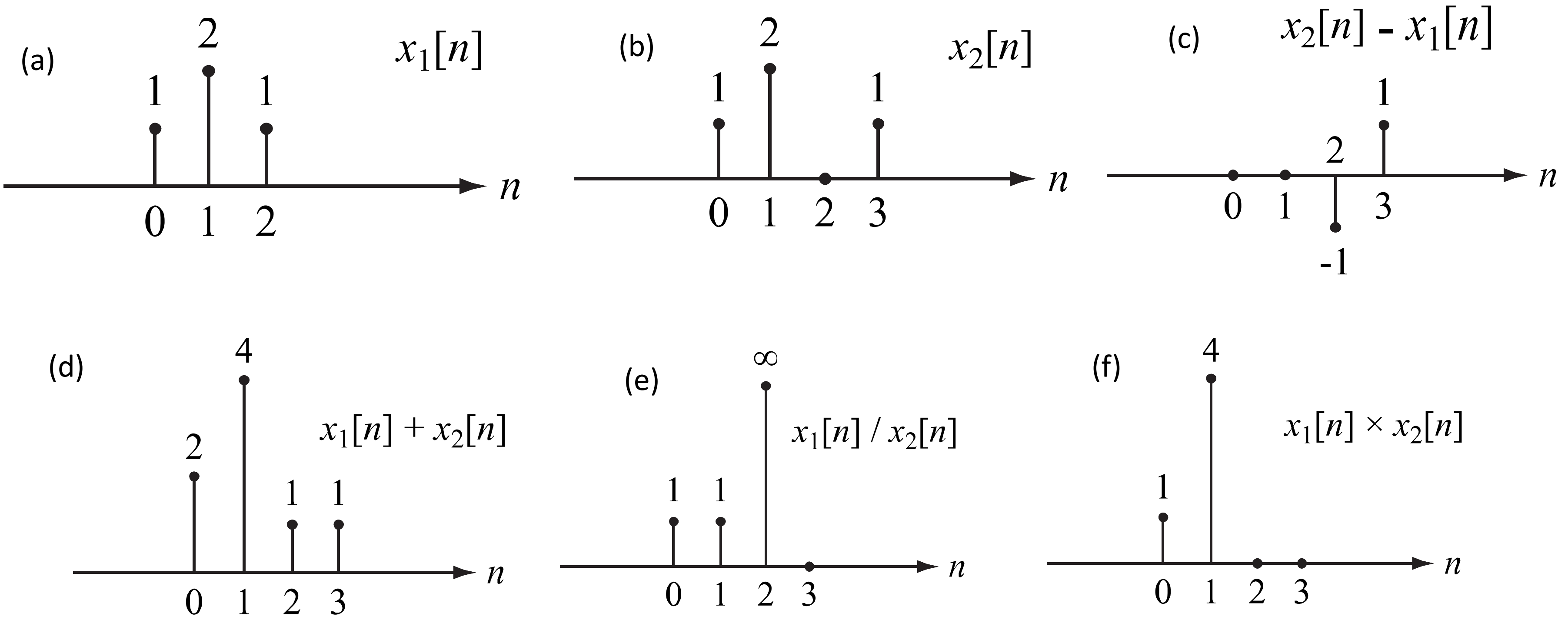


- A sequence can be represented by scaled and shifted impulses.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

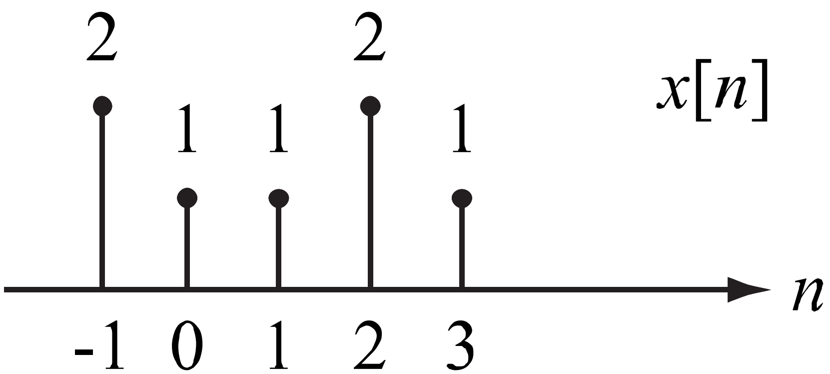
Arithmetic Operation

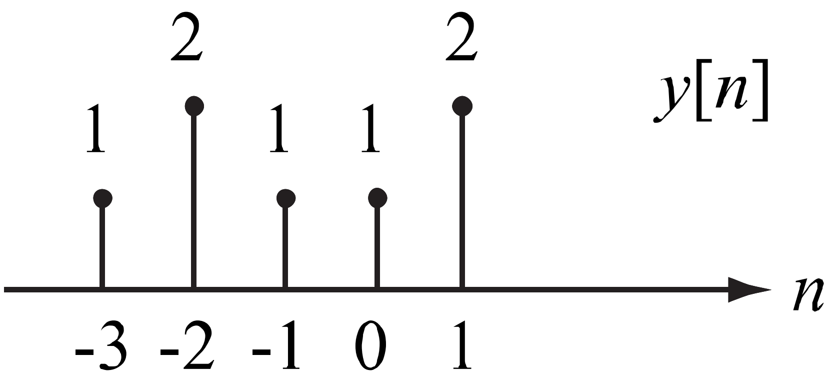
- For addition, subtraction, multiplication, division operations, the operations are made on each of the sample of similar n



Folding Operation

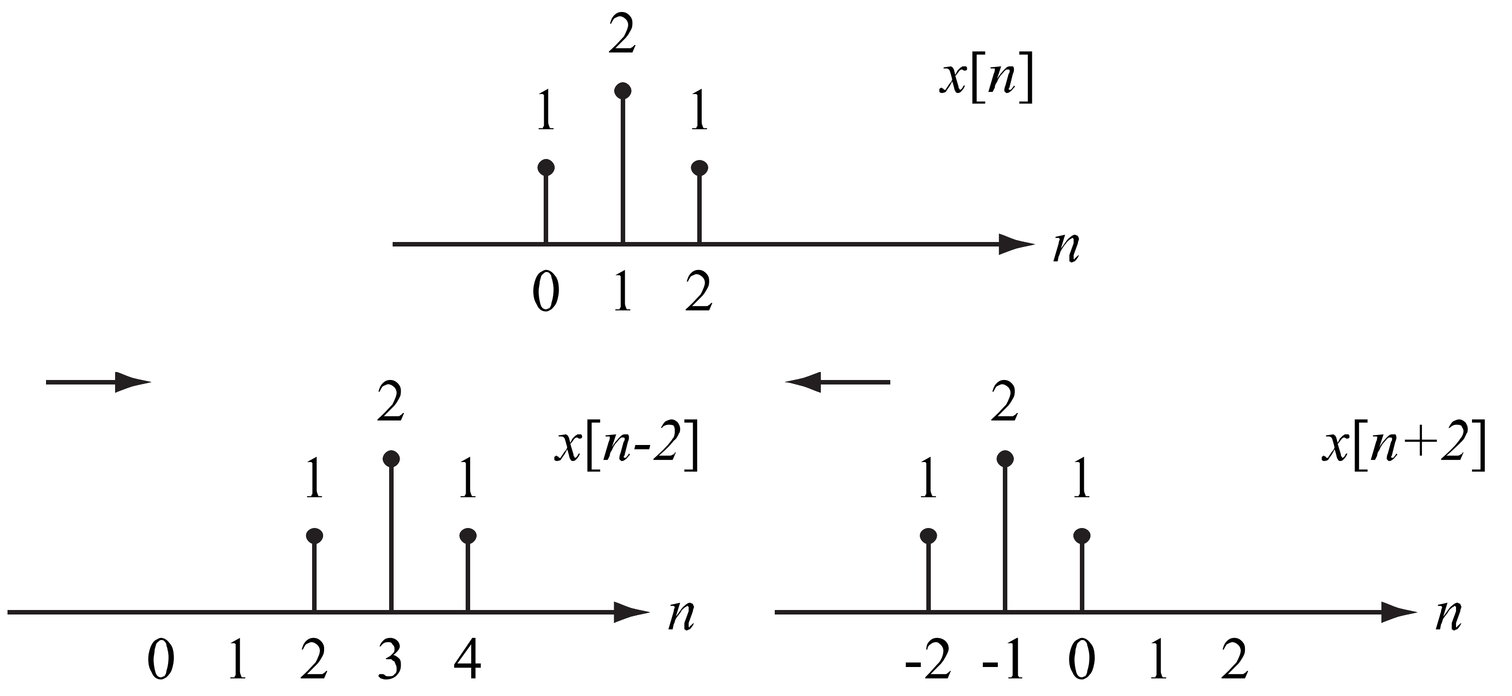
- Fold signal at $n = 0$ where $y[n] = x[-n]$



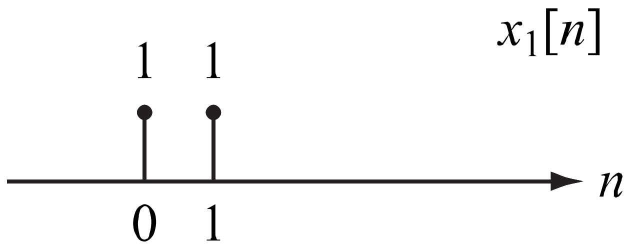


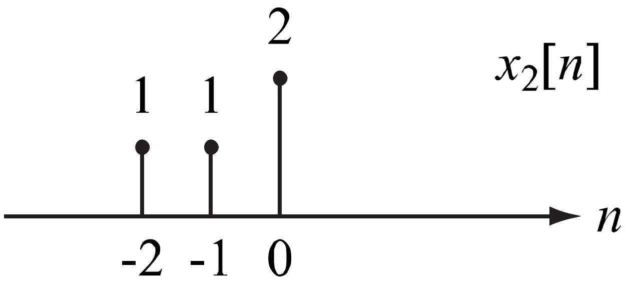
Delay Operation

Shift the signal to the right or left by n_d samples $y[n] = x[n - n_d]$



Quiz 2





Find:

1) $y[n] = x_1[-n] + x_2[n]$
2) $y[n] = x_1[n] + x_2[n-3]$
3) $y[n] = x_2[-n-2]$
4) $y[n] = x_1[n] + x_2[-2n]$
5) $y[n] = x_2[n] - x_1[n]$
6) $y[n] = u[-n] + x_1[n]$
7) $y[n] = \delta[n-3] + x_2[-n] + x_1[n+2]$
8) $y[n] = \delta[n] \times x_2[n]$
9) $y[n] = u[n] \times x_2[n]$
10) $y[n] = u[-n+2] - x_1[n]$
11) $y[n] = x_2[n] + x_2[n-1]$

References

1. John G. Proakis, Dimitris K Manolakis, “Digital Signal Processing: Principle, Algorithm and Applications”, Prentice-Hall, 4th edition(2006).
2. Sanjit K. Mitra, “Digital Signal Processing-A Computer Based Approach”, McGraw-Hill Companies, 3rd edition (2005).
3. Alan V. Oppenheim, Ronald W. Schafer, “Discrete-Time Signal Processing”, Prentice-Hall, 3rd edition (2009).