Shallow Foundation

MUHAMMAD AZRIL HEZMI
AHMAD SAFUAN AB RASHID
KAMARUDIN AHMAD
Shallow Foundation

- Shallow foundation is a foundation whose depth below the surface, $z$, is equal to or is less than its least dimension, $B$. $z \leq B$.

- Type of shallow foundation:
  
a) Pad footing
  
b) Strip/Continuous footing
  
c) Raft/ Mat foundation
Type of shallow foundation

a) Pad footing

Generally an individual foundation designed to carry a single column load although there are occasions when a pad foundation supports two or more columns.
Type of shallow foundation

b) *Strip / Continuous footing*

Often termed a continuous footing this foundation has a length significantly greater than its width. It is generally used to support a series of columns or a wall.
Type of shallow foundation

c) Raft / Mat foundation

This is a generic term for all types of foundations that cover large areas. A raft foundation is also called as a mat foundation.
Factors in the design

- Adequate depth
- Limiting settlement
- Safe against shear failure
Adequate depth

- The depth of footing must be sufficient to prevent any changes in surface conditions, horizontal loads and strong overturning moments.

- To prevent *frost action* and *volume change effect*, the depth of footing should more than 1.2 m and 1.5 m respectively.
Limiting settlement

- Guidelines to limiting settlement by *Skempton* and *MacDonald, 1956*:
  - Sand - Maximum total settlement *40 mm*
  - Clay - Maximum total settlement *60 mm*
Safe against shear failure

- Shear failure occurs when the soil divides into separate blocks or zones which move fully or partially and tangentially with respect to each other, along slip surfaces.
- Conventionally, the *factor of safety* to use in design against shear failure is more than *3.0*.
Modes of Shear Failure

- **General shear failure**
  - This occurs when a clearly defined slip surface forms under the footing and develops outward towards one or both sides and eventually to the ground surface.
Modes of Shear Failure

- *Local shear failure*
  - Significant vertical movement may take place before any noticeable development of shear plane occurs.
Modes of Shear Failure

- *Punching shear failure*
  - This is a downward movement of the foundation caused by soil shear failure only occurring along the boundaries of the wedge of soil immediately below the foundation.
Bearing capacity terms

- **Ultimate bearing capacity, \( q_u \) or \( q_f \)**
  - The value of the contact pressure between the foundation and the soil which will produce shear failure in the soil.

- **Net ultimate bearing capacity, \( q_{nett} \)**
  - Net loading at which ground fails in shear.
  - \( q_{nett} = q_u - \gamma D \)
Bearing capacity terms

- **Safe bearing capacity,** $q_{safe}$, $q_a$ or $q_{all}$
  - Net ultimate bearing capacity divided with factor of safety ($FS$) plus the term $\gamma D$.

  $$ q_{safe} = \frac{q_{nett}}{FS} + \gamma D $$

- **Allowable bearing capacity**
  - The maximum allowable net loading intensity on the soil allowing for both shear and settlement effects.
Bearing capacity analysis

• Prandtl’s analysis
  • Prandtl (1921) proposed the following general equation for $q_u$:
    
    $$q_u = c \cot \Phi \left[ \exp \left( \pi \tan \Phi \right) \tan^2 \left( 45^0 + \pi/2 \right) - 1 \right]$$

  • For a surface footing with $\Phi = 0$, Prandtl obtained
    
    $$q_u = 5.14c$$
Bearing capacity analysis

**Terzaghi’s analysis**

- *Terzaghi (1943)* produced a formula for $q_u$ which allows for the effects of cohesion and friction between the base of the footing and the soil and is also applicable to shallow and surface foundations.
  - Strip footing
    \[
    q_u = cN_c + \gamma z N_q + 0.5 \gamma BN \gamma
    \]
  - Square footing
    \[
    q_u = 1.3cN_c + \gamma z N_q + 0.4 \gamma BN \gamma
    \]
  - Rectangular footing
    \[
    q_u = cN_c \left( 1 + 0.3 \frac{B}{L} \right) + \gamma z N_q + 0.5 \gamma BN \gamma \left( 1 - 0.2 \frac{B}{L} \right)
    \]
  - Circular footing
    \[
    q_u = 1.3cN_c + \gamma z N_q + 0.3 \gamma BN \gamma
    \]
- The coefficients $N_c$, $N_q$ and $N_q$ depend upon the soil’s angle of shearing resistance and can be obtained from next figure.
Bearing capacity analysis

- Graph showing the relationship between φ (in degrees) and values of $N_c, N_q,$ and $N_f$.

- Table with values for $N_c, N_q,$ and $N_f$ at different values of φ.

<table>
<thead>
<tr>
<th>φ</th>
<th>0°</th>
<th>5°</th>
<th>10°</th>
<th>15°</th>
<th>20°</th>
<th>25°</th>
<th>30°</th>
<th>35°</th>
<th>40°</th>
<th>45°</th>
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</thead>
<tbody>
<tr>
<td>$N_c$</td>
<td>5.7</td>
<td>7.3</td>
<td>9.6</td>
<td>12.9</td>
<td>17.7</td>
<td>25.1</td>
<td>37.2</td>
<td>57.8</td>
<td>95.7</td>
<td>172</td>
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<tr>
<td>$N_q$</td>
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<td>2.7</td>
<td>4.4</td>
<td>7.4</td>
<td>12.7</td>
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<td>41.4</td>
<td>81.3</td>
<td>173</td>
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<tr>
<td>$N_f$</td>
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<td>0.5</td>
<td>1.2</td>
<td>2.5</td>
<td>5.0</td>
<td>9.7</td>
<td>19.7</td>
<td>42.4</td>
<td>100</td>
<td>298</td>
</tr>
</tbody>
</table>
Bearing capacity analysis

- A rectangular foundation, 2m x 4m is to be founded at a depth of 1 m below the surface of soft clay with unit weight, $\gamma = 20$ kN/m$^3$. If the soil parameters are $c = 24$ kN/m$^2$ and $\Phi = 0^\circ$, determine the ultimate bearing capacity of the foundation.
Bearing capacity analysis

- From Terzaghi’s chart,

\[ N_c = 5.7, \quad N_q = 1 \quad \text{and} \quad N_v = 0 \]

\[ q_u = [(24)(5.7) \{1+(0.3)(2/4)\}] + [(20)(1)(1)] + [0] \]

\[ = 177.3 \text{ kN/m}^2 \]
Bearing capacity analysis

- A strip footing is 2 m wide and is founded at depth of 3 m in a soil of unit weight 19.3 kN/m$^3$ with $c = 10$ kN/m$^2$ and $\Phi = 25^0$. Using factor of safety = 3.0, determine the values of safe bearing capacity of the foundation.
Bearing capacity analysis

- From Terzaghi’s chart,
- $N_c = 25$, $N_q = 13$ and $N_v = 10$
- $q_u = [(10)(25)] + [(19.3)(3)(13)] + [(0.5)(19.3)(2)(10)] = 1200 \text{ kN/m}^2$
- $q_{\text{nett}} = q_u - \gamma D$
  \[= 1200 - [(19.3)(3)]\]
  \[= 1142.1 \text{ kN/m}^2\]
- $q_{\text{safe}} = q_{\text{nett}} / FS + \gamma D$
  \[= (1142.1)/(3) + (19.3)(3)\]
  \[= 439 \text{ kN/m}^2\]
Bearing capacity analysis

- A square foundation is 1.5 m x 1.5 m in plan. The soil supporting the foundation has a $c = 15.2 \text{ kN/m}^2$ and $\Phi = 20^0$. The unit weight of soil, $\gamma = 17.8 \text{ kN/m}^3$. Determine the design load on the foundation with a factor of safety 4. Assume the depth of the foundation is 1 m.
Bearing capacity analysis

From Terzaghi chart,

\[ N_c = 17.7, \quad N_q = 7.4 \quad \text{and} \quad N_\gamma = 5 \]

\[ q_u = [(1.3)(15.2)(17.7)] + [(17.8)(1)(7.4)] + [(0.4)(17.8)(1.5)(5)] \]

\[ = 535 \quad \text{kN/m}^2 \]

\[ q_{\text{nett}} = q_u - \gamma D \]

\[ = 535 - [(17.8)(1)] \]

\[ = 517.2 \quad \text{kN/m}^2 \]

\[ q_{\text{safe}} = q_{\text{nett}}/FS + \gamma D \]

\[ = (517.2)/ (4) + (17.8)(1) \]

\[ = 147.1 \quad \text{kN/m}^2 \]
Bearing capacity analysis

Design load,

\[ Q = q_{\text{safe}} \times \text{base area} \]

\[ = [(147)] [(1.5) (1.5)] \]

\[ = 330.75 \text{ kN} \]
Bearing capacity analysis

- Determine the breadth of a strip footing required to carry an inclusive load of 550 kN per metre run at a depth of 1.5 m in a fine soil with $c = 90 \text{kN/m}^2$ and unit weight of soil, $\gamma = 19 \text{kN/m}^3$. Assume the factor of safety is 3.
Bearing capacity analysis

\[ q_{\text{safe}} = \frac{Q}{\text{base area}} \]

\[ = \frac{550}{[(B) (1)]} \]

\[ = \frac{550}{B} \]

From Terzaghi chart,

\[ N_c = 5.7, \ N_q = 1 \text{ and } N_y = 0 \]

\[ q_u = [(90)(5.7)] + [(19)(1.5)(1)] + [0] \]

\[ = 541.5 \text{ kN/m}^2 \]

\[ q_{\text{nett}} = q_u - \gamma D \]

\[ = 541.5 - [(19) (1.5)] \]

\[ = 513 \text{ kN/m}^2 \]

\[ q_{\text{safe}} = \frac{q_{\text{nett}}}{FS} + \gamma D \]

\[ = \frac{(513)}{(3)} + (19) (1.5) \]

\[ = 199.5 \text{ kN/m}^2 \]
Bearing capacity analysis

Therefore,

\[ 550 / B = 200 \]

\[ B = 550 / 200 \]

\[ B = 2.75 \text{ m} \]

*Use breadth of a strip footing = 2.8 m*
Bearing capacity analysis

- **Skempton’s analysis**

- *Skempton (1951)* using *Terzaghi’s analysis* by showing that for a cohesive soil (\(\Phi = 0\)), the value of \(N_c\) can be estimated from *Skempton chart* or can be obtained from the formula

\[
N_c = 5 \left( 1 + 0.2 \frac{B}{L} \right) \left( 1 + 0.2 \frac{z}{B} \right)
\]

- with a limiting value of \(N_c = 7.5 (1 + 0.2B/L)\).
Bearing capacity analysis

$N_c$ Vs depth (after Skempton, 1951)
Bearing capacity analysis

- A rectangular foundation, 2m x 4m is to be founded at a depth of 1 m below the surface of soft clay with unit weight, $\gamma = 20 \text{kN/m}^3$. If the soil parameters are $c = 24 \text{kN/m}^2$ and $\Phi = 0^0$, determine the ultimate bearing capacity of the foundation.
Bearing capacity analysis

- From *Skempton chart*,

\[ N_q = 1 \text{ and } N_v = 0 \]

\[ = 5 \left[ \left( \frac{1}{2} + \frac{0.2}{4} \right) \left( \frac{1}{2} + \frac{0.2}{2} \right) \right] \]

\[ = 6.05 \]

\[ q_u = \left[ 24 \left( \frac{6.05}{2} \right) \left( \frac{1}{2} + \frac{0.3}{4} \right) \right] + \left[ 20 \left( \frac{1}{2} \right) \right] + [0] \]

\[ = 186.9 \text{ kN/m}^2 \]
Meyerhof’s analysis
Meyerhof’s analysis

- **Meyerhof (1963)** proposed the following general equation for $q_u$:

\[
q_u = cN_c S_c I_c D_c + \gamma z N_q S_q I_q D_q + 0.5 \gamma B N \gamma S \gamma I \gamma D \gamma
\]

where:

- $N_q = \tan^2 (45^0 + \Phi/2) \exp (\tan \Phi)$
- $N_c = (N_q - 1) \cot \Phi$
- $N_\gamma = (N_q - 1) \tan 1.4 \Phi$
- $D_c = 1 + 0.4(z/B)$
- $D_q = 1 + 2 \tan \Phi (1 - \sin \Phi)^2 (z/B)$
- $D_\gamma = 1$
- $I_c = I_q = (1 - \alpha/90^0)^2$
- $I_\gamma = (1 - \alpha/\Phi)^2$
Meyerhof’s analysis

<table>
<thead>
<tr>
<th>$\phi(^\circ)$</th>
<th>$N_c$</th>
<th>$N_q$</th>
<th>$N_\gamma$</th>
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<td>50</td>
<td>266.88</td>
<td>319.06</td>
<td>758.10</td>
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Meyerhof’s analysis

A rectangular foundation, 2m x 4m is to be founded at a depth of 1 m below the surface of soft clay with unit weight, γ = 20 kN/m$^3$. If the soil parameters are $c = 24$ kN/m$^2$ and $\Phi = 0^\circ$, determine the ultimate bearing capacity of the foundation. (F.S. = 3)
Meyerhof’s analysis

\( N_c = 5.14, N_q = 1 \) and \( N_\gamma = 0 \)

\[
S_c = 1 + \frac{B}{L} \cdot \frac{N_q}{N_c} \\
= (1) + \left(\frac{2}{4}\right) \left(\frac{1}{5.14}\right) = 1.1
\]

\[
S_q = 1 + \frac{B}{L} \tan \phi \\
= (1) + \left(\frac{2}{4}\right) (\tan 0^\circ) = 1
\]

\[
S_\gamma = 1 - 0.4 \frac{B}{L} \\
= (1) - \left(\frac{0.4}{2/4}\right) = 0.8
\]
Meyerhof’s analysis

\[ D_c = 1 + 0.4(z/B) \]
\[ = (1) + [(0.4) (1/2)] = 1.2 \]

\[ D_q = 1 + 2 \tan \Phi (1 - \sin \Phi)^2 (z/B) \]
\[ = (1) + [(2) (\tan 0^\circ) (1 - \sin 0^\circ)^2 (1/2)] = 1 \]

\[ D_\gamma = 1 \]
Meyerhof’s analysis

Due to no inclination load, therefore $\alpha = 0^0$

\[ I_c = I_q = (1 – \alpha/90^0)^2 \]
\[ = (1 – 0/90)^2 \]
\[ = 1 \]

\[ I_\gamma = (1 – \alpha/\pi)^2 \]
\[ = (1 – 0/\pi)^2 \]
\[ = 1 \]

\[ q_u = cN_c S_c I_c D_c + \gamma z N_q S_q I_q D_q + 0.5 \gamma B N_\gamma S_\gamma I_\gamma D_\gamma \]
\[ = 182.8 \text{ kN/m}^2 \]
Meyerhof’s analysis

\[ q_{\text{nett}} = q_u - \gamma D \]
\[ = 182.8 - [(20)(1)] \]
\[ = 162.8 \text{ kN/m}^2 \]

\[ q_{\text{safe}} = \frac{q_{\text{nett}}}{FS} + \gamma D \]
\[ = \frac{(162.8)}{(3)} + (20)(1) \]
\[ = 74.3 \text{ kN/m}^2 \]
Hansen’s analysis

- Hansen (1970) using Meyerhof’s analysis by suggested different value of $N_γ$ that can be obtained from the formula
- $N_γ = 1.5 (N_q - 1) \tan 1.1\Phi$
- which can be applies to a strip footing only.
Hansen’s analysis

- A strip footing is 2 m wide and is founded at depth of 3 m in a soil of unit weight 19.3 kN/m³ with $c = 10$ kN/m² and $\Phi = 25^0$. Using factor of safety $= 3.0$, determine the values of safe bearing capacity of the foundation.
Hansen’s analysis

From bearing capacity table,

\[ N_c = 20.72 \text{ and } N_q = 10.66 \]

\[ N_y = 1.5 (N_q - 1) \tan 1.1\Phi \]

\[ = 1.5 (10.66 - 1) \left[ \tan 1.1 (25) \right] \]

\[ = 7.54 \]

\[ S_c = 1 + \frac{B}{L} \cdot \frac{N_q}{N_c} \]

\[ \approx \infty \]

\[ = (1) + \left[ \frac{2}{10.66 / 20.72} \right] \]

\[ S_q = 1 + \frac{B}{L} \tan \phi \]

\[ = 1 \]

\[ S_y = 1 - 0.4 \frac{B}{L} \]

\[ = 1 \]
Hansen’s analysis

\[ D_c = 1 + 0.4(z/B) \]
\[ = (1) + [(0.4) (3/2)] \]
\[ = 1.6 \]

\[ D_q = 1 + 2 \tan f (1 - \sin \Phi)^2 (z/B) \]
\[ = (1) + [(2) (\tan 25^0) (1 - \sin 25^0)^2 (3/2)] \]
\[ = 1.8 \]

\[ D_v = 1 \]
Hansen’s analysis

Due to no inclination load, therefore: $\alpha = 0$

1. \( l_c = l_q = (1 - \alpha/90^0)^2 \)
   
   \[= (1 - 0/90)^2 \]
   
   \[= 1 \]

2. \( I_\gamma = (1 - \alpha/\Phi)^2 \)
   
   \[= (1 - 0/0)^2 \]
   
   \[= 1 \]

3. \( q_u = cN_c S_c I_c D_c + \gamma zN_q S_q I_q D_q + 0.5 \gamma BN_\gamma S_\gamma I_\gamma D_\gamma \)


   \[= 1588 \text{ kN/m}^2 \]
Hansen’s analysis

\[ q_{\text{nett}} = q_u - \gamma D \]
\[ = 1588 - [(19.3) (3)] \]
\[ = 1530 \text{ kN/m}^2 \]

\[ q_{\text{safe}} = \frac{q_{\text{nett}}}{FS} + \gamma D \]
\[ = \frac{(1530)}{(3)} + (19.3) (3) \]
\[ = 567.9 \text{ kN/m}^2 \]
Effect of ground water on bearing capacity

- If the water table is at or above the footing’s base, the soil’s submerged unit weight, $\gamma'$ (unit weight of soil, $\gamma_{sat}$ – unit weight of water, $\gamma_w$) should be used.

- If the water table is below the footing’s base, the water table assumed to have no effect and the soil’s full unit weight, $\gamma$ should be used.
Effect of ground water on bearing capacity

A strip footing of breadth 2.5 m is to be founded at a depth of 2 m in a well-drained compact sand having the unit weight, $\gamma_{\text{sat}} = 20 \text{ kN/m}^3$ with $c' = 0 \text{ kN/m}^2$ and $\Phi' = 35^0$. Assuming that the water table may rise to the surface and adopting a factor of safety of 3, determine the safe bearing capacity using Terzaghi’s analysis.
Effect of ground water on bearing capacity

\[ N_c = 57.8, \quad N_q = 41.4 \text{ and } N_\gamma = 42.4 \]

\[ q_u = cN_c + \gamma' zN_q + 0.5\gamma' B_N \gamma' \]

\[ \gamma' = \gamma_{sat} - \gamma_w \]

\[ q_u = [(0)] + [(20 - 9.81)(2)(41.4)] + [(0.5)(20 - 9.81)(2.5)(42.4)] \]

\[ = 1384 \text{ kN/m}^2 \]

\[ q_{nett} = q_u - \gamma'D \]

\[ = 1384 - [(20 - 9.81)(2)] \]

\[ = 1363.6 \text{ kN/m}^2 \]

\[ q_{safe} = \frac{q_{nett}}{FS} + \gamma'D \]

\[ = \frac{(1363.6)}{(3)} + (20 - 9.81)(2) \]

\[ = 474.9 \text{ kN/m}^2 \]
Effect of eccentric loading on bearing capacity

- If the foundation is subjected to eccentric load, the effective length, $L'$ and effective width, $B'$ should be used in the calculation.
- $L' = L - 2e; \ B' = B - 2e$
Effect of eccentric loading on bearing capacity

A continuous footing is 1.8 m wide and is founded at a depth of 0.75 m in a clay soil of unit weight, $\gamma = 20 \text{ kN/m}^3$ with $c = 30 \text{ kN/m}^2$ and $\Phi = 0^\circ$. The foundation is to carry a vertical line load which will act at a distance of 0.5 m from the centre. Determine the value of safe bearing capacity using Meyerhof’s analysis by taking factor of safety = 3.
Effect of eccentric loading on bearing capacity

From bearing capacity table,

\[ N_c = 5.14, \quad N_q = 1 \quad \text{and} \quad N_γ = 0 \]

Eccentricity of line load,

\[ e = 0.5 \text{ m} \]

Effective width of footing,

\[ B' = B - 2e \]
\[ = 1.8 - [(2)(0.5)] \]
\[ = 0.8 \text{ m} \]

\[ S_c = 1 + \frac{B}{L} \cdot \frac{N_q}{N_c} = (1) + [(0.8/20.72)(10.66/20.72)] = 1 \]

\[ S_q = 1 + \frac{B}{L} \tan \phi = 1 \]

\[ S_γ = 1 - 0.4 \frac{B}{L} \]
Effect of eccentric loading on bearing capacity

\[ D_c = 1 + 0.4(z/B) \]
\[ = (1) + [(0.4) (0.75/0.8)] \]
\[ = 1.17 \]

\[ D_q = 1 + 2 \tan f (1 - \sin \Phi)^2 (z/B) \]
\[ = (1) + [(2) (\tan 0^0) (1 - \sin 0^0)^2 (1/0.8)] \]
\[ = 1 \]

\[ D_y = 1 \]
Effect of eccentric loading on bearing capacity

Due to no inclination load, therefore $\alpha = 0^\circ$

\[ I_c = I_q = (1 - \alpha/90^\circ)^2 \]
\[ = (1 - 0/90)^2 \]
\[ = 1 \]

\[ I_\gamma = (1 - \alpha/\Phi)^2 \]
\[ = (1 - 0/0)^2 \]
\[ = 1 \]

\[ q_u = cN_c S_c I_c D_c + \gamma z N_q S_q I_q D_q + 0.5 \gamma B N \gamma S \gamma I \gamma D \gamma \]

\[ = [(30)(5.14)(1)(1)(1.17)] + [(20)(0.75)(1)(1)(1)] + [0] \]
\[ = 195.4 \text{ kN/m}^2 \]
Effect of eccentric loading on bearing capacity

\[ q_{\text{nett}} = q_u - \gamma D \]
\[ = 195.4 - [(20) (0.75)] \]
\[ = 180.4 \text{ kN/m}^2 \]

\[ q_{\text{safe}} = \frac{q_{\text{nett}}}{FS} + \gamma D \]
\[ = \frac{(165.4)}{(3)} + (20) (0.75) \]
\[ = 75 \text{ kN/m}^2 \]
Effect of eccentric loading on bearing capacity

- A foundation is 3 m wide and 9 m long is to be founded at a depth of 1.5 m in a deep deposit of dense sand. The angle of shearing resistance of the sand is 35° and its unit weight is 19 kN/m³. The foundation is to carry a vertical line load which will act at a distance of 0.5 m from the centre. Determine the safe bearing capacity using Meyerhof’s analysis if it subjected to a vertical line load of 220 kN per meter run, together with a horizontal line load of 50 kN per meter run acting at the base of the foundation. Use factor of safety 3.
Effect of eccentric loading on bearing capacity

From bearing capacity table,

\[ N_c = 46.12, \quad N_q = 33.3 \quad \text{and} \quad N_\gamma = 45.23 \]

Eccentricity of line load, \( e = 0.3 \text{ m} \)

Effective width of footing,

\[ B' = B - 2e \]

\[ = 3 - [(2) (0.3)] \]

\[ = 2.4 \text{ m} \]

\[ S_c = 1 + \frac{B}{L} \cdot \frac{N_q}{N_c} \]

\[ = 1 + \frac{2.4}{9} \cdot \frac{33.3}{46.12} = 1.19 \]

\[ S_q = 1 + \frac{B}{L} \cdot \tan \phi \]

\[ = 1 + \frac{2.4}{9} \cdot \tan 35^0 = 1.19 \]

\[ S_\gamma = 1 - 0.4 \cdot \frac{B}{L} \]

\[ = 1 - [(0.4) (2.4/9)] = 0.89 \]
Effect of eccentric loading on bearing capacity

\[ D_c = 1 + 0.4(z/B) \]

\[ = (1) + [(0.4) (1.5/2.4)] \]

\[ = 1.25 \]

\[ D_q = 1 + 2 \tan \Phi (1 - \sin \Phi)^2 (z/B) \]

\[ = (1) + [(2) (\tan 35^0) (1 - \sin 35^0)^2 (1.5/2.4)] \]

\[ = 1.16 \]

\[ D_y = 1 \]
Effect of eccentric loading on bearing capacity

The foundation is effectively acted upon by an inclined load whose angle to a vertical load, therefore

\[
\alpha = \tan^{-1}\left(\frac{\text{horizontal load}}{\text{vertical load}}\right)
\]

\[
\alpha = \tan^{-1}\left(\frac{50}{220}\right)
\]

\[
\alpha = 12.8^0
\]

\[
l_c = l_q = (1 - \alpha/90^0)^2
\]

\[
= (1 - 12.8/90)^2
\]

\[
= 0.74
\]

\[
l_g = (1 - \alpha/\Phi)^2
\]

\[
= (1 - 12.8/35)^2
\]

\[
= 0.4
\]

\[
q_u = c N_c S_c I_c D_c + \gamma z N_q S_q I_q D_q + 0.5 \gamma B N \gamma S \gamma I \gamma D \gamma
\]

\[
= [0] + [(19)(1.5)(33.3)(1.19)(0.74)(1.16)] + [(0.5)(19)(2.4)(45.23)(0.89)(0.4)(1)]
\]

\[
= 1336.6 \text{ kN/m}^2
\]
Effect of eccentric loading on bearing capacity

\[ q_{\text{nett}} = q_u - \gamma D \]
\[ = 1336.6 - [(19) (1.5)] \]
\[ = 1308 \text{kN/m}^2 \]

\[ q_{\text{safe}} = q_{\text{nett}} / \text{FS} + \gamma D \]
\[ = (1308) / (3) + (19) (1.5) \]
\[ = 464.5 \text{ kN/m}^2 \]
Settlement

Total settlement: 50 mm for sand; 75 mm for clay.

Differential settlement: 12 mm while the angular distortion 1/500.

Two types of settlement:

Immediate settlement: as soon as the load is applied.

Consolidation settlement: process of pore water pressure dissipation from soil.
Settlement

Elastic settlement (immediate settlement) (Harr, 1966)

\[ S_e = \frac{Bq_o}{E} (1 - \mu_s^2) \alpha \]

where:
B = width of footing
\( q_o \) = nett pressure from footing, kN/m²
\( E_s \) = modulus of elasticity of soil, kN/m²
\( \mu \) = possion’s ratio of soil
\( \alpha \) = factor depend on footing flexibility
Example Problem 4.1

A flexible foundation of size $4 \times 2$ m carrying a uniform load of 100 kPa is lying on a compressible (medium soft) clay layer of infinite thickness. Determine the average immediate settlement under this foundation.

Solution

For a medium soft clay layer: $E_s$ is estimated at 40 MPa, $\nu = 0.45$ (Table 4.3)

For flexible foundation $L/B = 2$, then $\alpha = 1.30$

$$S_i = \frac{B q_o}{E} (1 - \nu^2) \alpha = \frac{2 \times 100 \times (1 - 0.45)^2 \times 1.30}{40,000} = 5.2 \text{mm}$$
Example Problem 4.2

A foundation of size $4 \times 2$ m carrying a uniform load of 100 kPa is embedded at a depth of 1 m on a compressible clay layer of 5 m thick. A hard stratum lies below the clay layer. Determine the average immediate settlement under the corner of this foundation if the undrained modulus of the clay is 40 MN/m$^2$.

Solution

Foundation is embedded at 1 m, then the thickness of compressible layer below the footing is 4 m. Then:

$$\frac{H}{B} = \frac{4}{2} = 2, \quad \frac{L}{B} = 2 \quad \text{Table 4.2, } \alpha = 0.29$$

Immediate settlement

$$S_i = \frac{Bq_o}{E} \left(1-\nu^2\right)\alpha = \frac{2 \times 100 \times (1-0.5^2) \times 0.29}{40,000} = 10 \text{ mm}$$
Settlement

Consolidation settlement (primary settlement) (normally consolidation)

\[ S_c = m_v \Delta \sigma' H \]

where

- \( m_v \) = coefficient of volume compressibility
- \( m_v \) = stress increment
- \( H \) = soil thickness
- \( \Delta \sigma \) = soil thickness
Settlement

Consolidation settlement (primary settlement) (normally consolidation)

\[ S_c = \frac{C_c H_o}{1 + e_o} \log \left( \frac{\sigma'_o + \Delta \sigma}{\sigma'_o} \right) \]

where

- \( C_c \) = compression index
- \( H_o \) = soil thickness
- \( e_o \) = initial void ratio
  - = effective vertical stress
  - = stress increment
Example Problem 4.5

A sand layer of thickness 10 m overlays a clay layer of 8 m thick as shown in Figure P4.5. A sand layer exists at the bottom of the clay. Given $m_v = 0.83 \text{ m}^2/\text{MN}$, (a) calculate the consolidation settlement due to the decrease of ground water level from soil surface to a depth of 4 m; (b) What is the settlement of the sand layer if given $C_c = 0.50$ and $e_o = 0.95$ (the soil is normally consolidated).

\[
\begin{align*}
-18 & \quad \text{sand} \\
-14 & \quad \gamma_{sat} = \gamma_b = 20 \text{ kN/m}^3 \\
-10 & \quad \gamma_{sat} = 20 \text{ kN/m}^3 \\
-4 & \quad \text{Final GWT}
\end{align*}
\]

$H = 8 \text{ m}$
Solution

a. Find the increase in effective stress at the middle of the clay layer due to the decrease in ground water table

initial condition: \( \sigma'_o = (10 \times 18.5) + (4 \times 20) - (14 \times 9.8) \)
\( \sigma'_o = 185 + 80 - 137.2 \)
\( \sigma'_o = 127.8 \text{ kN/m}^2 \)

final condition: \( \sigma'_i = (10 \times 18.5) + (4 \times 20) - (10 \times 9.8) \)
\( \sigma'_i = 185 + 80 - 98 \)
\( \sigma'_i = 167 \text{ kN/m}^2 \)

Change in stress \( \Delta \sigma = \sigma'_i - \sigma'_o = 167 - 127.8 = 39.2 \text{ kN/m}^2 \)
Thus, \( S_c = m_y \Delta \sigma H \)
\[ = 0.83 \text{ m}^2/\text{MN} \times 0.0392 \text{ MN/m}^2 \times 8 \text{ m} \]
\[ = 0.26 \text{ m} \]
\[ = 26 \text{ cm or 260 mm}. \]

b. Use formula
\[ S_c = C_c \frac{H}{1 + e_o} \log \frac{\sigma'_o + \Delta \sigma}{\sigma'_o} \]
\[ S_c = 0.50 \frac{8}{1 + 0.95} \log \frac{167}{127.8} = 0.24 \text{ m} = 240 \text{ mm}. \]
Settlement of Over-consolidated Soil

\[ S_c = C_r \frac{H}{1+e_o} \log \frac{\sigma'_o + \Delta \sigma}{\sigma'_o} \]

\[ S_c = C_r \frac{H}{1+e_o} \log \frac{\sigma'_c}{\sigma'_o} + C_c \frac{H}{1+e_o} \log \frac{\sigma'_o + \Delta \sigma}{\sigma'_c} \]
A clay layer of thickness 3 m is overlain by a sand layer of thickness 6 m. The clay layer overlays another layer of sand. A footing of size 4×4 m was constructed at depth of 1.5 m on the sand layer giving an additional stress of 12 kN/m². Based on laboratory test, the clay layer has an initial void ratio of 1.03, pre consolidation pressure of 140 kPa, compression index 
\( C_c \) of 0.275 and 
\( C_r \) of 0.0575. Calculate the consolidation settlement of the clay layer due to footing load.

\[ \text{sand } \gamma_{sat} = \gamma_b = 18.3 \text{ kN/m}^3 \]

\[ \text{clay } \gamma_{sat} = 19 \text{ kN/m}^3 \]

\[ H = 3 \text{m} \]
Solution

Find the initial stress at the middle of clay layer $\sigma'_o$

$$\sigma'_o = (2 \times 18.3) + (4 \times (18.3 - 9.8)) + (1.5 \times 9.2)$$
$$\sigma'_o = 84.36 \text{ kN/m}^2$$

Given $\sigma'_c = 140 \text{ kN/m}^2$  \( \sigma'_o < \sigma'_c \) \( \Rightarrow \) over-consolidated clay

Given $\Delta\sigma = 12 \text{ kN/m}^2$  \( \Rightarrow \sigma'_o + \Delta\sigma = 84.36 + 12 = 96.36 < \sigma'_c \)

$$S_e = C_r \frac{H}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}$$

$$= 0.0575 \frac{3}{1 + 1.03} \log \frac{96.36}{84.36}$$

$$S_e = 0.005 \text{ m} = 5 \text{ mm}$$
Granular fill is placed on an extended area to consolidate a soft clay layer of 8 m thick. The unit weight of fill ($\gamma_f$) is 20 kN/m$^3$ and the average thickness of the fill is 3.5 m. Ground water level coincide the ground surface. The saturated unit weight of the clay layer is 18 kN/m$^3$. The results of consolidation test is shown in Table P4.7. (a) Construct the $e - \log p'$ curve and (b) use the curve to evaluate compression index $C_c$, recompression index $C_r$, and the pre consolidation pressure ($\sigma'_c$) (c) Calculate the settlement of the clay layer due to the weight of the fill.

Table P4.7

<table>
<thead>
<tr>
<th>Stress (kPa)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>80</th>
<th>160</th>
<th>320</th>
<th>640</th>
<th>160</th>
<th>40</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void ratio</td>
<td>2.855</td>
<td>2.802</td>
<td>2.793</td>
<td>2.769</td>
<td>2.631</td>
<td>2.301</td>
<td>1.939</td>
<td>1.576</td>
<td>1.314</td>
<td>1.375</td>
<td>1.464</td>
<td>1.589</td>
</tr>
</tbody>
</table>
Plot the $e - \log p'$ curve (Figure P4.7) to get $C_c$, $C_r$, and $\sigma'_c$

$$C_c = \frac{2.68 - 1.53}{\log\left(\frac{400}{40}\right)} = 1.15 \quad \sigma'_c = 38 \text{ kPa}$$

$$C_r = \frac{1.15 - 1.13}{\log\left(\frac{100}{10}\right)} = 0.21 \quad \text{Void ratio corresponds to } \sigma'_c \text{ is 2.68}$$

(c) Find the initial stress at the middle of clay layer $\sigma'_o$

$$\sigma'_o = 4 \times (18 - 9.8) = 32.8 \text{ kN/m}^2$$

Given $\sigma'_c = 38 \text{ kN/m}^2 \quad \sigma'_o < \sigma'_c \rightarrow \text{the soil is over-consolidated}$

Increase in stress at depth of 4 m due to the weight of fill

Surcharge load due to fill is $3.5 \times 20 = 70 \text{ kN/m}^2$

$$\sigma'_o + \Delta \sigma = 32.8 + 70 = 102.8 \text{ kN/m}^2 \quad > \sigma'_c$$
Find settlement

\[ S_c = C_r \frac{H}{1+e_o} \log \frac{\sigma'_c}{\sigma'_o} + C_c \frac{H}{1+e_o} \log \frac{\sigma'_o + \Delta \sigma}{\sigma'_c} \]

\[ S_c = 0.21 \frac{8}{1+2.855} \log \frac{38}{32.8} + 1.15 \frac{8}{1+2.68} \log \frac{102.8}{38} \]

\[ S_c = 0.0375 + 1.41 = 1.45 \text{ m} \]