UNIVERSITI TEKNOLOGI MALAYSIA
SEMESTER II, SESSION 2014/2015
SSCE/SSE 2193: ENGINEERING STATISTICS

Chapter 7 Exercises

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1. A new type of spring is being tested. Ten weights x_i are hung and the spring length y_i is measured for each of the weights. The weights are measured in pounds and the lengths are measured in inches. The following are the results;

Weight, x	1.5	1.8	2.0	2.1	2.4	2.5	2.8	3.0	3.1	3.3
Length, y	2.4	2.7	2.9	3.1	3.3	3.4	3.7	4.0	4.2	4.5

Assume the weight and the spring length are normally distributed.

   (a) Fit a linear regression model for the data.
   (b) Predict the spring length when the weight is 2.95.
   (c) Can we make a conclusion that there exists a linear relationship between the weight and the length at significance level 0.01?
   (d) Measure the strength of the relationship between weight and spring length.

2. An auto manufacturing company wanted to investigate how the price of one of its car models depreciates with age. They took a sample of twelve cars of this model and collected the following information on the ages (in years) and prices (in RM) of these cars. Assume the age and the price are normally distributed.

Age	Price (in thousand)
6	48
3	96
8	15
2	120
6	52
4	89
8	18
5	66
9	20
3	95
4	81
5	67

   (a) Find the estimated regression line to fit the data using the method of least squares.
   (b) If the car is 7 years old, what is the depreciated price?
   (c) Do the data support the existence of a linear relationship between age and price? Test using $\alpha = 0.05$.
   (d) Find the Pearson correlation coefficient. What can you infer from the value?

3. For many chemicals, the amount that will dissolve in a given liter of water depends on the temperature. The number of grams of a certain chemical dissolved per liter of water (y) in terms of the temperature, x (in ° C) are observed. The results are tabulated as follows. Assume that number of grams of a certain chemicals dissolved and temperature are normally distributed.

x	1	2	3	4	5	6	7	8
y	0.2	0.23	0.31	0.34	0.38	0.4	0.45	0.52
x	10	12	15	18	22	24	30	34
y	0.6	0.7	0.8	0.85	0.88	0.92	0.96	0.99

(a) Find a regression line relating the number of grams of a certain chemicals dissolved per liter of water to temperature.

(b) Test for significance of regression using $\alpha = 0.05$. What conclusions can you draw?

(c) Estimate the number of grams of a certain chemicals dissolved if the temperature is $26^0 C$.

(d) Find the Pearson correlation coefficient and comment on your answer.

4. An estimate of a linear model $Y = \alpha + \beta x + \epsilon$ obtained in an experiment is $\hat{y} = 4.9997 + 0.2046x$ with $n = 20$, $r = 0.95$ and $Var(\hat{\beta}) = 0.0248$ where $\hat{\beta}$ is the estimate of β and r is the correlation coefficient.

   (a) Predict the value of Y if x is 1.5.

   (b) What does the value $r = 0.95$ show?

   (c) At significance level $\alpha = 0.01$, test the following hypotheses:

$$H_0 : \beta = 0.3$$
$$H_1 : \beta < 0.3$$

5. An estimate of a linear model $Y = \alpha + \beta x + \epsilon$ obtained in an experiment to study the relationship between the number of pounds of fertilizer (x) and the yield of tomatoes in bushels (y) is $\hat{y} = 5.6 + 0.07x$ with $n = 12$, $r = 0.21$ and $Var(\hat{\beta}) = 0.127$ where $\hat{\beta}$ is the estimate of β and r is the correlation coefficient.

   (a) What are the slope and intercept of the least squares line?

   (b) Predict the number of tomatoes yield if 25 pounds of fertilizer was used.

   (c) What does the value $r = 0.21$ says?

   (d) At significance level $\alpha = 0.05$, test the following hypotheses:

$$H_0 : \beta = 0$$
$$H_1 : \beta \neq 0$$

6. The following table shows the mean speeds of 12 motorcycles and the amount of traffic fines (RM) they pay during a period of a month.

Mean Speed (km/h)	100	120	115	112	108	105	116	121	125	118	115	122
Traffic fines (RM)	300	350	320	315	310	305	340	360	380	345	320	330

   Assuming that both mean speeds and the traffic fines distribute normally,

   (a) Obtain the linear regression of the fines against the mean speeds.

   (b) The mean speed of a motorcycle is 106 km/h. What is the estimated fine for that motorcyclist in a month?

   (c) At the significant level of 0.05, test the hypothesis that there exists a positive relation of the fines against the mean speeds.

   (d) Find the Pearson correlation index between the mean speeds and fines.

7. The following table lists the measurements of the air velocity and evaporation coefficient of burning fuel droplets in an impulse engine:

Air Velocity (cm/sec)	**Evaporation Coefficient (mm^2/sec)**
20	1.80
60	3.50
100	3.70
140	5.60
180	7.50
220	7.80
260	9.80
300	11.60
340	13.70
380	16.50
420	18.60
460	19.50

(a) Fit a straight line to these data by using the method of least squares.

(b) Estimate the evaporation of a droplet when the air velocity is 190 cm/sec.

(c) Test whether evaporation coefficient of burning fuel droplets in an impulse engine is positively related to the measurements of the air velocity at $\alpha = 0.10$ significance level.

(d) Find the Pearson correlation coefficient. Give your comment.

8. A research department in a university wants to find out if the starting monthly salaries (in RM100) of the recently university graduates in engineering is related to their CGPA. The excel output is as follows. Assume that the data is normally distributed.

Regression Statistics						
Multiple R	0.68714828					
R Square	0.47217276					
Adjusted R Square	0.43157067					
Standard Error	3.10422785					
Observations	15					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	1	112.0623361	112.0623	11.62927	0.004650548	
Residual	13	125.2709972	9.636231			
Total	14	237.3333333				
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	8.42692782	7.153116349	1.178078	0.259882	-7.026440504	23.8803
CGPA	7.74267638	2.270465267	3.410172	0.004651	2.837634387	12.64772

(a) Find the estimated regression line to fit the above data.

(b) Predict the starting monthly salary if the CGPA is 3.6.

(c) Does the data support the existence of a linear relationship between starting salaries and CGPA? Test using $\alpha = 0.05$.

(d) Find the Pearson correlation coefficient. What can you infer form the value?

9. A manufacturing company bought a new cutting tool from company A and wanted to investigate the useful life (in hours) related to the speed at which the tool is operated. The Excel output follows for useful life of the tool (in hours) and speed (meters per minutes)

Regression Statistics	
Multiple R	0.93391062
R Square	0.87218904
Adjusted R Square	0.86420085
Standard Error	0.54725308
Observations	18

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	32.69933611	32.6993	109.1849	1.48547E-08
Residual	16	4.791775	0.29949		
Total	17	37.49111111			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	8.32975	0.395577303	21.0572	4.32E-13	7.491163585	9.168336
speed	-0.085775	0.008208796	-10.4492	1.49E-08	-0.103176871	-0.06837

(a) Build a linear model between useful life and speed.

(b) Predict the useful life if the speed is 55 m/mins.

(c) Test on the validity of the model build in part (a). Use $\alpha = 0.01$.

(d) Find the correlation. Interpret the value.

10. The following output from Excel gives information on the engine powers x (in kilowatt) and the maximum speed y (km/hour) for 12 racing cars.

Regression Statistics						
Multiple R	0.74264483					
R Square	0.55152134					
Adjusted R Square	0.50667348					
Standard Error	9.79131186					
Observations	12					
ANOVA						
	df	SS	MS	F	Significance F	
Regression	1	1178.969	1178.969	12.29761	0.005661885	
Residual	10	958.6979	95.86979			
Total	11	2137.667				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	-1.7987195	45.03925	-0.03994	0.96893	-102.1524239	98.55498
Power	2.37935174	0.678498	3.506794	0.005662	0.867564666	3.891139

(a) Find the least square estimates of the regression line for the engine power against the maximum speed.

(b) What does the estimate of β imply?

(c) What is the predicted maximum speed if the engine power is 72 kilowatt?

(d) Is there any evidence that the data strongly suggest a linear association between the engine power and the maximum speed at the $\alpha = 0.01$ significance level.

(e) Find the correlation between the engine power and the maximum speed. Explain your answer.

Answers

1.	*a.* $\hat{\alpha} = 0.6623$, $\hat{\beta} = 1.1256$	*b. 3.98*	*c. Reject* H_0	*d. 0.9939*
2.	*a.* $\hat{\alpha} = 143.731$, $\hat{\beta} = -15.202$	*b. 37.317*	*c. Reject* H_0	*d.* -0.9859
3.	*a.* $\hat{\alpha} = 0.2757$, $\hat{\beta} = 0.0255$	*b. Reject* H_0	*c.* 0.9387	*d.* 0.9502
4.	*a. 5.3066*	*c. Accept* H_0		
5.	*a.* $\hat{\alpha} = 5.6$, $\hat{\beta} = 0.07$	*b. 3.85*	*d. Accept* H_0	
6.	*a.* $\hat{\alpha} = 2.8144$, $\hat{\beta} = 2.8622$	*b. 306.2076*	*c. Reject* H_0	*d. 0.8742*
7.	*a.* $\hat{\alpha} = 0.0016$, $\hat{\beta} = 0.0415$	*b. 7.8866*	*c. Reject* H_0	*d. 0.9901*

8. *a.* $Starting\ Monthly\ Salary = 8.4269 + 7.7427\ CGPA$
b. RM36.3 hundreds, or RM3630
c. Yes because Significance $F < 0.05$*, or P-value for CGPA coefficient* < 0.05
d. $r =$ Multiple R $= 0.6871$*; moderately strong positive linear correlation*

9. *a.* $Life = 8.32975 - 0.085775\ Speed$
b. 3.6121 hours
c. Yes because Significance $F < 0.01$*, or P-value for Speed coefficient* < 0.01
d. $r =$ Multiple R $= 0.9339$*; very strong positive linear correlation between Useful Life and Speed.*

10. *a.* $Max.\ Speed = -1.7987 + 2.3794\ power$
b. $\hat{\beta}$ *implies that a unit increase in power would lead to about 2.3794 units increase in Maximum Speed.*
c. 169.5146 *km/h*
d. Yes because Significance $F < 0.01$*, or P-value for Power coefficient* < 0.01
e. $r =$ Multiple R $= 0.7426$*; moderately strong positive linear correlation or association between Maximum Speed and Power.*