

SEMESTER 2, SESSION 2014/2014

SSCE2193 Engineering Statistics

Supplementary Exercise 2: Sampling Distributions

1. a) A blood test was conducted to determine the proportion of cattles infected by foot and mouth disease (FMD) from Jeram Ecofarm. It was found that 25% of the cattle have traces of FMD. If a random sample of 50 cattles is selected, what is the probability that
- i. not less than 35% has FMD. [Ans: 0.0708]
 - ii. between 22% and 28% has FMD. [Ans: 0.2586]
- b) A company XYZ produces two types of cables, type A and type B. Type A has a mean breaking strength of 4000 N and standard deviation of 300 N. Type B has a mean breaking strength of 4050 N and standard deviation of 200 N. 100 cables of type A and 50 cables type B are randomly selected. What is the probability that the difference between mean breaking strengths for both types of cable is less than 50 N? [Ans: 0.4925]

(Final, Sem. 2, 07/08)

2. Two independent experiments are being run in which two different types of paints are compared. 18 specimens are painted using type A and type B, and the drying time in hours is recorded on each specimen. The population standard deviations are both known to be 1.0 hour. Assuming the mean drying time is equal for the two types of paints, find the probability that the
- i. mean drying time of type A is smaller than the mean drying time of type B? [Ans: 0.5]
 - ii. mean drying time of type A will have at least one hour more than the mean drying from type B? [Ans: 0.0013]

(Final, Sem 1, 12/13)

3. From a local telecommunication office database, it is found that 45% and 30% of households in Town A and B respectively subscribe to Streamyx. 200 households from Town A and 250 from Town B were selected at random and interviewed.
- i. What is the probability that the sample proportion of households from Town A who subscribe to Streamyx exceeds the sample proportion of those from Town B by at least 16%. [Ans: 0.4129]
 - ii. Suppose the number of interviewed households from Town B decreases to n. Find n such that the probability of more than $(0.38 - 12n)100\%$ of interviewed households from Town B subscribing to Streamyx is 0.015. [Ans: 154.525]

(Final, Sem 2, 09/10)

4. a) A city is planning to build a manufacturing factory. According to a survey by a local newspaper, 53% of men while 42% of women in the city favour the construction of the factory. Suppose that a random sample of 50 men and a random sample of 50 women from

the city were taken. Find the probability that the difference between the sample proportions of both sexes in the city who favor the construction of the factory is less than 0.2.

- b) A random sample X of size 40 is taken from a Poisson distribution with mean 54. Another random sample Y of size 35 is taken from a Poisson distribution with mean 58. What is the probability that the sample mean X exceeds the sample mean Y ?

(Final, Sem 2, 10/11)

5. The average speed of a very large population of imported cars driven on a stretch of North South highway is 145 km/h with a standard deviation of 20 km/h.

- i. 100 imported cars driven on a stretch of North-South highway are randomly drawn from this population. Find the probability that the sample average speed exceeds 150 km/h.

[Ans: 0.00621]

- ii. How many imported cars need to be randomly selected so that the probability of the sample average speed being less than 141 km/h is 0.0139?

[Ans: 121]

(Final, Sem 1, 11/12)

6. Weekly losses of worker-hours due to accidents in industrial plants before and after the operation of a certain safety program are assumed to follow normal distributions with the following means and standard deviations (in hours):

	Before	After
Mean	80	65
Standard Deviation	12	16

Ten industrial plants are selected at random. Based on sample means of weekly losses of worker-hours due to accidents before and after the operation of the safety program in these industrial plants, find the probability that the safety program is effective. [Ans: 0.9912]

(Final, Sem 2, 12/13)

7. If all possible samples of size 16 are drawn from a normal population with mean equals 50 and standard deviation equals 5, what is the probability that a sample mean \bar{X} will fall in the interval from $\mu_{\bar{X}} - 1.96\sigma_{\bar{X}}$ to $\mu_{\bar{X}} - 0.4\sigma_{\bar{X}}$?

(Final, Sem 2, 11/12)

8. In a production front line, a sample of 15 items is selected at random every hour to be checked by a quality control (QC) inspector. If more than 2 items in the sample selected at any hour are defective, the production will be stopped immediately for machine adjustment. The probability that an item is defective is 5%. Assume that the QC starts his work at 8 a.m. and the items are independent.

- (i) One sample is checked at 9 a.m. What is the probability that the machine needs an adjustment?

- (ii) Hence, find the probability that the fourth check made is the 2nd time the machine has to be stopped for adjustment.

(Final, Sem 2, 11/12)

9. a) The length of stay for foreign engineering students in Malaysian higher learning institutions may due to several factors such as type of registration, study completion time and financial support. A study is conducted in a local university college shows that the length of stay of a foreign engineering student from country A is approximately normally distributed with mean 1050 days and standard deviation of 25 days. A foreign engineering student from country A is selected at random, what is the probability that her/his stay exceeds three years? Assume 1 year = 365 days. [Ans: 0.0359]
- b) Lifetime of a head projector light bulb follows normal distribution with mean $\mu = 28$ hours and standard deviation σ hours.
- A random sample of size $n = 25$ is drawn from the population of light bulbs. If $\sigma = 0.5$ hours, what is the probability that the sample mean, \bar{X} will exceed $\mu + 0.5\sigma$? [Ans: 0.00621]
 - A random sample of size n is to be drawn from that population. Determine the value of n if, after that sample has been drawn, we want the difference between the sample mean, \bar{X} and the population mean, μ is less than 0.25σ with probability 0.95 [i. e. $P(|\bar{X} - \mu| < 0.25\sigma) = 0.95$]. [Ans: $n \approx 64$]

(Test 1, Sem 1, 12/13)

10. From a study by an independent agency, 10% of housing developers in state A fail to complete a housing project within a specified time period. In order to verify this, a sample of 60 housing developers from state A is selected randomly.
- Find the probability that 4% of housing developers in state A fail to meet the completion time. [Ans: 0.0520]
 - From past record, 12% of housing developers in state B fail to meet the completion time. If the same sample size is taken, find the probability that housing developers in state B have better performance than housing developers in state A. [Ans: 0.3632]

(Test 1, Sem 2, 12/13)

11. A random sample of 100 engineering students was chosen from Univesiti Teknologi Malaysia City Campus, Kuala Lumpur. 40% of the students in the university work part time. The university authority wishes to know the following:
- What percentage of the engineering students in the sample can be expected to have part time jobs? [Ans: 0.4]
 - How much does the sample proportion vary from the population proportion? [Ans: 4.9%]
 - The probability of observing a sample proportion that is greater than 0.5. [Ans: 0.0162]

(Test 1, Sem 1, 08/09)

12. Transnational fleet of buses uses two brands of tyres Rockstone and Bridgelane. It is known that the mean distance travelled before the tyres wear out is 36,400 km for Rockstone tyre with standard deviation of 195 km while the mean distance travelled before the tyre wear out is 36,300 km for Bridgelane tyre with standard deviation of 254 km. A random sample of 25 tyres of Rockstone and 36 tyres of Bridgelane are taken where both random samples are assumed to come from normal distributions.

$$\bar{X}_A \sim N\left(36400, \frac{195^2}{25}\right) \text{ and } \bar{X}_B \sim N\left(36300, \frac{254^2}{36}\right)$$

$$\bar{X}_A - \bar{X}_B \sim N(100, 3313.1111)$$

- a. What is the probability that the difference between the mean distance travelled before the tyres wear out is at least 250 km? [Ans: 0.0045]
- b. What is the probability that the mean distance travelled before the tyres wear out is longer for Rockstone than that of Bridgelane? [Ans: 0.9591]

(Test 1, Sem 1, 08/09)

13. From previous record, 85% candidates passed the driving license test at Driving Learning Centre. The test is held every month. 80 candidates take the test in September. What is the probability that

- i. exactly 85% of the candidates passed the test? [Ans: 0.1974]
- ii. less than 10% failed the test? [Ans: 0.0793]

(Test 1, Sem 1, 12/13)

14. Astro electronics inspect weld strength in their welding procedure. Weld strength is measured by a pull test to destruction. Previous records indicate that the mean weld strength is 5.42 with a standard deviation of 1.4.

- a. Assuming that the weld strength is normally distributed, calculate the probability of weld strength being not more than 5. [Ans: 0.3821]
- b. A random sample of 40 such items are selected,
- i. obtain the sampling distribution of the mean weld strength.
- ii. find the new sample size if the probability that the mean weld strength will be less than 4.82 is 0.99. [Ans: 29]

(Test 1, Sem 2, 12/13)

15. Given a random sample of X_1, X_2, \dots, X_{40} , taken from a population which is Poisson distribution with mean 3.5. What is the probability that the sample mean is between 3.4 and 4.3, inclusive? [Ans: 0.6296]

(Test 1, Sem 1, 09/10)

16. The breaking strength X of a certain rivet used in a machine engine has a mean of 5000 psi and standard deviation of 400 psi. A random sample of n rivets is taken. What sample size n would be necessary in order to have $P(4900 < \bar{X} < 5000) = 0.4987$. [Ans: 144]

(Test 1, Sem 1, 09/10)

17. Intravenous fluid bags are filled by an automated filling machine. Assume that the fill volumes of the bags are normal random variables with a standard deviation of 0.10 fluid ounces.

- What is the standard deviation of the sample mean fill volume of 25 bags? [Ans: 0.02]
- If the mean fill volume of the bags is 6.16 ounces, what is the probability that the sample mean fill volume of 25 bags is more 6.10 ounces? [Ans: 0.9987]
- Find the new mean fill volume if the probability that the sample mean fill volume of 25 bags below 6.11 ounces is 0.04. [Ans: 6.145]

(Test 1, Sem 1, 10/11)

18. A manufacturing of semiconductor chips produces 2% defective chips. Assume that the chips are independent and a random sample of 500 chips is selected. What is the probability that

- more than 1% chips are defective. [Ans: 0.9251]
- at least 98.5% chips are non-defective. [Ans: 0.2611]

(Test 1, Sem 1, 10/11)

19. A random variable X , representing the number of strawberries in a strawberry puff, has the following probability distribution:

x	4	5	6	7
$P(X = x)$	0.2	0.4	0.3	0.1

- Show that the mean and variance for X are 5.3 and 0.81 respectively.
- Next, determine the mean and variance for the sample mean, \bar{X} from random samples of 36 strawberry puffs. [Ans: 5.3, 0.0225]
- Hence, find the probability that the average number of strawberries in 36 strawberry puffs will be less than 5.5. [Ans: 0.9082]

(Test 1, Sem 1, 11/12)

20. SJF Corporation manufactures CDs. The machine that is used to make these CDs is known to produce 6% defective CDs. A quality control inspector selects a sample of 100 CDs every

week and inspects them for being good or defective. If 8% or more of the CDs in the sample are defective, the process is stopped and the machine is readjusted. What is the probability that, based on a sample of 100 CDs, the process will be stopped to readjust the machine?

[Ans: 0.2643(from stat. table)/0.2638(from calculator)]

(Test 1, Sem 1, 11/12)

21. A technician has purchased 36 resistors from vendor 1 and 40 resistors from vendor 2. Let X_1 represents the vendor 1 observed resistances, which are assumed to be normally and independently distributed with mean 40 ohms and standard deviation 2.0 ohms. Similarly, let X_2 represents the vendor 2 observed resistances, which are assumed to be normally and independently distributed with mean 42 ohms and standard deviation of 3.0 ohms.

a. Determine the sampling distribution for $\bar{X}_1 - \bar{X}_2$.

b. Find the probability that the difference between the mean resistance for sample 1 and the mean resistance for sample 2 is not more than 0.3. [Ans: 0.99996]

(Test 1, Sem 1, 11/12)
