SSCE2193 Engineering Statistics

Supplementary Exercise 1: Probability Distributions

1.
   a. Find a formula for the probability distribution of the random variable *X* representing the outcomes when a dice is rolled once.
   b. A consignment of 10 printer units contains 3 defective units. A computer centre makes a random purchase of 5 of the units. If *x* is the number of defective units purchased by the computer centre, find the probability distribution of *X*.
   c. The probability distribution of *X*, the number of imperfections per 100 meters of a wire cable in continuous rolls of uniform width, is given by

x	0	1	2	3	4	5 and more
f(x)	0.32	0.25	0.18	0.11	0.08	0.06

   Construct the cumulative distribution of *X*.

   (SSCE 2193, Final, Sem. 1, 13/14)

2. In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The percentage that a person carries the gene is 10.5%.

   a. 25 volunteers are chosen randomly. What is the probability that at least two volunteers have the gene that can increase the risk for the disease?
   b. What is the probability that four or more people have to be tested until two with the gene are detected?
   c. How many people are expected to be tested before two with the gene are detected?

   (SSCE 2193, Final, Sem. 1, 12/13)

3. The time between telephone short messages system (SMS) received by TeCel Mobile SMS services is exponentially distributed with a mean of 10 seconds.
   a. What is the probability that there are not more than four SMSs within one minute?
   b. If there is no SMS received after the first minute of starting operation, what is the probability that the network provider has to wait for another 40 seconds for the first SMS?

   (SSCE 2193, Final, Sem. 1, 12/13)

4. Optical fibers of a certain type are cut into lengths of 1 km and collected together into a bundle of 20 fibers. The probability of an individual fiber being perfect is 0.94. Find the probability that
   a. at least 18 of the fibers in a bundle are perfect.
   b. less than 4 of the fibers in a bundle are faulty.

   (SSCE 2193, Test 1, Sem. 2, 12/13)

5. A server on a computer website can handle up to 8 hits per minute before there is a clear slow in response time. If hits arrive randomly at a rate of 240 per hour, what is the probability that a slow response occur in one minute period?

(SSCE 2193, Test 1, Sem. 2, 12/13)

6. The time (in years) to failure of an inverter drive system in a washing machine follows an exponential distribution with mean 10. Find the probability that the inverter drive system lifetime
   a. is between 10 and 15 years.
   b. exceeds 12 years given it has exceeded 8 years.

(SSCE 2193, Test 1, Sem. 2, 12/13)

7. In a production front line, a sample of 15 items is selected at random every hour to be checked by a quality control (QC) inspector. If more than 2 items in the sample selected at any hour are defective, the production will be stopped immediately for machine adjustment. The probability that an item is defective is 5%. Assume that the QC starts his work at 8 a.m. and the items are independent.
   a. One sample is checked at 9 a.m. What is the probability that the machine needs an adjustment?
   b. Hence, find the probability that the fourth check made is the 2nd time the machine has to be stopped for adjustment.

(SSE 2193, Final, Sem. 2, 11/12)

8. Aliyaa is a basketball player from Sekolah Tun Fatimah, Johor Bahru. She is a 70% throw shooter which means her probability of making a free throw is 0.70. During National Boarding School tournament, calculate the following probability that
   a. Aliyaa makes her third free throw on her fifth shot.
   b. Aliyaa makes her first free throw on her fifth shot.

(SSCE 2193, Test 1, Sem. 1, 10/11)

9. An exponential distribution is a continuous probability distribution which describes a process in which events occur continuously and independently at a constant average rate. If a random variable X has an exponential distribution with mean 60,
   a. write down the probability density function for X.
   b. Hence, calculate that the probability that X is less than 56.

(SSCE 2193, Test 1, Sem. 1, 10/11)

10. Medical employees at a hospital in Johor Bahru have determined that the average time between patient arrivals at the emergency room is exponentially distributed with a mean time of 11 minutes. On a given day, it has been 11 minutes since a patient has arrived. What is the probability that the patient will arrive within the next 5 minutes.

(SSCE 2193, Test 1, Sem. 1, 10/11)

11. A quality assurance officer makes a regular check on batches that consist of 30 coil springs each, from a production process, for test of conformance to customer requirement. The mean number of nonconforming coil springs in a batch is 3. Assume that the number of nonconforming coil springs in a batch follows a binomial distribution.

a. The officer checked one batch. What is the probability that he found at least 2 coil springs that are nonconforming?

b. A batch is rejected if at least two coil springs in the batch are nonconforming. What is the probability that the fifth batch checked by the officer is the second batch that is rejected?

<div align="right">(SSCE 2193, Test 1, Sem. 1, 11/12)</div>

12. The time between arrivals of buses at a particular bus stop in a suburban area is exponentially distributed with a mean of 15 minutes.

a. What is the probability that a passenger has to wait longer than 1 hour for a bus at the bus stop?

b. Suppose a passenger has already been waiting at the bus stop for one hour, what is the probability that a bus arrives within the next 15 minutes?

c. Determine T such that the probability that the passenger waits less than T minutes is 0.5.

<div align="right">(SSCE 2193, Test 1, Sem. 1, 11/12)</div>

13. An experiment is conducted to select a suitable catalyst for the production of a dispersant for cleaning oil spill in Straits of Malacca. Suppose Ir. Aziz, the chemical engineer randomly selected three catalysts for testing from 10 different proposed catalysts. Four of the catalysts have high acidity level and six of the catalysts have low acidity level. Calculate the probability that not more than one high acidity level catalyst is selected.

<div align="right">(SSCE 2193, Test 1, Sem. 1, 12/13)</div>

14. Potholes on a highway can be a serious problem and are in constant need of repair. With a particular type of terrain and make of concrete, past experience suggests that, on the average, 2 potholes per kilometer after a certain amount of usage. It is assumed that the Poisson process applies to the random variable for the number of potholes.

a. What is the probability that there will be between 3 and 9 potholes in a given section of 5 km.

b. What is the probability that there will be more than 3 km section before the next pothole is found.

<div align="right">(SSCE 2193, Test 1, Sem. 1, 12/13)</div>

15. A corporation buys motors for electric fans from company M that guarantees 95% of its motors are non-defective and it will replace all defective motors at no cost. The motors are received in large shipments. Assume that motor selections are independent events.

a. What is the probability that the eighth motor selected from a shipment is the third motor found to be defective?

b. The quality control department at the corporation randomly selects 20 motors from each shipment and inspects them for being good or defective. If the sample contains more than two defective motors, the entire shipment is rejected. What is the probability that a given shipment of motors received by the corporation will be rejected?

<div align="right">(SSCE 2193, Test 1, Sem. 1, 12/13)</div>

16. Despite various efforts taken by the quality control department, the aluminum rod made at XYZ Company always contains several defects. A certain type of aluminum rod made at this company contains an average of 2.2 defects per 100 meters.
    a. Let a random variable X denotes the number of defects that will occur in this 100-meter aluminum rod. Write the probability distribution of X.
    b. Find the probability that no defect will be found in a piece of 300 meters of this aluminum rod.
    c. Find the probability that the number of defects in a given 200-meter piece of this aluminum rod will be more than 5.
    d. Find the mean and standard deviation of the probability distribution developed in part (c).

    (SSE 2193, Final, Sem. 1, 11/12)

17. The transmission on a model of a specific hybrid car has a warranty for 80,000 km. It is known that the life of such a transmission has a normal distribution with a mean of 150,000 km and a standard deviation of 27,000 km.
    a. What percentage of the transmission will fail before the end of the warranty period?
    b. What percentage of the transmission will be good for more than 200,000 km?

    (SSE 2193, Final, Sem. 1, 11/12)

18. A company has an inspection system for the batches of laptops purchased from vendors. A batch typically contains 20 laptops. In the inspection system, a random sample of 5 laptops are selected and tested. Suppose there are 2 faulty laptops in the batch of 20, what is the probability that all laptops are in good conditions?

    (SSCE 2193, Test 1, Sem. 2, 11/12)

19. The number of failures for a certain machine is a Poisson random variable with a mean of 1 per 100 samples. If the machine usually processes 500 samples per day, find the expected number of failures per day. Based on that value, what is the probability that the number of failures per day is at least 10?

    (SSCE 2193, Test 1, Sem. 2, 11/12)

20. The lifetime of a particular energy saving light bulbs, also known as Compact Fluorescent Lamps (CFLs), has an exponential distribution with mean lifetime 5000 hours. A CFL is selected at random.
    a. Find the probability that the CFL is still working after 6000 hours.
    b. Given that it is still working after 6000 hours, what is the probability that it will fail to work before 7000 hours?
    c. Another CFL from a different brand is selected and the chance that it fails before 6000 hours is 35%. Determine its mean lifetime.

    (SSCE 2193, Test 1, Sem. 2, 11/12)

Answers:

1. a. $P(X = x) = \frac{1}{6}$; $x = 1, 2, 3, 4, 5, 6$

   b. $P(X = x) = \frac{\binom{3}{x}\binom{7}{5-x}}{\binom{10}{5}}$; $x = 0,1,2,3$

x	0	1	2	3
$P(X = x)$	0.0833	0.4167	0.4167	0.0833

c. The cumulative distribution:

x	$x < 0$	$0 \le x < 1$	$1 \le x < 2$	$2 \le x < 3$	$3 \le x < 4$	$4 \le x < 5$	$x \ge 5$
$F(x)$ $=P(X \le x)$	0	0.32	0.57	0.75	0.86	0.94	1

2. a. 0.7543 b. 0.9692 c. ≈ 10
3. a. 0.2851 b. 0.9817
4. a. 0.8850 b. 0.9710
5. 0.0214
6. a. 0.1447 b. 0.6703
7. a. 0.0362 b. 0.0037
8. a. 0.1852 b. 0.0057
9. b. 0.6068
10. 0.3653
11. a. 0.8163 b. 0.0165
12. a. 0.0183 b. 0.6321 c. 10.4 minutes
13. 2/3
14. a. 0.3225 b. 0.00248
15. a. 0.00203 b. 0.0755
16. b. 0.0014 c. 0.2801 d. mean = 4, std. dev. = 2.0976
17. a. 0.48% b. 3.216%
18. 0.5526
19. 5; 0.0318
20. a. 0.3012 b. 0.1813 c. 13928.13 hours