

UNIVERSITI TEKNOLOGI MALAYSIA
SEMESTER 2, SESSION 14/15
SSCE 2193 ENGINEERING STATISTICS

Supplementary Exercise 4: Tests of Hypotheses

1. The deflection temperature under load for two different types of plastic pipe is being investigated. Two random samples with size 8 and 9 pipe specimens are tested, and the deflection temperature observed are reported here (in °C):

Type 1 25 30 15 19 30 24 10 12
Type 2 20 22 15 8 25 12 14 10 9

Assume that the deflection temperature follows normal distribution. A technician claims that the mean deflection temperature for Type 1 is greater than that of Type 2. Is there sufficient evidence to support the claim at 5% significance level?

[Ans: $T_{test} = 1.6687$, Fail to reject H_0 , no evidence to support the claim at 5% sig level]

(Final, Sem 1, 12/13)

2. A technician suspects that a soft drink filling machine has a problem. If the machine is in good condition, the machine should have a standard deviation of at most 0.3ml. A technician takes a random sample of 10 cans and finds that the mean is 326ml and the standard deviation is 0.2ml. Conduct a hypothesis testing with 5% level of significance. What conclusion can the technician make?

[Ans: $\chi^2_{test} = 4$, Fail to reject H_0 , the machine still in good condition]

(Final, Sem 1, 12/13)

3. a) The number of accidents that occurs in a factory are recorded for 120 days and the data is given in the following table.

Number of accidents, (x)	0	1	2	3	4	≥ 5
Number of days, (f)	25	45	33	12	4	1

Can we conclude that the number of accidents per day follows Poisson distribution?

[Hint: $\bar{x} = \frac{\sum fx}{\sum f}$]

[Ans: $\chi^2_{test} = 2.0697$, Fail to reject H_0 , follows Poisson distribution]

- b) An advertisement agency is conducting a study to determine if there exist any differences with respect to reader recall among 3 kinds of magazine advertisements. One advertisement is humorous, the second is quite technical and the third is a pictorial comparison of

competing brands. Appropriate random sampling, response validations are taken and conducted to determine how well participants remembered each advertisement on a national level. Participants selecting the correct advertisements are labeled as Remembered and those unable to select the correct advertisements are labeled Not Remembered.

	<i>Types of Advertisements</i>		
	Humorous	Technical	Comparison
Remembered	25	10	7
Not Remembered	73	93	108
Total	98	103	115

Conduct a 5% significance level test to determine if there are any differences with respect to reader recall among the 3 kinds of magazines advertisements.

[Ans: $\chi^2_{test} = 20.66$, Reject H_0 , The recalling ability is not the same for the 3 advertisements]

(Final, Sem 1, 12/13)

4. Sodium content in organic cornflakes depends on fertilization method. A new fertilization method has been developed. It is found that amongst 850 farmers who applied the new method, 830 were satisfied. Do the above data provide strong evidence to indicate that the proportion of dissatisfied farmers exceeds 2%? Use $\alpha = 0.01$.

[Ans: $Z_{test} = 0.7350$, Fail to reject H_0 , no evidence that the proportion of dissatisfied farmers exceeds 2%]

(Final, Sem 2, 12/13)

5. Two machines are used for filling plastic bottles with a certain net volume.

- a. Quality engineer A found that 46 of 100 output from Machine 1 meet the specification while 36 of 75 output from Machine 2 meet the specification. Could engineer A conclude that both machines produce the same proportion of the output that meets specification? Use $\alpha = 0.05$.

[Ans: $Z_{test} = -0.2624$, Fail to reject H_0 , both machines produce the same proportion of output]

- b. On the other hand, quality engineer B suspects that both machines fill to different mean net volumes. A random sample of 10 bottles is taken from the output of each machine and the following information is obtained (in milliliters):

	Machine 1	Machine 2
Mean	473.58	463.32
Standard Deviation	0.92	0.75

Assume the net volume is normally distributed.

- i. Test at $\alpha = 0.05$ if the variability of the net volumes filled by both machines are the same.

[Ans: $f_{test} = 1.504$, Fail to reject H_0 , $\sigma_1^2 = \sigma_2^2$]

- ii. Use the result you obtained in part b.i., does the above information provide an evidence to support engineer B's suspicion? Test at $\alpha = 0.05$.

[Ans: $t_{test} = 27.33$, Reject H_0 , support Engineer B's suspicion]

(Final, Sem 2, 12/13)

6. Tensile strength measured in thousand psi of aluminum alloy, denoted as X , is claimed to follow a normal distribution with mean 68.72 and standard deviation 2. To test this claim, the strength was measured on 50 specimens of a new aluminum alloy and the data are tabulated as follows:

Strength, X	Frequency	P_i
$X < 65$	5	0.0314
$65 \leq X < 67$	9	0.1635
$67 \leq X < 69$	14	P_3
$69 \leq X < 71$	11	0.3172
$71 \leq X < 73$	7	0.1109
$X \geq 73$	4	P_6

- i. Assuming the claim is true, four of the corresponding probabilities P_i were computed for $i=1, 2, 4$, and 5 as shown in the above table. Compute the other two probability values P_3 and P_6 .

[Ans: $P_3 = 0.3608$, $P_6 = 0.0162$]

- ii. Hence, do we have enough evidence from the data to reject the claim at a level of significance $\alpha=0.01$?

[Ans: $\chi^2_{test} = 7.6470$, do not have enough evidence to reject the claim that the tensile strength follow the distribution]

(Final, Sem 2, 12/13)

7. A manager of a car service centre intends to plan his spare parts based on the problems faced by his customers. He is curious if the types of vehicles are dependent on the types of problems occurred. The following table shows a random sample of 430 vehicles with the corresponding problems.

Types of problems	Types of vehicles			Row Total
	Car	MPV	SUV	
Mechanical	140	94	48	282
Electrical	72	45	31	148
Column Total	212	139	79	430

- i. State appropriate hypothesis statements.
- ii. Next, do the above data provide enough evidence at 5% level of significance to support the manager's curiosity?

[Ans: $\chi^2_{test} = 1.0909$, Fail to reject H_0 , The type of vehicles and problems are independent]

(Final, Sem 2, 12/13)

8. a. Oxide layers on semiconductor wafers are etched in a mixture of gases to achieve the desired thickness. Two different mixtures of gases are under consideration and there is interest in determining if one is similar to the other in terms of variability. Samples of 26 layers are etched with each of the two mixtures, with standard deviation of 1.85 angstroms observed for the first mixture and 2.02 angstroms for the second mixture. Can we conclude that there exist similarities in terms of variability at significance level $\alpha = 0.05$?

[Ans: $0.4484 < f_{test} = 0.8388 < 2.23$, fail to reject H_0 . There exists similarities in terms of variability at $\alpha = 0.05$]

- b. An experiment was performed to compare the abrasive wear of two different materials. 12 pieces of material A were tested by exposing each piece to a machine. 10 pieces of material B were similarly tested. In each case, the depth of wear were observed. The samples of material A gave an average wear of 85 units and standard deviation of 4, while the samples of material B gave an average of 81 and standard deviation of 5. Can we conclude at level of significance $\alpha = 0.05$ that the abrasive wear of material A exceeds that of material B by more than 2 units? Assume the populations to be approximately normal with equal variances.

[Ans: $t_{test} = 1.0432 < 1.725$, fail to reject H_0 . No evidence that the abrasive wear of Material A exceeds that of Material B by more than 2 units at $\alpha = 0.05$]

(Final, Sem 2, 07/08)

9. a. In the last two years, MESRA Airlines has observed the mode of payment by customers as follows:

Cash, (C)	41%
Credit Card, (CC)	26%
Debit Card, (DC)	24%
Other, (O)	9%

To encourage customers paying by cash, the airline offers special discount for cash payment. The following table lists the frequency distribution of the mode of payment for a sample of 300 customers after the discount went into effect.

Mode of payment	C	CC	DC	O
Number of customer	120	102	55	23

Test at 1% significance level whether the distribution of mode of payment changed after the discount went into effect.

[Ans: $\chi^2_{test} = 12.064$, reject H_0 . The distribution of the mode of payments changed after the discount went into effect at $\alpha = 0.01$]

- b. Three lecturers handled the same subject for the past two years. The following table gives the distribution of grades for their students in randomly selected sections.

	Lectures		
	1	11	111
Grades 'C' and above	18	44	20
Below 'C'	25	36	15

Use the homogeneity test at 2.5% significance level to test the null hypothesis that the proportion of grades are the same for the three lecturers.

[Ans: $\chi^2_{test} = 2.4297$, fail to reject H_0 . The proportions of grades are homogeneous (the same) for the three lecturers at $\alpha = 0.025$]

(Final, Sem 2, 07/08)

10. A restaurant located in a commercial district of Kuala Lumpur has developed an improved process for serving customers during an hour lunch period. The waiting time (defined as the time a customer enters the restaurant until he is served by a waitress) of all customers during this hour has been recorded over a period of one week. A random sample of 15 customers is selected and the results (in minutes) are as follows:

4.21 5.55 3.02 5.13 4.77
 2.34 3.54 3.20 4.50 6.10
 0.38 5.12 6.46 6.19 3.79

- (a) At the 0.05 level of significance, is there evidence that the population mean waiting time is less than 5 minutes?

[Ans: $t_{test} = -1.6866$, fail to reject H_0 , there is no evidence that the population mean waiting time is less than 5 minutes at $\alpha = 0.05$]

- (b) What assumption about the population distribution is needed in order to conduct the test in part (a) above?
 (c) A customer walks into the restaurant during a lunch hour and asks a waitress how long he can expect to wait. On the basis of the result in part (a), help the waitress to answer the customer's question.

(Final, Sem 2, 09/10)

11. Computer Anxiety Rating Scale (CARS) measures an individual's level of computer anxiety, on a scale from 20 (no anxiety) to 100 (the highest level of anxiety). A research at a local university administered CARS to 47 randomly selected engineering students. An objective of the study is to determine whether there is a significant difference between the level of

computer anxiety experienced by male and female students. They record the following results:

	Males	Females
\bar{x}	40.26	36.85
s	13.35	9.42
n	21	26

- (a) Assume CARS measures are normally distributed, test at the 0.05 level of significance if there is any evidence of a difference in the variances of computer anxiety levels experienced by male and female engineering students.

[Ans: $f_{test} = 2.0084$, fail to reject H_0 , there is no evidence that the variability of the computer anxiety experienced by male and female engineering students are different]

- (b) Based on part (a), state and compute the appropriate test statistic that you should use in testing whether there is a significant difference in mean CARS for male and female engineering students.

[Ans: $t_{test} = 1.0253$]

(Final, Sem 2, 09/10)

12. A criminologist conducted a survey to determine whether the incidence of certain types of crime varied from one part of a large city to another. The particular crimes of interest were assault, burglary, and homicide. The following table shows the numbers of crimes committed in two areas of the state of Johor during past year.

District	Type of crime		
	Assault	Burglary	Homicide
Mersing	60	70	30
Johor bahru	80	90	70

- (a) Can we conclude from these data at the 0.05 level of significance that the occurrence of these types of crimes is dependent on the district?

[Ans: $\chi^2_{test} = 5.5807$, fail to reject H_0 , don't have enough from the data to conclude that the occurrence of the types of crime are dependent on city districts]

- (b) Is there any changes in your conclusion in (a) if the probability of Type I error is 0.10? Justify your answer.

[Ans: $\chi^2_{0.1,2} = 4.605$, reject H_0 , the occurrence of the types of crimes is dependent]

(Final, Sem 2, 09/10)

13. It is believed that the number of errors per page of a statistics text book follows the Poisson distribution with the mean number of errors per page is λ . The following data show the number of pages with the corresponding number of errors over 170 pages.

Number of errors	0	1	2	3	≥ 4
Number of pages	70	50	25	15	10

(a) Find an estimate for the distribution's unknown parameter, λ

[Ans: 1.088]

(b) Test the belief at the significance level $\alpha = 0.025$.

[Ans: $\chi^2_{test} = 11.9251$, reject H_0 , we can conclude that the number of errors per page does not follow a Poisson distribution with mean λ]

(Final, Sem 2, 09/10)

14. A traffic engineer conducted a study on the number of speed limit violation among Malaysian drivers. He posted a speed limit of 50 km/h and monitored the speed of 100 randomly selected vehicles and found that 27 of them violated the speed limit. The speed limit was raised to 70 km/h and he monitored the speed of 100 randomly selected vehicles and found 19 vehicles violated the speed limit.

Conclude on whether there is a difference in the proportions of vehicles violating the speed limit, before and after the speed limit was raised at $\alpha = 0.05$.

(Final, Sem 2, 10/11)

15. In semiconductor manufacturing, wet chemical etching is used to remove silicon from the backs of wafers prior to metallization. Two different etching solutions have been compared, using two random samples of 8 wafers. The observed etch rates are given in the following table

Observation Number	Solution 1	Solution 2
1	91.50	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21
5	91.79	97.19
6	89.07	97.04
7	94.72	91.07
8	89.21	92.75

Suppose that the two samples are independent and come from populations which are normally distributed. Also, assume that their population variances are equal.

- Is there any difference between the mean etching rates of the two solutions. Use $\alpha = 0.05$.
- The variability in the measurement of etching rates is a critical characteristic of the solutions. From the data above, test whether the assumption that the population variances are equal is true. Use $\alpha = 0.05$.

(Final, Sem 2, 10/11)

16. A study is conducted to compare the rainfall pattern of two main regions, the West coast and East coast in Peninsular Malaysia based on the values of their descriptive statistics. A random sample of 5 stations in the East coast and six stations in the West Coast give the results based on the rainfall intensity values recorded as below:

East		West	
Station	Rainfall (mm/day)	Station	Rainfall (mm/day)
Kota baru	19	Petaling jaya	11
Dungun	17	Gombak	10
Kemaman	20	Ipoh	8
Pekan	19	Setiawan	7
Kuantan	15	Port Dickson	12
		Malacca	11

- (a) Assume that the rainfall intensity values of stations in each region are normally distributed, test at the 0.05 level of significance if there is any evidence of a difference in the variances of rainfall values observed at both regions.

[Ans: $f_{test} = 1.0628$, fail to reject H_0 , no strong evidence of a difference in the variances of rainfall values observed at both regions]

- (b) Based on part (a), state and compute the appropriate test statistic that you should use in testing whether the mean rainfall intensity values recorded over the East Coast is greater than those values recorded over the West coast.

[Ans: $t_{test} = 6.859$]

(Final, Sem 1, 11/12)

17. A production manager of factory AZ claimed that the machine that is used to make bottle of fruit juices in the factory is supposed to mix 35% strawberries, 30% kiwi, 25% mango and 10% oranges. A study was done and found that a container with 500 bottles was found to have 180 strawberries, 160 kiwi, 120 mango and 40 oranges.

- (a) Can we accept the claim made by the production manager at the 0.5% level of significance?

[Ans: $\chi^2_{test} = 3.0096$, fail to reject H_0 , no evidence to reject the claim made by the production manager]

- (b) Is there any change in your conclusion if the probability of Type I error is 0.10? Justify your answer.

[Ans: $\chi^2_{0.1,3} = 6.251$]

(Final, Sem 1, 11/12)

18. A researcher claims that at least 15% of all football helmets have manufacturing flaws that could potentially cause injury to the wearer. A sample of 250 helmets revealed that 20 helmets contained such defects.

- i. Test the researcher's claim at 5% level of significance and state the conclusion whether the finding supports the claim.

- ii. Based on your conclusion in (i) what error could you have possibly made?

(Final, Sem 2, 11/12)

19. A study is conducted to compare the standard unleaded petrol prices between Australia and United States. A random sample of five states in Australia and six states in United States were selected and the records shown as below:

Australia	Price (Pence per Litre)	United States	Price (Pence per Litre)
Adelaide	97	Las Vegas	46
Brisbane	98	Miami	48
Melbourne	96	Washington DC	47
Perth	91	Dallas	45
Sydney	97	Los Angeles	51
		Seattle	51

Assume that unleaded petrol prices follow a normal distribution.

- (i) Based on the sample, would you conclude that the two population variances differ at 5% level of significance?
(ii) Using your result in part (i), test the mean standard unleaded petrol price of Australia is higher than that of United States at 1% level of significance.

(Final, Sem 2, 11/12)

20. A manufacturer of electronic devices used in biomedical equipment claims that the proportion of defective of the devices is less than 3%. A random sample of 150 such electronic devices shows 3 are defective. Do the data provide sufficient evidence to support the manufacturer's claim? Use $\alpha = 0.01$.

[Ans: $Z_{test} = 1.24$, fail to reject H_0 , the number of defect departures in US is tolerable]

(Test 2, Sem 1, 12/13)

21. A manufacturer of sprinkler systems use for fire protection in office buildings claims that the true average system-activation temperature is $130^\circ F$. A sample of $n = 49$ systems, when tested, yields a sample average activation temperature of $131.08^\circ F$. If the distribution of activation temperature is normal with standard deviation, $\sigma = 1.5^\circ F$, does the result contradict the manufacturer's claim at significance level $\alpha = 0.01$?

[Ans: $Z_{test} = 0.5040$, fail to reject H_0 , The data does not give strong support to the claim that the true average differs from the design value of $130^\circ F$ at $\alpha = 0.01$]

(Test 2, Sem 1, 12/13)

22. The mass of cheese tarts delivered into bags by a machine is known to be normally distributed. Prior to a minor overhaul of the machine, the contents (in grams) of a random sample of six bags have a sample mean of 151.8 and a sample standard deviation of 0.632.

After a minor overhaul, the contents of a random sample of twelve bags were measured with a sample mean of 150.6 and a sample standard deviation of 0.740. Suppose the probability of making Type I error is 0.05, test the hypothesis that the minor overhaul has had no effect on the mean mass of cheese tarts delivered by the machine. Assume that the population variances are equal.

[Ans: $T_{test} = 3.390$, reject H_0 , There is evidence that that the minor overhaul has affected the mean mass of cheese tarts delivered by the machine]

(Test 2, Sem 2, 12/13)

23. An investigation was conducted to study the dust content in the flue gases of two types of solid-fuel boilers. 16 boilers of type A and 16 boilers of type B were used under identical fuelling and extraction conditions. Over a similar period, the following quantities (in grams) of dust were deposited in similar traps inserted in each of the flues, with the following summary:

	Type A	Type B
Mean	63.9	52.9
Variance	193.3	81.8

- a. Assuming that these independent samples come from normal populations, test the variability of the dust content from type A boiler exceeds those of type B at $\alpha=0.05$.

[Ans: $F_{test} = 2.363$, fail to reject H_0 , There is no evidence that variability of the dust content from type A boiler exceeds that of type B]

- b. If $\alpha=0.01$, will you change the conclusions you have made in (a)?

(Test 2, Sem 2, 12/13)

24. The following table shows a frequency distribution for the number of emergency calls per day received by a general hospital for 100 days.

Number of calls	Number of days
$0 \leq X < 3$	5
$3 \leq X < 6$	30
$6 \leq X < 9$	48
$9 \leq X < 12$	15
$X \geq 12$	2

Based on the above data, can we conclude the number of calls received by the hospital follows a Poisson distribution with mean 6 calls per day at 0.05 level of significance?

[Ans: $\chi^2_{test} = 3.7865$, fail to reject H_0 , the number of calls received by the hospital follows a Poisson distribution with mean 6 calls per day]

(Test 2, Sem 2, 12/13)

25. A new modification in the process of manufacturing bearings is introduced to reduce the number of defective bearings. A random sample of 60 bearings from the new modification

process was selected and 45 non-defective bearings were found. Another random sample of 70 bearings from the old process was selected and 35 non-defective bearings were found.

- a. State the best estimates for the proportion of non-defective bearings produced from
- i. the new modification process

[Ans: 0.75]

- ii. the old process

[Ans: 0.50]

- b. Conduct a hypothesis test at 5% significance level and conclude whether the new modification process is more effective than the old process. Assume that the variables are normally distributed.

[Ans: $Z_{test} = 2.921$, reject H_0 . There is evidence that the new modification process is more effective compared to the old method]

(Test 2, Sem 2, 11/12)

26. A sample of 60 construction workers from Country A earns an average of RM 300 per week with a standard deviation of RM16, while a sample of 50 construction workers from Country B earn an average of RM280 per week with a standard deviation of RM18. Assume that the variables are normally distributed.

- a. Test at 2% significance level if significant difference exists between the average earning of construction worker from Country A and that from Country B.

[Ans: $Z_{test} = 6.101$, reject H_0 . There exist a significant difference between the average earning of construction worker from Country A and that from Country B]

- b. Without repeating the procedure in (a), will you change your conclusion at $\alpha = 0.03$ significance level. Justify your answer.

(Test 2, Sem 2, 11/12)

27. A manufacturer of bulbs claims that the variance of the span of life of the bulbs produced by his company is 320 hours. The inspection department selected a random sample of 15 bulbs and found the variance as 210 hours². At $\alpha = 0.05$, do you agree to the manufacturer's claim? Assume that the variable is normally distributed.

[Ans: Since $5.629 < \chi^2_{test} < 26.119$, fail to reject H_0 , agree to the manufacturer's claim]

(Test 2, Sem 2, 11/12)

28. An aircraft undergoes a complete scheduled annual inspection at five standard time intervals labelled A, B, C, D and E with failure rates at these intervals are known to follow a Crow-AMSAA model as 29.8%, 20.4%, 17.9%, 16.5% and 15.4%. A set of grouped test data was carried out and the observed number of failures from all five intervals is recorded in the table below:

Intervals	A	B	C	D	E
Observed number of failures	13	16	5	8	7

Test at 5% significance level to see whether the grouped test data fits a Crow-AMSAA model.

[Ans: $\chi^2_{test} = 5.4434$, fail to reject H_0 , The grouped test data fits the Crow-AMSAA model]

(Test 2, Sem 2, 11/12)

29. A company wants to establish that the mean life of its newly rechargeable Ni-MH batteries, when used in a cordless phone, is over 228 days. The data consist of the life lengths of batteries in 72 different cordless phones.

a. Formulate the null and alternative hypotheses.

b. What is the conclusion to your test if $\bar{x} = 233$ and $s = 47$ days? Assume $\alpha = 0.05$.

[Ans: $Z_{test} = 0.9027$, fail to reject H_0 , the stated claim that $\mu > 228$ is not substantiated]

c. Based on your decision in (b), what error could you have possibly made?

(Test 2, Sem 1, 11/12)

30. To compare the effectiveness of isometric and isotonic exercise methods, 24 obese female adults are included in an experiment: 12 are selected at random and assigned to one exercise method; the remaining 12 are assigned to the other exercise method. After eight weeks, the reductions in abdomen measurements are recorded in centimeters, and the following results are obtained.

	Isometric method	Isotonic method
Mean	3.2	4.6
Standard deviation	1.5	2.1

Assume that the reductions in abdomen measurements are normally distributed with equal population variances. Do the data support the claim that the isometric method is more effective? Test with $\alpha = 0.05$.

[Ans: $t_{test} = -1.8793$, fail to reject H_0 , the stated claim that the isometric method is more effective is not significantly substantiated]

(Test 2, Sem 1, 11/12)

31. A researcher wishes to test $H_0: \pi = 0.50$ versus $H_1: \pi > 0.50$, where π is the true proportion of university engineering students who graduate on time.

a. State the rejection region and the test statistic for a large sample test having $\alpha = 0.05$.

[Ans: $Z_{test} = 6.0249$, reject H_0 , the claim of true proportion of university engineering students who graduate on time is strongly supported by the data]

b. If 93 out of a random sample of 120 students graduated on time, what does the test conclude?

(Test 2, Sem 1, 11/12)

32. In a study of the television viewing habits of children, a developmental psychologist selects a random sample of 300 children. Each child is asked to indicate the amount of time spent watching television. Results are shown in the table given below.

Number of hours	≤ 1	2	3	4	5	≥ 6
Number of children	40	34	67	78	46	35

- a. If X is a random variable representing the number of hours children watching television, test the hypothesis that X follows the Poisson distribution with mean 4 hours per child at $\alpha = 0.01$.

[Ans: $\chi^2_{rest} = 29.073$, reject H_0 , the random variable X follows the Poisson distribution with mean 4 hours]

- b. Without repeating the above procedure, will you change your conclusion in (a.) if α increases to 5%?

(Test 2, Sem 1, 11/12)