DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
UNIVERSITI TEKNOLOGI MALAYSIA

SSH 1033 MATHEMATICAL METHODS II TUTORIAL 3

1. Use the ratio test to determine whether the series converges, diverges or inconclusive.

(a) $\sum_{k=1}^{\infty} \frac{3^k}{k!}$.
(b) $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$.
(c) $\sum_{k=2}^{\infty} \frac{1}{5k}$.
(d) $\sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k$.
(e) $\sum_{k=1}^{\infty} \frac{k!}{k^3}$.
(f) $\sum_{k=1}^{\infty} \frac{k}{k^2+1}$.

2. Use the root test to determine whether the series converges, diverges or inconclusive.

(a) $\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$.
(b) $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$.
(c) $\sum_{k=1}^{\infty} \frac{k}{5^k}$.
(d) $\sum_{k=1}^{\infty} \left(1+e^{-k}\right)^k$.

3. Use any appropriate test to determine whether the series converges.

(a) $\sum_{k=1}^{\infty} \frac{2^k}{k^3}$.
(b) $\sum_{k=1}^{\infty} \frac{1}{k^2}$.
(c) $\sum_{k=1}^{\infty} \frac{7^k}{k!}$.
(d) $\sum_{k=1}^{\infty} \frac{1}{2k+1}$.
(e) $\sum_{k=1}^{\infty} \frac{k^2}{5^k}$.
(f) $\sum_{k=1}^{\infty} \frac{k!\,10^k}{3^k}$.
(g) $\sum_{k=1}^{\infty} \frac{k^{50}}{e^{-k}}$.
(h) $\sum_{k=1}^{\infty} \frac{k^2}{k^2+1}$.
(i) $\sum_{k=1}^{\infty} k\left(\frac{2}{3}\right)^k$.
(j) $\sum_{k=1}^{\infty} k^k$.
(k) $\sum_{k=1}^{\infty} \frac{1}{k\ln k}$.
(l) $\sum_{k=1}^{\infty} \frac{2^k}{k^3+1}$.
(m) $\sum_{k=1}^{\infty} \left(\frac{4}{7k-1}\right)^k$.
(n) $\sum_{k=1}^{\infty} \frac{(k!)^2\,2^k}{(2k+2)!}$.
(o) $\sum_{k=1}^{\infty} \frac{1}{1+\sqrt{k}}$.
(p) $\sum_{k=1}^{\infty} \frac{k!}{k^k}$.

4. Show that the following series converges.

(a) $1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} + \cdots$.

(b) $1 + \frac{1\cdot 3}{3!} + \frac{1\cdot 3\cdot 5}{5!} + \frac{1\cdot 3\cdot 5\cdot 7}{7!} + \cdots$.

(c) $\frac{2!}{1} + \frac{3!}{1\cdot 4} + \frac{4!}{1\cdot 4\cdot 7} + \frac{5!}{1\cdot 4\cdot 7\cdot 10} + \cdots$.

5. (a) Show: $\lim_{k\to+\infty} (\ln k)^{1/k} = 1$.

[Hint: Let $y = (\ln x)^{1/x}$ and find $\lim_{k\to+\infty} (\ln k)^{1/k}$].

(b) Use the result in part (a) and the root test to show that $\sum_{k=1}^{\infty} \frac{\ln k}{3^k}$ converges.

(c) Show that the series converges using the ratio test.

6. Prove that the following series converges by the comparison test.

(a) $\sum_{k=1}^{\infty} \frac{1}{3^k + 5}$.

(b) $\sum_{k=1}^{\infty} \frac{2}{k^4 + k}$.

(c) $\sum_{k=1}^{\infty} \frac{1}{5k^2 - k}$.

(d) $\sum_{k=1}^{\infty} \frac{k}{8k^3 + 2k^2 - 1}$.

(e) $\sum_{k=1}^{\infty} \frac{2^k - 1}{3^k + 2k}$.

(f) $\sum_{k=1}^{\infty} \frac{5\sin^2 k}{k!}$.

7. Prove that the following series diverges by the comparison test.

(a) $\sum_{k=1}^{\infty} \frac{3}{k - \frac{1}{4}}$.

(b) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+8}}$.

(c) $\sum_{k=1}^{\infty} \frac{9}{\sqrt{k}+1}$.

(d) $\sum_{k=2}^{\infty} \frac{k+1}{k^2 - k}$.

(e) $\sum_{k=1}^{\infty} \frac{k^{4/3}}{8k^2 + 5k + 1}$.

(f) $\sum_{k=1}^{\infty} \frac{k^{-1/2}}{2 + \sin^2 k}$.

8. Use the limit comparison test to determine the series converges and diverges.

(a) $\sum_{k=1}^{\infty} \frac{4k^2 - 2k + 6}{8k^7 + k - 8}$.

(b) $\sum_{k=1}^{\infty} \frac{1}{9k + 6}$.

(c) $\sum_{k=1}^{\infty} \frac{5}{3^k + 1}$.

(d) $\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$.

(e) $\sum_{k=1}^{\infty} \frac{1}{(8k^2 - 3k)^{1/3}}$.

(f) $\sum_{k=1}^{\infty} \frac{1}{(2k+3)^{17}}$.

9. Use any appropriate test to determine whether the series converges or diverges.

(a) $\sum_{k=1}^{\infty} \frac{1}{k^3+2k+1}$.

(b) $\sum_{k=1}^{\infty} \frac{1}{(k+3)^{2/5}}$.

(c) $\sum_{k=1}^{\infty} \frac{1}{9k-2}$.

(d) $\sum_{k=1}^{\infty} \frac{\ln k}{k}$.

(e) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^3+1}$.

(f) $\sum_{k=1}^{\infty} \frac{4}{2+3^k k}$.

(g) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}}$.

(h) $\sum_{k=1}^{\infty} \frac{2+(-1)^k}{5^k}$.

(i) $\sum_{k=1}^{\infty} \frac{2+\sqrt{k}}{(k+1)^3-1}$.

(j) $\sum_{k=1}^{\infty} \frac{4+|\cos k|}{k^3}$.

(k) $\sum_{k=1}^{\infty} \frac{1}{4+2^{-k}}$.

(l) $\sum_{k=1}^{\infty} \frac{\sqrt{k}\ln k}{k^3+1}$.

10. Use the limit comparison test to investigate convergence of $\sum_{k=1}^{\infty} \frac{(k+1)^2}{(k+2)!}$.

11. Use the limit comparison test to investigate convergence of the series $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots$.

12. Prove that $\sum_{k=1}^{\infty} \frac{1}{k!}$ converges by comparison with a suitable geometric series.

13. Use the alternating series test to determine whether the series converges or diverges.

(a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k+1}$.

(b) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{3^k}$.

(c) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{3k+1}$.

(d) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+4}{k^2+k}$.

(e) $\sum_{k=1}^{\infty} (-1)^{k+1} e^{-k}$.

(f) $\sum_{k=3}^{\infty} (-1)^{k+1} \frac{\ln k}{k}$.

14. Use the ratio test for absolute convergenve to determine whether the series converges absolutely or diverges.

(a) $\sum_{k=1}^{\infty} \left(-\frac{3}{5}\right)^k$.

(b) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{2^k}{k!}$.

(c) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3^k}{k^2}$.

(d) $\sum_{k=1}^{\infty} (-1)^k \frac{k}{5^k}$.

(e) $\sum_{k=1}^{\infty} (-1)^k \frac{k^3}{e^k}$.

(f) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^k}{k!}$.

15. Classify the series as absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k}$.

(b) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{4/3}}$.

(c) $\sum_{k=1}^{\infty} \frac{(-4)^k}{k^2}$.

(d) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$.

(e) $\sum_{k=1}^{\infty} \frac{\cos k\pi}{k}$.

(f) $\sum_{k=3}^{\infty} (-1)^k \frac{\ln k}{k}$.

(g) $\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{k+2}{3k-1}\right)^k$.

(h) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2+1}$.

(i) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+2}{k(k+3)}$.

(j) $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k^2}{k^3+1}$.

(k) $\sum_{k=1}^{\infty} \sin \frac{k\pi}{2}$.

(l) $\sum_{k=1}^{\infty} \frac{\sin k}{k^3}$.

(m) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k \ln k}$.

(n) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k(k+1)}}$.

(o) $\sum_{k=2}^{\infty} \left(-\frac{1}{\ln k}\right)^k$.

(p) $\sum_{k=1}^{\infty} \frac{k \cos k\pi}{k^2+1}$.

Answers Tutorial 2

1. (a) $S_4 = 312/125$; $S_n = 5/2\,[1 - (1/5)^n]$ (b) $S_4 = 1/3$; $S_n = 1/2 - 1/(n+2)$

2. All converge except (e) and (f). (a) 1/4 (b) 4/7 (c) 8/9 (d) 6 (g) 1/2 (h) 3/4

4. (a) – 1/3 (b) 1/6

5. converges ; 1

9. (a) 4/9 (b) 1 (c) 532/99 (d) 159/999 (e) 44663/99000

10. $S_n = -\ln(n+1)$; diverges

11. 70 m

14. (a) $\{x : -1 < x < 1\}$ (b) $\{x : x < -2 \text{ or } x > 2\}$ (c) $\{x : x > 0\}$
(d) $\{x : -\infty < x < \infty\}$

16. (a) 4/3 (b) – 3/4 (c) -1/36 (d) 5

18. (a), (b), (c), (d), (g), (h), (i), (j), (k), (m), (n), (o). diverge

Answers Tutorial 3

1. (a), (d) converge, (b), (e) diverge, (c), (f) inconclusive
2. (c) converges, (a), (b) diverge (d) inconclusive
3. (b), (c), (e), (i), (m), (n), (p) converge; others diverge
8. (a), (c), (f) converge; others diverge
9. (a), (e), (f), (h), (i), (j), (l) converge; others diverge
10. converges 11. diverges 13. all converges except (c)
14. (c), (f) diverge; others converge absolutely
15. (b), (d), (g), (h), (l), (o) converge absolutely; (c), (k) diverge;
    others converge conditionally

Note : In your quest for some of the answers you may want to use the result

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \quad \text{and} \quad \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = e^{-1}.$$