Q1. (a) $\{2n + \ln(n)\}$

$\longrightarrow \lim_{n\to\infty} \{2n + \ln(n)\} = \infty$ (B1) — 1

(b) $\left\{\left(1 - \frac{2}{n}\right)^n\right\}$

$\longrightarrow \lim_{n\to\infty} \left(1 - \frac{2}{n}\right)^n$

$-$ let $y = \left(1 - \frac{2}{n}\right)^n \rightarrow$ (M1)

$\ln y = n \ln\left(1 - \frac{2}{n}\right)$

$= \dfrac{\ln\left(1 - \frac{2}{n}\right)}{\frac{1}{n}}$ — (A1)

taking limit $n\to\infty$ $y \approx \frac{0}{0}$

apply L'hopital rule,

$= \dfrac{\frac{1}{1-\frac{2}{n}} \cdot \cancel{\left(\frac{2}{n^2}\right)}}{-\cancel{\left(\frac{1}{n^2}\right)}}$ — (M1) — 4

taking $\lim_{n\to\infty} \rightarrow$ $= \dfrac{2}{\frac{2}{n} - 1}$

$= -1$

$\ln y = -2$

$y = e^{-2}$ # — (A1)

(c) $\lim_{n\to\infty} \left[\dfrac{2n^3 - n^2 + 8n}{-5n^3 + 7}\right]$

apply L'hopital rule. $= \lim_{n\to\infty} \left[\dfrac{6n^2 - 2n + 8}{-15n^2}\right]$ — (M1)

$= \lim_{n\to\infty} \left[\dfrac{12n - 2}{-30n}\right] = \lim_{n\to\infty} \left[\dfrac{12}{-30}\right]$ (M1) $= -\dfrac{12}{5}$ # (A1) — 3

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Q2. Using sandwich theorem, $\left[n^2 e^{\sin(1/n)} \right]$

$$\Rightarrow \quad -1 \le \sin(1/n) \le 1$$

$$e^{-1} \le e^{\sin(1/n)} \le e^{1}$$

$$n^2 e^{-1} \le n^2 e^{\sin(1/n)} \le n^2 e^{1}$$

M1 A1

$$\lim_{n\to\infty} n^2 e^{-1} = \cancel{\lim_{n\to\infty} \frac{e^{-1}}{1/n^2}} = \infty$$

$$\lim_{n\to\infty} n^2 e^{1} = \infty$$

M1 A1

Using sandwich theorem $\lim_{n\to\infty} \left(n^2 e^{\sin(1/n)}\right) = \infty$ — A1

Q3. Show that $\frac{3}{1.3} + \frac{3}{3.5} + \frac{3}{5.7} + \frac{3}{7.9} + \dots = \frac{3}{2}$

$1, 3, 5, 7 \rightarrow a = 1, \; d = 2 \qquad a + (n-1)d \Rightarrow 1 + (n-1)2$
$= 1 + 2n - 2$
$= 2n - 1$ — M1

$3, 5, 7, 9 \rightarrow a = 3, \; d = 2 \qquad \Rightarrow 3 + (n-1)2$
$= 2n + 1$

A1

$$\sum_{n=1}^{\infty} \frac{3}{(2n-1)(2n+1)} = 3 \sum_{n=1}^{\infty} \left(\frac{1}{(2n-1)(2n+1)} \right) = \frac{3}{2} \sum_{n=1}^{\infty} \left(\frac{1}{2n-1} - \frac{1}{2n+1} \right)$$

$$\frac{1}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

$1 = A(2n+1) + B(2n-1)$

$n = 0 \qquad 1 = A - B \qquad A = 1 - \frac{1}{2} = \frac{1}{2}$

$A = 1 + B$

$n = 1 \qquad 1 = 3A + B$

M1 A1

$\Rightarrow 1 = 3(1+B) + B \Rightarrow -2 = 4B \Rightarrow B = -\frac{1}{2}$

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$k \quad k-1$

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$$\longrightarrow \frac{3}{2}\left[\sum_{n=1}^{\infty}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)\right]$$

$$\left(1 - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \dots \quad \text{M1}$$

$$\left(\frac{1}{2k-4-1} - \cancel{\frac{1}{2k-4+1}}\right) + \left(\cancel{\frac{1}{2k-2-1}} - \cancel{\frac{1}{2k-2+1}}\right) + \left(\cancel{\frac{1}{2k-1}} - \frac{1}{2k+1}\right)$$

$$\lim_{k \to \infty} \frac{3}{2}\left(1 - \frac{1}{2k+1}\right)$$

$$= \frac{3}{2} \quad \text{A1}$$

#

Q4. $$\sum_{n=0}^{\infty}\left(\frac{2}{3^n} + \frac{2}{5^n}\right)$$

$$= 2\sum_{n=0}^{\infty}\left(\frac{1}{3^n} + \frac{1}{5^n}\right) \quad \text{M1} \quad \text{A1}$$

$$= 2\left[\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n\right] = 2\left(\frac{3}{2} + \frac{5}{4}\right)$$

$$= 11/2 \quad \text{A1}$$

geometric

$a = 1, \; r = 1/3$ $\qquad$ $a = 1, \; r = 1/5$

Since $r = 1/3$ & $r = 1/5$ are less than 1, thus the geometric series is converges. — B1

$$\frac{1}{1 - 1/3} = \frac{3}{2} \qquad \frac{1}{1 - 1/5} = \frac{5}{4} \quad \#$$

M1 $\qquad$ M1

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Q5. $\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{1+n^2}$

let $f(x) = \frac{\tan^{-1} x}{1+x^2}$

By using integral test.

$\int_1^{\infty} \frac{\tan^{-1}(x)}{1+x^2}\,dx$ — let $u = \tan^{-1} x$, $du = \frac{1}{1+x^2}\,dx$ — M1

$\lim_{h\to\infty} \int_1^{h} u\,du$ — A1

$\lim_{h\to\infty} \frac{u^2}{2}\Big|_1^{n} = \lim_{n\to\infty} \frac{(\tan^{-1} n)^2}{2}\Big|_1^{n}$ — M1 A1

$= \lim_{n\to\infty} \left[\frac{(\pi/2)^2}{2}\right]$

$= \frac{\pi^2}{4} \cdot \frac{1}{2} = \frac{\pi^2}{8}$ ∴ convergent — A1
#

Q6. $|a_n| = \frac{2^n}{n!}$ — M1

$|a_{n+1}| = \frac{2^{n+1}}{(n+1)!}$

$\frac{|a_{n+1}|}{|a_n|} = \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$ M1 A1

$= \frac{2}{n+1}$

$\lim_{n\to\infty} \frac{2}{n+1} = 0$ A1

By using ratio test we got $|a_n| = a_n$ is converges

Therefore $\sum_{n=1} (-1)^{n+1} \cdot \frac{2^n}{n!}$ is abs. convergent

B1

#