SSCM1033 Mathematical Methods II   Test 1 (15%)

Instruction: Answer all the questions.   Time: 1 hour 15 minutes

1. Find the limit of each of the following sequence,

   a) $\{2n + \ln(n)\}$

   b) $\left\{\left(1 - \dfrac{2}{n}\right)^n\right\}$

   c) $\left\{\dfrac{2n^3 - n^2 + 8n}{-5n^3 + 7}\right\}$

                                                                            [8 marks]

2. Using Sandwich theorem, find the limit of sequence $\left\{n^{-2}e^{\sin\left(\frac{1}{n}\right)}\right\}$.

                                                                            [5 marks]

3. Show that $\dfrac{3}{1\cdot 3} + \dfrac{3}{3\cdot 5} + \dfrac{3}{5\cdot 7} + \dfrac{3}{7\cdot 9} + \ldots = \dfrac{3}{2}$.

                                                                            [6 marks]

4. Show that $\displaystyle\sum_{n=0}^{\infty}\left[\dfrac{2}{3^n} + \dfrac{2}{5^n}\right]$ is a geometric series. Hence, determine the series converges or diverges. If

   it converges, find its sum.

                                                                            [6 marks]

5. Use any appropriate test to determine whether the following series converges, diverges or inconclusive,

$$\sum_{n=1}^{\infty}\frac{\tan^{-1}(n)}{1+n^2}.$$

                                                                            [5 marks]

6. Classify the series, $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\dfrac{2^n}{n!}$ as absolutely convergent, conditionally convergent or divergent.

                                                                            [5 marks]