

UNIVERSITI TEKNOLOGI MALAYSIA
SSCE 1993 ENGINEERING MATHEMATICS II
TUTORIAL 1

- 1.** Determine the domain and range for each of the following functions.

(a) $z = \sqrt{x^2 + y}$

(b) $z = \frac{1}{x - y^2}$

(c) $z = \frac{x}{x^2 + y}$

(d) $z = \ln(x - y)$

(e) $z = e^{x/(y-1)}$

- 2.** Sketch the contours (level curves projected on the xy -plane) of the following functions for $z = 0$, $z = 2$ and $z = 4$:

(a) $z = y^2 - x^2$

(b) $z = \frac{y}{x^2}$

(c) $z = xy$

(d) $z = \sqrt{xy - 1}$

(e) $z = 4x^2 + y^2$

- 3.** Determine the domains and ranges for the functions; sketch a few contours (level curves projected on the xy -plane) as well as the graphs.

(a) $z = 4 - x^2 - y^2$

(b) $z = \sqrt{x^2 + 9y^2}$

(c) $z = 9x^2 + y^2$

(d) $z = \sqrt{4 - x^2 - y^2}$

(e) $z = 4 + x^2 + y^2$

- 4.** Sketch the following surfaces in three dimensions.

(a) $x^2 + z^2 = 4$

(b) $z = 4 - y^2$

(c) $z = 5 - y$

(d) $z = 2$

(e) $y = x^2$

- 5.** Sketch the level surfaces for the functions.

(a) $w = 4x^2 + y^2 + 4z^2$, $c = 1$ (b) $w = z - x^2 - y^2 + 4$, $c = 7$

(c) $w = z - \sqrt{x^2 + y^2}$, $c = 1$, (d) $w = x^2 + y^2$, $c = 4$

- 6.** Find the first partial derivatives of the following functions.

(a) $z = \ln(2x + 3y)$

(b) $z = \sin(xy)$

(c) $z = x \cos(xy)$

(d) $s = \frac{rt}{r+t}$

(e) $w = \frac{x + y^2}{z}$

(f) $z = \frac{2}{y^2 - x}$

(g) $w = x \tan(y + 3z)$

(h) $w = \sin(xyz^3)$

(i) $w = \frac{1}{x^2 + y^2 + z^2}$

(j) $w = \frac{x}{\sin(2x + 3y + z^2)}$

7. (a) Given $z = \sqrt{x^2 + y^2}$, find $\frac{\partial z}{\partial x} \Big|_{(3,4)}$
- (b) Given $w = x \sin(xyz)$, find $\frac{\partial w}{\partial x}$ at $(1, \frac{1}{2}, \pi)$
- (c) Given $z = \frac{2x}{x + \cos y}$, find $\frac{\partial z}{\partial y} \Big|_{(1,\pi/2)}$
8. (a) Given $\sqrt{x^2 + y^2 + z^2} = 1$, find $\frac{\partial z}{\partial y}$
- (b) Given $yz^2 + y^2 + x^3 = xy + \sin(xz)$, find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$
- (c) Given $z^2 + yz = \ln(x+z)$, find $\frac{\partial z}{\partial y}$
9. (a) Given $f(x,y) = x \ln(xy)$, find f_{xy} and f_{yx}
- (b) Given $w = x^2 \cos\left(\frac{z}{y}\right)$, find $w_{xx}, w_{xz}, w_{yz}, w_{xy}, w_{yy}$ and w_{zz}
- (c) Given $z = e^x \sin y$, find z_{xx}, z_{xy}, z_{yyx} and z_{xyx}
- (d) If $z = \sin(3x + 2y)$, show that $3\frac{\partial^2 z}{\partial y^2} - 2\frac{\partial^2 z}{\partial x^2} = 6z$
10. (a) Given $z = f(xy)$, show that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$
- (b) If $v = f(x^2 + y^2)$, find $\frac{\partial v}{\partial y}, \frac{\partial v}{\partial x}$
- (c) Given $V = f(x - ct) + g(x + ct)$ where c is a constant, show that
- $$\frac{\partial^2 V}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2}.$$
11. Estimate the change in the value of $f(x, y, z) = 2xy^2z^3$ when (x, y, z) changes from $P(1, -1, 2)$ to $Q(0.99, -1.02, 2.02)$.
12. Use partial derivative to find the change in the value of $f(x, y) = x^2y - xy^2$ as (x, y) changes from $(1, 1)$ to $(1.02, .98)$.
13. Find the rate of change in the volume of a cylinder with radius 8 cm and height 12 cm if the radius increases at the rate of 0.2 cm/s while the height decreases at the rate of 0.5 cm/s.
14. Let $i = V/R$, find the error in calculating i if the error in computing V is 1 volt and R is 0.5 ohm at $V = 250$ volt and $R = 50$ ohm.

15. The length, width and height of a rectangular box increases at the rate of 1 cm/s, 2 cm/s and 3 cm/s respectively. Calculate the rate of increase in the diagonal of the box when the length is 2 cm, width is 3 cm and height 6 cm.
16. The dimensions of a closed rectangular box are measured as 3 meter, 4 meter and 5 meter with a possible error of 100/192 cm in each case. Use partial derivatives to approximate the maximum error in calculating the value of
- the surface area of the box, and
 - the volume of the box
17. The flow rate of gas through a pipe is given by $V = cd^{1/2}T^{-5/6}$ with c constant, d is diameter of the pipe and T the absolute temperature of the gas. The value of d is measured with a maximum error of 1.6% while the error in T is 0.36%. Find the maximum error in calculating V .
18. A box with height h has a square base with length x . The error in measuring the side of the base is 1% whereas that for the height is 2%. Approximate the maximum error in calculating the volume.
19. The total resistance R for three components with resistances R_1 , R_2 and R_3 connected in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If R_1 , R_2 and R_3 are measured with the maximum percentage error of 1% for each measurements, use partial derivatives to approximate the maximum error in calculating R .

In questions 20 – 23, use chain rule to find dw/dt

20. $w = x^3 - y^3$; $x = \frac{1}{t+1}$, $y = \frac{t}{t+1}$
21. $w = \ln(u+v)$; $u = e^{-2t}$, $v = t^3 - t^2$
22. $w = r^2 - s \tan v$; $r = \sin^2 t$, $s = \cos t$, $v = 4t$
23. $w = x^2y^3z^4$; $x = 2t + 1$, $y = 3t - 2$, $z = 5t + 4$
24. Find the first partial derivatives for the following functions using the chain rule.
- $z = x^2 + y^2$; $x = r \cos \theta$, $y = r \sin \theta$
 - $z = \ln(x+y)$; $x = u + v$, $y = u - v$
 - $z = x^2y^2 - x + 2y$; $x = \sqrt{u}$, $y = uv^3$
 - $z = r^3 + s + v$; $r = xe^y$, $s = ye^x$, $v = x^2y$
 - $z = pq + qw$; $p = 2x - y$, $q = x - 2y$, $w = -2x + 2y$

25. Given $w = 3xy^2z^3$; $y = 3x^2 + 2$, $z = \sqrt{x-1}$, find dw/dx by using the chain rule.
26. Given $w = \cos(uv)$; $u = xyz$, $v = \frac{\pi}{4(x^2 + y^2)}$, find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial z}$ at $x = 1$, $y = 1$ and $z = 1$.

In each of questions 27–30 the equation defines implicitly function of two variables $z = f(x, y)$. Find $\partial z/\partial x$ and $\partial z/\partial y$.

27. $z^2 + z \sin(xy) = 0$

28. $2xz^3 - 3\ln(yz^2) + x^2y^2 + 4z = 0$

29. $xe^{yz} - 2ye^{xz} + 3ze^{xy} = 1$

30. $x + y^2 + z + \sin z = \pi$

31. Find $\frac{d^2z}{d\theta^2}$ for the following functions using the chain rule.

- (a) $z = x^2 + y^2$; $x = \cos \theta$, $y = \sin \theta$
 (b) $z = \ln(x^2y)$; $x = e^\theta$, $y = \theta^2$

32. Determine the critical points for the following functions and use the second derivative test to classify each of the critical points as a local maximum, a local minimum or a saddle point:

- (a) $f(x, y) = xy + 2x - 3y - 2$
 (b) $f(x, y) = 2x^2 - 2xy + 3y^2 - 4x - 8y + 20$
 (c) $f(x, y) = x^3 + y^3 - 3xy$
 (d) $f(x, y) = e^x \sin y$
 (e) $f(x, y) = (x-1)(y-1)(x+y-1)$
 (f) $f(x, y) = 4xy - x^4 - y^4$

33. Session 2013/14 Sem II

- (a) Given a three variables function $w = f(x, y, z) = x^2 + 4y^2 + 9z^2$.
- (i) Find the domain and range of the function. **(2 marks)**
 (ii) Sketch the level surface of the function for $w = 1$. **(3 marks)**
- (b) (i) If $z = xy + xe^{y/x}$, show that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = xy + z. \quad \text{(4 marks)}$$

- (ii) Find $\partial z/\partial x$ if z as a function of x and y is defined implicitly by $\ln(x + yz) = 1 + xy^2z^3$. **(3 marks)**
- (c) Find the critical points of $f(x, y) = 2x^2 - 4xy + y^3 + 2$ and use the second derivative test to classify each point as a local maximum, a local minimum or a saddle point. **(8 marks)**

34. Session 2014/15 Sem I

- (a) Sketch the graph of the level surface of the function

$$f(x, y, z) = z - x^2 - y^2 + 2x + 1$$

when $f(x, y, z) = 2$. (5 marks)

- (b) The flow of blood in an arteriole measured in cm^3/sec is given by

$$F = \frac{\pi P R^4}{8kL},$$

where L is the length of the arteriole, R is the radius in centimeters, P is the difference in pressure between the two ends of the arteriole in dyne-sec/cm² which is a constant and k is a constant. Use partial derivative to find the maximum percentage error in calculating F if the maximum error in measuring L is 1% and error in R is 2%. (7 marks)

- (c) Find all critical points of $f(x, y) = x^2 + y^3 + 6xy - 7x - 7y$ and use the second derivative test to classify each as a relative maximum, a relative minimum or a saddle point. (8 marks)

35. Session 2002/03 Sem I

- (a) Sketch the graph of the level surface of the function

$$p(x, y, z) = z - x^2 - y^2 + 2y + 1$$

when $p(x, y, z) = 2$. (5 marks)

- (b) Find $\partial z / \partial x$, if $z = f(x, y)$ is defined implicitly as

$$xye^x = 7 + \sin(xyz). \quad (4 \text{ marks})$$

- (c) If $w = f(x - y, y - z, z - x)$, show that

$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (5 \text{ marks})$$

- (d) By using the partial derivatives, estimate the maximum percentage error in evaluating $T = 2\pi\sqrt{L/g}$, if the percentage error in estimating L and g are 0.5% and 0.1% respectively. (6 marks)

36. Session 2002/03 Sem II

- (a) Sketch the graph of the level surface of the function

$$w(x, y, z) = z + x^2 + y^2 + 2$$

when $w(x, y, z) = 3$. (4 marks)

- (b) The total resistant R of a parallel circuit with resistors having resistance x, y and z is

$$\frac{1}{R} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}.$$

If the percentage error in measuring the resistance x, y and z is 2% respectively, use partial derivative to find the maximum percentage error in calculating R . **(6 Marks)**

- (c) A cuboid without it's cover with a volume of 500 m^3 is to be build by using the minimum amount of aluminium sheet. Find out the dimension of the cuboid. **(10 marks)**

37. Session 2005/06 Sem I

- (a) If z is a function of x and y and is defined implicitly by $x^2 + y^2 + z^2 = 1$, show that

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z - \frac{1}{z} \quad \text{(6 marks)}$$

- (b) Let $z = f(x, y)$, where $x = t + \cos t$, and $y = e^t$. Find dz/dt at $t = 0$ given that $f_x(1, 1) = 4$ and $f_y(1, 1) = -3$. **(6 marks)**

- (c) The height of a right circular cone is increasing at the rate of 0.2 cm s^{-1} and the radius of the base is decreasing at the rate of 0.3 cm s^{-1} . Find the rate of change of the volume when the height and the radius are 15 cm and 10 cm respectively. **(8 marks)**

38. Session 2006/07 Sem I

- (a) If $f(x, y) = \cos(y^2 + 2x)$, show that $f_{xy} = -4y \cos(y^2 + 2x)$. Hence evaluate $f_{xy}(2, 1)$. **(4 marks)**

- (b) If $f(x, y)$ is a function of x, y and $z = xy + f(x^2 + y^2)$, show that

$$y \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = y^2 - x^2. \quad \text{(5 marks)}$$

- (c) Use the chain rule to find the value of $\partial u / \partial r$ at point $(\sqrt{\pi}, \sqrt{\pi}, 1)$, given that $u = z \sin(xy)$, $x = r + s$, $y = r - s$, and $z = r^2 + s^2$. **(5 marks)**

- (d) An open rectangular box is to have a volume of 32 m^3 . Find the dimensions that will make the surface area minimum. **(6 marks)**

39. Session 2007/08 Sem II

- (a) Sketch the level surface for the function

$$T(x, y, z) = 65 - x^2 - y^2 - z$$

at $T(x, y, z) = 29$. **(5 marks)**

- (b) Let $f(x, y, z) = x^2 + y^2 + z^2$, where $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$. Use chain rule to find $\partial f / \partial \rho$ in terms of ρ . **(6 marks)**
- (c) A closed rectangular box is to be build with the volume equal to 36 m^3 . The material for the bottom of the box cost RM 1 per m^2 while the top and sides cost RM 8 per m^2 . Find the dimensions of the box with minimum cost. **(9 marks)**

40. Session 2007/08 Sem II

- (a) Sketch the level surface of the function

$$w(x, y, z) = x^2 + y^2 - 2y + z^2$$

when $w(x, y, z) = 3$. **(4 marks)**

- (b) The relationship between the current (I), voltage (V), and the resistance (R) is given by $I = V/R$. By using the partial derivatives, estimate the maximum percentage error in evaluating I , if the percentage error in estimating V and R are 2% and 1% respectively. **(6 marks)**

- (c) Obtain the local extrema and saddle point (if exists) for the function

$$f(x, y) = 2x^4 + y^2 - 12xy. \quad \textbf{(10 marks)}$$

41. Session 2007/08 Sem II

- (a) If $z = \frac{x+y}{\sqrt{x^2+y^2}}$, find $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$. **(5 marks)**
- (b) Let $z = f(x, y)$, where $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$. Use chain rule to find $\partial z / \partial u$ and $\partial z / \partial v$. Hence show that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} \quad \textbf{(6 marks)}$$

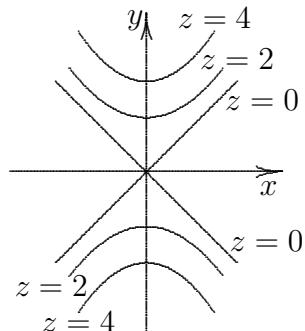
- (c) The volume V of a right circular cone is given in terms of its semi-vertical angle α and the radius r of its base by the formula

$$V = \frac{1}{3}\pi r^3 \cot \alpha.$$

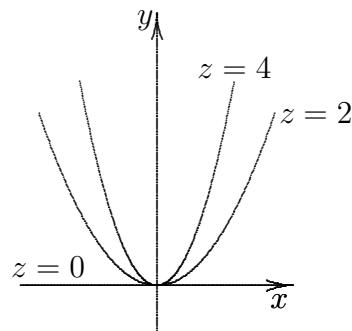
The radius r and the angle α are given to be 6 m and $\frac{1}{4}\pi$ rad, subject to the errors of 0.05 m and 0.005 radian respectively. Find the greatest percentage error in calculating the value of the volume. **(9 marks)**

ANSWERS TO TUTORIAL 1

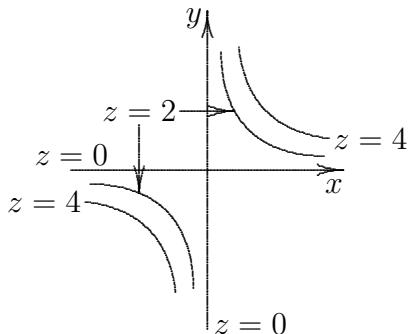
- 1.** (a) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty, x^2 + y \geq 0\}$,
 Range = $\{z : 0 \leq z < \infty\}$.
 (b) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty, x - y^2 \neq 0\}$,
 Range = $\{z : -\infty < z < \infty\}$.
 (c) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty, x + y^2 \neq 0\}$,
 Range = $\{z : -\infty < z < \infty\}$.
 (d) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty, x > y\}$,
 Range = $\{z : -\infty < z < \infty\}$.
 (e) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty, y \neq 1\}$,
 Range = $\{z : 0 < z < \infty\}$.

2. (a)

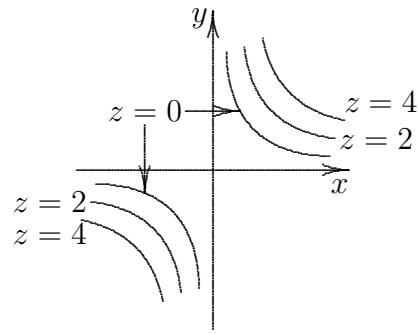
(b)



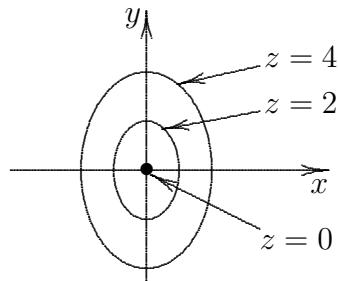
(c)



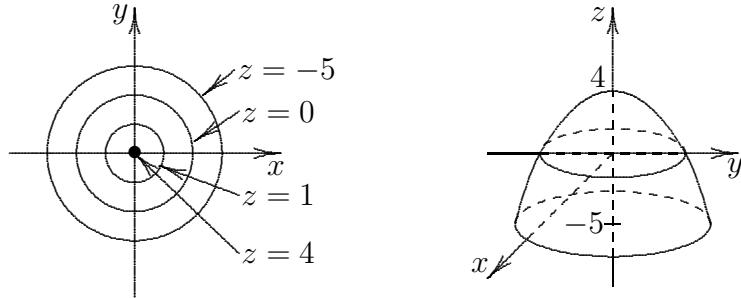
(d)



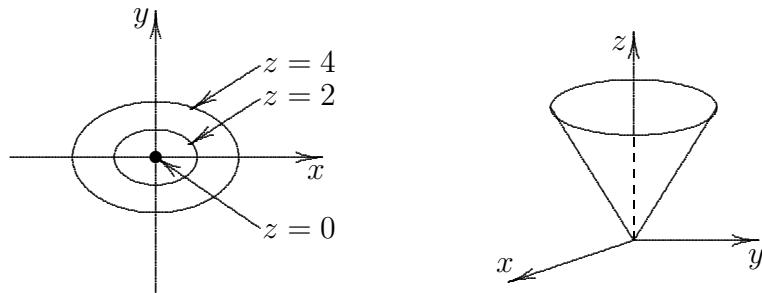
(e)



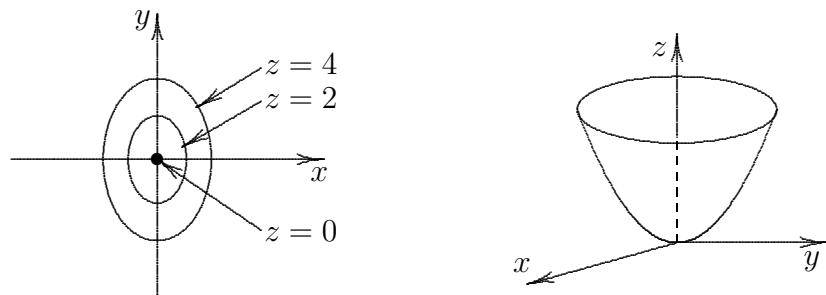
3. (a) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty\}$
 Range = $\{z : -\infty < z \leq 4\}$



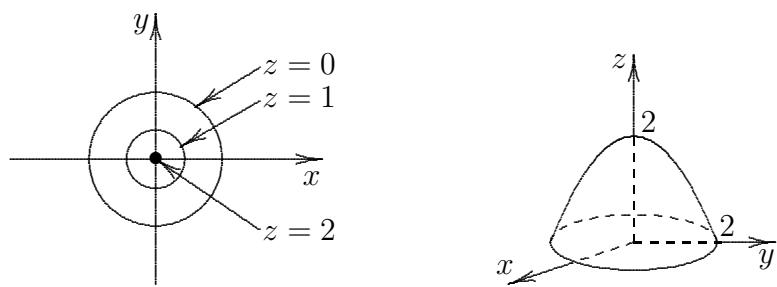
- (b) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty\}$
 Range = $\{z : 0 \leq z < \infty\}$



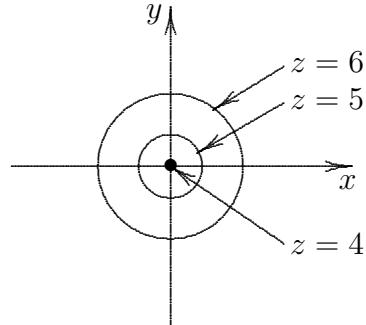
- (c) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty\}$
 Range = $\{z : 0 \leq z < \infty\}$



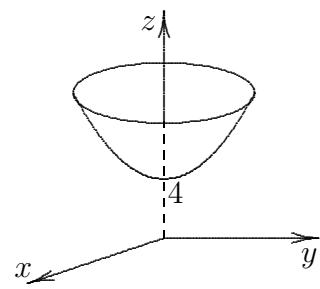
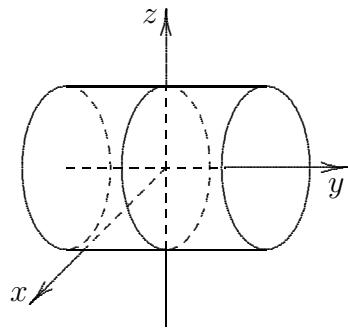
- (d) Domain = $\{(x, y) : 0 \leq x^2 + y^2 \leq 4\}$
 Range = $\{z : 0 \leq z < 2\}$



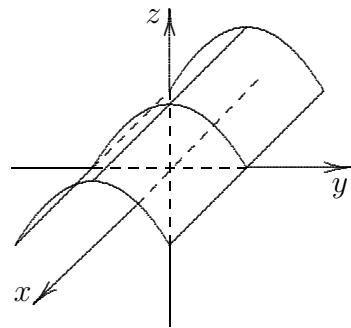
- (e) Domain = $\{(x, y) : -\infty < x < \infty, -\infty < y < \infty\}$
 Range = $\{z : 4 \leq z < \infty\}$



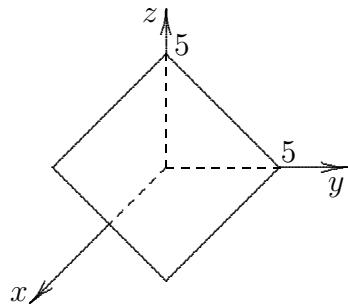
4. (a)

(b) Graph for $z \geq 0$ 

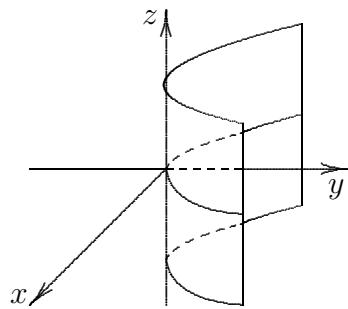
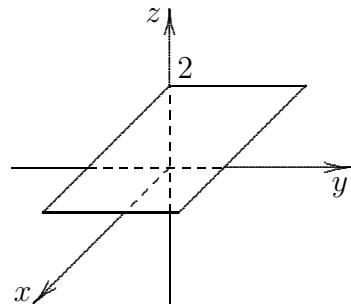
(c) Graph in the first octant



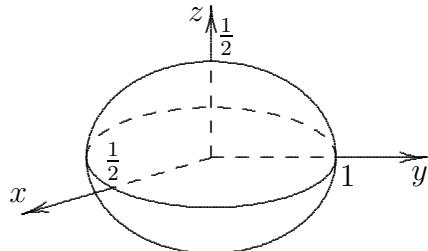
(d) Graph in the first octant



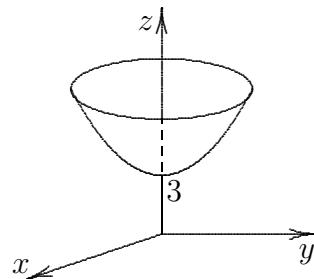
(e)



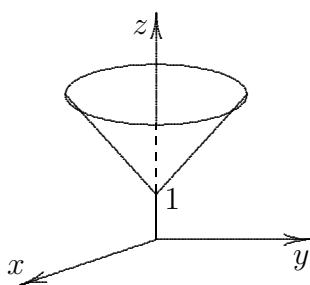
5. (a)



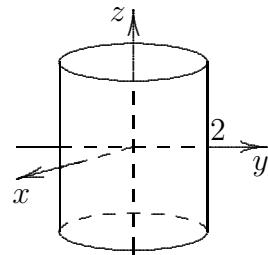
(b)



(c)



(d)



6. (a) $z_x = \frac{2}{(2x+3y)}; \quad z_y = \frac{3}{(2x+3y)}$

(b) $z_x = y \cos(xy); \quad z_y = x \cos(xy)$

(c) $z_x = \cos(xy) - xy \sin(xy); \quad z_y = -x^2 \sin(xy)$

(d) $s_r = \frac{t^2}{(r+t)^2}; \quad s_t = \frac{r^2}{(r+t)^2}$

(e) $w_x = \frac{1}{z}; \quad w_y = \frac{2y}{z}; \quad w_z = -\frac{(x+y^2)}{z^2}$

(f) $z_x = \frac{2}{(y^2-x)^2}; \quad z_y = \frac{-4y}{(y^2-x)^2}$

(g) $w_x = \tan(y+3z); \quad w_y = x \sec^2(y+3z); \quad w_z = 3x \sec^2(y+3z)$

(h) $w_x = yz^2 \cos(xyz^3); \quad w_y = xz^3 \cos(xyz^3); \quad w_z = 3xyz^2 \cos(xyz^3)$

(i) $w_x = \frac{-2x}{(x^2+y^2+z^2)^2}; \quad w_y = \frac{-2y}{(x^2+y^2+z^2)^2};$
 $w_z = \frac{-2z}{(x^2+y^2+z^2)^2}$

(j) $w_x = \frac{\sin(2x+3y+z^2) - 2x \cos(2x+3y+z^2)}{\sin^2(2x+3y+z^2)};$
 $w_y = \frac{-3 \cos(2x+3y+z^2)}{\sin^2(2x+3y+z^2)};$

$$w_z = \frac{-2z \cos(2x+3y+z^2)}{\sin^2(2x+3y+z^2)}$$

8. (a) $\frac{\partial z}{\partial y} = -\frac{y}{x}$

(b) $\frac{\partial z}{\partial x} = \frac{z \cos(xz) + y - 3x^2}{2yz - x \cos(xz)}$; $\frac{\partial z}{\partial y} = \frac{x - 2y - z^2}{2yz - x \cos(xz)}$

(c) $\frac{\partial z}{\partial y} = \frac{z^2 + xz}{1 - 2x^2 - 2xz - xy - yz}$

9. (a) $f_{xy} = \frac{1}{y} = f_{yx}$

(b) $w_{xx} = 2 \cos\left(\frac{z}{y}\right); \quad w_{xz} = -\frac{2x}{y} \sin\left(\frac{z}{y}\right)$
 $w_{yz} = \frac{x^2}{y^2} \left[\frac{z}{y} \cos\left(\frac{z}{y}\right) + \sin\left(\frac{z}{y}\right) \right]$
 $w_{xy} = \frac{2xz}{y^2} \sin\left(\frac{z}{y}\right)$
 $w_{yy} = -\frac{zx^2}{y^3} \left[\frac{z}{y} \cos\left(\frac{z}{y}\right) - 2 \sin\left(\frac{z}{y}\right) \right]; \quad w_{zz} = -\frac{x^2}{y^2} \cos\left(\frac{z}{y}\right)$

(c) $z_{xx} = e^x \sin y; \quad z_{xy} = e^x \cos y$

$$10. \text{ (b)} \quad \frac{\partial v}{\partial y} = 2yf'(x^2 + y^2); \quad \frac{\partial v}{\partial x} = 2xf'(x^2 + y^2);$$

11. 0.96 12. 0.04 13. 64π cm³/second

14. -0.03 15. $\frac{26}{7}$ 16. (i) 0.25 m^2 (ii) $\frac{47}{192} \text{ m}^3$

17. 1.1% 18. 4% 19. 4.1%

20. $\frac{dw}{dt} = -\frac{3(x^2 + y^2)}{(t+1)^2}$ 21. $\frac{dw}{dt} = \frac{3t^2 - 2t - 2e^{-2t}}{u+v}$

$$22. \frac{dw}{dt} = 4r \sin t \cos t - \tan v \sin t - 4s \operatorname{sek}^2 v$$

23. $\frac{dw}{dt} = 4xy^3z^4 + 9x^2y^2z^4 + 20x^2y^3z^3$

$$23. \frac{dw}{dt} = 4xy^3z^4 + 9x^2y^2z^4 + 20x^2y^3z^3$$

24. (a) $\frac{dz}{l} = 2x \cos \theta + 2y \sin \theta; \quad \frac{dz}{lo} =$

$$(b) \frac{dz}{d\theta} \equiv \frac{2}{a\theta}; \quad \frac{d\theta}{dz} \equiv 0$$

$$\frac{du}{dz} = \frac{x+y}{2xy^2-1} \quad \text{dr}$$

$$(c) \frac{du}{dz} = \frac{v^o(2x-y+2)}{2\sqrt{u}}; \quad \frac{dr}{dz} = 6uv^o(x-y+1)$$

$$(d) \frac{dx}{dy} = 2r^2 e^y + ye^x + 4vxy; \quad \frac{dy}{dx} = 3r^2 xe^y + e^x + 2vx^2$$

$$(e) \frac{dz}{dx} = p + w; \quad \frac{dz}{dy} = q - 2(p + w)$$

25. $\frac{dw}{dx} = 3y^2z^3 + 36x^2yz^3 + \frac{9xy^2z^2}{2\sqrt{x-1}}$

$$26. \quad \frac{dw}{dx} = 0; \quad \frac{dw}{dz} = -\frac{\pi}{8} \sin\left(\frac{\pi}{8}\right)$$

$$27. \frac{\partial z}{\partial x} = -\frac{yz \cos(xy)}{2z + \sin(xy)}; \quad \frac{\partial z}{\partial y} = -\frac{xz \cos(xy)}{2z + \sin(xy)}$$

28. $\frac{\partial z}{\partial x} = -\frac{z(z^3 + xy^2)}{(3xz^3 + 2z - 3)}$; $\frac{\partial z}{\partial y} = -\frac{z(2x^2y^2 - 3)}{2y(3xz^3 + 2z - 3)}$

$$29. \frac{\partial z}{\partial x} = -\frac{e^{yz} - 2yze^{xz} + 3zye^{xy}}{xye^{yz} - 2xye^{xz} + 3e^{xy}}, \quad \frac{\partial z}{\partial y} = -\frac{xze^{yz} - 2e^{xz} + 3xze^{xy}}{xye^{yz} - 2xye^{xz} + 3e^{xy}}$$

$$30. \quad \frac{\partial z}{\partial x} = -\frac{1}{\cos z}; \quad \frac{\partial z}{\partial y} = -\frac{2y}{\cos z}$$

31. (a) $\frac{\partial^2 z}{\partial \theta^2} = 0$ (b) $\frac{\partial^2 z}{\partial \theta^2} = -\frac{2}{\theta^2}$

- 32.** (a) $(3, -2, 4)$ saddle point (b) $(2, 2, 0)$ local minimum

- (c) $(0, 0, 0)$ saddle point; $(1, 1, -1)$ local minimum

- (d) no critical point

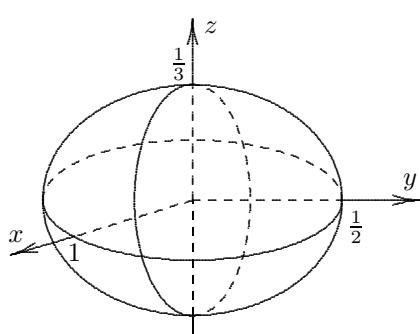
- (e) $(1, 1, 0), (0, 1, 0), (1, 0, 0), (1, -1, 0)$ saddle point

- (f) $(1, 1, 2)$, $(-1, -1, 2)$ local maximum; $(0, 0, 0)$ saddle point

33. (a) (i) Domain = $\{x, y, z\} : -\infty < x < \infty, -\infty < y < \infty, -\infty < z < \infty\}$

Range = $\{w : 0 \leq w < \infty\}$

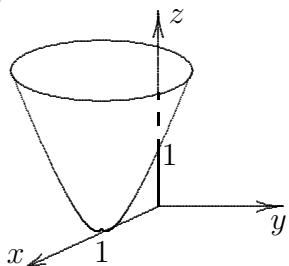
(ii)



$$(b) \text{ (ii)} \frac{\partial z}{\partial x} = \frac{1 - xy^2z^3 - y^3z^4}{y - 3x^2y^2z^2 - 3xy^3z^3}.$$

- (c) $(0, 0, 2)$ saddle point; $\left(\frac{4}{3}, \frac{4}{3}, \frac{22}{27}\right)$ local minimum.

34. (a)

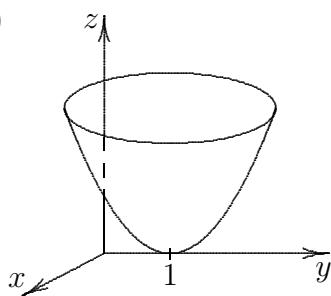


(1) 007

(c) $\left(\frac{1}{2}, 1, -\frac{21}{4}\right)$ saddle point;

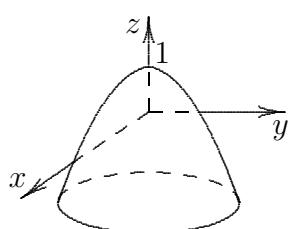
$\left(-\frac{23}{2}, 5, -\frac{1207}{4}\right)$ local minimum.

35. (a)



(b) $z_x = \frac{-z \cos(xyz) + e^x}{x \cos(xyz)}$;
 (d) 0.3%.

36. (a)



(b) 2%;
 (c) $x = 10, y = 10, z = 5$, area = 300 m^2 .

37. (b) $z_t = z_x x_t + z_y y_t = 1$;

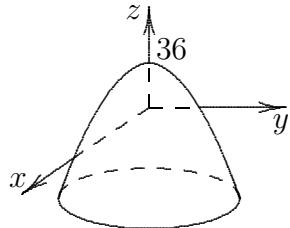
(c) $v_t = 23\frac{1}{3}\pi \text{ cm}^3\text{s}^{-1}$.

38. (a) $-4 \cos 5 = -1.135$;

(c) $u_r = yz \cos(xy) + xz \cos(xy) + 25 \sin(xy) = -2\sqrt{\pi}$.

(d) $z = \frac{32}{xy}, s = \frac{64}{x} + \frac{64}{y} + xy, x = 4, y = 4, z = 2$.

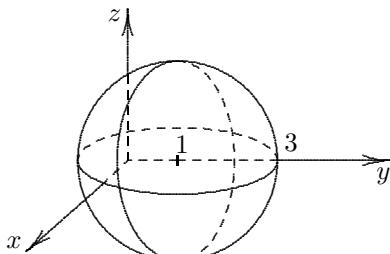
39. (a)



(b) 2ρ .

(c) $z = \frac{36}{xy}; \text{Cost} = 9xy + \frac{576}{x} + \frac{576}{y}; x = 4; y = 4; z = 2.25$.

Minimum cost = RM432.

40. (a) $x^2 + (y - 1)^2 + z^2 = 2^2$: sphere, center $(0, 1, 0)$, radius = 2.

(b) 3%.

(c) $(0, 0, 0)$ saddle point; $(3, 18, -162)$ & $(-3, -18, -162)$ minimum points.41. (a) $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0$. (c) 3.5%.