# Total differential is for ...

## Approximation Problems :-

1) the change of $z$
2) the incremental of $z$ or
3) an error of $z$

for $z = f(x,y)$

$\Downarrow$ 'have some meaning'

$$\boxed{\Delta z}$$

where

exact change

$$\boxed{\Delta z \approx df}$$

total differential

$$\boxed{df = f_x \, dx + f_y \, dy}$$

# TYPES OF APPROXIMATION

(i) Absolute Changes / absolute error / absolute decrement / absolute increment in $z$ i.e $\Delta z$

$$\boxed{\Delta z \approx df}$$

(ii) Maximum error / Maximum change / Maximum increment in $z$

$$\boxed{\|\Delta z\| \approx \|df\|}$$

Noticed that, $\|a+b\| \leq \|a\| + \|b\|$

(iii) Relative Changes / Relative error / Relative Increment in $z$

$$\boxed{\frac{\Delta z}{z} \approx \frac{df}{z}}$$

and

$\|a-b\| = \|a+(-b)\|$
$\leq \|a\| + \|-b\|$
$\leq \|a\| + \|b\|$

(iv) Percentage in error / Percentage in change, Percentage in increment in $z$ i.e $\frac{\Delta z}{z} \times 100\%$

$$\boxed{\frac{\Delta z}{z} \times 100\% \approx \frac{df}{z} \times 100\%}$$

(v) Maximum relative error in $z$ i.e $\left\|\frac{\Delta z}{z}\right\|$

$$\boxed{\left\|\frac{\Delta z}{z}\right\| \approx \left\|\frac{df}{z}\right\|}$$

(vi) Maximum Percentage Error in $z$ i.e $\left\|\frac{\Delta z}{z}\right\| \times 100\%$

$$\boxed{\left\|\frac{\Delta z}{z}\right\| \times 100\% \approx \left\|\frac{df}{z}\right\| \times 100\%}$$

4. The radius and height of a right circular cone are measured with errors of at most 3% and 2% respectively. Use increment to approximate the maximum possible percentage error in computing the volume of the cone using these measurements and the formula $V = \frac{1}{3}\pi R^2 H$.

Solution : we are given that

$$\left|\frac{\Delta R}{R}\right| \times 100\% \leq 3\%$$

&

$$\left|\frac{\Delta H}{H}\right| \times 100\% \leq 2\%$$

or

$$\left|\frac{\Delta R}{R}\right| \leq 0.03 \quad \& \quad \left|\frac{\Delta H}{H}\right| \leq 0.02$$

The partial derivatives of $V$ are

(c) $V_R = \frac{2}{3}\pi R H$ and $V_H = \frac{1}{3}\pi R^2$

So, the change of $V$ is approximated by

$$\Delta V \approx dV$$
$$\approx V_R \, dR + V_H \, dH$$
$$\approx \left(\frac{2}{3}\pi R H\right) dR + \left(\frac{1}{3}\pi R^2\right) dH$$

Dividing by the volume $V = \frac{1}{3}\pi R^2 H$, we obtain

$$\frac{\Delta V}{V} \approx \frac{\left(\frac{2}{3}\pi R H\right) dR + \left(\frac{1}{3}\pi R^2\right) dH}{\frac{1}{3}\pi R^2 H}$$
$$\approx 2\frac{dR}{R} + \frac{dH}{H}$$

So that, $\left|\frac{\Delta V}{V}\right| \approx \left|2\frac{dR}{R} + \frac{dH}{H}\right|$

Hence, $\left|\frac{\Delta V}{V}\right| \leq 2\left|\frac{dR}{R}\right| + \left|\frac{dH}{H}\right|$

$$\therefore \left| \frac{\Delta V}{V} \right| \leq 2(0.03) + (0.02)$$

$$\leq 0.08 \quad \#$$

Thus, the maximum percentage error in computing the volume $V$ is approximately 8 %.