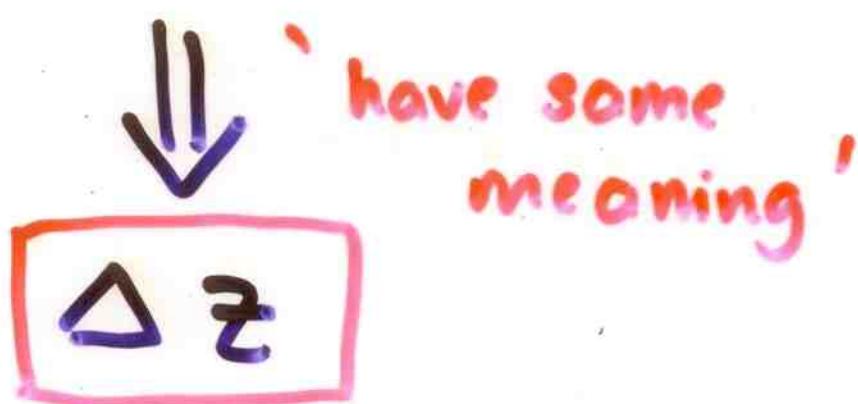


# Total differential is for ... Approximation Problem S :-

- 1) the change of  $z$
- 2) the incremental of  $z$  or
- 3) an error of  $z$

for  $z = f(x,y)$



where

exact change  $\Delta z \approx df$  total differential

$$df = f_x dx + f_y dy$$

# TYPES OF APPROXIMATION

absolute

(i) Absolute changes / error /  
absolute decrement / increment  
in  $z$  i.e  $\Delta z$

$$\Delta z \approx df$$

(ii) Maximum error / Maximum  
change / Maximum increment in  $z$

$$||\Delta z|| \approx ||df||$$

Noticed that,  
 $||a+b|| \leq$   
 $||a|| + ||b||$

(iii) Relative changes / Relative error/  
relative increment in  $z$

$$\frac{\Delta z}{z} \approx \frac{df}{z}$$

and

$$\begin{aligned} ||a-b|| &= ||a+(-b)|| \\ &\leq ||a|| + ||-b|| \\ &\leq ||a|| + ||b|| \end{aligned}$$

(iv) Percentage in error / Percentage in change, Percentage in increment in  $\bar{z}$  i.e.  $\frac{\Delta \bar{z}}{\bar{z}} \times 100\%$

$$\frac{\Delta \bar{z}}{\bar{z}} \times 100\% \approx \frac{df}{\bar{z}} \times 100\%$$

(v) Maximum relative error in  $\bar{z}$  i.e.  $\left\| \frac{\Delta \bar{z}}{\bar{z}} \right\|$

$$\left\| \frac{\Delta \bar{z}}{\bar{z}} \right\| \approx \left\| \frac{df}{\bar{z}} \right\|$$

(vi) Maximum Percentage Error in  $\bar{z}$  i.e.  $\left\| \frac{\Delta \bar{z}}{\bar{z}} \right\| \times 100\%$

$$\left\| \frac{\Delta \bar{z}}{\bar{z}} \right\| \times 100\% \approx \left\| \frac{df}{\bar{z}} \right\| \times 100\%$$

4. The radius and height of a right circular cone are measured with errors of at most 3% and 2% respectively. Use increment to approximate the maximum possible percentage error in computing the volume of the cone using these measurements and the formula  $V = \frac{1}{3}\pi R^2 H$ .

**Solution :** We are given that

$$\left| \frac{\Delta R}{R} \right| \times 100 \% \leq 3 \%$$

$$\left| \frac{\Delta H}{H} \right| \times 100 \% \leq 2 \% \quad \text{or}$$

$$\left| \frac{\Delta R}{R} \right| \leq 0.03 \quad \text{and} \quad \left| \frac{\Delta H}{H} \right| \leq 0.02$$

The partial derivatives of  $V$  are

$$V_R = \frac{2}{3}\pi R^2 H \text{ and } V_H = \frac{1}{3}\pi R^2$$

So, the change of  $V$  is approximated by

$$\Delta V \approx dV$$

$$\approx V_R dR + V_H dH$$

$$\approx \left(\frac{2}{3}\pi R^2 H\right) dR + \left(\frac{1}{3}\pi R^2\right) dH$$

Dividing by the volume  $V = \frac{1}{3}\pi R^2 H$ , we obtain

$$\frac{\Delta V}{V} \approx \frac{\left(\frac{2}{3}\pi R^2 H\right) dR + \left(\frac{1}{3}\pi R^2\right) dH}{\frac{1}{3}\pi R^2 H}$$

$$\approx 2 \frac{dR}{R} + \frac{dH}{H}$$

So that,

$$\left| \frac{\Delta V}{V} \right| \approx \left| 2 \frac{dR}{R} + \frac{dH}{H} \right|$$

$$\text{Hence, } \left| \frac{\Delta V}{V} \right| \leq 2 \left| \frac{dR}{R} \right| + \left| \frac{dH}{H} \right|$$

$$\therefore \left| \frac{\Delta V}{V} \right| \leq 2(0.03 + 0.02) \\ \leq 0.08 *$$

thus, the maximum percentage error  
in computing the volume  $V$  is  
approximately 8 % .