LOCAL EXTREMA

HOW DO WE FIND THE LOCAL EXTREMA?

DEFINITION

LET S BE THE DOMAIN OF *F* SUCH THAT *C* IS AN ELEMENT OF S.

THEN,

1) *F(C)* IS A **LOCAL MAXIMUM** VALUE OF *F* IF THERE EXISTS AN INTERVAL *(A,B)* CONTAINING *C*

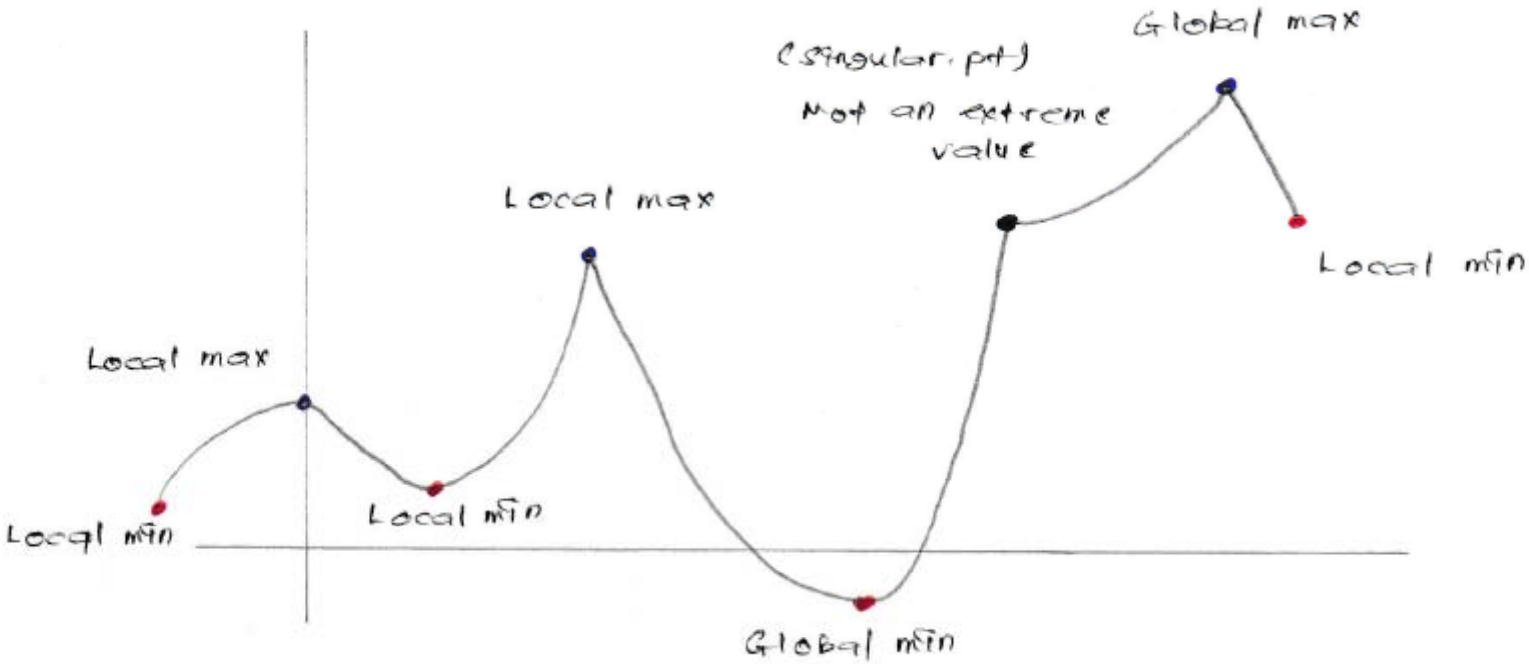
SUCH THAT *F(C)* IS THE MAXIMUM VALUE OF *F* ON *(A,B)*∩S.

2) *F(C)* IS A **LOCAL MINIMUM** VALUE OF *F* IF THERE EXISTS AN INTERVAL *(A,B)* CONTAINING *C*

SUCH THAT *F(C)* IS THE MINIMUM VALUE OF *F* ON *(A,B)*∩S.

3) F(C) IS A **LOCAL EXTREME VALUE** OF *F* IF IT IS EITHER A LOCAL MAXIMUM OR LOCAL MINIMUM

VALUE



* global max or min points are same as max or min absolute points.

* Global max points : points on graph with highest y-value.

* Local min / max points are "peaks" and "valley" points.

EX 1 Determine local maximum and minimum points for :

$$y = 2x^2 - 5x + 3$$

$$y' = 4x - 5 = 0$$

$$x = \frac{5}{4}$$

then, substitute $x = \frac{5}{4}$ into the equation:

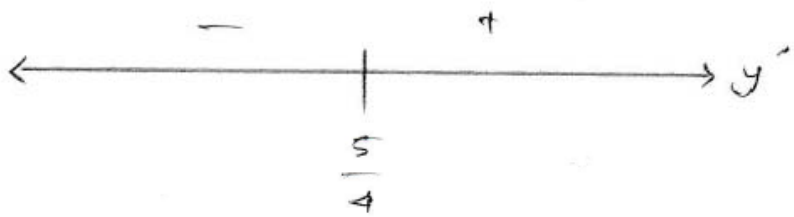
$$y\left(\frac{5}{4}\right) = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 3$$

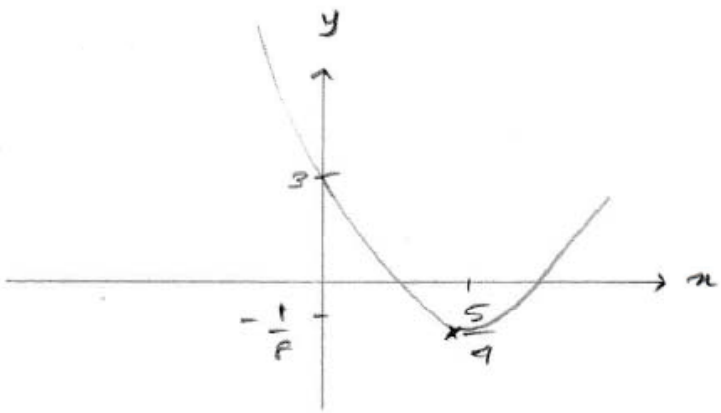
$$= \frac{25}{8} - \frac{25}{4} + 3$$

$$= -\frac{25}{8} + 3 = -\frac{1}{8}$$

$\because$ No max points

min. point $= \left(\frac{5}{4}, -\frac{1}{8}\right)$





EX 2 Find all local maximum and minimum points for:

$f(x) = \frac{1}{2}x + \sin x$ on $[0, 2\pi]$

$f'(x) = \frac{1}{2} + \cos x = 0$ (no singular points)

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Then, substitute the value of x into the equation:

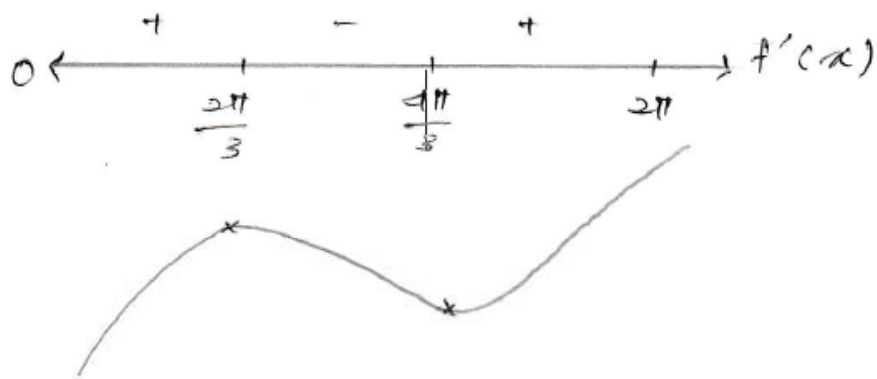
$f(x) = \frac{1}{2}x + \sin x$

$f\left(\frac{2\pi}{3}\right) = \frac{1}{2}\left(\frac{2\pi}{3}\right) + \sin\left(\frac{2\pi}{3}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

$$f\left(\frac{4\pi}{3}\right) = \frac{1}{2}\left(\frac{4\pi}{3}\right) + \sin\left(\frac{4\pi}{3}\right) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

then max point : $\left(\frac{2\pi}{3}, \frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$

min point : $\left(\frac{4\pi}{3}, \frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$



Theorem: Second Derivative Test

- Let f' and f'' exist at every point on the interval (a,b) containing c and $f'(c) = 0$.
- 1) If $f''(c) < 0$, then $f(c)$ is a local maximum.
- 2) If $f''(c) > 0$, the $f(c)$ is a local minimum

Second-Derivative Test for Functions of Two Variables

- Suppose $(x0, y0)$ is a point where "$f(x0, y0) = 0$. Let
- $D = fxx(x0, y0)fyy(x0, y0) - [fxy(x0, y0)]2$
- If $D > 0$ and $fxx(x0, y0) > 0$, then $f(x, y)$ has a local minimum at $(x0, y0)$.
- If $D > 0$ and $fxx(x0, y0) < 0$, then $f(x, y)$ has a local maximum at $(x0, y0)$.
- If $D < 0$, then $f(x, y)$ has a saddle point at $(x0, y0)$
- If $D = 0$, then we cannot say what happens at $(x0, y0)$

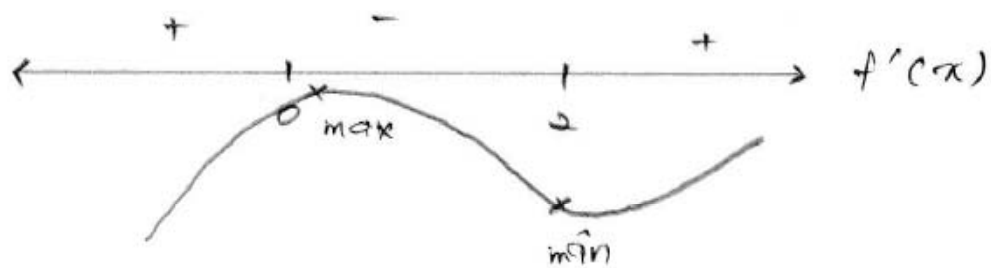
EX 3 Find all critical points for:

$f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0, 2$ (no singular points)



$f''(x) = 6x - 6$

$f''(0) = -6 < 0 \Rightarrow$ concave down at $x=0 \Rightarrow$ (max)

$f''(2) = 12 - 6 = 6 > 0 \Rightarrow$ concave up at $x = 2 \Rightarrow$ (min)

For critical points:

$f(x) = x^3 - 3x^2 + 1$

$f(0) = 1$

$f(2) = 8 - 3(4) + 1 = -3$

Then the critical points is:

$(0, 1)$ for local max

$(2, -3)$ for local min

EX 4 Find local and global extrema for:

$$y = x^2 + \frac{1}{x^2} \quad \text{on } [-2, 2]$$

note: there's a VA at $x = 0$ (we expect all derivatives to also be undefined at $x = 0$)

$$y' = 2x - \frac{2}{x^3} = 0$$

$$\frac{2x^4 - 2}{x^3} = 0$$

$$2x^4 - 2 = 0$$

$$x^4 = 1$$

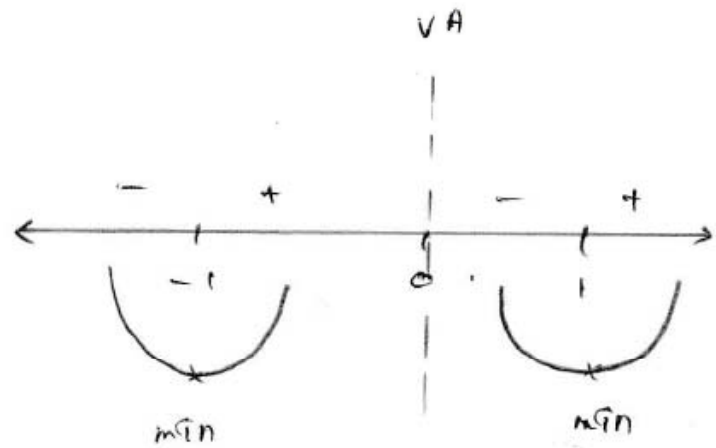
$$x = \pm 1$$

Test: $x = -2$, $\frac{+}{-}$ $\qquad$ $x = \frac{1}{2}$, $\frac{-}{+}$

$x = -\frac{1}{2}$ $\quad \frac{-}{-}$ $\qquad$ $x = 1000$, $\frac{+}{+}$

$$y'' = 2 + \frac{(-2)(-3)}{x^4}$$

$$= \frac{2x^4 + 6}{x^4} > 0$$



on $[-2, 2]$

$$y = x^2 + \frac{1}{x^2}$$

$$y(\pm 1) = 1 + 1 = 2$$

$y(\pm 1) = 1 + 1 = 2$

$y(\pm 2) = 4 + \frac{1}{4} = \frac{17}{4}$

min point : $(-1, 2)$

$(1, 2)$

endpoints : $(-2, 4\frac{1}{4})$

$(2, 4\frac{1}{4})$

$\therefore$ no global max (because graph goes up to ∞)

· global min points $(\pm 1, 2)$