SSCM 1033   Test 2(25%)

Time: 1 hour 30 min. Answer **all** questions.

1. A right circular cylinder is measured with an error of 2% in radius and 3% in height. What is the maximum percentage error in calculating its volume?   [5 marks]

2. A cuboid having the dimension 30 cm, 40 cm and 50 cm is heated so that each sides elongated by $\frac{1}{8}$ cm. By using partial derivative, estimate the increase in total surface area of the cuboid.   [5 marks]

3. If $z = \cos(5x + 3y)$, show that $5\dfrac{\partial^2 z}{\partial y^2} - 2\dfrac{\partial^2 z}{\partial x^2} = 5z$ [8 marks]

4. If $y^2 = 1 + \sin x$, show that

$$2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + y^2 = 1.$$

Hence deduce the expansion of $\sqrt{(1 + \sin x)}$ in ascending power of x up to x^3.

[8 marks]

5. Find the Maclaurin's Series for e^x upto the term x^4 and hence approximate the following integral,

$$\int_0^{0.3} e^{-x^4}\, dx$$

correct to 4 decimal places.

[**8 marks**]

6. Obtain the local extremums and saddle point (if exists) for the function

$$f(x, y) = 6x^2 - 2x^3 + 6y^2 + 6xy.$$

[**8 marks**]

7. Given that the distance from the point $(1, 3)$ to the circle, $x^2 + y^2 = 90$ is $d = \sqrt{(x-1)^2 + (y-3)^2}$. By using the method of Lagrange multipliers obtain the smallest value of d.

[**8 marks**]