Universiti Teknologi Malaysia
Department of Science Mathematics
Semester 1 2015/2016

SSCM1033 Mathematical Methods II Test 2 (25%)
Instruction: Answer all the questions. Time: 1 hour 30 minutes

1. The Maclaurin series for $\sin x$ to terms in x^5, is

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \ldots.$$

Use this information to evaluate $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$.

[6 marks]

2. Given $f(x) = \ln x$. Find the first three non-zero terms the Taylor series generated by f at $x = 1$. Use your series to approximate the value of the integral

$$\int_{1.1}^{1.3} \frac{\ln x}{x-1}\,dx$$

correct to five decimal places. [10 marks]

3. Let $z = f(x, y)$. Then, find $\frac{\partial z}{\partial y}$ if $\sin(xz) + y^2 + z = 2$. [5 marks]

4. Find $\frac{dw}{dt}$ in terms of x, y, z and t if $x = 2t + 1$, $y = 3t - 2$, $z = 5t + 4$ and $w(x, y, z) = x^2y^3z^4$.

[5 marks]

5. Determine the local extrema and saddle points (if any) of the function

$$f(x, y) = 2y^3 - 6xy - x^2.$$ [10 marks]

6. Find the rate of change of P given by $P = h^2 + rh$ where r is increasing at the rate 0.2cm/s and h is decreasing at the rate 0.3cm/s when the dimensions of r is 10cm and h is 11cm.

[7 marks]

7. A cuboid with the dimensions of 30cm, 40cm and 50cm was heated and each of the sides expanded to $\frac{1}{16}$ cm. Use partial derivatives to approximate the error in the measurement of the total volume of the cuboid.

[7 marks]