Assignment 1, sem II 2015/2016

1. Given the system of differential equations

$$\begin{aligned} \dot{x} &= 3x + 4y \\ \dot{y} &= 4x - 3y. \end{aligned}$$

   Find the fixed points and determine their nature. Sketch the nullclines and the signs of $\dot{x}$ and $\dot{y}$ on the nullclines and various regions determined by them. Sketch the phase portrait of the system.

2. For each of the system of differential equations given

   i)

$$\begin{aligned} \dot{x} &= x^2 - y^2 \\ \dot{y} &= xy - 1. \end{aligned}$$

   ii)

$$\begin{aligned} \dot{x} &= -x + y \\ \dot{y} &= xy - 1. \end{aligned}$$

   (a) Determine the fixed points.

   (b) Determine the nullclines and the sign of $\dot{x}$ and $\dot{y}$ in various regions of the plane.

   (c) Using the information from part(a) and (b), sketch by hand a rough phase portrait of the system.

3. Find the Hamiltonian for the given system and sketch the phase portrait.

   (a)

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x - x^2. \end{aligned}$$

   (b)

$$\begin{aligned} \dot{x} &= y + x^2 - y^2 \\ \dot{y} &= -x - 2xy. \end{aligned}$$

4. Investigate the stability of the origin for the system

$$\begin{aligned} \dot{x} &= -y - x^3 \\ \dot{y} &= x - y^3. \end{aligned}$$

using the Lyapunov function $V(x, y) = x^2 + y^2$

5. Given a two preys and one predator equations

$$\begin{aligned} \dot{x} &= \alpha xz + \beta xy - \gamma x \\ \dot{y} &= \delta y - \epsilon xy \\ \dot{z} &= \mu z(\nu - z) - \chi(xz). \end{aligned}$$

Find the coexistence fixed point of the system and analyze the stability of the fixed point using the Routh-Hurwitz criteria.

6. Describe some of the features for the following set of polar differential equation

$$\begin{aligned} \dot{r} &= r(1-r)(2-r)(3-r) \\ \dot{\theta} &= -1. \end{aligned}$$

7. By following the flow across the square with coordinates at $(1, 1)$, $(1, -1)$, $(-1, 1)$ and $(-1, -1)$ centered at the origin, prove the system

$$\begin{aligned} \dot{x} &= -y + x\cos(\pi x) \\ \dot{y} &= x - y^3. \end{aligned}$$

has a stable limit cycle.