Assignment 2:

Chapter 4 & 5 (PARTIAL DERIVATIVE AND ITS APPLICATIONS)
TOTAL MARKS: 40

1. Given $z = f(x, y)$, $x = rs$ and $y = r$. Show that

   i. $\dfrac{\partial z}{\partial r} = \dfrac{\partial z}{\partial x}s + \dfrac{\partial z}{\partial y}.$

   ii. $\dfrac{\partial^2 z}{\partial r^2} = s^2\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x \partial y} + s\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2}.$

   [7 MARKS]

2. A quantity S which is in terms of u and v is given as $S = u^2 - uv$. The maximum errors in measuring u and v are respectively 0.3 and 0.5. Estimate the maximum error in calculating the value of S when $u = 5$ and $v = 8$.

   [7 MARKS]

3. Let $z = f(x, y)$ where $x = t + \cos t$ and $y = e^t$. Find $\dfrac{dz}{dt}$ at $t = 0$ given that $f_x(1,1) = 4$ and $f_y(1,1) = -3$.

   [5 MARKS]

4. Find local maximum, local minimum and saddle points of

   $$f(x, y) = 2x^2 - 4xy + y^4 + 2$$

   if they do exist, by using the partial derivative tests.

   [7 MARKS]

5. Find the minimum and maximum values of the function $f(x, y, z) = 3x + 4y$ on the circle $x^2 + y^2 = 1$ by using Lagrange Multipliers

   [7 MARKS]

6. Use Lagrange Multiplier Method to find the maximum and minimum values of $f(x, y, z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 120$.

   [7 MARKS]