Universiti Teknologi Malaysia
Department of Mathematical Sciences
Semester I, 2016/17

SSCM 1033: MATHEMATICAL METHODS II   Test 2(25%)   Time: $1\frac{1}{2}$hr

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ANSWER ALL QUESTIONS

1. Let z be defined implicitly as a function of x and y by the equation

$$z^2 = e^{zx} - \sin(xy).$$

Find $\dfrac{\partial z}{\partial x}$.

[5 marks]

2. Obtain $\dfrac{\partial p}{\partial r}$ by using the chain rule if

$$p = u^2 \sin(vw), \quad u = xye^{rs}, \quad v = s\ln(xyr), \quad w = ys + r.$$

[5 marks]

3. A cuboid having dimensions 20 cm, 30 cm and 60 cm is heated so that each sides is elongated by $\dfrac{1}{20}$ cm. By using partial derivatives, estimate the increase in total surface area of the cuboid.

[7 marks]

4. Use total differentials to approximate the maximum percentage error in calculating $P = \dfrac{ws^3}{r}$ if the maximum percentage error in r,s, and w are 3.2%, 2.3% and 1.6%, respectively.

[7 marks]

5. Find all local extrema and saddle points, if any, on the graph of

$$z = 4xy - y^4 - x^4$$

[7 marks]

6. Use Lagrange Multiplier to find the dimensions of the rectangular box with the largest volume if the total surface area is given as 64cm^2

[7 marks]

7. Evaluate

$$\int_0^{\pi}\int_{\frac{y}{3}}^{\frac{\pi}{3}} \frac{\sin x}{x} \quad dx\,dy$$

[6 marks]

8. Use polar coordinates to evaluate

$$\iint_R \frac{1}{1+x^2+y^2} \quad dA,$$

where R is the region bounded by two circles r_1 and r_2, which are centred at the origin and $\theta_1 \leq \theta \leq \theta_2$.

[6 marks]

TOTAL: 50 MARKS

1. Let z be defined implicitly as a function of x and y by the equation $z^2 = e^{zx} - \cancel{\#} \sin(xy)$. Find $\frac{\partial z}{\partial x}$. 5m

$$F(x,y,z) = z^2 - e^{zx} + \cancel{\#} \sin(xy)$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

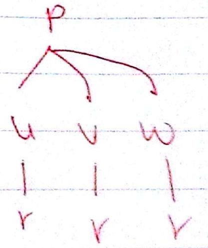
$$= -\frac{(-ze^{zx} + \cancel{z}y\cos(xy))}{2z - xe^{zx} + \cancel{\quad}}$$

#.

2. Obtain $\frac{\partial p}{\partial r}$ by using the chain rule if

$p = u^2 \sin(vw)$, $u = xye^{rs}$, $v = \sin(xyr)$, $w = ys + r$

5m

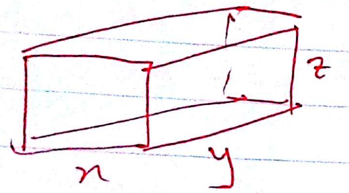


$$\frac{\partial p}{\partial r} = P_u U_r + P_v V_r + P_w W_r$$

$$= 2u\sin(vw)\, sxye^{rs} + wu^2\cos(vw)\frac{s\cancel{xy}}{\cancel{xy}r} + vu^2\cos(vw)$$

✱.

3. A cuboid having dimensions 20cm, 30 cm and 60 cm is heated so that each sides is elongated by $\frac{1}{20}$ cm. By using partial derivatives, estimate the ~~increase in~~ change total surface area of the cuboid. (5m)



$x = 20$, $dx = \frac{1}{20}$

$y = 30$, $dy = 1/20$

$z = 60$, $dz = 1/20$

$A(x,y,z) = 2xy + 2yz + 2xz$

$A_x = 2(y+z)$

$A_y = 2(x+z)$

$A_z = 2(x+y)$

$dA = A_x\,dx + A_y\,dy + A_z\,dz$

$= 2(y+z)\left(\frac{1}{20}\right) + 2(x+z)\left(\frac{1}{20}\right) + 2(x+y)\left(\frac{1}{20}\right)$

$= \frac{1}{10}(y+z + x+z + x+y)$

$= \frac{1}{10}(2y + 2z + 2x)$

$= \frac{1}{5}(y+z+x) = \frac{1}{5}(20+30+60) = \frac{110}{5} = 22$

4) Use total differentials to approximate the maximum percentage error in calculating $P = \frac{ws^3}{r}$ if the maximum percentage error in r, s and w are 3.2%, 2.3% and 1.6%, respectively.

$P = \frac{ws^3}{r}$, $\left\|\frac{dP}{P}\right\| \times 100\%$?

$$\left|\frac{dP}{P}\right| = \left|\frac{P_w\,dw + P_s\,ds + P_r\,dr}{P}\right|$$

$$= \left|\frac{\frac{s^3}{r}\,dw + \frac{3ws^2}{r}\,ds + -\frac{ws^3}{r^2}\,dr}{\frac{ws^3}{r}}\right| \leq \left|\frac{dw}{w}\right| + \left|3\frac{ds}{s}\right| + \left|-\frac{dr}{r}\right|$$

$$\left|\frac{dP}{P}\right| \times 100\% \leq \left|\frac{dw}{w}\right| \times 100\% + 3\left|\frac{ds}{s}\right| \times 100\% + \left|\frac{dr}{r}\right| \times 100\%$$

$$\leq 1.6\% + 3(2.3)\% + 3.2\%$$

$$\leq 11.7\%$$

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~~4~~. Find all local extrema and saddle points, if any, on the graph of

$$z = 4xy - y^4 - x^4 \qquad (5m)$$

$z_x = 4y - 4x^3 = 0 \Rightarrow y = x^3$

$z_y = 4x - 4y^3 = 0 \Rightarrow x = y^3$

$x = x^9$

$x - x^9 = 0$

$x(1 - x^8) = 0$

$x = 0, \quad x = 1, \quad x = -1$

$\downarrow \qquad \downarrow \qquad \downarrow$

$y = 0 \quad y = 1 \quad y = -1$

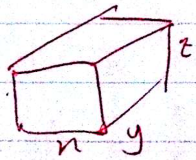
$z_{xx} = -12x^2$ $\qquad z_{xy} = 4$

$z_{yy} = -12y^2$

$D: 144x^2y^2 - 16.$

Point	x	y	z_{xx}	D	Type	z
A	0	0	0	-16	Saddle	0
B	1	1	-12	>0	Max	$4-1-1=2$
C	-1	-1	-12	>0	Max	$4-1-1=2.$

6) Use Langrange multipliers to find the dimensions of the rectangular box with the largest volume if the total surface area is given as 64 cm^2.



$$f(x,y,z) = xyz.$$

$$A(x,y,z) = 2xy + 2xz + 2yz = \cancel{64}\ 32.$$

$$\underbrace{xy + xz + yz}_{g(x,y,z)} = \cancel{16}.32.$$

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$$\left.\begin{aligned} f_x &= \lambda g_x \\ f_y &= \lambda g_y \\ f_z &= \lambda g_z \\ g(x,y,z) &= 16. \end{aligned}\right\} \quad \left.\begin{aligned} yz &= \lambda(y+z) \\ xz &= \lambda(x+z) \\ xy &= \lambda(x+y) \end{aligned}\right\} \quad \begin{aligned} xyz &= \lambda x(y+z) \quad \text{— (1)} \\ xyz &= \lambda y(x+z) \quad \text{— (2)} \\ xyz &= \lambda z(x+y) \quad \text{— (3)} \end{aligned}$$

$$xy + xz + yz = \cancel{16}.\ 32 \quad \text{— (4)}$$

(1) = (2) : $\lambda x(y+z) = \lambda y(x+z)$

$$\lambda(\cancel{xy} + xz) - \lambda(\cancel{yx} + yz) = 0$$

$$\lambda(xz - yz) = 0 \Rightarrow \lambda = 0, \quad z(x-y) = 0$$

$$x - y = 0 \Rightarrow \boxed{x = y}$$

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(2) = (3) : $\lambda y(x+z) = \lambda z(x+y)$

$$\lambda(yx + \cancel{yz}) - \lambda(zx + \cancel{zy}) = 0$$

$$\lambda yx - \lambda zx = 0$$

$$\lambda(yx - zx) = 0 \Rightarrow x(y-z) = 0$$

$$\boxed{y = z}$$

$$x^2 + x^2 + x^2 = \cancel{16}.\ 32$$

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$$3x^2 = \cancel{16}\ 32$$

$$x^2 = \frac{\cancel{16}\ 32}{3}$$

$$x = \pm\sqrt{\frac{\cancel{16}\ 32}{3}}$$

$$\therefore x = y = z = \sqrt{\frac{32}{3}} \text{ cm} \ \#.$$

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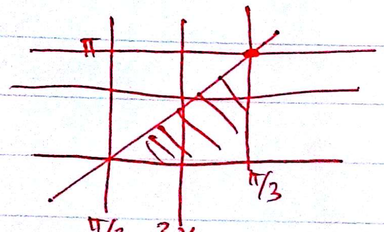
7) Evaluate the double integral by reversing its order

$$\int_0^{\pi}\int_{y/3}^{\pi/3} \frac{\sin x}{x}\,dx\,dy$$

$x = y/3 \Rightarrow y = 3x$



$x \left\langle \begin{matrix} \pi/3 \\ y/3 \end{matrix} \right.$

$y \left\langle \begin{matrix} \pi \\ 0 \end{matrix} \right.$

$$\int_0^{\pi/3}\int_0^{3x} \frac{\sin x}{x}\,dy\,dx = \int \frac{\sin x}{x}\left[y\right]_0^{3x} dx$$

$$= \int 3\sin x\,dx$$

$$= -3\left[\cos x\right]_0^{\pi/3}$$

$$= -3\left[\cos \pi/3 - \cos 0\right]$$

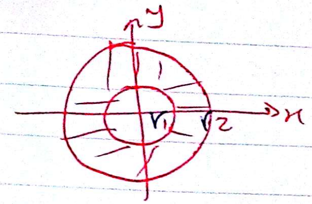
$$= -3\left(\frac{1}{2} - 1\right) = \frac{3}{2}$$

8) Use polar coordinates to evaluate

$$\iint_R \frac{1}{1+x^2+y^2}\, dA$$

where R is the region bounded by $r_1 \le r \le r_2$; $\theta_1 \le \theta \le \theta_2$

$$\int_0^{2\pi}\int^{2} \frac{1}{1+r^2}\cdot r\,dr\,d\theta$$



$u = 1+r^2$

$du = 2r$

$r\,dr = d$

$$= \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} \frac{1}{u}\cdot\frac{du}{2}\,d\theta$$

$$= \frac{1}{2}\int\left[\ln u\right] d\theta$$

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$$= \frac{1}{2}\int\left[\ln(1+r^2)\right]_{r_1}^{r_2} d\theta$$

$$= \frac{1}{2}\left(\ln(1+r_2^2) - \ln(1+r_1^2)\right)\int d\theta$$

$$= \frac{1}{2}\left[\ln(1+r_2^2) - \ln(1+r_1^2)\right]\left[\theta\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2}\left[\ln(1+r_2^2) - \ln(1+r_1^2)\right]\left(\theta_2 - \theta_1\right) \divideontimes$$