

**DEPARTMENT OF MATHEMATICAL SCIENCES,  
FACULTY OF SCIENCE  
UNIVERSITI TEKNOLOGI MALAYSIA  
SEMESTER 2 1718**

**SSCE 1993**

Answer **all** questions.

**ASSIGNMENT 2 UTMSPACE KL**

1. Evaluate the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \sin\left(\frac{y^3 + 1}{2}\right) dy dx$$

by reversing the order of integration.

[5 marks]

2. Evaluate

$$\int_0^2 \int_0^{\sqrt{1-(y-1)^2}} \left(\frac{2}{y}\right) dx dy$$

by first changing it to polar coordinates.

[6 marks]

3. A solid is bounded by the surfaces  $z = 2 - x^2$  and  $z = x^2$  for  $0 \leq y \leq 3$ .  
Find the mass of the solid when the density  $\delta(x, y, z) = x + y$ .

[7 marks]

4. Find the volume of a solid that lies inside the cylinder  $x^2 + y^2 = 4$  bounded above by  $z = x^2 + y^2 + 6$  and below by  $z = 4 - x^2 - y^2$ .

[6 marks]

5. Use spherical coordinates to evaluate

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z^2 dz dy dx.$$

[6 marks]

$$\begin{aligned}
 ① \int_0^1 \int_{\frac{y}{2}}^1 \sin\left(\frac{y^3+1}{2}\right) dy dx &= \int_0^1 \int_{\frac{y}{2}}^1 \sin\left(\frac{y^3+1}{2}\right) dx dy \\
 &= \int_0^1 \pi r^2 \sin\left(\frac{y^3+1}{2}\right) dy \\
 &= \int_0^1 y^2 \sin\left(\frac{y^3+1}{2}\right) dy \\
 &= -\frac{2}{3} \cos\left(\frac{y^3+1}{2}\right) \Big|_0^1 \\
 &= -\frac{2}{3} [\cos(1) - \cos\frac{1}{2}]
 \end{aligned}$$

$$\begin{aligned}
 ② \int_0^2 \int_0^{\sqrt{1-(y-1)^2}} \frac{2}{y} dx dy &= \int_0^{\frac{\pi}{2}} \int_0^{\frac{2 \sin \theta}{r \sin \theta}} r dr d\theta \\
 &= \int_0^{\frac{\pi}{2}} \frac{2}{\sin \theta} [r]_0^{2 \sin \theta} d\theta \\
 &= \int_0^{\frac{\pi}{2}} \frac{4 \sin \theta}{\sin \theta} d\theta = 4 \int_0^{\frac{\pi}{2}} \theta d\theta = 2\pi
 \end{aligned}$$

$$\begin{aligned}
 x &= \sqrt{1-(y-1)^2} \\
 x^2 &= 1-(y-1)^2 \\
 x^2 + (y-1)^2 &= 1 \\
 x^2 + y^2 - 2y &= 1 \\
 y^2 - 2y &= 2m\theta \\
 r &= 2m\theta
 \end{aligned}$$

$$\begin{aligned}
 ③ \text{ mass} &= \iiint_G \rho(x, y, z) dV = \iiint_G (x+y) dV \\
 &= \int_{-1}^1 \int_0^{2-x^2} \int_0^2 (x+y) dz dx dy \\
 &= \int_{-1}^1 \int_0^{2-x^2} \left[ xy + \frac{y^2}{2} \right]_0^2 dz dx \\
 &= \int_{-1}^1 \int_0^{2-x^2} \left( 3x + \frac{9}{2} \right) dz dx = \int_{-1}^1 \left( 3x + \frac{9}{2} \right) \left[ z \right]_{x^2}^{2-x^2} dx \\
 &= \int_{-1}^1 \left( 3x + \frac{9}{2} \right) \left[ 2 - 2x^2 \right] dx = \int_{-1}^1 \left( 9 + 6x - 9x^2 - 6x^3 \right) dx \\
 &= \left[ 9x + 3x^2 - 3x^3 - \frac{6x^4}{4} \right]_{-1}^1 = 1.2
 \end{aligned}$$

$$\begin{aligned}
 ④ \text{ Vol} &= \int_0^{2\pi} \int_0^r \int_0^{r^2} r dz dr d\theta \\
 ⑤ \boxed{6} &= \int_0^{2\pi} \int_0^r \int_{r^2}^{r^2+6} dr dz d\theta \\
 &= \int_0^{2\pi} \int_0^r (2r^2 + 2r) dr d\theta \\
 &= \int_0^{2\pi} \left[ \frac{r^4}{2} + r^2 \right]_0^r d\theta \\
 &= \int_0^{2\pi} 12 d\theta = 24\pi \text{ unit}^3
 \end{aligned}$$

$$\begin{aligned}
 ⑥ &\text{ limits } \theta = \frac{\pi}{4}, \mu = \frac{1}{\sqrt{2}} \\
 &\theta = 0, \mu = 1 \\
 &u = \cos \phi \\
 &du = -\sin \phi d\phi \\
 &\boxed{6} \quad \text{Diagram showing a quarter of a sphere of radius } r \text{ in spherical coordinates.} \\
 &\text{Volume element } dV = (\rho \cos \phi)^2 \sin \phi d\rho d\phi d\theta \\
 &= \int_0^{\pi} \int_0^{\frac{\pi}{4}} \int_0^1 (\rho \cos \phi)^2 \sin \phi d\rho d\phi d\theta \\
 &= \int_0^{\pi} \int_0^{\frac{\pi}{4}} \left[ \frac{\rho^3}{3} \right]_0^1 \cos^2 \phi d\phi d\theta \\
 &= \frac{32}{15} \int_0^{\pi} \int_0^{\frac{\pi}{4}} u^2 (-du) d\theta \\
 &= \frac{32}{5} \int_0^{\pi} \left[ -\frac{u^3}{3} \right]_0^{\frac{1}{\sqrt{2}}} d\theta \\
 &= \frac{32}{5} \left[ \frac{1}{3(2\sqrt{2})} + \frac{1}{3} \right] \pi = \frac{16(2\sqrt{2}-1)\pi}{15\sqrt{2}}
 \end{aligned}$$

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