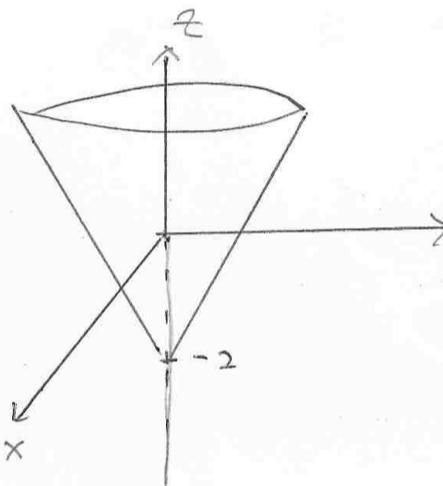
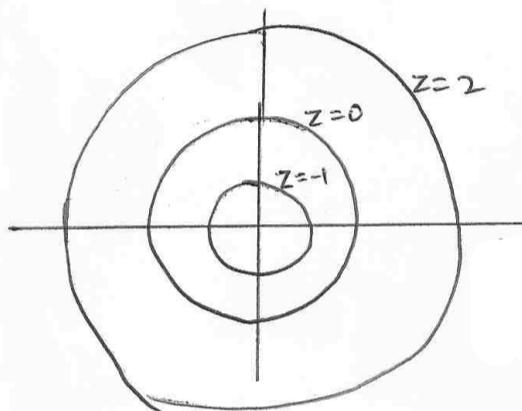


$$(2) z = -2 + \sqrt{x^2+y^2}$$

$$z = -1, \quad -2 + \sqrt{x^2+y^2} = -1 \Rightarrow x^2+y^2 = 1$$

$$z = 0 \quad -2 + \sqrt{x^2+y^2} = 0 \Rightarrow x^2+y^2 = 2^2$$

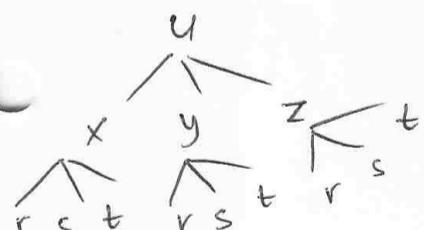
$$z = 2 \quad -2 + \sqrt{x^2+y^2} = 2 \Rightarrow x^2+y^2 = 4^2$$



[6]

$$(3) u = x^4y + y^2z^3, \quad x = rs e^{it}, \quad y = rs^2 e^{it}, \quad z = r^2 s \sin t$$

$$\begin{aligned} \frac{\partial u}{\partial s} &= U_x X_s + U_y Y_s + U_z Z_s \\ &= (4x^3y) re^{it} + (x^4 + 2yz^3)(2rs e^{it}) \\ &\quad + 3y^2z^2(r^2 s \sin t) \end{aligned}$$



$$\text{when } r=2, s=1, t=0, \quad \begin{aligned} x &= (2)(1)e^0 = 2 \\ y &= (2)(1)e^0 = 2 \\ z &= (2)(1)\sin 0 = 0 \end{aligned}$$

$$\therefore \text{When } r=2, s=1, \text{ and } t=0, \quad \begin{aligned} \frac{\partial u}{\partial s} &= 4(2)^3(2)(2)e^0 + (2^4 + 2(2)(0))(2(2)(1)e^0) \\ &\quad + 3(2)^2(0)^2() \end{aligned}$$

$$= 128 + 64 + 0$$

$$= 192$$

[6]

$$(4) \quad xyz = 4y^2z^2 + 2e^{xy} \ln x \quad F_x =$$

$$xyz - 4y^2z^2 - 2e^{xy} \ln x = 0 \quad F_y =$$

$$F_z =$$

$$\therefore F = xyz - 4y^2z^2 - 2e^{xy} \ln x$$

M1

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{[yz - (2ye^{xy} \ln x + 2e^{xy}(\frac{1}{x}))]}{xy - 8y^2z}$$

M1

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{[xz - 8yz^2 - 2xe^{xy} \ln x]}{xy - 8y^2z}$$

6

$$(5) \quad V = \pi r^2 h \quad r=6, h=10, dr=0.2, dh=-0.1$$

$$\begin{aligned} dV &= V_r dr + V_h dh \\ &= 2\pi rh(0.2) + \pi r^2(-0.1) \\ &= 2\pi(6)(10)(0.2) + \pi(6)^2(-0.1) \\ &= 24\pi - 3.6\pi \\ &= \underline{\underline{20.4\pi}} \\ &\approx 64.088 \end{aligned}$$

6

$$(6) \quad f(x,y) = x^3 - 3xy + y^3$$

$$f_x = 3x^2 - 3y \Rightarrow 3x^2 - 3y = 0 \quad \therefore y = x^2$$

$$f_y = -3x + 3y^2 \Rightarrow -3x + 3y^2 = 0$$

$$\begin{aligned} -3x + 3(x^2)^2 &= 0 \\ x(-3 + 3x^3) &= 0 \end{aligned}$$

$$\therefore x = 0, x = 1$$

Critical points are
(0,0) and (1,1)

$$G(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 6x & -3 \\ -3 & 6y \end{vmatrix} = 36xy - 9$$

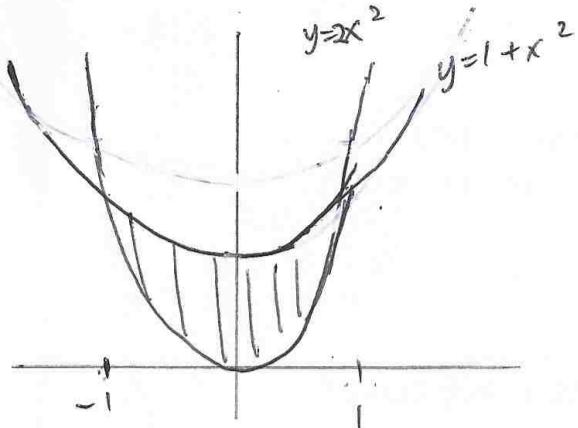
$$G(0,0) = 36(0)(0) - 9 = -9 \Rightarrow (0,0) \text{ is saddle point}$$

$$G(1,1) = 36(1)(1) - 9 = 27 (> 0)$$

$$f_{xx}(1,1) = 6 \Rightarrow (1,1) \text{ is local minimum}$$

8

(7)



$$2x^2 = 1 + x^2$$

$$x^2 = 1 \Rightarrow x = 1, -1$$

$$\therefore \iint_R (x+2y) dA$$

$$2x^2 \leq y \leq 1+x^2 \\ -1 \leq x \leq 1$$

$$= \int_{-1}^1 \int_{2x^2}^{1+x^2} (x+2y) dy dx$$

$$= \int_{-1}^1 [xy + y^2]_{2x^2}^{1+x^2} dx$$

$$= \int_{-1}^1 (x + x^3 + 1 + 2x^2 + x^4 - 2x^3 - 4x^4) dx$$

$$= \int_{-1}^1 (-3x^4 - x^3 + 2x^2 + x + 1) dx$$

$$= \left[-\frac{3}{5}x^5 - \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + x \right]_{-1}^1$$

$$= \frac{32}{15}$$

8

$$= 2.13$$

TOTAL: 45 Marks

$$= -\frac{3}{5} - \frac{1}{4} + \frac{2}{3} + \frac{1}{2} + 1$$

$$= -\frac{3(3) + 2(5)}{15} + \frac{-1 + 2 + 4}{4}$$

$$= \frac{-9 + 10}{15} + \frac{5}{4}$$

$$= \frac{1}{15} + \frac{5}{4}$$

$$= \frac{4 + 15(5)}{60} = \frac{79}{60} = 1.32$$