

## LISTS OF FORMULAE

Trigonometric	Hyperbolic
$\cos^2 x + \sin^2 x = 1$ $1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \operatorname{cosec}^2 x$ $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$ $= 2 \cos^2 x - 1$ $= 1 - 2 \sin^2 x$ $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$ $2 \sin x \cos y = \sin(x + y) + \sin(x - y)$ $2 \sin x \sin y = -\cos(x + y) + \cos(x - y)$ $2 \cos x \cos y = \cos(x + y) + \cos(x - y)$	$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\cosh^2 x - \sinh^2 x = 1$ $1 - \tanh^2 x = \operatorname{sech}^2 x$ $\coth^2 x - 1 = \operatorname{cosech}^2 x$ $\sinh 2x = 2 \sinh x \cosh x$ $\cosh 2x = \cosh^2 x + \sinh^2 x$ $= 2 \cosh^2 x - 1$ $= 1 + 2 \sinh^2 x$ $\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$ $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$ $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$ $\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

Logarithm	Inverse Hyperbolic
$a^x = e^{x \ln a}$ $\log_a x = \frac{\log_b x}{\log_b a}$	$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}), -\infty < x < \infty$ $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), x \geq 1$ $\tanh^{-1} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right), -1 < x < 1$

Differentiations	Integrations
$\frac{d}{dx}[k] = 0, \quad k \text{ constant}$	$\int k dx = kx + C$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}[\ln x ] = \frac{1}{x}$	$\int \frac{dx}{x} = \ln x  + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x dx = -\cos x + C$
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x dx = \sin x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}[\cot x] = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x dx = -\cot x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x dx = \sec x + C$
$\frac{d}{dx}[\operatorname{cosec} x] = -\operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}[e^x] = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}[\cosh x] = \sinh x$	$\int \sinh x dx = \cosh x + C$
$\frac{d}{dx}[\sinh x] = \cosh x$	$\int \cosh x dx = \sinh x + C$
$\frac{d}{dx}[\tanh x] = \operatorname{sech}^2 x$	$\int \operatorname{sech}^2 x dx = \tanh x + C$
$\frac{d}{dx}[\coth x] = -\operatorname{cosech}^2 x$	$\int \operatorname{cosech}^2 x dx = -\coth x + C$
$\frac{d}{dx}[\operatorname{sech} x] = -\operatorname{sech} x \tanh x$	$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$
$\frac{d}{dx}[\operatorname{cosech} x] = -\operatorname{cosech} x \coth x$	$\int \operatorname{cosech} x \coth x dx = -\operatorname{cosech} x + C$
$\frac{d}{dx} \ln \sec x + \tan x  = \sec x$	$\int \sec x dx = \ln \sec x + \tan x  + C$
$\frac{d}{dx} \ln \operatorname{cosec} x + \cot x  = -\operatorname{cosec} x$	$\int \operatorname{cosec} x dx = -\ln \operatorname{cosec} x + \cot x  + C$

Differentiations of Inverse Functions	Integrations Resulting in Inverse Functions
$\frac{d}{dx}[\sin^{-1} u] = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx},  u  < 1.$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C.$
$\frac{d}{dx}[\cos^{-1} u] = \frac{-1}{\sqrt{1-u^2}} \cdot \frac{du}{dx},  u  < 1.$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C.$
$\frac{d}{dx}[\tan^{-1} u] = \frac{1}{1+u^2} \cdot \frac{du}{dx}.$	$\int \frac{dx}{ x \sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C.$
$\frac{d}{dx}[\cot^{-1} u] = \frac{-1}{1+u^2} \cdot \frac{du}{dx}.$	$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, a > 0.$
$\frac{d}{dx}[\sec^{-1} u] = \frac{1}{ u \sqrt{u^2-1}} \cdot \frac{du}{dx},  u  > 1.$	$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, x > 0.$
$\frac{d}{dx}[\cosec^{-1} u] = \frac{-1}{ u \sqrt{u^2-1}} \cdot \frac{du}{dx},  u  > 1.$	$\int \frac{dx}{a^2-x^2}$ $= \begin{cases} \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) + C, &  x  < a, \\ \frac{1}{a} \coth^{-1}\left(\frac{x}{a}\right) + C, &  x  > a. \end{cases}$
$\frac{d}{dx}[\sinh^{-1} u] = \frac{1}{\sqrt{u^2+1}} \cdot \frac{du}{dx}$	$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a} \operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C,$ $0 < x < a.$
$\frac{d}{dx}[\cosh^{-1} u] = \frac{1}{\sqrt{u^2-1}} \cdot \frac{du}{dx},  u  > 1.$	$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a} \operatorname{cosech}^{-1}\left \frac{x}{a}\right  + C,$ $0 < x < a.$
$\frac{d}{dx}[\tanh^{-1} u] = \frac{1}{1-u^2} \cdot \frac{du}{dx},  u  < 1.$	
$\frac{d}{dx}[\coth^{-1} u] = \frac{1}{1-u^2} \cdot \frac{du}{dx},  u  > 1.$	
$\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-1}{u\sqrt{1-u^2}} \cdot \frac{du}{dx}, 0 < u < 1.$	
$\frac{d}{dx}[\operatorname{cosech}^{-1} u] = \frac{-1}{ u \sqrt{1+u^2}} \cdot \frac{du}{dx}, u \neq 0.$	

**Arc Length:**

Parametric $x = f(t),$ $y = g(t)$	$\mathcal{L} = \int_{t=t_0}^{t=t_1} \sqrt{(x'(t))^2 + (y'(t))^2} dt$
Cartesian $y = f(x)$	$\mathcal{L} = \int_{x=a}^{x=b} \sqrt{1 + (f'(x))^2} dx$
Cartesian $x = g(y)$	$\mathcal{L} = \int_{y=c}^{y=d} \sqrt{1 + (g'(y))^2} dy$
Polar $r = f(\theta)$	$\mathcal{L} = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$

**Area of Surface Revolution:**

Type of equation	Revolve around X-axis	Revolve around Y-axis
Polar $r = f(\theta)$	$S_x = \int_a^b 2\pi r \sin \theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$	$S_y = \int_a^b 2\pi r \cos \theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$
Parametric $x = f(t),$ $y = g(t)$	$S_x = \int_{t_1}^{t_2} 2\pi y(t) \sqrt{(x'(t))^2 + (y'(t))^2} dt$	$S_y = \int_{t_1}^{t_2} 2\pi x(t) \sqrt{(x'(t))^2 + (y'(t))^2} dt$
Cartesian $y = f(x)$	$S_x = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} dx$	$S_y = \int_a^b 2\pi x \sqrt{1 + (f'(x))^2} dx$
Cartesian $x = g(y)$	$S_x = \int_c^d 2\pi y \sqrt{1 + (g'(y))^2} dy$	$S_y = \int_c^d 2\pi g(y) \sqrt{1 + (g'(y))^2} dy$