DERIVATION OF PIM SHAPE FUNCTION

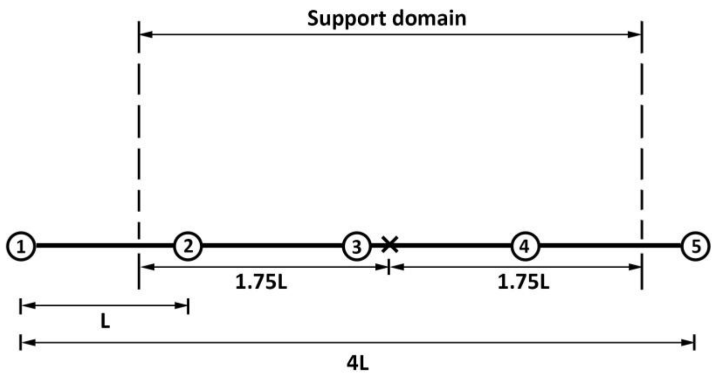

Figure B.1 : Domain of a flow.

Figure 1 shows a domain of a flow. As shown, it contains five nodes which is uniformly distributed for simplification. The length of the domain is 5L. The size of support domain is decided as 3.5L, obtained by taking $\alpha_s = 3.5$. A support domain centered at a point of interest $x = 2.1L$ is graphically shown in the figure. Based on the figure, three nodes are included in the present support domain. For such an arrangement, a quadratic polynomial function is chosen, thus

$$Q = a_1 + a_2 x + a_3 x^2 \tag{B.1}$$

Evaluating Eq. (B.1) at nodal location and prescribing the corresponding degree of freedoms give

$$\begin{aligned} a_1 + a_2 L + a_3 L^2 &= Q_2 \\ a_1 + 2a_2 L + 4a_3 L^2 &= Q_3 \\ a_1 + 3a_2 L + 9a_3 L^2 &= Q_4 \end{aligned} \tag{B.2}$$

which can be given in matrix form as

$$\begin{bmatrix} 1 & L & L^2 \\ 1 & 2L & 4L^2 \\ 1 & 3L & 9L^2 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{Bmatrix} Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} \tag{B.3}$$

or

$$[P|_n]\{a\} = \{Q\} \tag{B.4}$$

where $[P|_n]$ is the moment matrix of the system. Using shape functions, N_i and degree of freedom, Q_i, the flow rate can also be given as

$$Q = N_1Q_1 + N_2Q_2 + N_3Q_3 \tag{B.5}$$

or

$$Q = N_iQ_i \tag{B.6}$$

or

$$Q = \{N_i\}^T\{Q_i\} \tag{B.7}$$

Since Eqn. (B.1) and (B.5) represent the same variable distribution, both should give the same numerical value of the variable when evaluated at the same location. Therefore, if both equations are evaluated at the point of interest (i.e $x = 2.1L$), the following is obtained.

$$N_1|_{pi}Q_1 + N_2|_{pi}Q_2 + N_3|_{pi}Q_3 = a_1 + a_2x|_{pi} + a_3x^2|_{pi} \tag{B.8}$$

w $|_{pi}$ refers the evaluation of variable at point of interest. Based on this, it can be stated that $N_1|_{pi},\ N_2|_{pi},\ N_3|_{pi}$ are the numerical values of the shape function, $N_1,\ N_2,\ N_3$ at the location of the point of interest respectively.

Eqn. (B.8) can also be given as

$$\{N_1|_{pi}\quad N_2|_{pi}\quad N_3|_{pi}\}^T \begin{Bmatrix} Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \{1\quad x|_{pi}\quad x^2|_{pi}\}^T \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \tag{B.9}$$

Inserting Eqn. (B.3) into Eqn. (B.9), we obtain

$$\{N_1|_{pi}\quad N_2|_{pi}\quad N_3|_{pi}\} \begin{bmatrix} 1 & L & L^2 \\ 1 & 2L & 4L^2 \\ 1 & 3L & 9L^2 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \{1\quad x|_{pi}\quad x^2|_{pi}\}^T \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \tag{B.10}$$

Simplification and rearrangement then

$$\begin{bmatrix} 1 & L & L^2 \\ 1 & 2L & 4L^2 \\ 1 & 3L & 9L^2 \end{bmatrix}^T \begin{Bmatrix} N_1|_{poi} \\ N_2|_{poi} \\ N_3|_{poi} \end{Bmatrix} = \begin{Bmatrix} 1 \\ x|_{poi} \\ x^2|_{poi} \end{Bmatrix} \tag{B.11}$$

By solving Eqn. (B.11), the values of the shape functions at $x = 2.1L$ are solved as

$$\begin{Bmatrix} N_1|_{pi} \\ N_2|_{pi} \\ N_3|_{pi} \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}^{-1} \begin{Bmatrix} 1.00 \\ 2.10 \\ 4.41 \end{Bmatrix} = \begin{Bmatrix} -0.0450 \\ 0.9900 \\ 0.0550 \end{Bmatrix}$$

Figure B.2 shows the plot of the solved values above whilst Figure B.3 shows the plot of N_1 for various numbers of points of interest. As can be seen from Figure B.3, the smoothness of the shape functions increases when the numbers of points of interest increases.

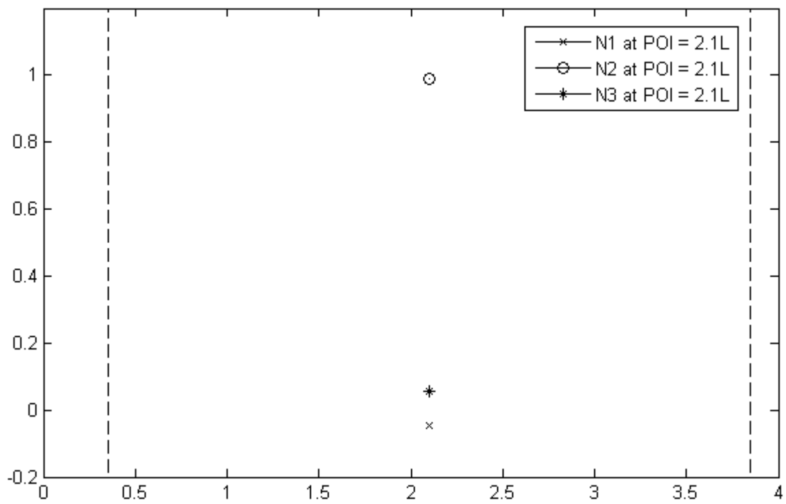

Figure B.2: Plots of PIM shape functions at a typical point of interest

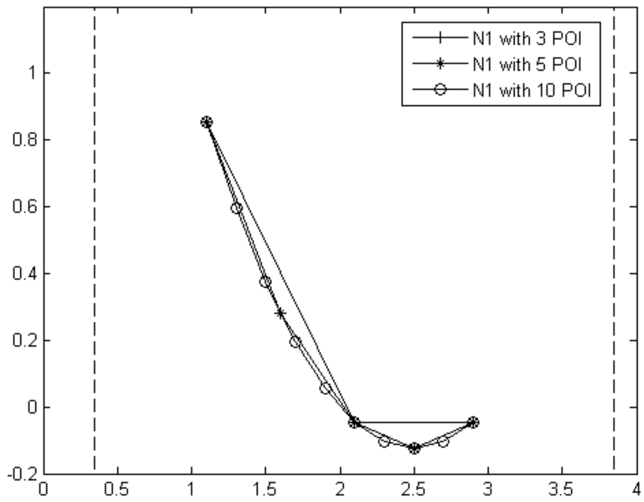

Figure B.3: Plots of PIM shape function N_1 using various numbers of points of interest