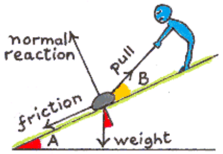
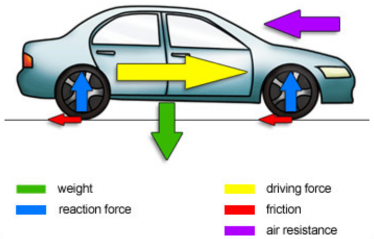
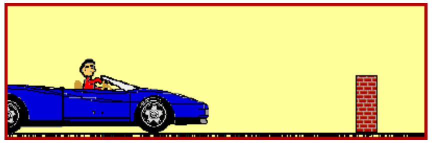
FACULTY OF CHEMICAL & ENERGY ENGINEERING

CHAPTER 2

STATICS OF PARTICLES







OBJECTIVES

- To show how to **add** forces and resolve them into components using the **parallelogram law**.
- To introduce the concept of the **free-body diagram** for a particle.
- To show how to solve **particle equilibrium problems** using the **equations of equilibrium**.

TOPIC OUTCOMES

It is expected that students will be able to:

- **add** forces and **resolve** them into components using the Parallelogram Law
- **determine** the vector's magnitude and direction
- **draw** a correct free body diagram
- **write** the equations of equilibrium corresponding to the free-body diagrams.
- **solve** the equilibrium equations.

2.1 INTRODUCTION

- Study the **effect of forces acting on particles**
  1. How to replace two or more forces acting on a particle by a **single force** having the same effect, resultant force.
  2. **Relations** which exist among the various forces acting on a particle in a **state of equilibrium** to determine some of the forces acting on the particle.
  3. **Resolution** of a force into components

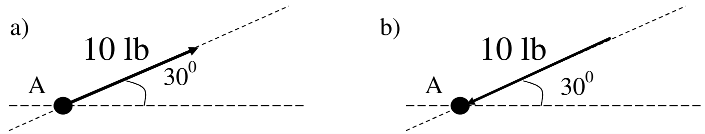
DEFINITIONS

SCALAR - A quantity characterized by a positive or negative number is called a scalar. Examples of scalars used in Statics are mass, volume or length.

VECTOR - A quantity that has both magnitude and a direction. Examples of vectors used in Statics are position, force, and moment.

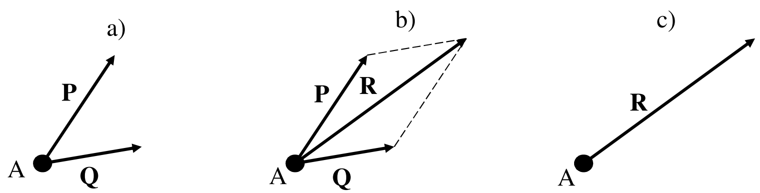
2.2 FORCES ON A PARTICLE. RESULTANT OF TWO FORCES

- **FORCE**: action of one body on another; characterized by its 1) *point of application, 2) magnitude* and *3) direction*
- **Direction** of a force: *line of action* and *sense* of the force
- **Line of action**: infinite straight line along which the force act; characterized by the angle it forms with some fixed axis
- **Sense** of the force: indicated by an arrowhead
- Force represented by a segment of that line



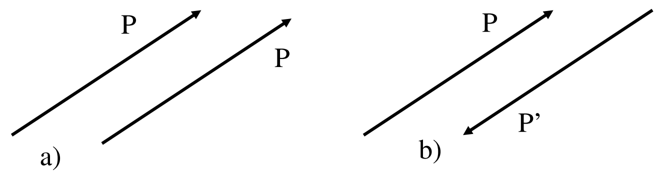
PARALLELOGRAM RULE

- Forces P and Q acting on a particle A may be **replaces** by a single force R; has the same of the particle ⇒ resultant force
- By constructing a parallelogram, the diagonal that passes through A represents the resultant ⇒ parallelogram law for the **additional of two forces**



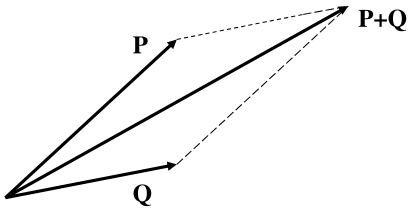
2.3 VECTOR

- **Vectors**: mathematical expression possessing **magnitude and direction**; which add according to parallelogram law.
- Two vectors; have the same magnitude and direction are said to be *equal* (a)
- Two vectors; have the same magnitude, parallel lines of action and opposite sense are *equal* and *opposite.* (b)



2.4 ADDITION OF VECTORS

- Vectors **add** according to the parallelogram law (**PL**)
- The sum of two vectors P and Q ⇒ attach the two vectors to the same point A and using PL; the diagonal passes through A represents the sum of vectors P and Q; denoted by P+Q
- The sum does not depend upon the order of the vectors; the addition of two vectors is **commutative: P + Q = Q + P**



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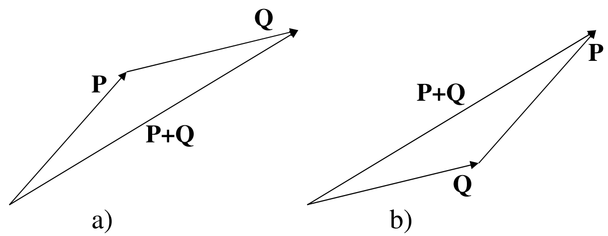
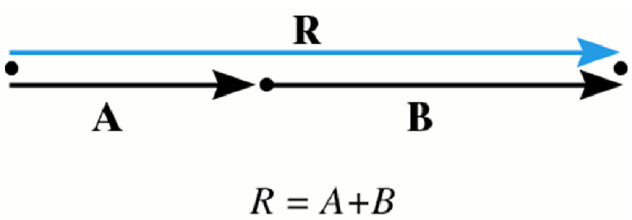
- **TRIANGLE RULE:** Alternate method for determining the sum of two vectors
- Draw only **half of PL**
- The sum of the two vectors; by arranging P and Q in ***tip-to-tail fashion*** by connecting the *tail of P* with the *tip of Q*



2.4.1 ADDITION OF COLLINEAR VECTORS



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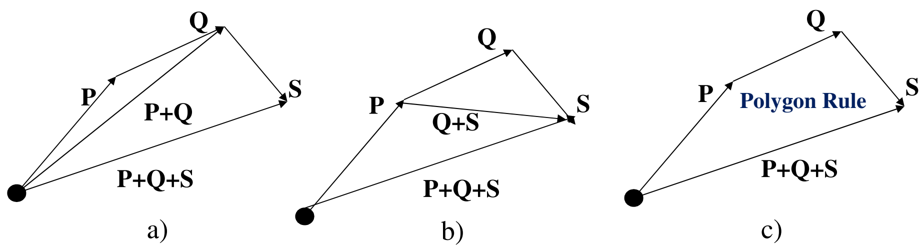
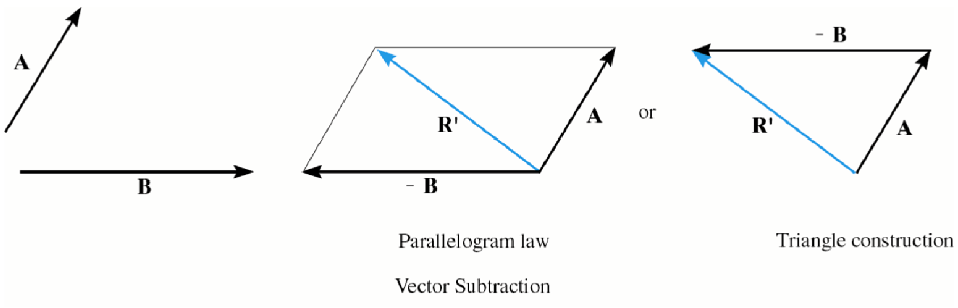
- Consider **sum of three or more vectors**
  - The sum of three vectors; P, Q and S; obtained by first adding the vectors P and Q, then adding the vector S to the vector P+Q.

$$P + Q + S = (P + Q) + S$$

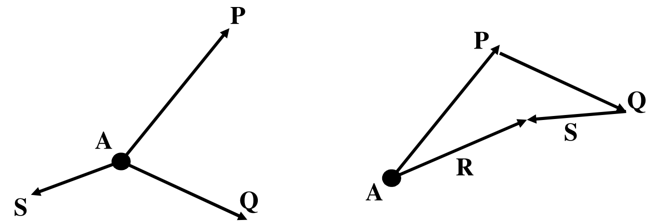


2.4.2 VECTOR SUBTRACTION

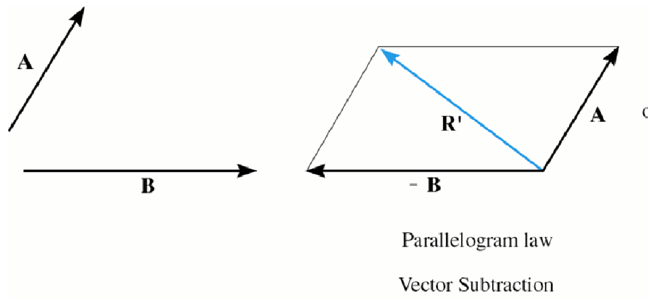


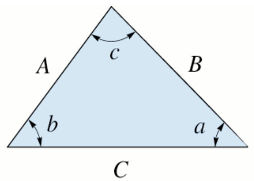
2.5 RESULTANT OF SEVERAL CONCURRENT FORCES

- Several forces contained in the same plane; a particle A acted upon by several coplanar forces
- The forces all pass through A; concurrent
  - The vectors representing the forces acting on A, added by the polygon rule
  - The vector R represents the resultant of the given concurrent forces; has the same effect on the particle A as the given forces



2.6.1 RESOLUTION OF A VECTOR





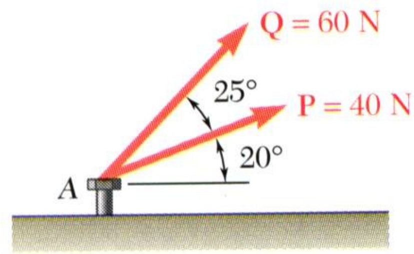
Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2\,A\,B\cos c}$$

Sample Problem 1

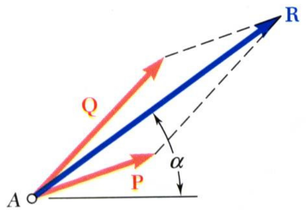


The two forces act on a bolt at *A*. Determine their resultant.

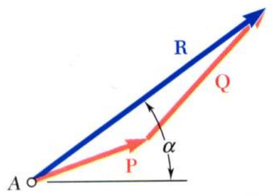
SOLUTION:

- Graphical solution - construct a parallelogram with sides in the same direction as **P** and **Q** and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the the diagonal.
- Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.

SOLUTION



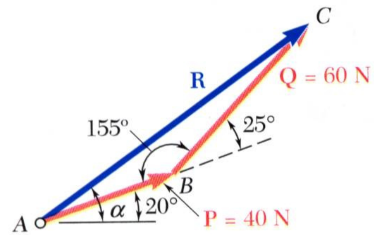
- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

SOLUTION

- Trigonometric solution - Apply the triangle rule. From the Law of Cosines,



$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

$$\boxed{R = 97.73\text{N}}$$

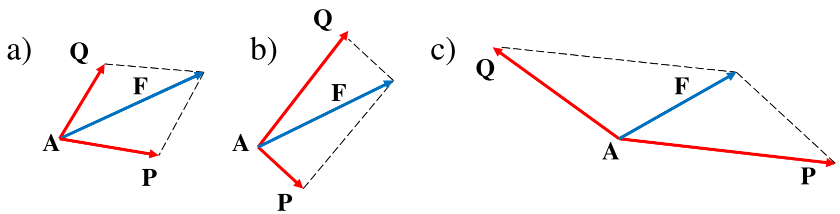
From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

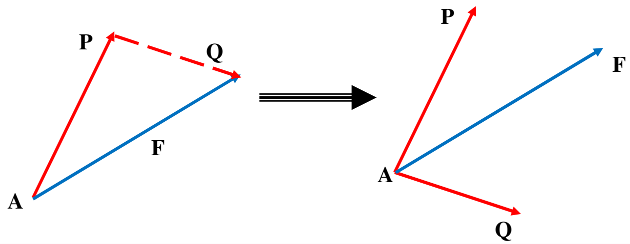
2.6 RESOLUTION OF A FORCE INTO COMPONENTS

- A single force F acting on a particle may be replaced by two or more forces; which have the same effect.
  - The forces are called *components*; the process of substituting them is called *resolving the force F into components*
- For each force F; exist an infinite number of possible set of components
  - Numbers of ways in which a given force F maybe resolved into two components



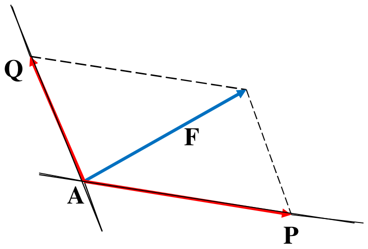
2.6.1 RESOLUTION OF A FORCE *continued….*

- **One of the two components; (if P is known)**
  - The second component; Q is obtained by applying the triangle rule; joining the tip P to the tip of F; the magnitude and direction of Q are determined graphically or by trigonometry
  - Once Q has been determined, both components of P and Q should apply at A



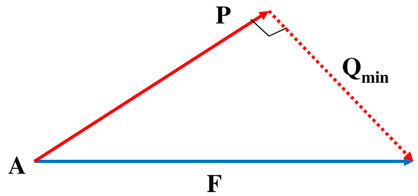
2.6.1 RESOLUTION OF A FORCE *continued….*

- **The line of action of each component is known**
  - The magnitude and sense of the components are obtained by applying the *parallelogram law* and drawing lines; through the tip of F parallel to the given lines of action
  - Two well defined components of P and Q; determined graphically or by trigonometry



2.6.1 RESOLUTION OF A FORCE *continued….*

- **The direction of one component may be known** while the magnitude of the other component is to be *as small as possible (minimum)*
- The appropriate triangle is drawn



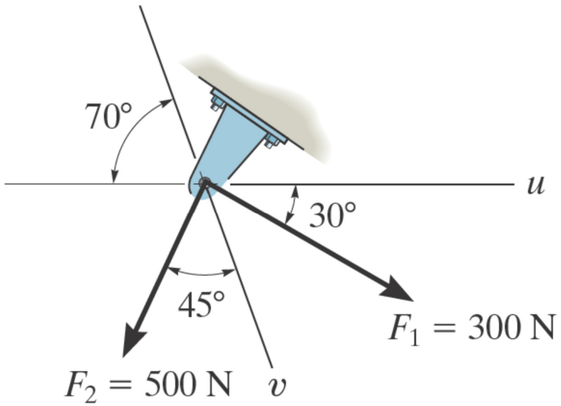
P and Q_{min} is perpendicular

Sample Problem 2

Resolved the forces into two components along axes *u* and axes *v*.

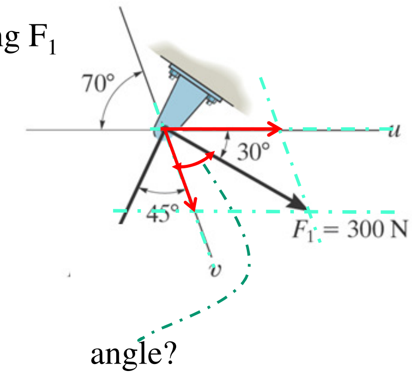
a) F_1

b) F_2



SOLUTION

a) Resolving F_1



$$\frac{F_{1u}}{\sin 40°} = \frac{300}{\sin 110°}$$

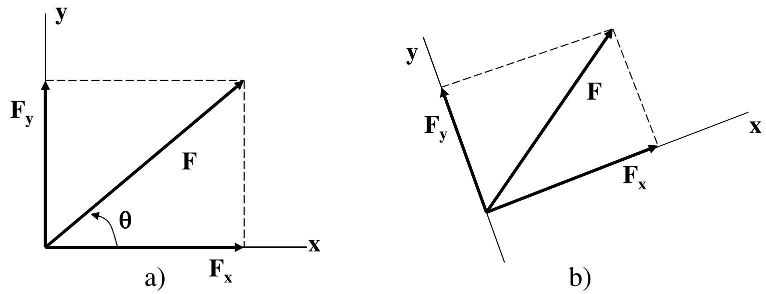
$$F_{1u} = 205 \text{ N}$$

$$\frac{F_{1v}}{\sin 30°} = \frac{300}{\sin 110°}$$

$$F_{1v} = 160 \text{ N}$$

2.7 RECTANGULAR COMPONENTS OF A FORCE

- Force, F resolved into a component **F_x along the x axis and F_y along the y axis**; the parallelogram drawn to obtain the two component is a rectangular
- The x and y axes may be chosen in **ANY two perpendicular directions**



2.7 RECTANGULAR COMPONENTS OF A FORCE *continued….*

- The magnitude of the components F_x and F_y;

$$F_x = F \cos\theta \qquad F_y = F \sin\theta$$

- Scalar component F_x is positive when the vector component F_x has the same sense as the x-axis; and negative when F_x has the opposite sense. Same goes to F_y component.
- The angle, θ, defined by;

$$\tan\theta = F_y / F_x$$

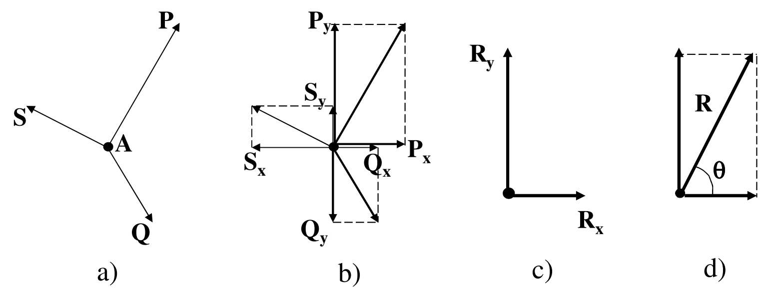
- The magnitude F of the force can be obtained using **Pythagorean theorem**;

$$F = \sqrt{F_x^2 + F_y^2}$$

2.8 ADDITION OF FORCES BY SUMMING *X* AND *Y* COMPONENTS

- When three or more forces are to be added; **practical trigonometry** solution may be **difficult**.
- An analytical solution; resolving each force into two rectangular components
- The **horizontal** comps.; added into a **single force R_x**; and the **vertical** comps. into a **single force R_y**.
- The forces R_x and R_y then be **added vectorially** into the resultant R of the given system

Continued…

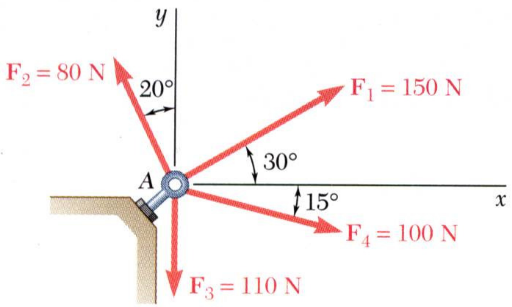


$$R_x = P_x + Q_x + S_x \qquad R_y = P_y + Q_y + S_y$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y$$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \tan\theta = R_y / R_x$$

SAMPLE PROBLEM 2

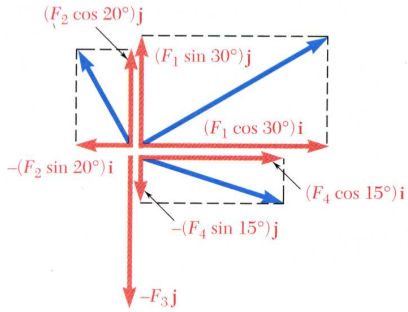


Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

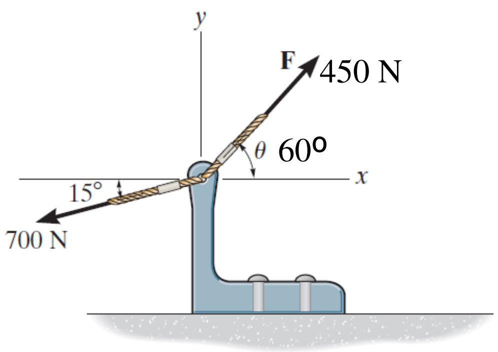
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

SOLUTION



IN CLASS HW

Determine the resultant of the force and its direction, measured counterclockwise from the positive x-axis.



2.9 EQUILIBRIUM OF A PARTICLE

- To introduce the **concept of the free-body diagram** for a particle.
- To show how **to solve** particle equilibrium problems using the equations of equilibrium.

2.9 EQUILIBRIUM OF A PARTICLE continued….

- When the resultant of all the forces acting on a particle is zero; *the particle is in equilibrium.*
- Particle; acted by two forces will be in equilibrium if the two forces have the same magnitude, same line of action and opposite sense (see Figure a)).
- Particle; acted by three forces or more will be in equilibrium if the resultant of all the forces is determined by polygon rule (tip-to-tail fashion) (see Figure b)).
- To express algebraically that a particle is in equilibrium; the two rectangular comps. R_x and R_y of the resultant are zero;

$$\sum F_x = 0 \qquad \sum F_y = 0$$

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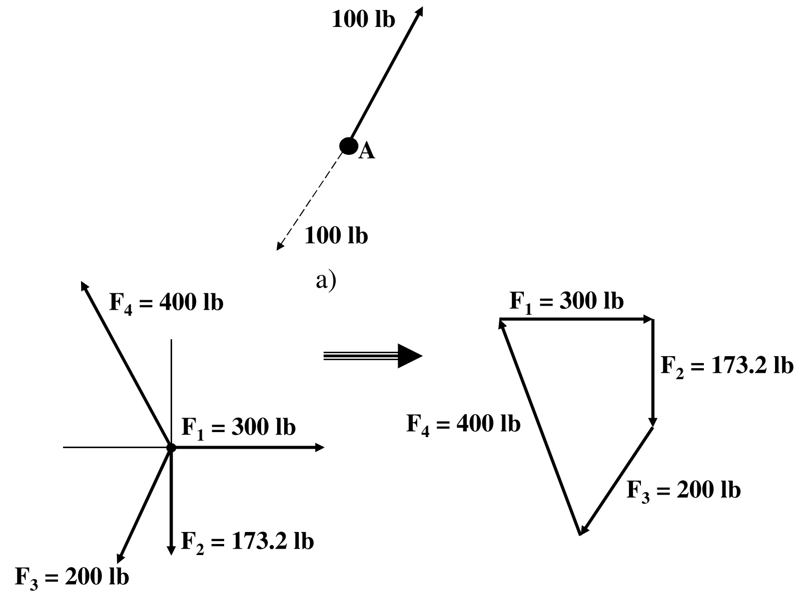


b)

2.10 NEWTON'S FIRST LAW OF MOTION

- **Newton's First law**:
  - If the **resultant forces** acting on a particle is **zero**, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion)
- As a **conclusion**; a particle in equilibrium either is at rest or is moving in a straight line with constant speed.

2.11 PROBLEMS INVOLVING THE EQUILIBRIUM OF A PARTICLE. FREE BODY DIAGRAM

- A problem in mechanics engineering is derived from an actual physical situation.
- A sketch showing the physical conditions of the problem is known as **space diagram** (see Fig. a) page 35).
- Done by choosing a significant particle and drawing a separate diagram showing this particle (in equilibrium) and all the forces acting on it ⇒ **free body diagram** (see Fig. b) page 35)**.**

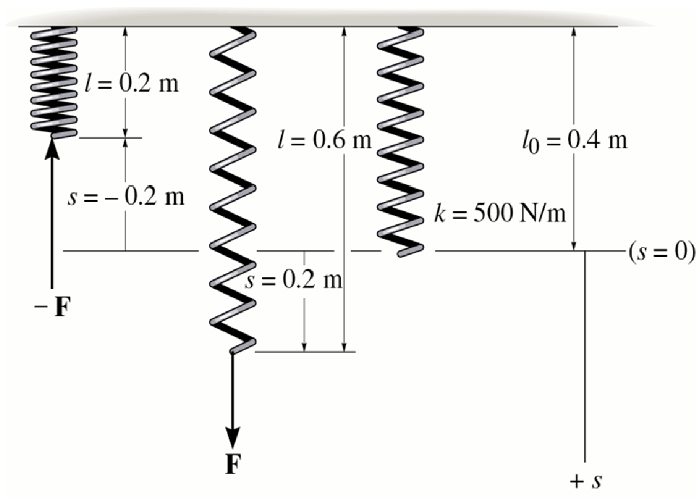
2.11.1 THE FREE-BODY DIAGRAM

- To apply equilibrium equations we must account for all **known** and **unknown** forces acting on the particle.
- The best way to do this is to **draw a free-body diagram** of the particle.
- This is a sketch that shows the particle "free" from its surroundings with all the forces acting on it.

TYPICAL SUPPORTS

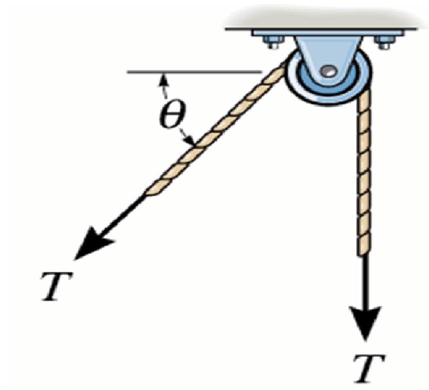
TYPICAL SUPPORTS
- SPRING
- CABLES & PULLEYS

SPRINGS



$$F = ks$$

CABLES AND PULLEYS

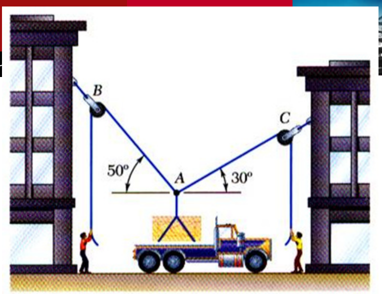


Cable is in tension

Cables are assumed to have negligible weight and they cannot stretch. They can only support tension or pulling *(you can't push on a rope)*. Pulleys are assumed to be frictionless. A continuous cable passing over a frictionless pulley must have tension force of a constant magnitude. The tension force is always directed in the direction of the cable.

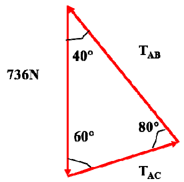
1) **Draw Outlined Shape** - Imagine the particle isolated or cut "free" from its surroundings
2) **Show All Forces** - Include "active forces" and "reactive forces"
3) **Identify Each Force** - Known forces labeled with proper magnitude and direction. Letters used for unknown quantities.

- Consider the 75-kg crate shown in the space diagram. The crate was lying between two buildings; and being lifted onto the truck. The crate was supported by a vertical cable; joined at A to two ropes pass over pulleys at B and C. It is desired to **determined** **the tension AB and AC.**



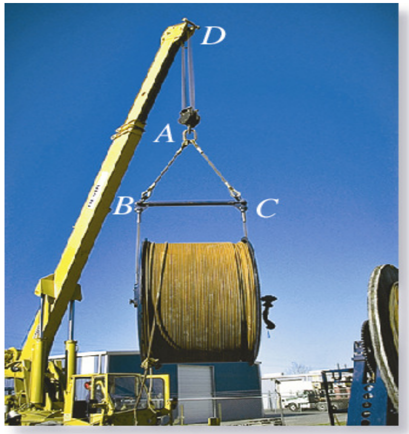
2 METHODS TO DETERMINE THE TENSIONS

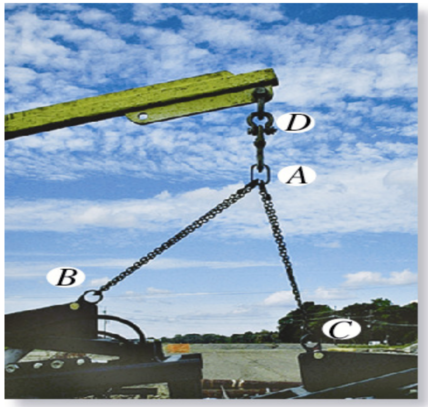
a) **By trigonometry law:**



b) **By Equilibrium Equation**:

- These equation may be solved for no more than two unknowns; similarly the force triangle law.
- Problems with more than two unknowns may be need more than one FBD to solve.





FORCE TYPES

- **Active Forces** - tend to set the particle in motion.
- **Reactive Forces** - result from constraints or supports and tend to prevent motion.

Revision: Procedure for Analysis

Free-Body Diagram

1. Establish the x, y axes in any suitable orientation.
2. Label all known and unknown force magnitudes and directions on the FBD.
3. The sense of an unknown force may be assumed.

Continued…

Equations of Equilibrium

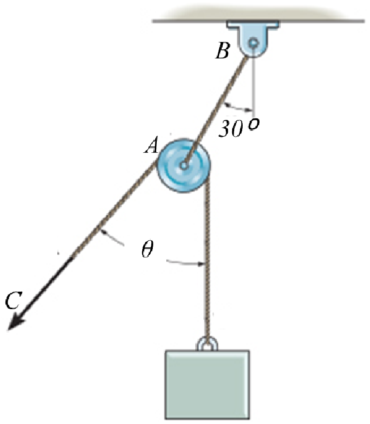
1. Apply equations of equilibrium.

$$\sum F_x = 0 \quad and \quad \sum F_y = 0$$

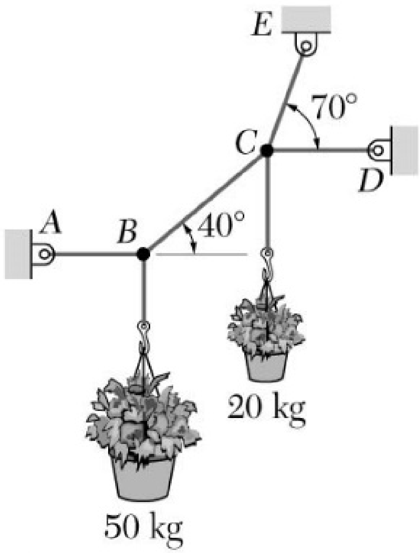
2. Components of force are positive if directed along a positive axis and negative if directed along a negative axis.
3. If solution yields a negative result the force is in the opposite sense of that shown on the FBD.

IN CLASS HW

The block has a weight of 20 lb and is being hoisted at uniform velocity. Determine the angle θ for equilibrium and the required force in each cord.



Two flower pots as in the figure, are stably suspended using a cable system. Calculate the tensions on cables AB, BC, CD and CE.



Determine the stretch in springs AC and AB for equilibrium of the 2-kg block. The springs are shown in the equilibrium position.