FRAME ANALYSIS

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Introduction

3D Frame: Beam, Column & Slab

Building Construction

3D Frame: Beam & Column ONLY

2D Frame Analysis

2D Frame Analysis
Introduction: Substitute Frame

2D Frame Analysis

Sub-frame: Roof Level

Sub-frame: Other Floor Level
Types of Frame

**Braced Frame**
- **NOT** contribute to the overall structural stability
- None lateral actions are transmitted to columns & beams
- Support **vertical actions** only

**Unbraced Frame**
- Contribute to the overall structural stability
- All lateral actions are transmitted to columns & beams
- Support **vertical & lateral actions**
(i) One-level Sub-frame

(ii) Two-points Sub-frame
(iii) Continuous Beam and One-point Sub-frame
Load Set 1: Alternate or adjacent spans loaded

- Alternate span carrying the design variable and permanent load \((1.35G_k + 1.5Q_k)\), other spans carrying only the design permanent loads \((1.35G_k)\)

- Any two adjacent spans carrying the design variable and permanent loads \((1.35G_k + 1.5Q_k)\), all other spans carrying only the design permanent load \((1.35G_k)\)
Combination of Actions

Load Set 1

Alternate Span Loaded

1.35G

1.5Qₖ

1.5Qₖ

Adjacent Span Loaded

1.5Qₖ

1.5Qₖ

1.35G
Combination of Actions

MALAYSIAN / UK NATIONAL ANNEX

Load set 2: All or alternate spans loaded

- All span carrying the design variable and permanent loads \((1.35G_k + 1.5Q_k)\)

- Alternate span carrying the design variable and permanent load \((1.35G_k + 1.5Q_k)\), other spans carrying only the design permanent loads \((1.35G_k)\)
Combination of Actions

Load Set 2

All Spans Loaded

Alternate Span Loaded
Example 1

ACTION ON BEAMS
Example 1

Level 1
Level 2
Level 3
Level 4
Level 5
Level 6
Level 7

6 m
6 m
6 m
6 m
6 m
6 m
6 m

1 m

7 @ 5 m = 35 m
2 @ 4 m = 8 m

6 @ 3.5 m = 21 m
4 m

Example 1
Example 1: Action on Beams

**Loading Data**

- Finishes, ceiling & services = 0.75 kN/m²
- Concrete density = 25 kN/m³
- Variable action = 4 kN/m²
- Partition = 0.50 kN/m²

**Structural Dimensions**

- Slab thickness, $h$ = 150 mm
- Beam size, $b \times h$ = 250 \times 600 mm
- Column size, $b \times h$ = 300 \times 400 mm
**Example 1: Action on Beams**

### Load on Slab

Slab self weight = 0.15 m $\times$ 25 kN/m$^3$ = 3.75 kN/m$^2$

Finishes, ceiling, services = 1.25 kN/m$^2$

**Characteristics permanent action, $g_k$** = 5.00 kN/m$^2$

Imposed load = 4.00 kN/m$^2$

Partition = 0.50 kN/m$^2$

**Characteristics variable action, $q_k$** = 4.50 kN/m$^2$

### Other Loads on Beam

Beam self weight = 0.25 m $\times$ (0.60 − 0.15) m $\times$ 25 kN/m$^3$ = 2.81 kN/m
Example 1: Action on Beams

\[ \frac{L_y}{L_x} = \frac{6}{4} = 1.50 \text{ (Case 2)} \]

\[ \frac{L_y}{L_x} = \frac{6}{4} = 1.50 \text{ (Case 1)} \]

\[ \frac{L_y}{L_x} = \frac{6}{4} = 1.50 \text{ (Case 2)} \]

\[ \frac{L_y}{L_x} = \frac{6}{5} = 1.20 \text{ (Case 2)} \]

\[ \frac{L_y}{L_x} = \frac{6}{5} = 1.20 \text{ (Case 1)} \]

\[ \frac{L_y}{L_x} = \frac{6}{5} = 1.20 \text{ (Case 2)} \]
Example 1: Action on Beams

\[ G_k = \text{Permanent action on slab + Beam self weight} \]
\[ = (0.47 \times 5.00 \times 4) + (0.42 \times 5.00 \times 5) + 2.81 = 22.71 \text{ kN/m} \]

\[ Q_k = \text{Variable action on slab} \]
\[ = (0.47 \times 4.50 \times 4) + (0.42 \times 4.50 \times 5) = 17.91 \text{ kN/m} \]

\[ G_k = \text{Permanent action on slab + Beam self weight} \]
\[ = (0.45 \times 5.00 \times 4) + (0.39 \times 5.00 \times 5) + 2.81 = 21.56 \text{ kN/m} \]

\[ Q_k = \text{Variable action on slab} \]
\[ = (0.45 \times 4.50 \times 4) + (0.39 \times 4.50 \times 5) = 16.88 \text{ kN/m} \]
Example 1: Action on Beams

\[ w_{\text{max}} = 1.35G_k + 1.5Q_k = 57.53 \text{ kN/m} \]
\[ w_{\text{min}} = 1.35G_k = 30.66 \text{ kN/m} \]
Example 2
ANALYSIS OF ONE-LEVEL SUB-FRAME
Example 2

- $2 \times 4 \text{ m} = 8 \text{ m}$
- $2 \times 4 \text{ m} = 8 \text{ m}$
- $7 \times 5 \text{ m} = 35 \text{ m}$
- $6 \text{ m}$
- $6 \text{ m}$
- $6 \text{ m}$

Levels:
- Level 1
- Level 2
- Level 3
- Level 4
- Level 5
- Level 6
- Level 7

Dimensions:
- $6 \times 3.5 \text{ m} = 21 \text{ m}$
- $4 \text{ m}$
- $1 \text{ m}$
- $6 \text{ m}$
- $6 \text{ m}$
- $6 \text{ m}$

- $4 \text{ m}$
Example 2: Analysis of One Level Sub-Frame

Analysis using LOAD SET 1

\[ w_{\text{max}} = 57.53 \text{ kN/m} \]
\[ w_{\text{min}} = 30.66 \text{ kN/m} \]

\[ w_{\text{max}} = 54.42 \text{ kN/m} \]
\[ w_{\text{min}} = 29.11 \text{ kN/m} \]

\[ w_{\text{max}} = 57.53 \text{ kN/m} \]
\[ w_{\text{min}} = 30.66 \text{ kN/m} \]
Example 2: Analysis of One Level Sub-Frame

**Structural Dimension**

**Beam:** 250 mm × 600 mm  
**Column:** 300 mm × 400 mm

**Moment of Inertia, \( I = bh^3/12 \)**

**Beam:**  
\[
\frac{250 \times 600^3}{12} = 4.5 \times 10^9 \ mm^4
\]

**Column:**  
\[
\frac{300 \times 400^3}{12} = 1.6 \times 10^9 \ mm^4
\]

**Stiffness, \( K = I/L \)**

**Beam:**  
\[
K_{AB} = K_{BC} = K_{CD} = \frac{4.5 \times 10^9}{6000} = 7.5 \times 10^5 \ mm^3
\]

**Column:**  
\[
K_{c,u} = \frac{1.6 \times 10^9}{3500} = 4.6 \times 10^5 \ mm^3
\]
\[
K_{c,l} = \frac{1.6 \times 10^9}{4000} = 4.0 \times 10^5 \ mm^3
\]
Example 2: Analysis of One Level Sub-Frame

Distribution Factor, \( F = \frac{K}{\sum K} \)

**Joint A & Joint D:**

\[
F_{AB, CD} = \frac{K_{AB, CD}}{(K_{AB, CD} + K_{C,u} + K_{C,l})} = 0.47
\]

\[
F_{C,u} = \frac{K_{C,u}}{(K_{AB, CD} + K_{C,u} + K_{C,l})} = 0.28
\]

\[
F_{C,l} = \frac{K_{C,l}}{(K_{AB, CD} + K_{C,u} + K_{C,l})} = 0.25
\]

**Joint B & Joint C:**

\[
F_{BA, CB} = \frac{K_{AB, BC}}{(K_{AB, BC} + K_{BC, CD} + K_{C,u} + K_{C,l})} = 0.32
\]

\[
F_{BC, CD} = \frac{K_{BC, CD}}{(K_{AB, BC} + K_{BC, CD} + K_{C,u} + K_{C,l})} = 0.32
\]

\[
F_{C,u} = \frac{K_{C,u}}{(K_{AB, BC} + K_{BC, CD} + K_{C,u} + K_{C,l})} = 0.19
\]

\[
F_{C,l} = \frac{K_{C,l}}{(K_{AB, BC} + K_{BC, CD} + K_{C,u} + K_{C,l})} = 0.17
\]
Example 2: Analysis of One Level Sub-Frame

CASE 1: Span 1 & 2 = $w_{\text{max}}$, Span 3 = $w_{\text{min}}$

Fixed End Moment

\[
M_{AB} = M_{BA} = \frac{wL^2}{12} = \frac{57.5 \times 6^2}{12} = 172.6 \text{ kNm}
\]

\[
M_{BC} = \frac{wL^2}{12} = \frac{54.4 \times 6^2}{12} = 163.3 \text{ kNm}
\]

\[
M_{CD} = \frac{wL^2}{12} = \frac{30.7 \times 6^2}{12} = 92.0 \text{ kNm}
\]
### Example 2: Analysis of One Level Sub-Frame

#### Moment Distribution Method

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tbody>
<tr>
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<td>0.3</td>
<td>0.3</td>
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</tbody>
</table>

The table above shows the distribution of moments for different levels in a sub-frame analysis using the Moment Distribution Method.
Example 2: Analysis of One Level Sub-Frame

Moment in the **COLUMN** at each joint

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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</thead>
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<tr>
<td>0.32</td>
<td>172.6</td>
<td>163.3</td>
<td>92.0</td>
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<td>-22.7</td>
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<td>0.17</td>
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</tr>
<tr>
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<td>-125.9</td>
<td>45.1</td>
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</tbody>
</table>
Example 2: Analysis of One Level Sub-Frame

Moment in the BEAM at each joint

<table>
<thead>
<tr>
<th></th>
<th>A</th>
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<th>D</th>
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<td>51.00</td>
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<td>0.47</td>
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<td>44.6</td>
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<td></td>
<td></td>
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</tbody>
</table>
Example 2: Analysis of One Level Sub-Frame

**Reaction Calculation at Support**

Solve for each span using equilibrium method of analysis:

\[
\begin{align*}
V_{AB} & = 155.1 \text{ kN} \\
V_{BA} & = 190.0 \text{ kN} \\
V_{BC} & = 170.3 \text{ kN} \\
V_{CB} & = 156.2 \text{ kN} \\
V_{CD} & = 105.5 \text{ kN} \\
V_{DC} & = 78.5 \text{ kN}
\end{align*}
\]
Example 2: Analysis of One Level Sub-Frame

Shear Force and Bending Moment Diagram

SFD (kN)

BMD (kNm)
Example 2: Analysis of One Level Sub-Frame

Do the same for Load CASE 2, CASE 3 & CASE 4

SELF STUDY !!!

CASE 2: Span 2 & 3 = $w_{\text{max}}$, Span 1 = $w_{\text{min}}$

- $w_{\text{min}} = 30.7 \text{ kN/m}$
- $w_{\text{max}} = 54.4 \text{ kN/m}$
- $w_{\text{max}} = 57.5 \text{ kN/m}$
Example 2: Analysis of One Level Sub-Frame

Do the same for Load CASE 2, CASE 3 & CASE 4

SELF STUDY !!!

CASE 3: Span 1 & 3 = $w_{max}$, Span 2 = $w_{min}$

$w_{max} = 57.5$ kN/m

$w_{min} = 29.1$ kN/m
Example 2: Analysis of One Level Sub-Frame

Do the same for Load CASE 2, CASE 3 & CASE 4

SELF STUDY !!!

CASE 4: Span 1 & 3 = $w_{min}$, Span 2 = $w_{max}$

CASE 4: Span 1 & 3 = $w_{min}$, Span 2 = $w_{max}$

$w_{min} = 30.7 \text{ kN/m}$

$w_{max} = 54.4 \text{ kN/m}$

$w_{min} = 30.7 \text{ kN/m}$
Example 2: Analysis of One Level Sub-Frame

Shear Force and Bending Moment Envelope

**SFD (kN)**
- Case 1: 161.9, 161.9, 155.1, 78.5
- Case 2: 170.3, 170.3, 156.2, 87.3
- Case 3: 190.0, 190.0, 183.2, 105.5
- Case 4: 95.6, 113.5, 200.4, 82.0, 82.0

**BMD (kNm)**
- Case 1: 87.3, 156.2, 190.0, 170.3
- Case 2: 142.3, 105.5, 104.5, 95.6
- Case 3: 184.5, 123.0, 200.4
- Case 4: 82.0, 82.0, 184.5

---

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Example 2: Analysis of One Level Sub-Frame

Shear Force and Bending Moment Envelope

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFD (kN)</td>
<td>161.9, 161.9</td>
<td>170.3, 170.3</td>
<td>190.0, 190.0</td>
</tr>
<tr>
<td>BMD (kNm)</td>
<td>55.9</td>
<td>156.2</td>
<td>183.2</td>
</tr>
</tbody>
</table>

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innovative ● entrepreneurial ● global
Example 3

ANALYSIS OF TWO FREE-JOINT SUB-FRAME
Example 3
Example 3: Analysis of Two Free-Joint Sub-Frame

CASE 1: Span 1 & 2 = \( w_{\text{max}} \)

**Stiffness, \( K = I/L \)**

**Beam:**
- \( K_{AB} = \frac{4.5 \times 10^9}{6000} = 7.5 \times 10^5 \ mm^3 \)
- \( K_{BC} = 0.5 \times \frac{4.5 \times 10^9}{6000} = 3.75 \times 10^5 \ mm^3 \)

**Column:**
- \( K_{c,u} = \frac{1.6 \times 10^9}{3500} = 4.6 \times 10^5 \ mm^3 \)
- \( K_{c,l} = \frac{1.6 \times 10^9}{4000} = 4.0 \times 10^5 \ mm^3 \)
Example 3: Analysis of Two Free-Joint Sub-Frame Distribution Factor, $F = K/\Sigma K$

**Fixed End Moment**

\[
M_{AB} = \frac{wL^2}{12} = \frac{57.5 \times 6^2}{12} = 172.6 \text{ kNm}
\]

\[
M_{BC} = \frac{wL^2}{12} = \frac{54.4 \times 6^2}{12} = 163.3 \text{ kNm}
\]

**Joint A:**

\[
F_{AB} = \frac{7.5}{(7.5+4.6+4.0)} = 0.47
\]

\[
F_{c,u} = \frac{4.6}{(7.5+4.6+4.0)} = 0.28
\]

\[
F_{c,l} = \frac{4.0}{(7.5+4.6+4.0)} = 0.25
\]

**Joint B:**

\[
F_{BA} = \frac{7.5}{(7.5+3.75+4.6+4.0)} = 0.38
\]

\[
F_{BC} = \frac{3.75}{(7.5+3.75+4.6+4.0)} = 0.19
\]

\[
F_{c,u} = \frac{4.6}{(7.5+3.75+4.6+4.0)} = 0.23
\]

\[
F_{c,l} = \frac{4.0}{(7.5+3.75+4.6+4.0)} = 0.20
\]

**Moment Distribution Method**

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<thead>
<tr>
<th>$M$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
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<tbody>
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<td>0.0</td>
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<td>-173.2</td>
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CASE 2: Span 1 = \( w_{\text{max}} \), Span 2 = \( w_{\text{min}} \)

\[ w_{\text{max}} = 57.5 \text{ kN/m} \]
\[ w_{\text{min}} = 29.11 \text{ kN/m} \]

Moment Distribution Method

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<tr>
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<table>
<thead>
<tr>
<th>Span 1</th>
<th>Span 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 m</td>
<td>6 m</td>
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</tbody>
</table>

A

B

C

Example 3: Analysis of Two Free-Joint Sub-Frame
Example 3: Analysis of Two Free-Joint Sub-Frame

 Reaction Calculation at Support

**CASE 1**

\[ w_{\text{max}} = 57.5 \, \text{kN/m} \]

\[ \begin{align*}
V_{AB} &= 156.2 \, \text{kN} \\
V_{BA} &= 189.0 \, \text{kN}
\end{align*} \]

**CASE 2**

\[ w_{\text{max}} = 57.5 \, \text{kN/m} \]

\[ \begin{align*}
V_{AB} &= 162.0 \, \text{kN} \\
V_{BA} &= 183.2 \, \text{kN}
\end{align*} \]
Example 3: Analysis of Two Free-Joint Sub-Frame

Shear Force and Bending Moment Diagram

**SFD (kN)**

- 162.0
- 156.2
- 183.2
- 189.0

**BMD (kNm)**

- 105.1
- 97.1
- 56.0
- 26.4
- 114.9
- 122.9
- 195.4
- 168.8
- 49.0
- 30.2

Span AB
Example 3: Analysis of Two Free-Joint Sub-Frame

CASE 1: Span 1 & 2 = $w_{\text{max}}$, Span 3 = $w_{\text{min}}$

- $w_{\text{max}} = 57.5 \text{ kN/m}$
- $w_{\text{max}} = 54.4 \text{ kN/m}$
- $w_{\text{min}} = 30.7 \text{ kN/m}$

Span BC
Example 3: Analysis of Two Free-Joint Sub-Frame

### Fixed End Moment

**Beam:**
- $M_{AB} = M_{BA} = \frac{wL^2}{12} = \frac{57.5 \times 6^2}{12} = 172.6 \text{ kNm}$
- $M_{BC} = M_{CB} = \frac{wL^2}{12} = \frac{54.4 \times 6^2}{12} = 163.3 \text{ kNm}$
- $M_{CD} = M_{DC} = \frac{wL^2}{12} = \frac{30.7 \times 6^2}{12} = 92.0 \text{ kNm}$

**Column:**
- $K_{AB} = K_{BC} = 0.5 \times \frac{4.5 \times 10^9}{6000} = 3.75 \times 10^5 \text{ mm}^3$
- $K_{BC} = \frac{4.5 \times 10^9}{6000} = 7.5 \times 10^5 \text{ mm}^3$
- $K_{C,u} = \frac{1.6 \times 10^9}{3500} = 4.6 \times 10^5 \text{ mm}^3$
- $K_{C,t} = \frac{1.6 \times 10^9}{4000} = 4.0 \times 10^5 \text{ mm}^3$

### Distribution Factor, $F = \frac{K}{\Sigma K}$

**Joint B:**
- $F_{BA} = \frac{3.75}{(3.75+7.5+4.6+4.0)} = 0.19$
- $F_{BC} = \frac{7.5}{(3.75+7.5+4.6+4.0)} = 0.38$
- $F_{C,u} = \frac{4.6}{(3.75+7.5+4.6+4.0)} = 0.23$
- $F_{C,t} = \frac{4.0}{(3.75+7.5+4.6+4.0)} = 0.20$

**Joint C:**
- $F_{CB} = \frac{7.5}{(3.75+7.5+4.6+4.0)} = 0.38$
- $F_{CD} = \frac{3.75}{(3.75+7.5+4.6+4.0)} = 0.19$
- $F_{C,u} = \frac{4.6}{(3.75+7.5+4.6+4.0)} = 0.23$
- $F_{C,t} = \frac{4.0}{(3.75+7.5+4.6+4.0)} = 0.20$
Example 3: Analysis of Two Free-Joint Sub-Frame

**Moment Distribution Method**

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<table>
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</thead>
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<tr>
<td>-27.0</td>
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<td>0.0</td>
</tr>
<tr>
<td>136.7</td>
<td>-14.5</td>
<td>-105.6</td>
</tr>
</tbody>
</table>
CASE 2: Span 2 = \( w_{\text{max}} \), Span 1 & 3 = \( w_{\text{min}} \)

\[ w_{\text{max}} = 30.7 \text{ kN/m} \]
\[ w_{\text{max}} = 54.4 \text{ kN/m} \]
\[ w_{\text{min}} = 30.7 \text{ kN/m} \]
Example 3: Analysis of Two Free-Joint Sub-Frame

**Fixed End Moment**

\[
M_{AB} = M_{BA} = \frac{wL^2}{12} = \frac{30.7 \times 6^2}{12} = 92.0 \text{ kNm}
\]
\[
M_{BC} = M_{CB} = \frac{wL^2}{12} = \frac{54.4 \times 6^2}{12} = 163.3 \text{ kNm}
\]
\[
M_{CD} = M_{DC} = \frac{wL^2}{12} = \frac{30.7 \times 6^2}{12} = 92.0 \text{ kNm}
\]

**Stiffness, \( K = I/L \)**

**Beam:** \( K_{AB} = K_{BC} = 0.5 \times \frac{4.5 \times 10^9}{6000} = 3.75 \times 10^5 \text{ mm}^3 \)

\[
K_{BC} = \frac{4.5 \times 10^9}{6000} = 7.5 \times 10^5 \text{ mm}^3
\]

**Column:** \( K_{c, u} = \frac{1.6 \times 10^9}{3500} = 4.6 \times 10^5 \text{ mm}^3 \)

\[
K_{c, l} = \frac{1.6 \times 10^9}{4000} = 4.0 \times 10^5 \text{ mm}^3
\]

**Distribution Factor, \( F = K/\Sigma K \)**

**Joint B:**

\[
F_{BA} = \frac{3.75}{(3.75+7.5+4.6+4.0)} = 0.19
\]
\[
F_{BC} = \frac{7.5}{(3.75+7.5+4.6+4.0)} = 0.38
\]
\[
F_{c, u} = \frac{4.6}{(3.75+7.5+4.6+4.0)} = 0.23
\]
\[
F_{c, l} = \frac{4.0}{(3.75+7.5+4.6+4.0)} = 0.20
\]

**Joint C:**

\[
F_{CB} = \frac{7.5}{(3.75+7.5+4.6+4.0)} = 0.38
\]
\[
F_{CD} = \frac{3.75}{(3.75+7.5+4.6+4.0)} = 0.19
\]
\[
F_{c, u} = \frac{4.6}{(3.75+7.5+4.6+4.0)} = 0.23
\]
\[
F_{c, l} = \frac{4.0}{(3.75+7.5+4.6+4.0)} = 0.20
\]
Example 3: Analysis of Two Free-Joint Sub-Frame

**Moment Distribution Method**

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</table>
Example 3: Analysis of Two Free-Joint Sub-Frame

Reaction Calculation at Support

CASE 1

$w_{max} = 54.4 \text{ kN/m}$

\[ V_{BC} = 175.3 \text{ kN} \]
\[ V_{CB} = 136.7 \text{ kN} \]

Solve for each span using **equilibrium method** of analysis:

\[ V_{BC} = 169.7 \text{ kN} \]
\[ V_{CB} = 156.8 \text{ kN} \]

CASE 2

$w_{max} = 54.4 \text{ kN/m}$

\[ V_{BC} = 146.6 \text{ kN} \]
\[ V_{CB} = 146.6 \text{ kN} \]

Solve for each span using **equilibrium method** of analysis:

\[ V_{CB} = 163.3 \text{ kN} \]
\[ V_{CB} = 163.3 \text{ kN} \]
Example 3: Analysis of Two Free-Joint Sub-Frame

Shear Force and Bending Moment Diagram

SFD (kN)

BMD (kNm)
Example 4

CONTINUOUS BEAM + ONE FREE-JOINT SUB-FRAME
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Moment of Inertia

\[ I_{beam} = \frac{bh^3}{12} = \frac{250 \times 600^3}{12} = 4.5 \times 10^9 \text{ mm}^4 \]

Stiffness, \( K = \frac{I}{L} \)

\[ K_{AB} = K_{CD} = 0.75 \times \frac{I_{beam}}{L} = 0.75 \times \frac{4.5 \times 10^9}{6000} = 5.63 \times 10^5 \text{ mm}^3 \]

\[ K_{BC} = \frac{I_{beam}}{L} = \frac{4.5 \times 10^9}{6000} = 7.5 \times 10^5 \text{ mm}^3 \]

Distribution Factor, \( F = \frac{K}{\Sigma K} \)

Joint A:

\[ F_{BA} = \frac{5.63}{(5.63+0)} = 1.00 \]

Joint B:

\[ F_{BA} = \frac{5.63}{(5.63+7.5)} = 0.43 \]

\[ F_{BC} = \frac{7.5}{(5.63+7.5)} = 0.57 \]

Joint C:

\[ F_{BC} = \frac{7.5}{(5.63+7.5)} = 0.57 \]

\[ F_{CD} = \frac{5.63}{(7.5+5.63)} = 0.43 \]

Joint D:

\[ F_{DC} = \frac{5.63}{(5.63+0)} = 1.00 \]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

CASE 1: Span 1 & 2 = $w_{\text{max}}$, Span 3 = $w_{\text{min}}$

$w_{\text{max}} = 57.5 \text{ kN/m}$

$w_{\text{max}} = 54.4 \text{ kN/m}$

$w_{\text{min}} = 30.7 \text{ kN/m}$

Span 1

A

6 m

Span 2

B

6 m

Span 3

C

6 m

D

Fixed End Moment, FEM

$M_{AB} = M_{BA} = \frac{wL^2}{12} = \frac{57.5 \times 6^2}{12} = 172.6 \text{ kNm}$

$M_{BC} = M_{CB} = \frac{wL^2}{12} = \frac{54.4 \times 6^2}{12} = 163.3 \text{ kNm}$

$M_{CD} = M_{DC} = \frac{wL^2}{12} = \frac{30.7 \times 6^2}{12} = 92.0 \text{ kNm}$
**Example 4: Continuous Beam + One Free-Joint Sub-Frame**

**Moment Distribution Method**

<table>
<thead>
<tr>
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<th>B</th>
<th>C</th>
<th>D</th>
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<td><strong>-217.9</strong></td>
<td><strong>137.2</strong></td>
<td><strong>-137.2</strong></td>
</tr>
</tbody>
</table>
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Reaction Calculation at Support

Solve for each span using equilibrium method of analysis:

\[ V_{AB} = 136.3 \, \text{kN} \]
\[ V_{BA} = 208.9 \, \text{kN} \]
\[ V_{BC} = 176.7 \, \text{kN} \]
\[ V_{CB} = 149.8 \, \text{kN} \]
\[ V_{CD} = 114.9 \, \text{kN} \]
\[ V_{DC} = 69.1 \, \text{kN} \]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Do the same for Load CASE 2, CASE 3 & CASE 4
SELF STUDY !!!

CASE 2: Span 2 & 3 = $w_{\text{max}}$, Span 1 = $w_{\text{min}}$

- $w_{\text{min}} = 30.7 \text{ kN/m}$
- $w_{\text{max}} = 54.4 \text{ kN/m}$
- $w_{\text{max}} = 57.5 \text{ kN/m}$

Span 1: 6 m
Span 2: 6 m
Span 3: 6 m
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Do the same for Load CASE 2, CASE 3 & CASE 4

SELF STUDY !!!

CASE 3: Span 1 & 3 = $w_{max}$, Span 2 = $w_{min}$

\[
\begin{align*}
&w_{max} = 57.5 \text{ kN/m} \\
&w_{min} = 29.1 \text{ kN/m}
\end{align*}
\]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Do the same for Load CASE 2, CASE 3 & CASE 4

SELF STUDY !!!

CASE 4: Span 2 = \( w_{\text{max}} \), Span 1 & 3 = \( w_{\text{min}} \)

\[
\begin{align*}
\text{Span 1} & : w_{\text{min}} = 30.7 \text{ kN/m} \\
\text{Span 2} & : w_{\text{max}} = 54.4 \text{ kN/m} \\
\text{Span 3} & : w_{\text{min}} = 30.7 \text{ kN/m}
\end{align*}
\]

FINALLY DO THE SFD & BMD ENVELOPE !!!
**Example 4: Continuous Beam + One Free-Joint Sub-Frame**

### Sub-Frame: Bending Moment in Column

**Fixed End Moment, FEM**

\[ M_{AB} = \frac{wL^2}{12} = \frac{57.5 \times 6^2}{12} = 172.6 \text{ kN}m \]

**Moment of Inertia, \( I = bh^3/12 \)**

- **Beam:**
  \[ I_{AB} = \frac{250 \times 600^3}{12^2} = 4.5 \times 10^9 \text{ mm}^4 \]

- **Column:**
  \[ I_{c,u} = \frac{300 \times 400^3}{12^2} = 1.6 \times 10^9 \text{ mm}^4 \]

**Stiffness, \( K = I/L \)**

- **Beam:**
  \[ K_{AB} = K_{BC} = K_{CD} = \frac{4.5 \times 10^9}{6000} = 7.5 \times 10^5 \text{ mm}^3 \]

- **Column:**
  \[ K_{c,u} = \frac{1.6 \times 10^9}{3500} = 4.6 \times 10^5 \text{ mm}^3 \]
  \[ K_{c,l} = \frac{1.6 \times 10^9}{4000} = 4.0 \times 10^5 \text{ mm}^3 \]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Sub-Frame: Bending Moment in Column

**Moment in upper column, \( M_{\text{upper}} \)**

\[
M_{\text{upper}} = FEM \times \frac{K_{c,u}}{(K_{c,u} + K_{c,l} + 0.5K_{AB})} = 172.6 \times \frac{4.6}{(4.6 + 4.0 + 0.5 \times 7.5)} = 64.3 \text{ kNm}
\]

**Moment in lower column, \( M_{\text{lower}} \)**

\[
M_{\text{upper}} = FEM \times \frac{K_{c,l}}{(K_{c,u} + K_{c,l} + 0.5K_{AB})} = 172.6 \times \frac{4.0}{(4.6 + 4.0 + 0.5 \times 7.5)} = 56.0 \text{ kNm}
\]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Sub-Frame: Bending Moment in Column

**Fixed End Moment, FEM**

\[
M_{BA} = \frac{wL^2}{12} = \frac{57.5 \times 6^2}{12} = 172.6 \text{ kNm}
\]

\[
M_{BC} = \frac{wL^2}{12} = \frac{29.1 \times 6^2}{12} = 87.3 \text{ kNm}
\]

\[
\therefore \Delta FEM = M_{AB} - M_{BC} = 85.3 \text{ kNm}
\]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Sub-Frame: Bending Moment in Column

Moment in upper column, $M_{\text{upper}}$

\[
M_{\text{upper}} = \Delta FEM \times \frac{K_{c,u}}{K_{c,u} + K_{c,l} + 0.5K_{AB} + 0.5K_{BC}} = 85.3 \times \frac{4.6}{(4.6 + 4.0 + 0.5 \times 7.5 + 0.5 \times 7.5)} = 24.2 \text{ kNm}
\]

Moment in lower column, $M_{\text{lower}}$

\[
M_{\text{upper}} = \Delta FEM \times \frac{K_{c,l}}{K_{c,u} + K_{c,l} + 0.5K_{AB} + 0.5K_{BC}} = 85.3 \times \frac{4.0}{(4.6 + 4.0 + 0.5 \times 7.5 + 0.5 \times 7.5)} = 21.2 \text{ kNm}
\]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Sub-Frame: Bending Moment in Column

Fixed End Moment, FEM

\[ M_{BC} = \frac{wL^2}{12} = \frac{29.1 \times 6^2}{12} = 87.3 \text{ kNm} \]

\[ M_{CD} = \frac{wL^2}{12} = \frac{57.5 \times 6^2}{12} = 172.6 \text{ kNm} \]

\[ \therefore \Delta FEM = M_{CD} - M_{BC} = 85.3 \text{ kNm} \]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Sub-Frame: Bending Moment in Column

Moment in upper column, $M_{\text{upper}}$

$$M_{\text{upper}} = \Delta FEM \times \frac{K_{c,u}}{(K_{c,u} + K_{c,l} + 0.5K_{BC} + 0.5K_{CD})} = 85.3 \times \frac{4.6}{(4.6 + 4.0 + 0.5 \times 7.5 + 0.5 \times 7.5)} = 24.2 \text{ kNm}$$

Moment in lower column, $M_{\text{lower}}$

$$M_{\text{upper}} = \Delta FEM \times \frac{K_{c,l}}{(K_{c,u} + K_{c,l} + 0.5K_{BC} + 0.5K_{CD})} = 85.3 \times \frac{4.0}{(4.6 + 4.0 + 0.5 \times 7.5 + 0.5 \times 7.5)} = 21.2 \text{ kNm}$$
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Sub-Frame: Bending Moment in Column

Column D

Fixed End Moment, FEM

\[ M_{DC} = \frac{wL^2}{12} = \frac{57.5 \times 6^2}{12} = 172.6 \text{ kNm} \]

\[ w_{\text{max}} = 57.5 \text{ kN/m} \]

\[ 0.5K_{CD} \]

\[ K_{c,u} \]

\[ K_{c,l} \]

\[ 4 \text{ m} \]

\[ 3.5 \text{ m} \]

\[ 6 \text{ m} \]
Example 4: Continuous Beam + One Free-Joint Sub-Frame

Sub-Frame: Bending Moment in Column

**Moment in upper column, \( M_{\text{upper}} \)\:**

\[
M_{\text{upper}} = FEM \times \frac{K_{c,u}}{(K_{c,u}+K_{c,l}+0.5K_{CD})} = 172.6 \times \frac{4.6}{(4.6+4.0+0.5\times7.5)} = 64.3 \text{ kNm}
\]

**Moment in lower column, \( M_{\text{lower}} \)\:**

\[
M_{\text{lower}} = FEM \times \frac{K_{c,l}}{(K_{c,u}+K_{c,l}+0.5K_{CD})} = 172.6 \times \frac{4.0}{(4.6+4.0+0.5\times7.5)} = 56.0 \text{ kNm}
\]
Wind Action

• Intensity of wind action on a structure related to:
  a) **Square of wind velocity**
  b) **Member dimensions that resist the wind**

• Wind velocity dependent on:
  a) **Geographical location**
  b) **Height of structure**
  c) **Topography area**
  d) **Roughness of the surrounding terrain**

• Response of the structure from the variable wind action:
  a) **Background component**
  b) **Resonant component**
Wind Action

Background Component

- **Static deflection** of the structure under wind pressure

Resonant Component

- **Dynamic vibration** of the structure under wind pressure
- Relatively small
- Normally treated using static method of analysis
Wind Action

Plan View

- Windward
- Wake
- Leeward
- Stagnation point
- Separation point
- Separation Zone

Side View

- Wind
- \( \alpha < 15^\circ \)
- (+)
- (-)

Effect of Wind on Buildings
## Calculation of Wind Pressure

**(MS 1553: 2002)**

<table>
<thead>
<tr>
<th>Simplified Method</th>
<th>Analytical Procedure</th>
<th>Wind Tunnel Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>♦ Building of rectangular in plan</td>
<td>♦ Rectangular buildings</td>
<td>♦ Complex buildings</td>
</tr>
<tr>
<td>♦ Height &lt; 15 m</td>
<td>♦ Height &lt; 200 m</td>
<td>♦ Roof span &lt; 100 mm</td>
</tr>
</tbody>
</table>
Appendix A, MS 1553: 2002

1. Determined *basic wind speed, $V_s$* (Figure A1, MS 1553: 2002)
2. Determine the *terrain/height multiplier, $M_{z,cat}$* (Table A1, MS 1553: 2002)
3. Determine *external pressure coefficient, $C_{p,e}$* for surface of enclose building (A2.3 and A2.4, MS 1553: 2002)
4. Determine *internal pressure coefficient, $C_{p,i}$* for surface of enclose building. Taken as $+0.6$ or $-0.3$ and shall be considered to determine the critical load requirements.
5. Design *wind pressure, $p$* (in Pa):

\[ p = 0.613 (V_s)^2 (M_{z,cat})^2 (C_{p,e} - C_{p,i}) \]
Wind Action: Simplified Method

Figure A1, MS 1553: 2002

Table 3.1 Wind speed (m/s) for various return period

<table>
<thead>
<tr>
<th>Station</th>
<th>$V_{20}$</th>
<th>$V_{50}$</th>
<th>$V_{100}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temerloh</td>
<td>25.1</td>
<td>27.4</td>
<td>29.1</td>
</tr>
<tr>
<td>Tawau</td>
<td>24.6</td>
<td>26.6</td>
<td>28.1</td>
</tr>
<tr>
<td>Subang</td>
<td>29.2</td>
<td>32.1</td>
<td>34.3</td>
</tr>
<tr>
<td>Sri Aman</td>
<td>27.6</td>
<td>30.3</td>
<td>32.4</td>
</tr>
<tr>
<td>Sitiawan</td>
<td>23.3</td>
<td>25.3</td>
<td>26.7</td>
</tr>
<tr>
<td>Sibu</td>
<td>27.0</td>
<td>29.3</td>
<td>31.0</td>
</tr>
<tr>
<td>Senai</td>
<td>26.9</td>
<td>29.1</td>
<td>30.7</td>
</tr>
<tr>
<td>Sandakan</td>
<td>23.4</td>
<td>25.8</td>
<td>27.7</td>
</tr>
<tr>
<td>Petaling Jaya</td>
<td>28.8</td>
<td>31.4</td>
<td>33.4</td>
</tr>
<tr>
<td>Muadzam Shah</td>
<td>22.6</td>
<td>24.4</td>
<td>25.3</td>
</tr>
<tr>
<td>Miri</td>
<td>26.9</td>
<td>29.0</td>
<td>30.5</td>
</tr>
<tr>
<td>Mersing</td>
<td>29.5</td>
<td>32.0</td>
<td>33.8</td>
</tr>
<tr>
<td>Melaka</td>
<td>26.7</td>
<td>29.4</td>
<td>31.3</td>
</tr>
<tr>
<td>Labuan</td>
<td>26.0</td>
<td>27.7</td>
<td>29.0</td>
</tr>
<tr>
<td>Kudat</td>
<td>27.1</td>
<td>29.1</td>
<td>30.6</td>
</tr>
<tr>
<td>Kuala Terengganu</td>
<td>25.5</td>
<td>27.2</td>
<td>28.5</td>
</tr>
<tr>
<td>Kuantan</td>
<td>27.5</td>
<td>29.8</td>
<td>31.6</td>
</tr>
<tr>
<td>Kluang</td>
<td>29.6</td>
<td>32.6</td>
<td>34.9</td>
</tr>
<tr>
<td>Kuala Krai</td>
<td>27.2</td>
<td>29.5</td>
<td>31.3</td>
</tr>
<tr>
<td>Kuching</td>
<td>29.5</td>
<td>32.6</td>
<td>34.9</td>
</tr>
<tr>
<td>Kota Bahru</td>
<td>30.0</td>
<td>32.4</td>
<td>34.2</td>
</tr>
<tr>
<td>Kota Kinabalu</td>
<td>28.3</td>
<td>30.5</td>
<td>32.2</td>
</tr>
<tr>
<td>Ipoh</td>
<td>30.6</td>
<td>33.5</td>
<td>35.7</td>
</tr>
<tr>
<td>Chuping</td>
<td>23.8</td>
<td>25.6</td>
<td>27.0</td>
</tr>
<tr>
<td>Cameron Highlands</td>
<td>25.2</td>
<td>28.8</td>
<td>30.0</td>
</tr>
<tr>
<td>Butterworth</td>
<td>24.6</td>
<td>26.4</td>
<td>27.7</td>
</tr>
<tr>
<td>Batu Embun</td>
<td>25.3</td>
<td>27.6</td>
<td>28.2</td>
</tr>
<tr>
<td>Bayan Lepas</td>
<td>25.6</td>
<td>27.5</td>
<td>28.9</td>
</tr>
<tr>
<td>Bintulu</td>
<td>23.9</td>
<td>25.6</td>
<td>28.9</td>
</tr>
<tr>
<td>Alor Setar</td>
<td>27.2</td>
<td>29.9</td>
<td>31.8</td>
</tr>
</tbody>
</table>

Basic Wind Speed

Zone I, $V_S = 33.5$ m/s
Zone II, $V_S = 32.5$ m/s
# Wind Action: Simplified Method

## Table A1. Terrain height multiplier, $M_{z,\text{cat}}$

<table>
<thead>
<tr>
<th>Height, $z$ (m)</th>
<th>$M_{z,\text{cat}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Terrain Category 1</td>
</tr>
<tr>
<td>3</td>
<td>0.99</td>
</tr>
<tr>
<td>5</td>
<td>1.05</td>
</tr>
<tr>
<td>10</td>
<td>1.12</td>
</tr>
<tr>
<td>15</td>
<td>1.16</td>
</tr>
</tbody>
</table>

**NOTE.** Terrain categories definitions:

a) Category 1: Exposed open terrain with few or no obstructions.
b) Category 2: Water surfaces, open terrain, grassland with few well scattered obstructions having height generally from 1.5 m to 10.0 m.
c) Category 3: Terrain with numerous closely spaced obstructions 3.0 m to 5.0 m high such as areas of suburban housing.
d) Category 4: Terrain with numerous large, high (10.0 m to 30.0 m high) and closely spaced obstructions such as large city centres and well-developed industrial complexes.
Wind Action: Simplified Method

A2.3 The external pressure coefficients, $C_{p,e}$, for windward wall shall be taken as 0.8. $C_{p,e}$ for leeward and side wall shall be as per Tables A2 and A3 respectively.

Table A2. External pressure coefficients $C_{p,e}$, for leeward wall

<table>
<thead>
<tr>
<th>$\alpha^*$</th>
<th>$d/b^*$</th>
<th>$C_{p,e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>1</td>
<td>-0.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-0.2</td>
</tr>
<tr>
<td>15°</td>
<td>All values</td>
<td>-0.3</td>
</tr>
<tr>
<td>20°</td>
<td></td>
<td>-0.4</td>
</tr>
<tr>
<td>25°</td>
<td></td>
<td>-0.5</td>
</tr>
</tbody>
</table>

* For intermediate values of $d/b$ and $\alpha$, use linear interpolation.

Table A3. External pressure coefficients $C_{p,e}$, for side walls

<table>
<thead>
<tr>
<th>Horizontal distance from windward edge</th>
<th>$C_{p,e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to 2h</td>
<td>-0.65</td>
</tr>
<tr>
<td>&gt;2h</td>
<td>-0.30</td>
</tr>
</tbody>
</table>
Wind Action: Simplified Method

A2.4 The external pressure coefficients, $C_{pe}$, for roofs shall be as per Tables A4, A5 and A6.

**Table A4.** For up-wind slope, $U$ and down-wind slope, $D$ for $\alpha<10^\circ$ and $R$ for gable roofs

<table>
<thead>
<tr>
<th>Roof type and slope</th>
<th>Horizontal distance from windward edge</th>
<th>External pressure coefficient, $C_{pe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross wind slopes for gable roofs, $R$</td>
<td>$h/d \leq 0.5^{**}$</td>
<td>$h/d \geq 1.0^{**}$</td>
</tr>
<tr>
<td>All $\alpha$</td>
<td>$\alpha &lt; 10^\circ$</td>
<td>0 to 1h</td>
</tr>
<tr>
<td></td>
<td>1h to 2h</td>
<td>-0.5, 0</td>
</tr>
<tr>
<td></td>
<td>$&gt; 2h$</td>
<td>-0.3, 0.2</td>
</tr>
</tbody>
</table>

**Table A5.** Up-wind slope, $U$, $\alpha \geq 10^\circ$

<table>
<thead>
<tr>
<th>Roof type and slope</th>
<th>Ratio $h/d$</th>
<th>External pressure coefficients, $C_{pe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up-wind Slope, $U$</td>
<td>$\alpha \geq 10^\circ$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\leq 0.25$</td>
<td>-0.7, -0.3</td>
</tr>
<tr>
<td></td>
<td>$0.5$</td>
<td>-0.9, -0.4</td>
</tr>
<tr>
<td></td>
<td>$\geq 1.0$</td>
<td>-1.3, -0.6</td>
</tr>
<tr>
<td></td>
<td>$0.8\sin\alpha$</td>
<td></td>
</tr>
</tbody>
</table>

**Table A6.** Down-wind slope, $D$, $\alpha \geq 10^\circ$ and $R$ for hip roofs

<table>
<thead>
<tr>
<th>Roof type and slope</th>
<th>Ratio $h/d$*</th>
<th>External pressure coefficient, $C_{pe}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross wind slopes for hip roof, $R$</td>
<td>$\alpha \geq 10^\circ$</td>
<td></td>
</tr>
<tr>
<td>Down-wind slopes, $D$</td>
<td>$\leq 0.25$</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>$0.5$</td>
<td>-0.5</td>
</tr>
<tr>
<td></td>
<td>$\geq 1.0$</td>
<td>-0.7</td>
</tr>
</tbody>
</table>

* Interpolation shall only be carried out on values of the same sign.
* For intermediate values of roof slopes and $h/d$ ratios, use linear interpolation.
Wind Action: Analytical Procedure

Section 2, MS 1553: 2002

Design ***wind pressure*** (in Pa):

\[ p = 0.613(V_{des})^2 C_{fig} C_{dyn} \]

- **Design Wind Speed**: \( V_{des} = V_{sit} l \)
- **Site Wind Speed**: \( V_{sit} = V_s M_d M_{z,cat} M_s M_h \)
- **Dynamic Coefficient**: \( C_{dyn} = 1.0 \text{ if } > 1 \text{ Hertz (unless the structure is wind sensitive)} \)
- **Fig Coefficient**: \( C_{fig} = C_{p,e} K_a K_c K_l K_p \)

If others, see Next
\[ V_{\text{des}} = \text{Design wind speed} \]
\[ l = \text{Importance factor (Table 3.2, MS 1553: 2002)} \]
\[ V_s = \text{Basic wind speed (33.5 m/s: Zone 1 & 32.5 m/s: Zone 2)} \]
\[ M_d = \text{Wind directional multiplier} = 1.0 \]
\[ M_{z,\text{cat}} = \text{Terrain/Height multiplier (Table 4.1, MS 1553: 2002)} \]
\[ M_s = \text{Shielding multiplier (Table 4.3, MS 1553: 2002)} = 1.0 \text{ if effects of shielding is ignored or not applicable} \]
\[ M_h = \text{Hill shape multiplier} = 1.0 \text{ except for particular cardinal direction in the local topographic zones} \]
\[ C_{\text{fig}} = \text{Aerodynamic shape factor} \]
\[ C_{p,e} = \text{External pressure coefficient (Table 5.2(a) & 5.2(b), MS 1553: 2002)} \]
\[ C_{\text{dyn}} = \text{Dynamic response factor} \]
\[ K_a = \text{Area reduction factor} = 1.0 \text{ (in most cases)} \]
\[ K_c = \text{Combination factor} = 1.0 \text{ (in most cases)} \]
\[ K_l = \text{Local pressure factor} = 1.0 \text{ (in most cases)} \]
\[ K_p = \text{Porous cladding reduction factor} = 1.0 \text{ (in most cases)} \]
Wind Action: Analytical Procedure

Section 2, MS 1553: 2002

\[ B_s = \frac{1}{1 + \sqrt{\frac{36(h - s)^2 + 64b_{sh}^2}{L_h}}} \]

\[ S = \frac{1}{\left[ 1 + \frac{3.5n_a h(1 + g_v l_h)}{V_{des}} \right] \left[ 1 + \frac{4n_a b_{0h}(1 + g_v l_h)}{V_{des}} \right]} \]

\[ C_{dyn} = \frac{1 + 2l_h \sqrt{g_v^2 B_s + \frac{g_R^2 S E_t}{\zeta}}}{(1 + 2g_v l_h)} \]

\[ E_t = \frac{0.47N}{(2 + N)^{5/6}} \]

\[ N = \frac{n_a L_h[1 + (g_v l_h)]}{V_{des}} \]
# Wind Action: Analytical Procedure

## Table 3.2 Importance factor, $I$

<table>
<thead>
<tr>
<th>Nature of Occupancy</th>
<th>Category of Structures</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buildings and structures that represent low hazard to human life in the event of failure such as agricultural facilities, temporary facilities and minor storage facilities.</td>
<td>I</td>
<td>0.87</td>
</tr>
<tr>
<td>All buildings and structure except those listed in category I, III and IV.</td>
<td>II</td>
<td>1.0</td>
</tr>
<tr>
<td>Buildings and structures where the primary occupancy is one in which more than 300 people congregate in one area.</td>
<td>III</td>
<td>1.15</td>
</tr>
</tbody>
</table>
| Essential buildings and structures  
  Hospital and medical facilities  
  Fire and police stations  
  Structures and equipment in civil defense  
  Communication centres and facilities for emergency response  
  Power stations and other emergency utilities  
  Defense shelter. | IV                     | 1.15 |
### Table 6.2 Values of fraction critical damping of structures (ξ)

<table>
<thead>
<tr>
<th>Stress levels and type of construction</th>
<th>Fraction of critical damping (ξ)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Serviceability limit states</strong></td>
<td></td>
</tr>
<tr>
<td>Steel frame</td>
<td>0.005 to 0.010</td>
</tr>
<tr>
<td>Reinforced or pre-stressed concrete</td>
<td>0.005 to 0.010</td>
</tr>
<tr>
<td><strong>Ultimate limit states</strong></td>
<td></td>
</tr>
<tr>
<td>Steel frame welded</td>
<td>0.02</td>
</tr>
<tr>
<td>Steel frame bolted</td>
<td>0.05</td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>0.05</td>
</tr>
</tbody>
</table>
## Wind Action: Analytical Procedure

### Table 4.1. Terrain/height multipliers for gust wind speeds in fully developed terrain. Serviceability limit state design and ultimate limit state

<table>
<thead>
<tr>
<th>Height (z) (m)</th>
<th>Multiplier ((M_{Z,cat}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Terrain Category 1</td>
</tr>
<tr>
<td></td>
<td>(0.99)</td>
</tr>
<tr>
<td>(5)</td>
<td>(1.05)</td>
</tr>
<tr>
<td>(10)</td>
<td>(1.12)</td>
</tr>
<tr>
<td>(15)</td>
<td>(1.16)</td>
</tr>
<tr>
<td>(20)</td>
<td>(1.19)</td>
</tr>
<tr>
<td>(30)</td>
<td>(1.22)</td>
</tr>
<tr>
<td>(40)</td>
<td>(1.24)</td>
</tr>
<tr>
<td>(50)</td>
<td>(1.25)</td>
</tr>
<tr>
<td>(75)</td>
<td>(1.27)</td>
</tr>
<tr>
<td>(100)</td>
<td>(1.29)</td>
</tr>
<tr>
<td>(150)</td>
<td>(1.31)</td>
</tr>
<tr>
<td>(200)</td>
<td>(1.32)</td>
</tr>
<tr>
<td>(250)</td>
<td>(1.34)</td>
</tr>
<tr>
<td>(300)</td>
<td>(1.35)</td>
</tr>
<tr>
<td>(400)</td>
<td>(1.37)</td>
</tr>
<tr>
<td>(500)</td>
<td>(1.38)</td>
</tr>
</tbody>
</table>

**NOTE.** For intermediate values of height \(z\) and terrain category, use linear interpolation.

### Table 4.3 Shielding multiplier, \(M_s\)

<table>
<thead>
<tr>
<th>Shielding parameter, (s)</th>
<th>Shielding multiplier, (M_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\leq1.5)</td>
<td>(0.7)</td>
</tr>
<tr>
<td>(3.0)</td>
<td>(0.8)</td>
</tr>
<tr>
<td>(6.0)</td>
<td>(0.9)</td>
</tr>
<tr>
<td>(\geq12.0)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>Normal suburban housing</td>
<td>(0.85)</td>
</tr>
</tbody>
</table>

**NOTE.** For intermediate values of \(s\), use linear interpolation.
# Wind Action: Analytical Procedure

## Table 5.2 Walls: External pressure coefficients, $C_{p,e}$ for rectangular enclosed buildings.

### Table 5.2 (a) Windward wall, W

<table>
<thead>
<tr>
<th>$h \leq 25.0 \text{ m}$</th>
<th>$h &gt; 25.0 \text{ m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For buildings on ground:</strong></td>
<td></td>
</tr>
<tr>
<td>0.8, when wind speed varies with height, or</td>
<td>0.8, when wind speed varies with height,</td>
</tr>
<tr>
<td>0.7, when speed is taken for $z = h$</td>
<td></td>
</tr>
<tr>
<td><strong>For elevated buildings:</strong></td>
<td></td>
</tr>
<tr>
<td>0.8 when wind speed is taken for $z = h$</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5.2 (b) Leeward wall, L

<table>
<thead>
<tr>
<th>$\alpha^*$</th>
<th>$d/b^*$</th>
<th>External pressure Coefficients, $C_{p,e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 10°</td>
<td>$\leq 1$</td>
<td>-0.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>$\geq 4$</td>
<td>-0.2</td>
</tr>
<tr>
<td>10°</td>
<td></td>
<td>-0.3</td>
</tr>
<tr>
<td>15°</td>
<td>All values</td>
<td>-0.3</td>
</tr>
<tr>
<td>20°</td>
<td></td>
<td>-0.4</td>
</tr>
<tr>
<td>$\geq 25^\circ$</td>
<td>$\leq 0.1$</td>
<td>-0.75</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>-0.625</td>
</tr>
<tr>
<td></td>
<td>$\geq 0.3$</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

* For intermediate values of $d/b$ and $\alpha$, use linear interpolation.
##Wind Action: Analytical Procedure##

<table>
<thead>
<tr>
<th>Height (z) m</th>
<th>Terrain Category 1</th>
<th>Terrain Category 2</th>
<th>Terrain Category 3</th>
<th>Terrain Category 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.17</td>
<td>0.21</td>
<td>0.27</td>
<td>0.34</td>
</tr>
<tr>
<td>5</td>
<td>0.16</td>
<td>0.20</td>
<td>0.27</td>
<td>0.34</td>
</tr>
<tr>
<td>10</td>
<td>0.16</td>
<td>0.18</td>
<td>0.24</td>
<td>0.34</td>
</tr>
<tr>
<td>15</td>
<td>0.15</td>
<td>0.18</td>
<td>0.22</td>
<td>0.34</td>
</tr>
<tr>
<td>20</td>
<td>0.15</td>
<td>0.17</td>
<td>0.21</td>
<td>0.34</td>
</tr>
<tr>
<td>30</td>
<td>0.14</td>
<td>0.16</td>
<td>0.20</td>
<td>0.34</td>
</tr>
<tr>
<td>40</td>
<td>0.13</td>
<td>0.16</td>
<td>0.19</td>
<td>0.30</td>
</tr>
<tr>
<td>50</td>
<td>0.13</td>
<td>0.15</td>
<td>0.19</td>
<td>0.27</td>
</tr>
<tr>
<td>75</td>
<td>0.12</td>
<td>0.14</td>
<td>0.18</td>
<td>0.25</td>
</tr>
<tr>
<td>100</td>
<td>0.11</td>
<td>0.13</td>
<td>0.17</td>
<td>0.23</td>
</tr>
<tr>
<td>150</td>
<td>0.10</td>
<td>0.12</td>
<td>0.15</td>
<td>0.21</td>
</tr>
<tr>
<td>200</td>
<td>0.09</td>
<td>0.11</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td>250</td>
<td>0.08</td>
<td>0.10</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>300</td>
<td>0.07</td>
<td>0.09</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>400</td>
<td>0.07</td>
<td>0.08</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>500</td>
<td>0.06</td>
<td>0.07</td>
<td>0.10</td>
<td>0.14</td>
</tr>
</tbody>
</table>

**NOTE:** For intermediate values of height, z and terrain category, use linear interpolation.
Example 5

WIND LOAD CALCULATION
Example 5: Wind Load Calculation

Location: Kuala Lumpur (Zone 1)
Terrain: Suburban Terrain for All Directions
Topography: Ground Slope Less than 1 in 20 for Greater than 5 km in All Directions
Construction: Reinforced Concrete
Sway Frequencies, \( n_a = n_c \): 0.2 Hertz
Mode Shape: Linear, \( k \): 1.0
Average Building Density: 160 kg/m³
Example 5: Wind Load Calculation

Using Analytical Procedure in Section 2, MS 1553: 2002

Design wind pressure, \( p = 0.613(V_{\text{des}})^2C_{\text{fig}}C_{\text{dyn}} \)

(1) Design wind speed, \( V_{\text{des}} = V_{\text{sit}}l = 32.5 \times 1.15 = 37.37 \text{ m/s} \)

where \( V_{\text{sit}} = V_sM_dM_{z,\text{cat}}M_sM_h = 33.5 \times 1.00 \times 0.97 \times 1.00 \times 1.00 = 32.5 \text{ m/s} \)

- \( V_s = \) Basic wind speed (Figure A.1) = 33.5 m/s
- \( M_d = \) Wind directional multiplier = 1.00
- \( M_{z,\text{cat}} = \) Terrain/Height multiplier (Table 4.1) = 0.97 by interpolation
- \( M_s = \) Shielding multiplier = 1.00
- \( M_h = \) Hill shape multiplier = 1.00

\( l = \) Importance factor (Table 3.2) = 1.15
Example 5: Wind Load Calculation

Table 4.1. Terrain/height multipliers for gust wind speeds in fully developed terrain. Serviceability limit state design and ultimate limit state

<table>
<thead>
<tr>
<th>Height (z)</th>
<th>Multiplier ($M_{z,cat}$)</th>
<th>Terrain Category 1</th>
<th>Terrain Category 2</th>
<th>Terrain Category 3</th>
<th>Terrain Category 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>≤3</td>
<td></td>
<td>0.99</td>
<td>0.85</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>1.05</td>
<td>0.91</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>1.12</td>
<td>1.00</td>
<td>0.83</td>
<td>0.75</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>1.16</td>
<td>1.05</td>
<td>0.89</td>
<td>0.75</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>1.19</td>
<td>1.08</td>
<td>0.94</td>
<td>0.75</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>1.22</td>
<td>1.12</td>
<td>1.00</td>
<td>0.75</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.80</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>1.24</td>
<td>1.16</td>
<td>1.04</td>
<td>0.85</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>1.25</td>
<td>1.18</td>
<td>1.07</td>
<td>0.90</td>
</tr>
<tr>
<td>75</td>
<td></td>
<td>1.27</td>
<td>1.22</td>
<td>1.12</td>
<td>0.98</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>1.29</td>
<td>1.24</td>
<td>1.16</td>
<td>1.03</td>
</tr>
<tr>
<td>150</td>
<td></td>
<td>1.31</td>
<td>1.27</td>
<td>1.21</td>
<td>1.11</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>1.32</td>
<td>1.29</td>
<td>1.24</td>
<td>1.16</td>
</tr>
<tr>
<td>250</td>
<td></td>
<td>1.34</td>
<td>1.31</td>
<td>1.27</td>
<td>1.20</td>
</tr>
<tr>
<td>300</td>
<td></td>
<td>1.35</td>
<td>1.32</td>
<td>1.29</td>
<td>1.23</td>
</tr>
<tr>
<td>400</td>
<td></td>
<td>1.37</td>
<td>1.35</td>
<td>1.32</td>
<td>1.28</td>
</tr>
<tr>
<td>500</td>
<td></td>
<td>1.38</td>
<td>1.37</td>
<td>1.35</td>
<td>1.31</td>
</tr>
</tbody>
</table>

NOTE: For intermediate values of height z and terrain category, use linear interpolation.
Example 5: Wind Load Calculation

<table>
<thead>
<tr>
<th>Nature of Occupancy</th>
<th>Category of Structures</th>
<th>( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buildings and structures that represent low hazard to human life in the event of failure such as agricultural facilities, temporary facilities and minor storage facilities.</td>
<td>I</td>
<td>0.87</td>
</tr>
<tr>
<td>All buildings and structures except those listed in category I, III and IV.</td>
<td>II</td>
<td>1.0</td>
</tr>
<tr>
<td>Buildings and structures where the primary occupancy is one in which more than 300 people congregate in one area.</td>
<td>III</td>
<td>1.15</td>
</tr>
</tbody>
</table>
| Essential buildings and structures  
  Hospital and medical facilities  
  Fire and police stations  
  Structures and equipment in civil defense  
  Communication centres and facilities for emergency response  
  Power stations and other emergency utilities  
  Defense shelter. | IV                     | 1.15   |
(2) Aerodynamic shape factor, $C_{fig} = C_{p,e}K_aK_cK_lK_p$

\[ \therefore C_{fig}(W) = C_{p,e}K_aK_cK_lK_p = 0.80 \]
\[ \therefore C_{fig}(L) = C_{p,e}K_aK_cK_lK_p = -0.25 \]

where $C_{p,e} =$ External pressure coefficient
Windward wall = 0.80 (for varying $z$)
Leeward wall = -0.25 *by interpolation* (for $d/b = 51 \text{ m}/18 \text{ m} = 2.8$)

$K_a =$ Area reduction factor = 1.00
$K_c =$ Combination factor = 1.00
$K_l =$ Local pressure factor = 1.00
$K_p =$ Porous cladding reduction factor = 1.00
### Example 5: Wind Load Calculation

#### Table 5.2 Walls: External pressure coefficients, $C_{p,e}$ for rectangular enclosed buildings.

**Table 5.2 (a) Windward wall, W**

<table>
<thead>
<tr>
<th>External pressure coefficients, $C_{p,e}$</th>
<th>$h \leq 25.0 \text{ m}$</th>
<th>$h &gt; 25.0 \text{ m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>For buildings on ground:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8, when wind speed varies with height,</td>
<td></td>
<td>0.8, when wind speed varies with height,</td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7, when speed is taken for $z = h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>For elevated buildings:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8 when wind speed is taken for $z = h$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 5.2 (b) Leeward wall, L**

<table>
<thead>
<tr>
<th>$\alpha'$</th>
<th>$\frac{d}{b'}$</th>
<th>External pressure Coefficients, $C_{p,e}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 10^\circ$</td>
<td>$\leq 1$</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>$\geq 1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\geq 4$</td>
<td></td>
</tr>
<tr>
<td>$10^\circ$</td>
<td></td>
<td>-0.5</td>
</tr>
<tr>
<td>$15^\circ$</td>
<td></td>
<td>-0.3</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td></td>
<td>-0.4</td>
</tr>
<tr>
<td>$\geq 25^\circ$</td>
<td>$\leq 0.1$</td>
<td>-0.75</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>-0.625</td>
</tr>
<tr>
<td></td>
<td>$\geq 0.3$</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

*For intermediate values of $\frac{d}{b}$ and $\alpha'$, use linear interpolation.*
Example 5: Wind Load Calculation

(3) Dynamic response factor, $C_{dyn} = \frac{1 + 2l_h}{\sqrt{1 + 2g_v l_h}} = 1.327$

where $l_h = \text{Turbulence intensity at } z = h \text{ (Table 6.1) = 0.205 by interpolation}$

$g_v = \text{Peak factor} = 3.7 \text{ given}$

$B_s = \text{Background factor} = \frac{1}{1 + \sqrt{\frac{36(h-s)^2 + 64b_{sh}^2}{L_h}}}$

$S = \text{Size reduction factor} = \frac{1}{1 + 3.5n_a h (1 + g_v l_h) + 4n_a b_{0h} (1 + g_v l_h)} V_{des}$

$E_t = \text{Spectrum of turbulence} = \frac{0.47N}{(2+N)^{5/6}} = 0.623$

$\xi = \text{Ratio of structural damping to critical damping (Table 6.2) = 0.05}$

$g_R = \text{Peak factor for resonance response} = \sqrt{2 \log_e (600 n_a)} = 3.09$
# Example 5: Wind Load Calculation

## Table 6.1 Turbulence intensity ($\xi$) ultimate limit state design and serviceability limit state design – all regions

<table>
<thead>
<tr>
<th>Height ($z$) m</th>
<th>Terrain Category 1</th>
<th>Terrain Category 2</th>
<th>Terrain Category 3</th>
<th>Terrain Category 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 3$</td>
<td>0.17</td>
<td>0.21</td>
<td>0.27</td>
<td>0.34</td>
</tr>
<tr>
<td>5</td>
<td>0.16</td>
<td>0.20</td>
<td>0.27</td>
<td>0.34</td>
</tr>
<tr>
<td>10</td>
<td>0.16</td>
<td>0.18</td>
<td>0.24</td>
<td>0.34</td>
</tr>
<tr>
<td>15</td>
<td>0.15</td>
<td>0.18</td>
<td>0.22</td>
<td>0.34</td>
</tr>
<tr>
<td>20</td>
<td>0.15</td>
<td>0.17</td>
<td><strong>0.21</strong></td>
<td>0.34</td>
</tr>
<tr>
<td>30</td>
<td>0.14</td>
<td>0.16</td>
<td>0.20</td>
<td>0.34</td>
</tr>
<tr>
<td>40</td>
<td>0.13</td>
<td>0.16</td>
<td>0.19</td>
<td>0.30</td>
</tr>
<tr>
<td>50</td>
<td>0.13</td>
<td>0.15</td>
<td>0.19</td>
<td>0.27</td>
</tr>
<tr>
<td>75</td>
<td>0.12</td>
<td>0.14</td>
<td>0.18</td>
<td>0.25</td>
</tr>
<tr>
<td>100</td>
<td>0.11</td>
<td>0.13</td>
<td>0.17</td>
<td>0.23</td>
</tr>
<tr>
<td>150</td>
<td>0.10</td>
<td>0.12</td>
<td>0.15</td>
<td>0.21</td>
</tr>
<tr>
<td>200</td>
<td>0.09</td>
<td>0.11</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td>250</td>
<td>0.08</td>
<td>0.10</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>300</td>
<td>0.07</td>
<td>0.09</td>
<td>0.12</td>
<td>0.17</td>
</tr>
<tr>
<td>400</td>
<td>0.07</td>
<td>0.08</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>500</td>
<td>0.06</td>
<td>0.07</td>
<td>0.10</td>
<td>0.14</td>
</tr>
</tbody>
</table>

**NOTE**: For intermediate values of height, $z$ and terrain category, use linear interpolation.
Example 5: Wind Load Calculation

Table 6.2  Values of fraction critical damping of structures (\( \xi \))

<table>
<thead>
<tr>
<th>Stress levels and type of construction</th>
<th>Fraction of critical damping (( \xi ))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Serviceability limit states</strong></td>
<td></td>
</tr>
<tr>
<td>Steel frame</td>
<td>0.005 to 0.010</td>
</tr>
<tr>
<td>Reinforced or pre-stressed</td>
<td></td>
</tr>
<tr>
<td>concrete</td>
<td>0.005 to 0.010</td>
</tr>
<tr>
<td><strong>Ultimate limit states</strong></td>
<td></td>
</tr>
<tr>
<td>Steel frame welded</td>
<td>0.02</td>
</tr>
<tr>
<td>Steel frame bolted</td>
<td>0.05</td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>0.05</td>
</tr>
</tbody>
</table>
### Example 5: Wind Load Calculation

$$p = 0.613 (V_{des})^2 C_{fig} C_{dyn} \text{ (N/m}^2)$$

<table>
<thead>
<tr>
<th>Level</th>
<th>Height (m)</th>
<th>$M_{z,cat}$ (Table 4.1)</th>
<th>$p_w$</th>
<th>$V_{des,z}$ (m/s)</th>
<th>$V_{des,h}$ (m/s)</th>
<th>$p_L$</th>
<th>$p_w - p_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 (Roof)</td>
<td>25.0</td>
<td>0.970</td>
<td>910</td>
<td>37.4</td>
<td>37.4</td>
<td>-284</td>
<td>1194</td>
</tr>
<tr>
<td>6</td>
<td>21.5</td>
<td>0.950</td>
<td>872</td>
<td>36.6</td>
<td>37.4</td>
<td>-284</td>
<td>1156</td>
</tr>
<tr>
<td>5</td>
<td>18.0</td>
<td>0.920</td>
<td>815</td>
<td>35.4</td>
<td>37.4</td>
<td>-284</td>
<td>1099</td>
</tr>
<tr>
<td>4</td>
<td>14.5</td>
<td>0.890</td>
<td>766</td>
<td>34.3</td>
<td>37.4</td>
<td>-284</td>
<td>1050</td>
</tr>
<tr>
<td>3</td>
<td>11.0</td>
<td>0.840</td>
<td>683</td>
<td>32.4</td>
<td>37.4</td>
<td>-284</td>
<td>967</td>
</tr>
<tr>
<td>2</td>
<td>7.5</td>
<td>0.790</td>
<td>601</td>
<td>30.4</td>
<td>37.4</td>
<td>-284</td>
<td>885</td>
</tr>
<tr>
<td>1</td>
<td>4.0</td>
<td>0.750</td>
<td>543</td>
<td>28.9</td>
<td>37.4</td>
<td>-284</td>
<td>827</td>
</tr>
</tbody>
</table>
Example 5: Wind Load Calculation

\[ p_L (kN/m^2) \]

- Level 7 (Roof): 0.910, 0.872, 0.766, 0.601
- Level 6
- Level 4
- Ground

\[ p_W (kN/m^2) \]

- Level 7 (Roof): 0.248
- Level 6
- Level 4
- Ground

\[ p_L (kN/m^2) \]

- Level 7 (Roof): 1.194
- Level 6: 1.156
- Level 4: 1.050
- Ground: 0.885

\[ Total (kN/m^2) \]

- Level 7 (Roof): 2.108
- Level 6: 2.212
- Level 4: 2.101
- Ground: 1.733
Frame Analysis for Lateral Actions

**Portal Method**
- Frame divided into *independent portals*
- Shear in each storey is assumed to be divided between the bay in proportion to their spans
- Shear in each bay is divided equally between columns

**Cantilever Method**
- Axial loads in column are assumed to be proportional to the distance from the frame’s centre of gravity
- Assumed column in a storey has equal cross-sectional area & point of contraflexure are located at midspan for all column & beams
Pinned beam-to-column connections
Frame is not stable when subjected to lateral forces
Frame Stability – Unbraced

Unstable Frame

Rigid beam-to-column connection

Pinned base

Rigid base
Frame Stability – Braced

Unstable Frame

Stable Frame

Pinned connections
Rigid core
Example 6

LATERAL LOAD ANALYSIS – CANTILEVER METHOD
Example 6: Lateral Load Analysis (Cantilever Method)

Design wind load for each floor level of sub-frame 3 = 1.2\(w_k\):

where \(w_k = \text{Total wind pressure, } p_{total} \times \text{Contact area}\)

- **Roof**: \(1.2 \times 1.194 \text{kN/m}^2 \times 4.50 \text{ m} \times 1.75 \text{ m} = 11.3 \text{kN}\)
- **Level 6**: \(1.2 \times 1.156 \text{kN/m}^2 \times 4.50 \text{ m} \times 3.50 \text{ m} = 21.8 \text{kN}\)
- **Level 5**: \(1.2 \times 1.099 \text{kN/m}^2 \times 4.50 \text{ m} \times 3.50 \text{ m} = 20.8 \text{kN}\)
- **Level 4**: \(1.2 \times 1.050 \text{kN/m}^2 \times 4.50 \text{ m} \times 3.50 \text{ m} = 19.8 \text{kN}\)
- **Level 3**: \(1.2 \times 0.967 \text{kN/m}^2 \times 4.50 \text{ m} \times 3.50 \text{ m} = 18.3 \text{kN}\)
- **Level 2**: \(1.2 \times 0.885 \text{kN/m}^2 \times 4.50 \text{ m} \times 3.50 \text{ m} = 16.7 \text{kN}\)
- **Level 1**: \(1.2 \times 0.827 \text{kN/m}^2 \times 4.50 \text{ m} \times 3.75 \text{ m} = 16.7 \text{kN}\)
Example 6: Lateral Load Analysis (Cantilever Method)

11.3 kN

21.8 kN

20.8 kN

19.8 kN

18.3 kN

16.7 kN

16.7 kN

x = 9 m

A

B

C

D

Roof

Level 6

Level 5

Level 4

Level 3

Level 2

Level 1

6 m

6 m

6 m

4 m
Example 6: Lateral Load Analysis (Cantilever Method)

### Column Axial Load

<table>
<thead>
<tr>
<th>Column</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from Centroid</td>
<td>9.00</td>
<td>3.00</td>
<td>3.00</td>
<td>9.00</td>
</tr>
<tr>
<td>Column Axial Load</td>
<td>3.0P</td>
<td>1.0P</td>
<td>1.0P</td>
<td>3.0P</td>
</tr>
</tbody>
</table>

**Diagram:**
- Roof: 3.5 m
- Level 6: 3.5 m
- Level 5: 3.5 m
- Level 4: 3.5 m
- Level 3: 3.5 m

**Legend:**
- Column A
- Column B
- Column C
- Column D

**Loading:**
- 11.3 kN
- 21.8 kN
- 20.8 kN
- 19.8 kN
- 18.3 kN

**Distances:**
- 6 m
- 6 m
- 6 m
- 4 m
Example 6: Lateral Load Analysis (Cantilever Method)

LEVEL K AND ABOVE
Example 6: Lateral Load Analysis (Cantilever Method)

Level K and Above

Axial Force in Columns

\[ \sum M @K = 0 \]
\[ (11.3 \times 1.75) + (1.0P \times 6) - (1.0P \times 12) - (3.0P \times 18) = 0 \]
\[ \therefore P = 0.33 \text{ kN} \]
Example 6: Lateral Load Analysis (Cantilever Method)

Shear Force in Beams & Columns

Consider sub-frame to the left of $F_1$:
\[ \sum F_v = 0: \quad F_1 - 0.99 = 0 \]
\[ \sum M@F_1 = 0: \quad (H_1 \times 1.75) - (0.99 \times 3) = 0 \]
\[ \therefore F_1 = 0.99 \text{ kN} \quad \therefore H_1 = 1.70 \text{ kN} \]

Consider sub-frame to the left of $F_2$:
\[ \sum F_v = 0: \quad F_2 - 0.99 - 0.33 = 0 \]
\[ \sum M@F_2 = 0: \quad (H_1 + H_2) \times 1.75 - (0.99 \times 9) - (0.33 \times 3) = 0 \]
\[ \therefore F_2 = 1.32 \text{ kN} \quad \therefore H_2 = 3.96 \text{ kN} \]

Consider sub-frame to the left of $F_3$:
\[ \sum F_v = 0: \quad F_3 - 0.99 - 0.33 + 0.33 = 0 \]
\[ \sum M@F_3 = 0: \quad (H_1 + H_2 + H_3) \times 1.75 - (0.99 \times 15) - (0.33 \times 9) + (0.33 \times 3) = 0 \]
\[ \therefore F_3 = 0.99 \text{ kN} \quad \therefore H_3 = 3.96 \text{ kN} \]

\[ \sum F_H = 0: \quad 11.3 - H_1 - H_2 - H_3 - H_4 = 0 \]
\[ \therefore H_4 = 1.68 \text{ kN} \]
Example 6: Lateral Load Analysis (Cantilever Method)

LEVEL L AND ABOVE
Example 6: Lateral Load Analysis (Cantilever Method)

Level L and Above

Axial Force in Columns

\[ \sum M @L = 0 \]

\[ (11.3 \times 5.25) + (21.8 \times 1.75) + (1.0P \times 6) - (1.0P \times 12) - (3.0P \times 18) = 0 \]

\[ \therefore P = 1.62 \text{ kN} \]
Example 6: Lateral Load Analysis (Cantilever Method)

Shear Force in Beams & Columns

Consider sub-frame to the left of $F_1$:

- $\sum F_v = 0$: $F_1 - 4.86 + 0.99 = 0$
- $\sum M@F_1 = 0$: $(H_1 + 1.70) \times 1.75 - (4.86 - 0.99) \times 3 = 0$

\[ \therefore F_1 = 3.87 \text{ kN} \]
\[ \therefore H_1 = 4.93 \text{ kN} \]

Consider sub-frame to the left of $F_2$:

- $\sum F_v = 0$: $F_2 - 4.86 + 0.99 - 1.62 + 0.33 = 0$
- $\sum M@F_2 = 0$:

\[ (H_1 + H_2) \times 1.75 + (1.70 + 3.96) \times 1.75 - (4.86 - 0.99) \times 9 - (1.62 - 0.33) \times 3 = 0 \]

\[ \therefore F_2 = 5.16 \text{ kN} \]
\[ \therefore H_2 = 11.52 \text{ kN} \]
Example 6: Lateral Load Analysis
(Cantilever Method)

Shear Force in Beams & Columns

Consider sub-frame to the left of $F_3$:

$\Sigma F_v = 0$: 

$F_3 - 4.86 + 0.99 - 1.62 + 0.33 + 1.62 - 0.33 = 0$ 

$\therefore F_3 = 3.87 \text{kN}$

$\Sigma M@F_3 = 0$: 

$(H_1 + H_2 + H_3) \times 1.75 + (1.70 + 3.96 + 3.96) \times 1.75 - (4.86 - 0.99) \times 15 - (1.62 - 0.33) \times 9$ 

$+ (1.62 - 0.33) \times 3 = 0$ 

$\therefore H_3 = 11.52 \text{kN}$

$\Sigma F_H = 0$: 

$33.1 - H_1 - H_2 - H_3 - H_4 = 0$ 

$\therefore H_4 = 5.13 \text{kN}$
Example 6: Lateral Load Analysis (Cantilever Method)

LEVEL $M$ AND ABOVE
Example 6: Lateral Load Analysis (Cantilever Method)

Level M and Above

\[ M @ L = 0 \]

\[(11.3 \times 8.75) + (21.8 \times 5.25) + (20.8 \times 1.75) + (1.0P \times 6) - (1.0P \times 12) - (3.0P \times 18) = 0 \]

\[ \therefore P = 4.16 \text{ kN} \]
Example 6: Lateral Load Analysis (Cantilever Method)

Shear Force in Beams & Columns

Consider sub-frame to the left of $F_1$:

$\sum F_v = 0$: \[ F_1 - 12.48 + 4.86 = 0 \]
$\sum M@F_1 = 0$: \[ (H_1 + 4.93) \times 1.75 - (12.48 - 4.86) \times 3 = 0 \]
\[ \therefore F_1 = 7.62 \text{ kN} \]
\[ \therefore H_1 = 8.13 \text{ kN} \]

Consider sub-frame to the left of $F_2$:

$\sum F_v = 0$: \[ F_2 - 12.48 + 4.86 - 4.16 + 1.62 = 0 \]
$\sum M@F_2 = 0$: \[ (H_1 + H_2) \times 1.75 + (4.93 + 11.52) \times 1.75 - (12.48 - 4.86) \times 9 - (4.16 - 1.62) \times 3 = 0 \]
\[ \therefore F_2 = 10.16 \text{ kN} \]
\[ \therefore H_2 = 18.93 \text{ kN} \]
Example 6: Lateral Load Analysis (Cantilever Method)

Consider sub-frame to the left of $F_3$:

\[
\Sigma F_v = 0: \quad \sum F_v = 0: \quad F_3 - 12.48 + 4.86 - 4.16 + 1.62 + 4.16 - 1.62 = 0 \quad \therefore F_3 = 7.62 \text{ kN}
\]

\[
\Sigma M@F_3 = 0: \quad \sum M@F_3 = 0: \\
(H_1 + H_2 + H_3) \times 1.75 + (4.93 + 11.52 + 11.52) \times 1.75 - (12.48 - 4.86) \times 15 - (4.16 - 1.62) \times 9 + (4.16 - 1.62) \times 3 = 0 \\
\therefore H_3 = 18.99 \text{ kN}
\]

\[
\Sigma F_H = 0: \quad \sum F_H = 0: \\
53.9 - H_1 - H_2 - H_3 - H_4 = 0 \\
\therefore H_4 = 7.85 \text{ kN}
\]
Example 6: Lateral Load Analysis (Cantilever Method)

Do the Following Level:

LEVEL N AND ABOVE
LEVEL P AND ABOVE
LEVEL Q AND ABOVE
LEVEL R AND ABOVE
Example 6: Lateral Load Analysis (Cantilever Method)

SHEAR FORCE IN BEAMS & COLUMNS
Example 6: Lateral Load Analysis (Cantilever Method)

\[
\begin{array}{ccc}
1.70 & 3.96 & 0.99 \\
3.87 & 5.16 & 3.96 \\
4.93 & 11.52 & 0.99 \\
7.62 & 10.16 & 1.68 \\
8.13 & 18.93 & 11.52 \\
18.99 & 7.62 & 5.13 \\
7.85 & 18.99 & 5.13 \\
\end{array}
\]
Example 6: Lateral Load Analysis (Cantilever Method)

BENDING MOMENTS IN BEAMS & COLUMNS
Example 6: Lateral Load Analysis (Cantilever Method)

Continues for Level N, P, Q, R
Example 7

ANALYSIS OF ONE LEVEL SUBFRAME – VERTICAL LOAD ONLY
Example 7: Analysis of One Level Sub-Frame – Vertical Load Only

- Level 1
- Level 2
- Level 3
- Level 4
- Level 5
- Level 6
- Roof

Dimensions:
- 6 m
- 4 m

Points:
- A
- B
- C
- D
Example 7: Analysis of One Level Sub-Frame – Vertical Load Only

CASE 1: $1.2G_k + 1.2Q_k$

Fixed End Moment

\[ M_{AB} = M_{BA} = \frac{wL^2}{12} = \frac{48.7 \times 6^2}{12} = 146.2 \text{ kNm} \]

\[ M_{BC} = \frac{wL^2}{12} = \frac{46.1 \times 6^2}{12} = 138.4 \text{ kNm} \]

\[ M_{CD} = \frac{wL^2}{12} = \frac{48.7 \times 6^2}{12} = 146.2 \text{ kNm} \]
### Example 7: Analysis of One Level Sub-Frame – Vertical Load Only

#### Moment Distribution Method

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<tr>
<th></th>
<th>43.82</th>
<th>-9.97</th>
<th>-9.97</th>
<th>-43.82</th>
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<tr>
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<td>0.27</td>
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<td>41.60</td>
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<td>1.3</td>
<td>-1.3</td>
<td>-0.9</td>
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<td>0.2</td>
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<td></td>
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<td>-146.9</td>
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<td>38.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>82.2</th>
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<tr>
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<td>-146.9</td>
<td>-8.7</td>
<td>8.7</td>
<td>-38.3</td>
</tr>
</tbody>
</table>
Example 7: Analysis of One Level Sub-Frame – Vertical Load Only

**Reaction Calculation at Support**

Solve for each span using **equilibrium method** of analysis:

\[ V_{AB} = 132.3 \text{ kN} \]
\[ V_{BA} = 160.1 \text{ kN} \]

\[ V_{BC} = 138.4 \text{ kN} \]
\[ V_{CB} = 138.4 \text{ kN} \]

\[ V_{CD} = 160.1 \text{ kN} \]
\[ V_{DC} = 132.3 \text{ kN} \]
Example 7: Analysis of One Level Sub-Frame – Vertical Load Only

Shear Force and Bending Moment Diagram

SFD (kN)

BMD (kNm)
Example 7: Analysis of One Level Sub-Frame – Vertical Load Only

Bending Moment due to Vertical Load \((1.2G_k + 1.2Q_k)\)

\[\begin{align*}
82.2 & \quad 43.8 \\
38.3 & \quad 97.5 \\
19.2 & \quad 4.4 \\
& \quad \vdots \\
82.2 & \quad 38.3
\end{align*}\]

\[\begin{align*}
165.6 & \quad 146.9 \\
60.6 & \quad 8.7 \\
4.4 & \quad 19.2 \\
& \quad \vdots \\
165.6 & \quad 38.3
\end{align*}\]

\[\begin{align*}
97.5 & \quad 43.8 \\
10.0 & \quad 8.7 \\
4.4 & \quad 19.2 \\
& \quad \vdots \\
97.5 & \quad 38.3
\end{align*}\]

\[\begin{align*}
+ \\
\text{Bending Moment due to Wind Load, } 1.2W_k \text{ (from your exercise)}
\end{align*}\]

\[\begin{align*}
\text{Combination of Vertical & Wind Load}
\end{align*}\]