Effects of Volume Fraction to the Strength and Ductility of a Small-Scaled Fibre Reinforced Concrete Beam

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Abstract. Due to advances in technology of materials, substitution of steel reinforcement with steel fibre becomes one of the possible solution for reducing the congestion of transverse reinforcement in concrete’s element structures. Previously, there are various researches done on laboratory work to investigate on mechanical behaviors such as tensile strength and flexural strength of steel fibre reinforced concrete beams. However, there is a limitation on the part of non-linear analysis modelling especially for concrete beams without steel reinforcement. Thus, validation of non-linear analysis data with the existing experimental data has to be done first followed by investigation on the effect of volume fraction of steel fibres on flexural strength of concrete beam. Non-linear Finite Element Analysis was carried out by using LUSAS V15 software. Three types of steel fibre with different length of 60mm, 50mm and 33mm were adopted in reinforced concrete beam with volume fraction of 0.25\%, 0.50\%, 0.75\% and 1.00\% for each fibre. In short, the flexural strength, stiffness and ductility begin to increase significantly when addition of steel fibre reach 0.75\% and 1.00\%. Long steel fibre perform better than short steel fibre in flexural, stiffness and ductility.

INTRODUCTION

Today, the performance and compressive strength of concrete keep on improved with the technological advancements until the application of high strength concrete in industries become common thing. However, with the increasing of compressive strength, the plain concrete still remains brittle, weak in tensile strength and may also lead to sudden failures. So, to prevent flexural failure by conventional method, sufficient reinforcement bars are added to concrete and casted to become reinforced concrete. Yet, when too much reinforcement bar is required, the situation of reinforcement become congested. Hence, due to advances in technology of materials, substitution of steel reinforcement with steel fibre becomes one of the possible solution for reducing the congestion of transverse reinforcement in concrete’s element structures.

However, with the addition of steel fibres, the flow ability of concrete mixture will surely be disturbed. To solve the problem on workability of concrete, SCC was chosen rather than normal plain concrete. Scientific and technologies keep on improving till creation of lots of finite element software to bring convenience and reduce the difficulties of human in every aspect notably engineering analyses. By implementing it, it brings convenience as compared with conventional prototypes testing which was time consuming and also labor intensive. So, in this research, FEA with application of a software, LUSAS had been adopted to carry out a nonlinear analysis of steel fibre reinforced concrete beam.

Steel fibre can be defined as discrete fibre with small diameter and short length which has been widely used over decades to enhance the post cracking characteristic of concrete. Normally, the performance of steel fibres is depended on its characteristics such as volume fraction, diameter, length, aspect ratio and types of steel fibres. Steel fibres are effective in cracking control of concrete. Although steel fibre only provides little resistance against crack initiation, it is proved to resist the propagation of crack effectively after the cracking of concrete.
begins. When the matrix with steel fibres start to crack, the steel fibres which cross over the crack will perform its pull out behavior and ensure stress generated between the cracks.

Generally, there are three stages included in FEA which are pre-processing, solution and post-processing. In pre-processing, user will develop a finite element model with finer mesh, geometry, material properties, support and loading. Finer mesh needed to subdivide a structure or component into smaller element. Then followed by solution with derivation of complex matrix equations from program for purpose to solve the engineering problems like stress and strain. Finally, for post-processing, user able to obtain a model with deformed shape, contour plots and graph for validation purpose of the solution. The behavior of concrete is always non-linear since concrete always achieve maximum strength value when concrete begins to crack. The stiffness of model is no longer constant and yet non-linear finite element analysis should be adopted.

Joaquim A O Barros & Figueiras, (2001) had developed a model for non-linear analysis of steel fibre concrete. To get the stress strain relationship for compression of concrete, the equations were proposed as below:

\[
\sigma_c = f_{cm} \frac{\varepsilon_c/\varepsilon_{ct}}{(1-p-q)+q(\varepsilon_c/\varepsilon_{ct})+p(\varepsilon_c/\varepsilon_{ct})^{(1-q)/p}} \quad (MPa) \quad (1)
\]

With,

\[
q = 1 - p - \left(\frac{E_c/\varepsilon_{ct}}{E_{cl}}\right), \quad p + q = 0, 1, \quad (1-q)/p > 0
\]

Where  
\( f_{cm} \) = compressive strength  
\( E_c \) = initial modulus of elasticity  
\( E_{ct} \) = modulus of elasticity at peak stress  
\( \varepsilon_{ct} \) = strain at peak stress  
\( \varepsilon_{ct0} = 2.2 \times 10^3 \) (strain at peak for plain concrete)

\[
E_{ct} = \frac{f_{cm}}{\varepsilon_{ct}} \quad (2)
\]

\[
\varepsilon_{ct} = \varepsilon_{ct0} + 0.0002W_f, \quad p = 1.0 - 0.919\exp(-0.394W_f) \quad (3)
\]

For ZP30/.50 steel fibre

\[
\varepsilon_{ct} = \varepsilon_{ct0} + 0.00026W_f, \quad p = 1.0 - 0.722\exp(-0.144W_f) \quad (4)
\]

For ZQ60/.80 steel fibre.

Neoclesous et al. (2006) had done experimental work regarding different types of fibre concretes. Based on the lab work result, a uni-axial stress strain model for concrete tensile has been proposed. Inverse analysis had been done by using experimental result for determining tensile behavior of steel fibre concrete. An assumption had been stated that the tensile strength of steel fibre concrete is same as that of plain concrete but the tension stiffening has been improved. Besides, a value of 0.19 factor was adopted for hooked end steel fibres.

![FIGURE 1. Tensile stress strain model for steel fibre concrete.](image_url)

The objectives of the study are; to implement constitutive model for SFRC into finite element model (LUSAS), to compare the numerical modelling with the case study of SFRC and to investigate the effect of steel fibres on flexural strength, stiffness and ductility of SFRC beam. In the research, normal strength concrete, M40
grade of concrete was used. Three types of hooked end steel fibres adopted in the modelling of this research. Two types of the steel fibre have similar diameter of 0.75mm but with different length of 60mm (SF60) and 50mm (SF50) respectively. Another types steel fibre (SF33) has 0.55mm of diameter and 33mm in length. The aspect ratio of SF60, SF50 and SF33 are 80, 67 and 60 respectively. Each types of steel fibres were added to concrete mix specimens at different volume fraction which are 0.25%, 0.50%, 0.75% and 1.0%, respectively. Another model was a plain concrete without the addition of steel fibres to act as a control model. The shape of the model was in prism with dimension of 150mm x 150mm x 750mm.

**METHODOLOGY**

Non-linear finite element analysis has been carried out in this present research. In this study, LUSAS V15 is used for the modelling of the specimens. The modelling by non-linear finite element analysis was carried out based on the experiment work by Sarbini (2014) and the volume fraction used was varied from 0% to 1% with increment of 0.25%. Then, the data from modelling will compared with the experimental results from case study by Sarbini (2014) for validation purpose.

Those parameters like compressive strength, tensile strength, elastic modulus, and Poisson ratio and stress-strain behavior were needed to further modelling analysis of steel fibre concrete to get the load deflection curve. The poisson ratio for plain concrete is taken as 0.2. For steel fibre concrete, the poisson ratio is within 0.18 and 0.22 which depends on the grade of concrete. In current research, poisson ratio is taken as average of upper boundary and lower boundary which equal to 0.2. After getting the load deflection curve, the flexural strength or modulus of rupture can be obtained.

Three types of steel fibre shown in Table 1 were adopted in concrete beam with different portion of steel fibre in 2D for investigation in this research study. Plain concrete model acted as a control model. The dimension of beam is shown in Figure 3. Based on Sarbini (2014) study, the beam is simply supported and there are two point loads acted on the beam. However, due to symmetry of the beam, only half left hand side beam was modelled. So, by considering only left span of the beam, so only a point load and a roller support in y direction were included in the model. The model is shown in Figure 2.

<table>
<thead>
<tr>
<th>Types</th>
<th>SF60</th>
<th>SF50</th>
<th>SF33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm)</td>
<td>60</td>
<td>50</td>
<td>33</td>
</tr>
<tr>
<td>Diameter (mm)</td>
<td>0.75</td>
<td>0.75</td>
<td>0.55</td>
</tr>
<tr>
<td>Aspect Ratio, L/D</td>
<td>80</td>
<td>67</td>
<td>60</td>
</tr>
</tbody>
</table>

**FIGURE 2.** Beam Dimension (mm).

Generally, concrete has two types of material phases, which are elastic and plastic. So, before cracking created, the model always behave in linear elastic pattern. Linear analysis means that there is a directly proportional relationship among stress and strain. The properties of materials like elastic modulus are constant and the deformations happen are quite small. However, after cracks were initiated, the model started to behave in plastic or non-linear pattern. Then, relationship between stress and strain become nonlinear and structure models will begin to experience larger deflection or deformation. Hence, non-linear analysis is vital in this concrete behavior’s modelling especially after the concrete starts to crack.

Since the models involve combination of steel fibres and concrete, it was considered as composite concrete. LUSAS Composite Plus was chosen as product for the analysis. After done drawing the beam, mesh attribute was used to separate a geometry of structure into small tiny portions called set of finite elements (mesh) with shared nodes joining them together (Figure 3). Quadrilateral was chosen as element shape while quadratic was chosen as interpolation order.

For geometry attribute (Figure 4), it was used to assign the thickness of the beam since the model is in 2D. The thickness value was applied as 150mm. Then, concrete with steel fibres was assigned as material. The elastic modulus for steel fibre concrete was assigned as 30 kN/mm² for elastic behavior of beam. This is because the types of steel fibre and volume fraction of steel fibres would not affect the elastic modulus of concrete. After cracking, plastic behavior occurs and non-linear analysis happens. For plastic or non-linear analysis, the compressive and tensile stress and strain value after the maximum point of stress was inserted for plastic behavior purpose.
Concrete was assigned as isotropic materials which mean has same value of properties in all directions. So, isotropic is assigned to materials so that plastic properties of materials can be inserted. Since it is steel fibre concrete to be analyzed, a modified von mises was adopted. Modified von mises is done by inserting the stress and strain value and will function only after yielding begins. The load was assigned to become nonlinear and transient control for nonlinear analysis purpose. The nonlinear load increment is set as automatic with increment of 2000N for subsequent iteration. After analysis has done, we can get the stress distribution pattern of concrete beam. Load versus deflection curve can also be plotted.

![FIGURE 3. Meshing of beam in 2D.](image)

![FIGURE 4. Loading and supports of beam thickness.](image)

**RESULTS AND DISCUSSIONS**

The comparison between the modelling result and experimental result will be made for validation purpose. Then, the results and trend of graphs from modelling regarding inclusion of different volume fraction of similar steel fibre to concrete beam will be discussed. Lastly, it will followed by the influence of aspect ratio of steel fibres on flexural strength of concrete beam based on modelling results.

According to Table 2, it is clearly noted that there is only slight increment in load applied and deflection with inclusion of 0.25% and 0.50% of steel fibre for all types of steel fibres. However, when it comes to 0.75%, every types of steel fibre concrete beams start to undergo greater deflection. Yet, the increment in load by 0.75% of SF60 and SF33 still insignificant When 1.00% of steel fibres were added, the increment in load applied were obvious for all kinds of steel fibres. The load carrying capacity of steel fibre concrete beam is the highest at 1.00% of volume fraction. So, based on the trend of the load deflection curve, the deflection of the concrete beam can be increased obviously with addition of 0.75% and 1.00% of steel fibres regardless type of steel fibres were adopted.

<table>
<thead>
<tr>
<th>Volume Fraction (%)</th>
<th>SF60</th>
<th>Experiment</th>
<th>Percentage Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Modelling</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>Pp 25.78, δp 0.42, fp 3.44</td>
<td>Pp 28.00, δp 0.81, fp 3.73</td>
<td>7.85</td>
</tr>
<tr>
<td>0.25</td>
<td>Pp 27.46, δp 0.93, fp 3.66</td>
<td>Pp 24.87, δp 0.83, fp 3.32</td>
<td>10.24</td>
</tr>
<tr>
<td>0.50</td>
<td>Pp 27.46, δp 0.86, fp 3.66</td>
<td>Pp 24.53, δp 0.75, fp 3.27</td>
<td>11.93</td>
</tr>
<tr>
<td>0.75</td>
<td>Pp 30.67, δp 3.02, fp 4.09</td>
<td>Pp 29.77, δp 2.86, fp 3.97</td>
<td>3.02</td>
</tr>
<tr>
<td>1.00</td>
<td>Pp 43.37, δp 2.45, fp 5.78</td>
<td>Pp 38.63, δp 2.33, fp 5.15</td>
<td>12.23</td>
</tr>
</tbody>
</table>

From Table 3 and 4, the types of steel fibre will has influence on the flexural strength of the SFRC. SF60 and SF50 were considered as longer steel fibre while SF33 was considered as shorter steel fibre. So, longer steel fibres can provide better flexural strength than shorter steel fibres. Although the SF50 has higher flexural strength than SF60 at small fraction of steel fibre (0.50% and 0.75%), but the flexural strength of SF60 is believed to increase significant and exceed SF50 when volume fraction exceed 1.00%. However, the scope is out of this research and yet to study.

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The crack generated will cause the steel fibre to slip little by little. For fibre slippage, it normally occurs on the shorter side of fibres which intersect the cracks (Sarbini, 2014). So, the fibre slippage is depended on the embedment length of steel fibre in concrete. When crack occurs, the longer steel fibre within the crack was embedded deeper into the concrete. In another word, longer steel fibre has greater contact length with concrete for purpose of development of higher bond stress and reduces the fibre slippage. Shorter fibre has limited length in contact with concrete for development of bond stress. Hence, longer steel fibre has greater improvement on flexural strength as compared with shorter steel fibre. In short, flexural strength of SFRC increases proportionally with the volume fraction of steel fibres (Figure 5).

![Flexural Strength vs Volume Fraction](image)

**FIGURE 5.** Comparison of Flexural Strength for SF60, SF50 and SF33.

Basically, the stiffness of beam can be obtained from load deflection curve by getting the slope or gradient of the curve. However, the plastic deformation will occur after exceeding the yield point. So, the gradient is only considered until the yield point. The deflection at yield point for every steel fibres are constant at value of 0.16mm and only varied in term of load applied. From the results in Figure 6, the addition of 0.25% and 0.50% give insignificant effect on stiffness of SFRC beam. Even at 0.50%, all three types of steel fibres only increase a little bit in stiffness. Again, when come to 0.75% and 1.00%, the stiffness increase obviously. The optimum volume fraction will be 1.00% which give highest stiffness. However, SF50 perform better than SF60 at optimum volume fraction. In short, beam stiffness will increase with increment of steel fibres.
Generally, the ductility of the beams can be determined based on the ductility ratio ($\mu$). All SFRC beams were observed to have higher ductility ratio than the control beam (without steel fibre). Higher ductility ratio of SFRC beam means that it has higher ductility after plastic deformation occurs. This showed that the addition of steel fibres is able to improve the ductility of the SFRC beams considerably. From Figure 7, the increase in ductility ratio is more significant when the volume fraction increased up to 0.75% and 1.00%. When compared among three types of steel fibres, SF60 produces greater ductility than SF50 and SF33 at optimum performance.

**CONCLUSIONS**

In this study, a finite element modelling had been presented for determination of the flexural strength of SFRC beam. Therefore, the main points alongside important conclusions are summarized here:

1. A constitutive model for SFRC had been successfully implement into finite element model by using LUSAS.
2. The data from numerical modelling from LUSAS agrees well with experimental data from case study of SFRC with maximum deviation of 13.71% in flexural strength.
3. The flexural strength, stiffness and ductility of SFRC increase with addition of steel fibres. The optimum volume fraction of SFRC for all types of steel fibres are 0.75% and 1.00%. SF60 and SF50 with longer steel fibre have greater improvement in flexural, stiffness and ductility than SF33 (short steel fibre).

**REFERENCES**