Chapter 3

Hydrostatics and Floatation

3.1 Archimedes Law of Floatation

Archimedes (born 287 B.C) Law states that

"***An object immersed in a liquid experience a lift equivalent to the mass of liquid the object displaces.***"

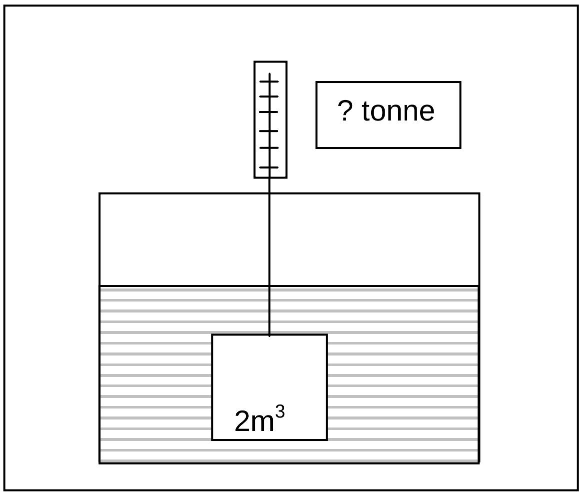
A man immersed in water for example will feel a weight reduction because part of the weight is supported by buoyancy. This buoyancy is equal to the weight of water displaced by his immersed body.

3.2 Reduction of Weight of Immersed Objects

The maximum buoyancy is when the object is fully immersed and this equal the total outside volume of the object multiplied by the density of the fluid. When maximum available buoyancy is less than the weight of the object, the object will sink. That is why an anchor will sink to the bottom. However the object will still feel the weight reduction.

Example 3.1:

Consider a cuboid having dimensions 1m x 1m x 2m. If it weighs 3 tonnes in air, what is its apparent weight in water density 1000 kg/m^3?



If the object is immersed in liquid, it will displace liquid around it equivalent to its external volume.

In this case, displaced volume = 1 x 1 x 2 = 2 m^3
This is the volume of liquid pushed aside by the cuboid.

Archimedes says that the weight of this object in liquid is reduced due to the support given by liquid on the object. The apparent weight equals the weight in air minus the reduction in weight of the object; or the buoyancy i.e.

Buoyancy = volume diplacement x density of liquid
= mass displacement
= $2m^3$ x 1000 kg/m^3
= 2000 kg
= 2 tonnes
= reduction in weight

Apparent weight = weight in air – buoyancy

Since the object weighs 3 tonne in air, it will apparently weigh only 1 tonne in water.

Exercise 3.1

Do similar calculations to find out the apparent weight in oil (density 0.85 tonne/m^3) and muddy water (density 1.3 tonne/m^3) and mercury (density 13,000 kg/m^3)

Fluid	**Density ( )**	**Fluid Support ( )**	**Apparent Weight ( )**
Oil			
Fresh Water			
Muddy Water			
Mercury			

What can be concluded about relationships between buoyancy of objects and the densities of fluids in which they are immersed?

3.3 What make a Ship Floats?

When the maximum available buoyancy is more than the weight of the object, the object will rise to the surface. It will rise to the surface until the weight of the object balances the buoyancy provided by its immersed portions. When the object is floating, its buoyancy is just enough to support its weight. At that point:

Total weight W = Buoyancy = Displaced volume x ρ_{liquid}

This principle explains why a steel or concrete ship can float. As long as the outer shell of the ship can provide enough volume to displace the surrounding water exceeding the actual weight of the ship, the ship will float. A floating ship is such that the total weight of its hull, machinery and deadweight equals to the weight of water displaced by its outer shell. If, while it is floating weights are added until the total weight exceeds the maximum buoyancy provided by the outer shell of the ship, the ship will sink.

3.4 Effect of Density

An object experiences buoyancy force equivalent to the weight of fluid it displaces. For a particular object, the buoyancy force will depend on the density of the fluid, since its volume is constant. This explains for example why a bather will feel more buoyant while swimming at sea compared to in the river or lake. Also, a floating object of constant weight will sink at a deeper draught in freshwater compared to in seawater.

Total weight W = Buoyancy = Displaced volume x ρ_{liquid}

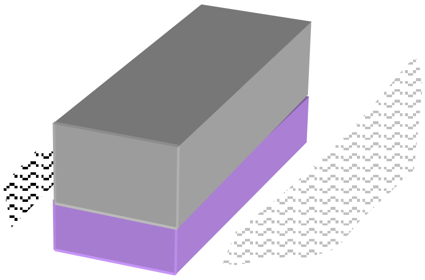
Since weight does not change, the buoyancy is also constant. So displaced volume will be inversely proportional to the density of fluid. For floating object, this will determine its level of sinkage or draught.

3.5 Some Simple Problems

The fact that a floating object displaces fluid equivalent to its weight can be used to solve a number of problems.

Total weight W = Buoyancy = Displaced volume x ρ_{water}

From this equation, we can obtain the weight of the object if we know the volume of water displaced. On the other hand, if we know its weight, we can work out its displaced volume.



Just to understand the concept, consider a floating box of dimension L x B x D, floating at a draught T.

CASE 1: We know its weight, we can find its draught

In this case, we know the weight of the object, we can find the displaced volume:

$$\text{Displaced volume} = \frac{W}{\rho_{water}}$$

i.e. for a box-shaped vessel:

Displaced volume = L x B x T
Hence draught T of the cuboid can be found.

Example 3.2

A cuboid shaped wooden block (L x B x D) 1.45m x 0.5m x 0.25m floats in water. If the block weighs 0.154 tonnes, find its draught if it floats in freshwater density 1.00 tonne/m^3.

Solution:

The weight of the block of 0.154 tonnes must be supported by displaced water i.e. the block must display 0.154 tonnes of water:

In fresh water,
Volume of displaced water $\nabla = L \times B \times T$
Weight of displaced water $\Delta = \nabla \times \rho_{FW}$
$\Delta = 1.45 \times 0.5 \times T \times \rho_{FW}$

This must equal 0.154 tonne
$1.45 \times 0.5 \times T \times \rho_{fw} = 0.154$ tonnes
$T = \underline{0.212 \text{ m}}$

> ***Exercise 3.2***
>
> Do similar calculations for salt water (density 1025 kg/m^3 and oil density 0.85 tonne/m^3)

CASE 2: If we know its draught, we can know its volume displacement, we can find its weight

If we know the draught of the cuboid, we can find its volume displacement and hence the weight of the object;

Say if we know its draught T, volume displacement = L x B x T
Weight = Buoyancy = Volume Displacement x ρ_{water}
Weight = $L \times B \times T \times \rho_{water}$

Example 3.3

A box barge length 100m breadth 20m floats at a draught of 5m in sea water 1.025 tonne/m^3. Find its weight.

Solution

While floating in sea water density 1.025 tonne/m^3:
Volume Displacement = $\nabla = L \times B \times T$
Weight of barge = Weight displacement, Δ
$W = \Delta = \nabla \times \rho_{salt\ water}$
$= 100 \times 20 \times 5 \times 1.025$
= **<u>10250 tonnes</u>**

Exercise 3.3

A block of wood length 5m, breadth 0.5m and depth 0.2m is floating in seawater at a draught of 0.1m. Find the weight of the block.

Exercise 3.4

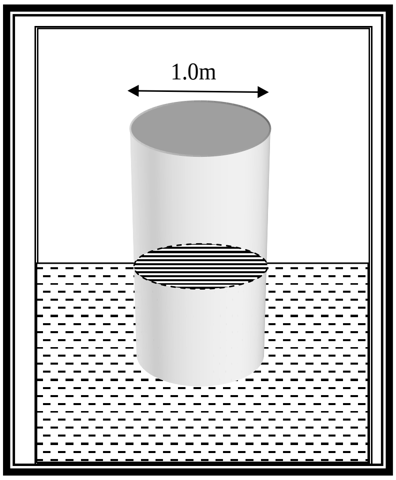
Find the new draught of the box in example 3.3 when it goes into river, water density 1.000 tonne/m^3. Also find a new draught if it is in sea water with density 1.100 tonne/m^3.

Exercise 3.5

A cylindrical container weighing 5 tonne floats with its axis vertical. If the diameter is 1.0m, find its draught in:

i. sea water

ii. oil of density 870 kg/ m^3.



Exercise 3.6

A cylindrical tank diameter 0.6m and mass 200kg floats with its axis vertical. Find its present draught in oil (ρ = 0.95 tonne/m^3).
Find the weight of cargo to be added to ensure it will float at a draught of 0.85m.

3.6 Hydrostatic Particulars

A floating object will be at a certain draught depending on the total weight of the object, density of water and the shape of the object. For a ship, the shape of the object has strong influence on the draught of the ship; the shape and draught have to provide enough buoyancy to support the ship.

When a ship is floating at a certain draught, we can find the mass displacement and weight of the ship if we can find its displaced volume ∇. Also we can know its waterplane area, calculate its TPC, KB, C_b etc. These particulars which are properties of the immersed part of the ship are called **hydrostatic particulars**. Examples of hydrostatic particulars are:

Δ , ∇, KB, LCB, A_w, BM_T, BM_L, TPC, C_B, C_P, C_M, C_W, LCF, MCTC, WSA

These particulars describe the characteristics of the underwater portion of ship at a particular draught. It is related to volumes, areas, centroids of volumes and areas and moments of volumes and areas of the immersed portion. If the ship is out of water, and draught becomes zero, the particulars ceased to exist.

As long as draught and trim is maintained, the size and shape of the underwater immersed parts of the ship remains the same. The volumes, areas and moments of areas and volumes remain the same. Once draught or trim changes, the particulars will also change.

This change in draught will normally occur due to changes in total weight of the ship, or if a force is applied to the ship to make it sink to a deeper draught.

Example 3.4

A box 2m x 1m (LxB) in sea water is floating at a draught of 0.3m.
Calculate its ∇, Δ, C_B, C_{WP} and TPC.

i. $\nabla = L \times B \times T = 2 \times 1 \times 0.3 = 0.6 m^3$
ii. $\Delta = L \times B \times T \times \rho = \nabla \times \rho = 0.6 \times 1.025 = 0.615$ tonnes

iii. $C_B = \dfrac{\nabla}{L \times B \times T} = \dfrac{0.6}{0.6} = 1.00$

iv. $C_{WP} = \dfrac{A_{wp}}{L \times B} = \dfrac{2 \times 1}{2 \times 1} = 1.00$

v. $TPC = \dfrac{A_{wp} \times \rho}{100} = \dfrac{2 \times 1 \times 1.025}{100} = 0.0205$

Exercise 3.7

Calculate the particulars at draught of 0.4, 0.5, 0.6 and 0.7m.

Exercise 3.8

Find hydrostatic particulars in sea water (∇, Δ,Awp,LCB, LCF,TPC) of a box barge with dimension L=100m, B=20m, at draughts of 1.0m, 3.0m, 5.0m, 7.0m, 9.0m. If the barge weighs 2300 tonne, what is its draught? If the barge is floating at a draught of 4m, what is its C_B?

It can be seen from Exercise 3.8 that for a box-shaped object at different draughts, the waterplane areas are constant. Hence, many hydrostatics particulars remain constant.

Exercise 3.9:

An empty cylindrical shaped tank is floating in sea water (density 1.025 t/m^3) at a draught of 8.0 m with its axis vertical. The external diameter of the tank is 12.0 m, internal diameter 11.0 m, thickness of base 1.0 m and the overall height is 16.0 meter. Its centre of gravity is 6 meter above its inner base.

Calculate:

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i. Find Hydrostatic particulars ∇, Awp, LCB, Cb, Cp, TPC, WSA at T=1, 2, 4, 6, 8m.

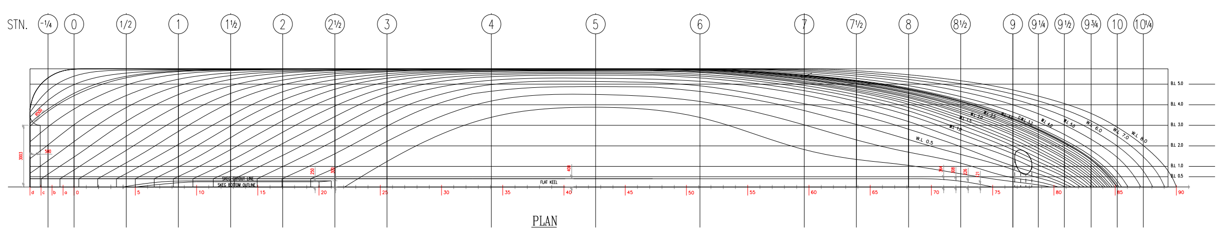
ii. Plot hydrostatic curves similar to page 19 showing all data.

iii. Final draught of the tank after 500 m^3 diesel oil (density 850 kg/m^3) is poured into the tank.

The second moment of area of a circle about its diameter is $\frac{\pi D^4}{64}$.

3.7 Hydrostatic Particulars of a Ship

Hydrostatic particulars of a real ship will be different. Consider the ship whose lines plan is shown below. At different draughts, the ship will have different waterplane areas, volumes and centroids. Hence, the hydrostatic particularly will vary as the draughts changes.



If areas, volumes, moments, centroids of the waterplanes and sections of the ships can be calculated, hydrostatic particulars of a ship can be obtained. These are calculated at the design stage, once the shape and size of the ship has been decided.

Exercise 3.10

A ship with length 100m, breadth 22m has the following volumes and areas at different waterlines. Calculate its Δ, C_B, C_W and TPC in saltwater density 1.025tonnes/m^3.

Draught (m)	Aw (m^2)	∇ (m^3)	Δ (tonnes)	Cb	Cw	TPC
			∇x ro	$\frac{\nabla}{LBT}$	$\frac{Aw}{(LB)}$	$\frac{Aw \times ro}{100}$
2	1800.0	3168.0				
4	2000.0	6547.2				
6	2100.0	10137.6				
8	2120.0	13728.0				
10	2130.0	17424.0				

The particulars can be presented in two forms, either as a set of curves or in tabular format. Table 3.1 shows a typical table of hydrostatic particulars while an example of hydrostatic curves is shown on page 18.

Table 3.1 Hydrostatic Particulars of *Bunga Kintan* LBP 100m

Draught (m)	Displacement (tones)	Cb	KB (m)	BM_T (m)	BM_L (m)	MCTC (tonne-m)	LCB (m from ⊗)	LCF (m from ⊗)
8.00	14820.00	0.72	4.07	3.66	180.00	190.00	2.50	2.00
7.50	13140.00	0.71	3.67	3.98	195.00	183.00	2.30	1.50
7.00	11480.00	0.70	3.26	4.46	219.00	180.00	2.00	0.70
6.50	9870.00	0.69	2.85	5.02	244.00	172.00	1.80	-0.06
6.00	8280.00	0.67	2.44	5.66	279.00	165.00	1.50	-1.00
5.50	6730.00	0.66	2.04	6.67	327.00	157.00	1.10	-2.00
5.00	5220.00	0.64	1.63	8.06	392.00	146.00	0.00	-3.00

3.8 Using Hydrostatic Curves and Tables

Hydrostatic curves and tables can be used to obtain all hydrostatic particulars of a ship once the draught or any one of the particulars is known.

Example 3.5

From MV Bulker hydrostatic Curves (pg18) at a draught of 7m, we can obtain displacement Δ = 31,000 tonnes, LCF = 2.0m forward of amidships and MCTC = 465 tonne-m etc. Also if we know the ship weighs 40,000 tonnes, its draught, TPC, MCTC, LCF and LCB can be obtained.

Exercise 3.11

Using MV Bulker Hydrostatic Curves, find displacement, LCB, LCF, TPC at draught of 9.5m. If 1500 tonnes is added to the ship, what is its new draught?

Hydrostatic tables can be used in a similar manner to obtain hydrostatic particulars once draught is known or to obtain draught and other particulars once the displacement or another particular is known. There is however a need to interpolate the table to obtain intermediate values.

HOMEWORK 1:

By using the hydrostatic particulars of Bunga Kintan shown in Table 3.1:

- **i. Draw full hydrostatic curves of the ship**
- **ii. Find values of displacement Δ , KB , LCB, BM_T, BM_L, MCTC, C_B, LCF of the ship if it is floating at a draught of 6.75m.**
- **iii. Find values of T, KB , LCB, BM_T, BM_L, MCTC, C_B, LCF of ship if the ship weighs 11,480 tonnes.**
- **iv. When the ship is floating at a draught of 5.5m, 3000 tonne cargo was added. What is its new draught?**

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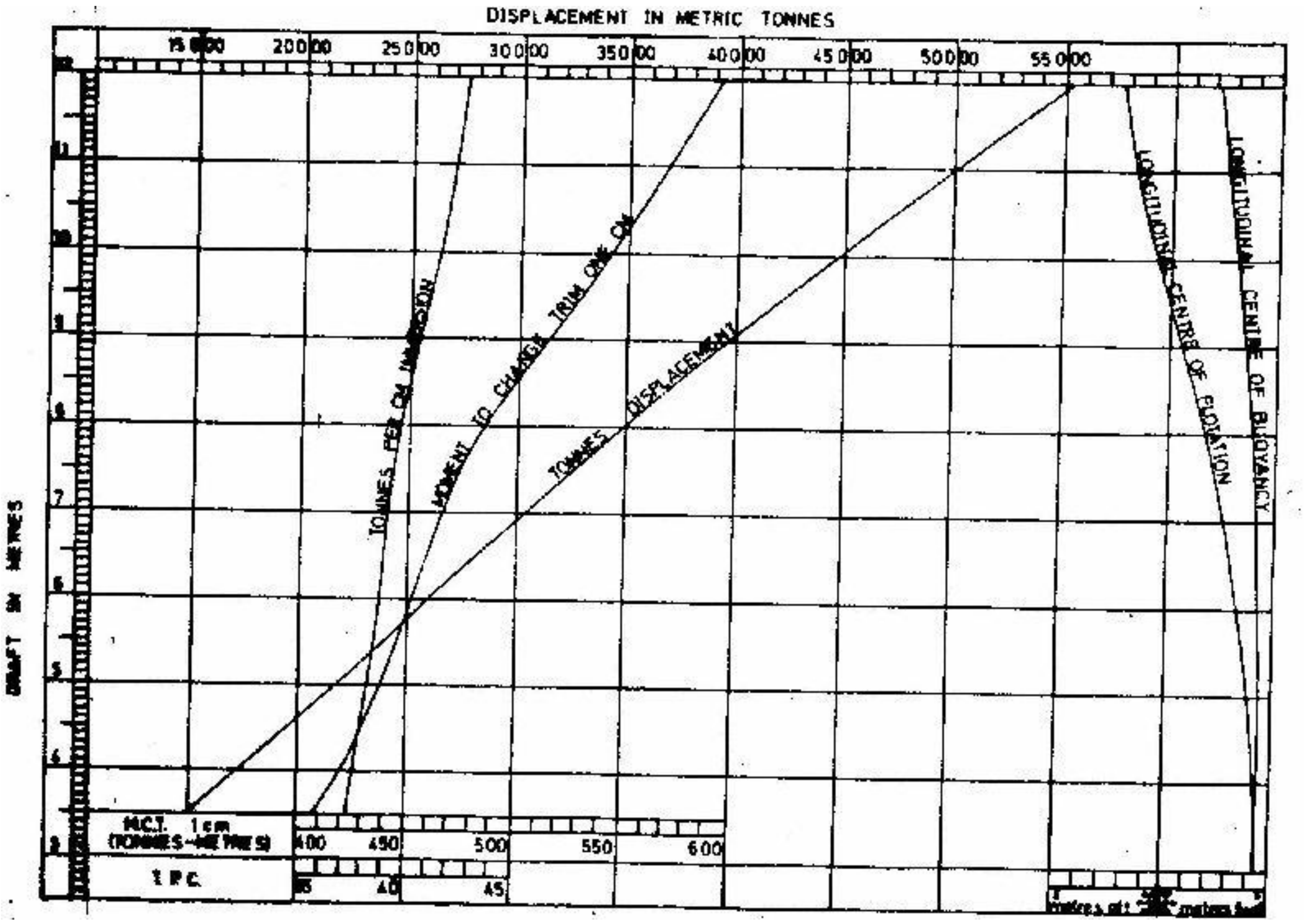
Exercise 3.11:

Calculate Δ , ∇, KB , LCB, A_W, TPC, C_B, C_P, C_M, C_W, LCF of a cylinder radius 1m floating with axis vertical at draughts of 1.0, 1.5, 2.0 and 2.5m.



MV Bulker Hyrostatic Curves

Chapter 4
Basic Stability Consideration

4.1 Introduction

One of the factor threatening the safety of the ship, cargo and crew is the lost or lack of stability of the vessel. Stability calculation is an important step in the design of the ship and during its operation. While designing the ship, the designers must be able to estimate or calculate to check whether the ship will be stable when constructed and ready to operate. For the ship's master, he must be able to load and stow cargo and handle the ship while ensuring that the ship will be stable and safe.

4.2 What is stability?

Stability is the tendency or ability of the ship to return to upright when displaced from the upright position. A ship with a strong tendency to return to upright is regarded as a stable vessel. On the other hand, a vessel is said to be not stable when it has little or no ability to return to the upright condition. In fact, an unstable ship may require just a small external force or moment to cause it to capsize.

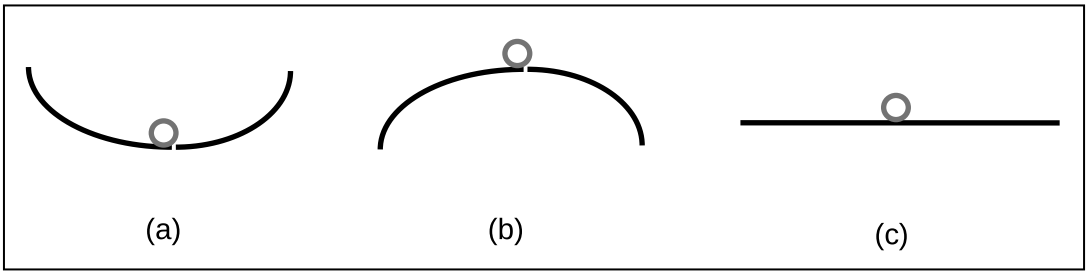


Figure 4.1

An analogy for stability is often given of the marble. In Figure 1 (a), the marble in the bowl will return to its original position at the bottom of the bowl is it is moved to the left or the right. This marble is in a condition called positively stable. A slight push on the marble which is put on an upside down bowl as in Figure 1 (b) will cause it to roll off, a condition equivalent to instability. A neutrally stable ship is analogous to a marble put on a flat surface, it will neither return nor roll any further.

4.3 Longitudinal and Transverse Stability

Ship initial stability can be seen from two aspects, longitudinally and transversely.

From longitudinal viewpoint, the effect of internal and external moments on ship's trim is considered. Important parameters to be calculated are trim and the final draughts at the perpendiculars of the ship. In any state, there is a definite relationship between trim, draughts and the respective locations of the centres of buoyancy and centre of gravity. The trim angle θ is rarely taken into consideration.
Transverse stability calculation considers the ship stability in the port and starboard direction. We are interested in the behaviour of the ship when external statical moment is applied such as due to wind, waves or a fishing net hanging from the side. The effect of internally generated moment such as movement of masses on-board transversely is also studied. An important relationship considered is that

between heeling and righting moments and the resulting angle of heel ϕ and its consequence on the safety of the boat.

This Chapter will focus on basic transverse stability particularly the relationships between the metacentre and the centre of gravity.

4.4 Basic Initial Stability: The role of GM

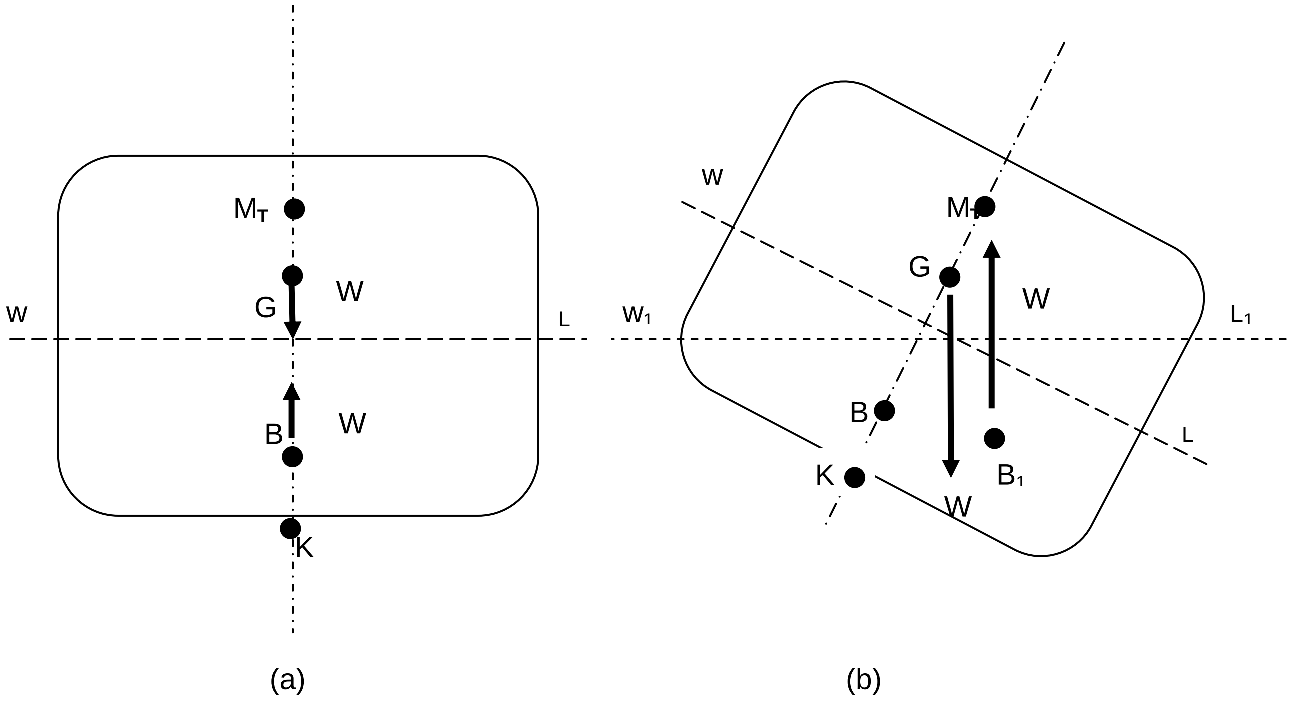


Figure 4.2

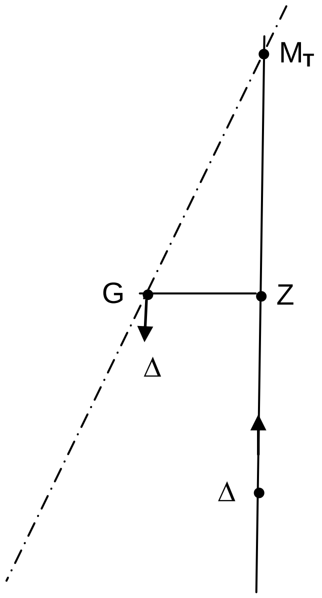
Consider the ship floats upright in equilibrium as in the above figure 4.2 (a). The weight of the ship equals its displacement and the centre of buoyancy is directly below the centre of gravity. When the ship is slightly disturbed from upright, the centre of buoyancy being centre of underwater volume moves to the right. The line of action of buoyancy vertically upward crosses the original centreline at the metacentre, M. Since G does not move, a moment is generated to turn the ship back to its original position. This moment is called the returning moment.

In this case, M was originally above G and we can see that the returning moment is positive. If M was below G i.e. GM negative, the returning moment will be negative hence the ship is unstable. If M is at G, then the ship is neutrally stable.

Righting moment is the real indication of stability i.e. the ability of the ship to return to oppose any capsizing moment and return the ship to upright position.

The larger the righting moment, the better stability is.

Consider the triangle shown below:

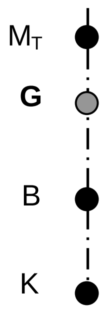


Righting moment = Δ x GZ

and GZ = $GM_T \sin \phi$

For any displacement, righting moment depends on GZ.

And GZ depends on GM. The bigger GZ, the bigger Righting Moment.



Relationships between K, B, G and M_T are important.

$$KM_T = KB + BM_T$$

$$KM_T = KG + GM_T$$

For any particular draught or displacement at low angle of heel, keel K and metacentre M are fixed. Therefore the values of KB, BM and hence KM are fixed, as can be obtained from hydrostatic particulars. Therefore the distance GM_T will only depend on the height of centre of gravity. In other words, to ensure a large GM_T, we can only 'control' KG.

4.5 Determining Centre of Gravity, Areas or Volumes of Composite Bodies

The above section has shown that the relative position of M and G are important in determining ship stability. Since M is constant for any particular draught, only G will finally determine the value of GM.

Before we go into the details of stability calculations, we have to consider how to determine the location of G. Consider a composite body consisting of two portions shown in Figure 4.3.

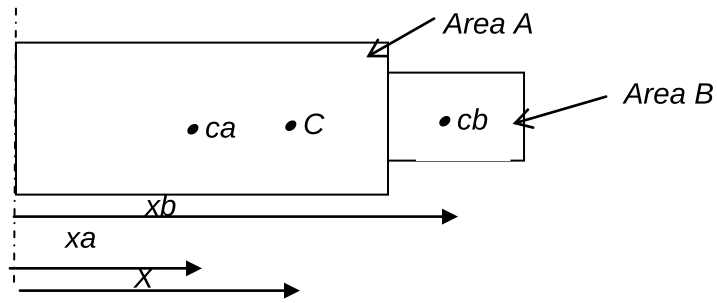


Figure 4.3

Distance of Centre of Composite to the reference axis:

$$X = \frac{A \times xa + B \times xb}{A + B}$$

i.e.,

$$X = \frac{\text{Total moment of area about the reference axis}}{\text{Total area}}$$

If the composite consists of volumes,

Centre of Volume

$$X = \frac{\text{Total moment of volume about the reference axis}}{\text{Total volume}}$$

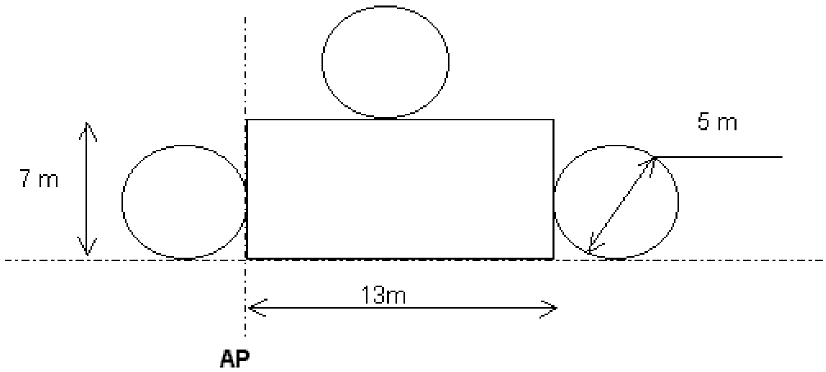
If the composite consists of weights,

Centre of Gravity

$$X = \frac{\text{Total moment of weight about the reference axis}}{\text{Total weight}}$$

Example 4.1

Find centre of area (from AP) for an object consisting of four components shown in the figure below.



Component	Area (m^2)	Distance from AP (m)	Moment of Area about AP (m^3)
1		-2.5	
2			
3			
4			
TOTAL			

Centroid from AP $= \frac{\text{Total moment of area about AP}}{\text{Total area}}$

= m

Example 4.2

A trimaran has three hulls and the respective volume displacements, LCB and KB are shown below. Find the total displacement, LCB and KB.

Hull	Volume Displacement (m^3)	Lcb (m aft of amidships)	Kb (m above keel)
Side 1	158.7	13.0	2.5
Main	1045.8	2.0	2.0
Side 2	158.7	13.0	2.5

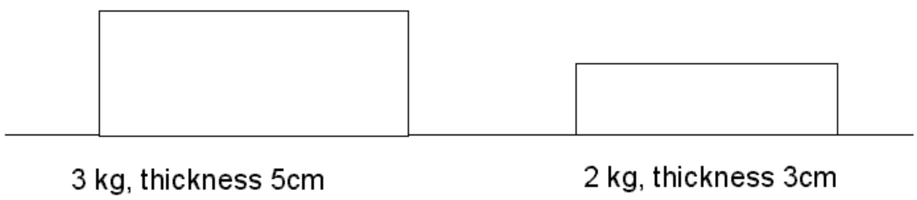
Hull	Volume Displacement (m^3)	lcb (m aft of amidships)	Moment about amidships (m^4)	Kb (m above keel)	Moment about keel (m^4)
Side 1	158.7	13.0		2.5	
Main	1045.8	2.0		2.0	
Side 2	158.7	13.0		2.5	
TOTAL					

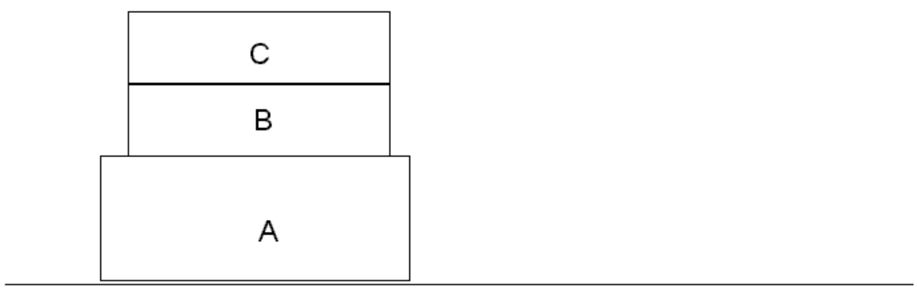
$$LCB = \frac{\text{Total moment about amidships}}{\text{Total Volume}} = 4.56 \text{ m aft of amidships}$$

$$KB = \frac{\text{Total moment about keel}}{\text{Total Volume}} = 2.12 \text{ m}$$

Example 4.3

A stack of weights consists of one 3kg weight and two 2kg weights. Find centre of gravity of the stack above the floor:





Item	Weight (kg)	CG above floor (cm)	Moment about Floor (kgcm)
Wt A			
Wt B			
Wt C			
JUMLAH			

Final CG = = cm

Example 4.4

A ship has three parts and the respective weights and Kg are as follows. Find the total weight and KG.

Part	Weight (tonnes)	Kg (m above keel)
Lightship	2000	5.5
Cargo 1	300	7.6
Cargo 2	500	2.5

Part	Weight (tonnes)	Kg (m above keel)	Moment about keel (tonne-m)
Lightship	2000	5.5	
Cargo 1	300	7.6	
Cargo 2	500	2.5	
TOTAL			

$$KG = \frac{\text{Total moment about Keel}}{\text{Total weight}} = \quad m$$

Example 4.5

A ship of 6,000 tonnes displacement has KG = 6 m and KM = 7.33 m. The following cargo is loaded:

1000 tonnes, Kg 2.5 m
500 tonnes, Kg 3.5 m
750 tonnes, Kg 9.0 m

The following cargo is then discharged:

450 tonnes of cargo Kg 0.6 m
And 800 tonnes of cargo Kg 3.0 m

Find the final GM.

Item	Weight (tonne)	Kg	Moment about keel (tonne-m)
Ship Loaded Cargo1 Cargo2 Cargo3	6000 1000 500 750	6.0 2.5 3.5 9.0	
Unloaded Cargo	-450 -800	0.6 3.0	

$$\text{Final KG} = \frac{\text{Final moment}}{\text{Final displacement}}$$

= ________

Final KG = m

Final KM = m
Final KG = m
Ans. Final GM = m

Homework 2

A box-shaped barge is floating in sea water at a draught of 5m. The extreme dimensions of the barge (L x B x D) are 12m x 11m x 10m. The wall and floor are 0.5m thick. Its centre of gravity is 4m above keel.

Calculate:

i. The displacement and GM_T of the empty barge.
ii. The barge is to be used to carry mud (density1500 kg/m^3). If the draught of the barge cannot exceed 7.5m, find the maximum volume of mud that can be loaded into the barge.
iii. For the barge loaded as in (ii), find its GM_T.

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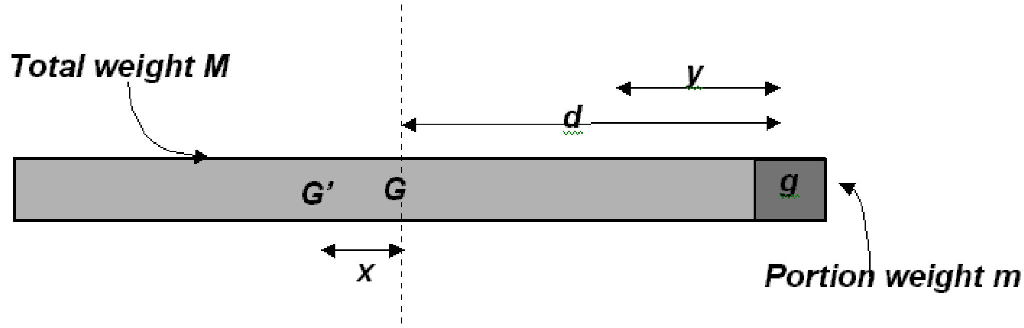
4.6 Movement of Centre of Areas, Volumes and Weights

When a portion is added or removed from an object, its centre moves.

Consider a homogenous object as shown below:



i. <u>If weight is moved a distance y:</u>

Centre of gravity moved $x = GG' = \frac{m \times y}{M}$

i.e. total moment divided by total weight

ii. <u>If weight m is removed:</u>

The remaining weight M-m

Movement of centre of gravity $x = GG' = \frac{m \times d}{M-m}$

i.e. total moment divided by remaining weight.

<u>Example 4.6</u>

A ship weighing 7000 tonnes is floating at the wharf. At that time, KM = 6.5 m and GM 0.5m. Find new GM when a 30 tonnes box is loaded at Kg 10.0m. Assume no change in KM.

Method 1:

Find rise in KG

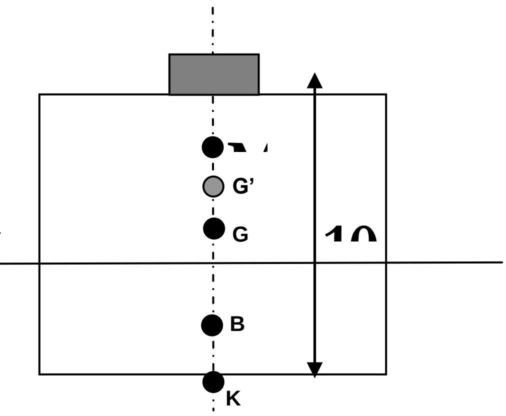
Original KG = KM - GM = m

Distance 30 tonnes box from original G = m

$GG' = \frac{30 \times 4.0}{7030} = 0.017m$

KG'= KG+ GG' = m

KM does not change, therefore, GM = = m



Method 2:

Find final KG using table of moment about keel

Portion	Mass (m)	Kg (m)	Moment about keel (tonne-m)
Ori. Ship	7000	6.0	
Box	30	10.0	
Total	7030		

KG = $\frac{\text{Sum of moment}}{\text{Sum of weight}}$

KG = m

GM = KM - KG

KM - KG = m

4.7 Hanging Weights, The Use Of Derricks And Cranes

The use of cranes and derricks will make the weights suspended. Suspended weights acts at the point of suspension. Therefore a weight that was initially located on the lower deck for example will instantly be transferred to the point of suspension at the instant the weight is lifted by the derrick. The centre of gravity KG will suddenly increase and since KM is constant, GM will reduce suddenly. If the rise in KG is more than the original GM, the net GM will be negative, leading to instability.

Example 4.7

A ship of 7,500 tonnes displacement is upright and has GM 0.20m and KM 6.5 m. A heavy cargo of 100 tonnes already on the lower deck (kg=2m) is to be unloaded using the ship's crane. When lifting the cargo crane head is 15 m above keel. What is GM during lifting. Comment of the safety of the operation.

Treat as if the weight is suddenly transferred from lower deck to the point of suspension, a distance of 15 meters. The KG will rise, and since KM constant, GM will be reduced.

Original KG = KM-GM= 6.5 – 0.2 = 6.3m

Rise in KG = $\frac{100 \times 13}{7,500}$

=0.173m

KG during lifting = KG + Rise = 6.473m

GM during lifting = KM- Kgnew = 6.5- 6.473 = 0.027m

4.8 Free Surface Correction

When free surface exists on board the ship, stability of ship is affected. The free surface gives rise to free surface moment which in effect reduce GM. The reduction is called Free Surface Correction (F.S.C).

FSC is calculated from the second moment of area of the surface of the fluid;

$$FSC = \frac{\text{Free surface moment}}{\text{Ship displacement}}$$

Free Surface Moment (FSM) = $i \times \rho_{fluid}$

Where i the second moment of area of the surface of the fluid and ρ_{fluid} is the density of the fluid being considered.

Once the FSC is known, the new reduced GM called GM_{fluid} is obtained

$$GM_{fluid} = GM_{solid} - FSC$$

It is important that free surface be avoided or at least minimised.

Note also that KG in ships having free surface is called KG_{fluid} and regarded increased by FSC.

$$KG_{fluid} = KG_{solid} + FSC$$

For tanks with a rectangular surface:

$$\text{Free surface moment} = \frac{1}{12} \times \text{tank length} \times \text{tank breadth}^3 \times \text{density of fluid}$$

$$\text{Free surface correction} = \frac{1}{12} \times \frac{\text{tank length} \times \text{tank breadth}^3 \times \text{density of fluid}}{\text{ship displacement}}$$

EXERCISE 4

1. Bunga Kintan (Hydrostatic data given on page 12) is floating at draught of 6.5m. If its KG is 6.8m, what is its GM?

2. A ship has a displacement of 1,800 tonnes and KG = 3m. She loads 3,400 tonnes of cargo (KG = 2.5 m) and 400 tonnes of bunkers (KG = 5.0m). Find the final KG. 2.84m

3. A ship sails with displacement 3,420 tonnes and KG = 3.75 m. During the voyage bunkers were consumed as follows: 66 tonnes (KG = 0.45 m) and 64 tonnes (KG =1 m). Find the KG at the end of the voyage.

4. A ship has displacement 2,000 tonnes and KG = 4m. She loads 1,500 tonnes of cargo (KG = 6m), 3,500 tonnes of cargo (KG = 5m), and 1,520 tonnes of bunkers (KG = 1m). She then discharges 2,000 tonnes of cargo (KG = 2.5 m) and consumes 900 tonnes of oil fuel (KG = 0.5 m.) during the voyage. If KM= 5.5m, find the final GM on arrival at the port of destination.

5. A ship arrives in port with displacement 6,000 tonnes and KG 6 m. She then discharges and loads the following quantities:

Discharge	1250 tonnes of cargo	KG 4.5 metres
	675 tonnes of cargo	KG 3.5 metres
	420 tonnes of cargo	KG 9.0 metres
Load	980 tonnes of cargo	KG 4.25 metres
	550 tonnes of cargo	KG 6.0 metres
	700 tonnes of bunkers	KG 1.0 metre
	70 tonnes of FW	KG 12.0 metres

During the stay in port 30 tonnes of oil (KG 1 m.) are consumed. If the final KM is 6.8 m., find the GM on departure.

6. A ship of 9,500 tonnes displacement has KM 9.5 m and KG 9.3 m. A load 300 tonnes on the lower deck (Kg 0.6 m) is lifted to the upper deck (Kg 11 m). Find the final GM.

7. A ship of 4,515 tonnes displacement is upright and has KG 5.4 m and KM 5.5 m. It is required to increase GM to 0.25m. A weight of 50 tonnes is to be shifted vertically for this purpose. Find the height through which it must be shifted.

8. A ship of 7,500 tonnes displacement has KG 5.8 m. and GM 0.5 m. A weight of 50 tonnes is added to the ship, location Kg = 11m and 7m from centreline to the starboard side. Find final location of G above keel and from the centreline. What is its new GM?

9. A ship has a displacement of 3,200 tonnes (KG = 3 m. and KM = 5.5 m.). She then loads 5,200 tonnes of cargo (KG = 5.2 m.). Find how much deck cargo having a KG = 10 m. may now be loaded if the ship is to complete loading with a positive GM of 0.3 metres.

10. A ship of 4,500 tonnes displacement is upright and has KG 5.4 m and KM 5.5 m. It is required to move a weight of 50 tonnes already on the deck (kg=6m) using the ship's derrick. The derrick head is 13 m above keel. Is it safe to do so?

11. A ship of 9,500 tonnes displacement and has KM 9.5 m and KG 9.3 m. The ship has two fuel tanks in double bottoms, rectangular shape each 20 x 5m containing bunker density 900 kg/m^3. Find GM_{fluid} when free surface exists in the tank.

12. Find Gm_{fluid} for the ship in question 11 but with one tank only, length 20m breadth 10m.

13. What happens to i when there are three tanks with b = 3.33m in question 11.

Appendix A

Second Moments of Areas

The second moment of an element of an area about an axis is equal to the product of the area and the square of its distance from the axis. Let dA in Figure A.1 represent an element of an area and let y be its distance from the axis AB

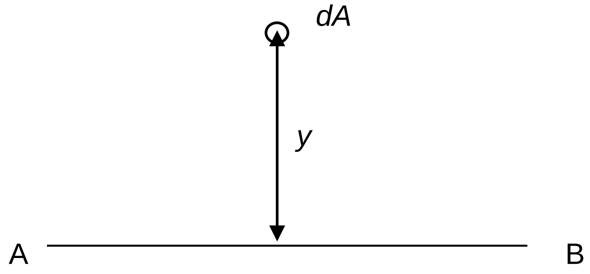


Fig. A.1

The second moment of the element about AB is equal to $dA \times y^2$

2. To find the second moment of a rectangle about an axis parallel to one of its sides and passing through the centroid

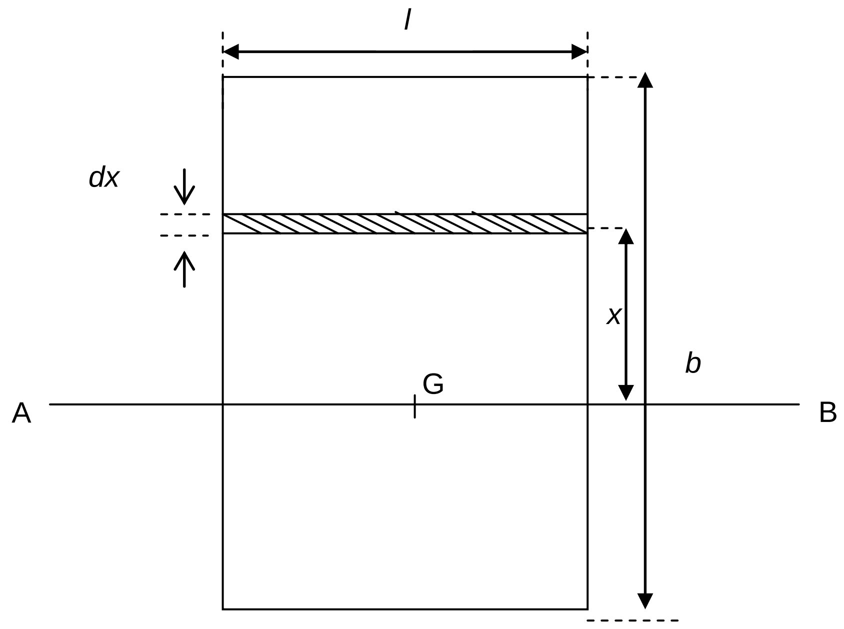


Fig. A.2

In Figure A.2, l represents the length of the rectangle and b represents the breadth. Let G be the centroid and let AB, an axis parallel to one of the sides, pass through the centroid.

Consider the elementary strip which is shown shaded in the figure. The second moment (i) of the strip about the axis AB is given by the equation:-

$$i = l\,dx \times x^2$$

Let I_{AB} be the second moment of the whole rectangle about the axis AB then:-

$$1_{AB} = \int_{-b/2}^{+b/2} l.\,x^2.dx$$

$$1_{AB} = l \int_{-b/2}^{+b/2} x^2.dx$$

$$= l\left[\frac{x^3}{3}\right]_{-b/2}^{+b/2}$$

$$1_{AB} = \frac{lb^3}{12}$$

3. To find the second moment of a rectangle about one of its sides.

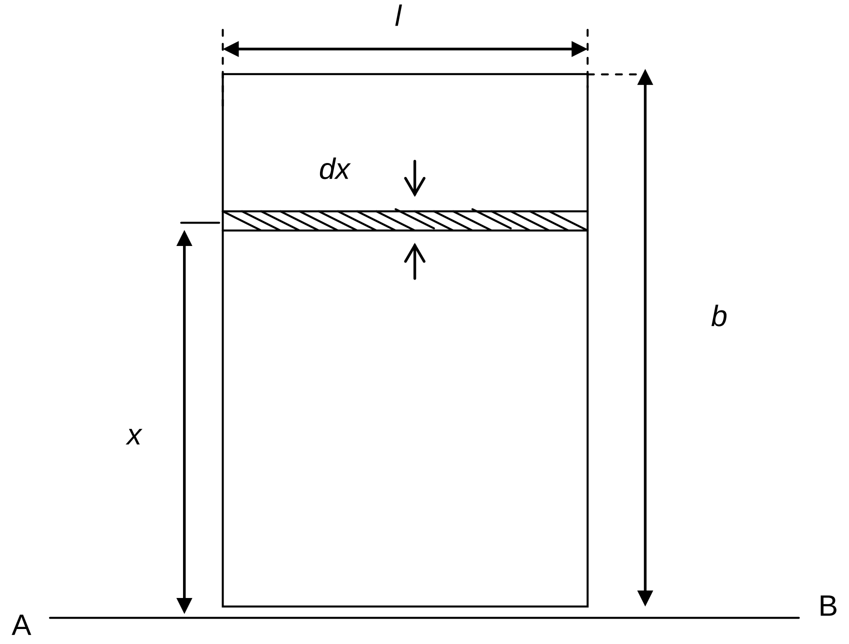


Fig. A.3

Consider the second moment (i) of the elementary strip shown in Figure A.3 about the axis AB.

$$i = l\,dx \times x^2$$

Let I_{AB} be the second moment of the rectangle about the axis AB. Then :-

$$1_{AB} = \int_{O}^{b} l.\,x^2.dx$$

$$= l\left[\frac{x^3}{3}\right]_O^b$$

or

$$1_{AB} = \frac{lb^3}{3}$$

4. The Theorem of Parallel Axes

The second moment of an area about an axis through the centroid is equal to the second moment about any other axis parallel to the first reduced by the product of the area and the square of the perpendicular distance between the two axes. Thus, in Figure A.4, if G represents the centroid of the area (A) and the axis OZ is parallel to AB then:-

$$I_{OZ} = I_{AB} - Ay^2$$

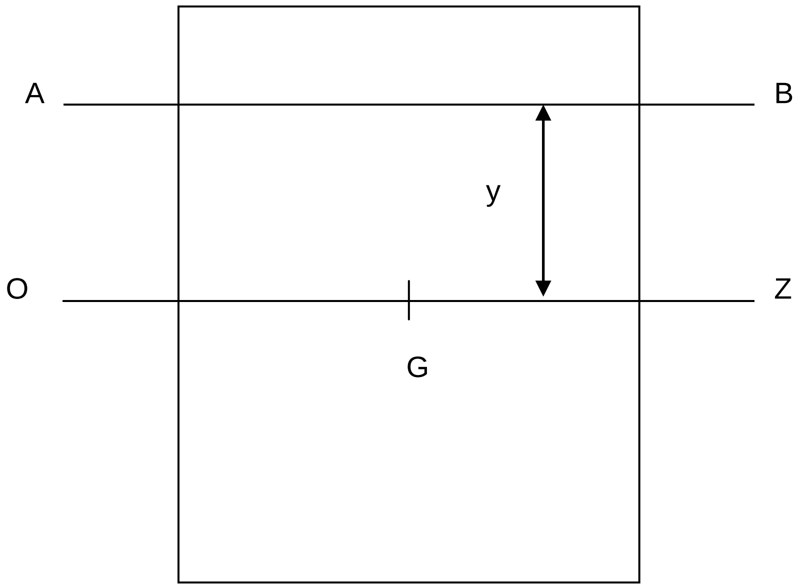


Fig. A.4

5. Second moment of area of a circle

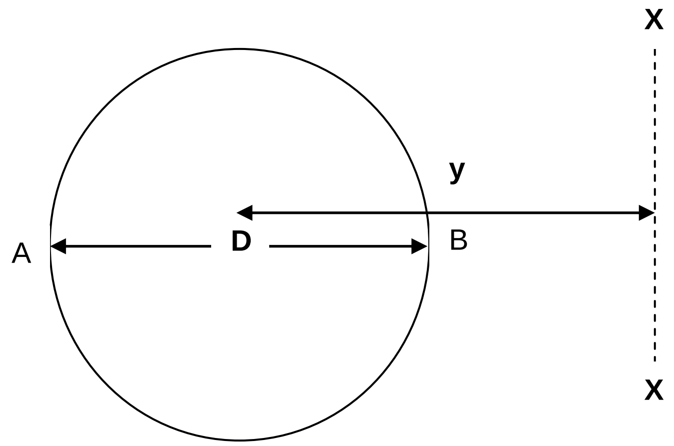


Fig. A.5

For circle, the second moment of area about an axis AB.

$$I_{AB} = \frac{\pi D^4}{64}$$

What is I_{XX}?

6. Applications.

Second moment of areas are used in calculations of BM_L and BM_T :

$$BM_L = \frac{I_F}{\nabla}$$

and

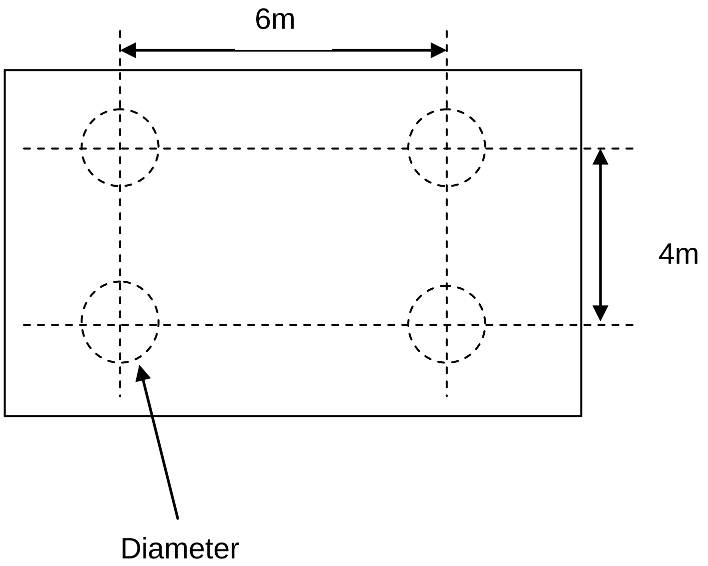
$$BM_T = \frac{I_T}{\nabla}$$

Where I_F is longitudinal second moment of area of the waterplane about the centre of floatation, I_T is transverse second moment of area about the centreline and ∇ is volume displacement.

EXERCISES

1. Find BM_L and BM_T of a box shaped barge 120m x 20m x 10m floating at a draught of 7m.
2. A cylinder of radius r = 10m is floating upright at draught of 6m in fresh water. Find its KM_L and KM_T.

3. A fish cage consists of a wooden platform placed on used oil drums with the following dimensions.



If the total weight of the structure is 3 tonnes, floating in sea water calculate:

i) draught
ii) KM_T
iii) KM_L

Homework 3:

A catamaran consists of two box-shaped hulls spaced 5m apart, centreline to centreline. Each hull measures (L x B x D) 10m x 0.5m x 1m. If its draught is 0.3m, find its :

i) ∇ and Δ
ii) KB
iii) BM_T
iv) Maximum allowable KG if GM minimum is 0.2m

Submission Date: _______________