

Chapter 4 Calculations of Ship Hydrostatic Particulars

4.1 The Importance of Hydrostatic Particulars

In the previous chapter, we have seen the importance of knowing the hydrostatic particulars of a vessel. If we have the hydrostatic particulars in the form of tables, curves, or our own direct calculation, we can obtain details about the ship in any particular condition. We can also determine or estimate what would happen when ship condition changes such as due to addition or removal of weights.

To draw hydrostatic curves or to make the table, we need to calculate the particulars. The hydrostatic particulars can be obtained only if we carry out calculations of area, volumes and moment at various draughts or water plane area. Using some known relationships, the particulars can be derived from areas, volumes and moments,

If the body has a uniform shape, such as cuboids, cones, spheres or prisms, calculation of areas, volumes and moments are easy. For example water plane areas, block coefficients, TPC, MCTC, KB and LCB of such objects can be found using simple formulae. We can easily obtain the particulars at any draught and if necessary plot the curves.

However not all ships have simple and uniform shapes as above. In fact, most ships have hull shapes which are varying in three directions. This makes it difficult to calculate hydrostatic particulars.

4.2 Methods to Calculate Areas

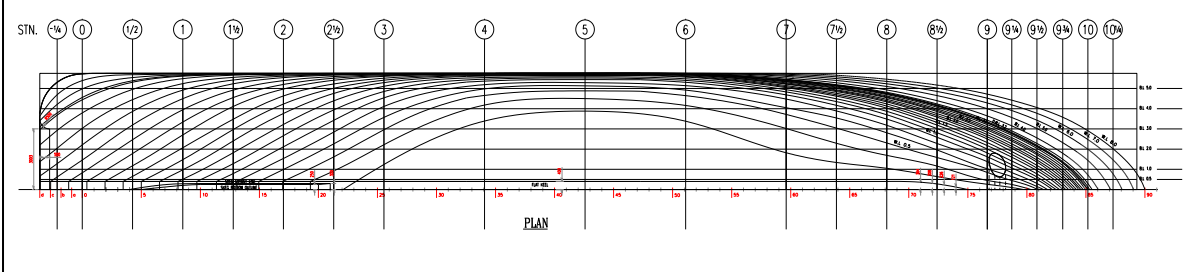


Figure 4.1 Typical Ship Half-Breadth Plan

Consider the shape of the ship whose body plan is shown in Figure 4.1. If we want to find the area of the section or water plane for example, we do not have simple methods. Similarly to find volume displacement or LCF will not be easy.

If we want calculate the water plane area of the ship in Figure 4.1 at a particular draught, we may use a few methods.

First is to plot the curve on a graph paper from where the area under the curve can be obtained by counting the squares. To improve accuracy, smaller boxes or triangles can be used. The method is tedious and its accuracy depends on the size of the smallest grid. To use this method, we need to plot the curve first; a disadvantage when sometimes we are only provided with offset data, i.e. half-breadth at various stations.

The second method is to use an equipment called the planimeter. This equipment can be used to measure the area of a shape drawn on paper. Again, this equipment can only be used only when hard copy of the waterline drawing is available. Moreover, similar to graphical method, planimeter requires a lot of man power.



Figure 3.2 A Planimeter

The third method is to use mathematical approximation. In this method, an attempt is made to represent the curve or shape by a mathematical expression. By using calculus, area and moments of the area bounded by the curve can be found by integration.

Mathematical methods are normally preferred for a number of reasons. First there is no need for a hard copy of the curves. Offset tables are normally available and the data can be used directly in the calculations. A very important feature of mathematical methods is the ability to make use of the technology offered by computers. The use of mathematical methods also enable us to obtain not only areas but all hydrostatic particulars. As we have seen in chapter 3, we need to calculate not only areas but also volumes, positions of centroids of waterplanes (LCF) and centroids of volumes (KB and LCB). In addition we require second moments of areas for calculations of MCTC and metacentric heights. Unlike graphical or planimeter methods, mathematical methods can easily be used to calculate these particulars.

A very important caution should be noted when using mathematical methods. The accuracy of the calculations will mainly depend on the degree of fit of the actual curve to the mathematical expression representing it.

4.3 Mathematical Methods

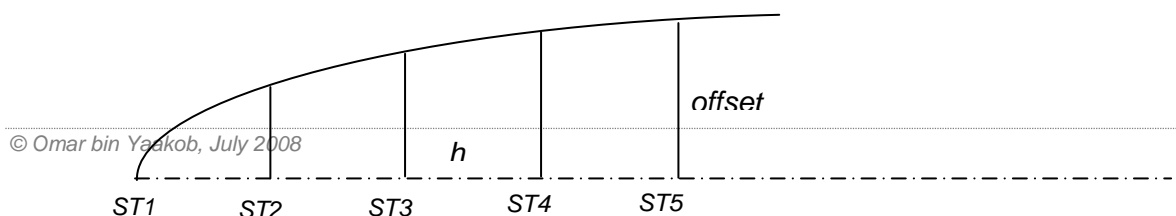


Figure 4.3 Waterline or Sectional Area Curve

Figure 4.3 shows a curve which may represent a half-waterplane area or a curve of sectional areas. A waterplane curve is represented by offsets made up of half-breadth at various stations. Stations are positions along the length of the ship and normally separated by a common-interval, h . To cater for the fast changing slopes of the curve at the stern and bow regions, half stations may be used.

To calculate the area, centroid and moment under such curve, its offsets and h are required. By assuming that the curve can be represented by a certain mathematical formulae, calculations can be made. A number of methods have been developed for these purpose such as Newton-Cottes, Tchebycheff, Trapezoidal and Simpson methods. In this course, we will concentrate on the two most popular methods; Trapezoidal and Simpson methods.

4.4 Trapezoidal Method

When a curve can be assumed to be represented by a set of trapezoids, the area under the curve can be calculated.

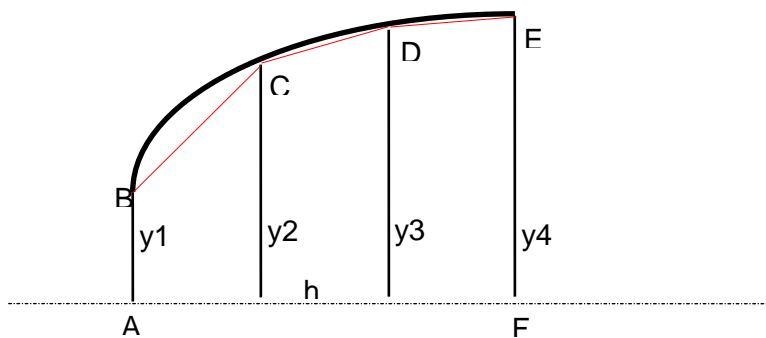


Figure 4.4 Waterline or Sectional Area Curve

In Figure 4.4, the area under the curve is the are area of trapezoid ABCDEF.

$$\begin{aligned}
 \text{Area} &= \frac{1}{2}(y_1 + y_2)h + \frac{1}{2}(y_2 + y_3)h + \frac{1}{2}(y_3 + y_4)h \\
 &= \frac{1}{2}h(y_1 + 2y_2 + 2y_3 + y_4)
 \end{aligned}$$

Exercise

1. Find the Trapezoidal formulae for curves made up of

- i) 6 offsets
- ii) 9 offsets
- iii) n offsets

2. The midship section of a chine vessel has the following offsets:

Draught(m)	0	0.25	0.5	0.75	1.0
Half-breadth (m)	0	0.6	1.0	1.5	1.9

Calculate its midship section coefficient at draught of 1.00m.

3. Find the water plane area of a ship LBP = 10m made up of the following offsets:

Station	0	1	2	3	4
Half-breadth (m)	0	0.3	1.0	1.2	1.1

Find its area, TPC and waterplane coefficient

4.5 Simpson Rules for Areas.

Simpson rule is the most popular method being used in ship calculations to calculate volumes, second moments of areas and centroid. This is because it is flexible, easy to use and its mathematical basis is easily understood.

Basically, the rule states that the ship waterlines or sectional area curves can be represented by polynomials. By using calculus, the areas, volumes, centroids and moments can be calculated. Since the separation between stations are constant, the calculus has been simplified by using multiplying factors or multipliers.

There are three Simpson rules, depending on the number and locations of the offsets.

4.5.1 Simpson First Rule

Simpson's First Rule

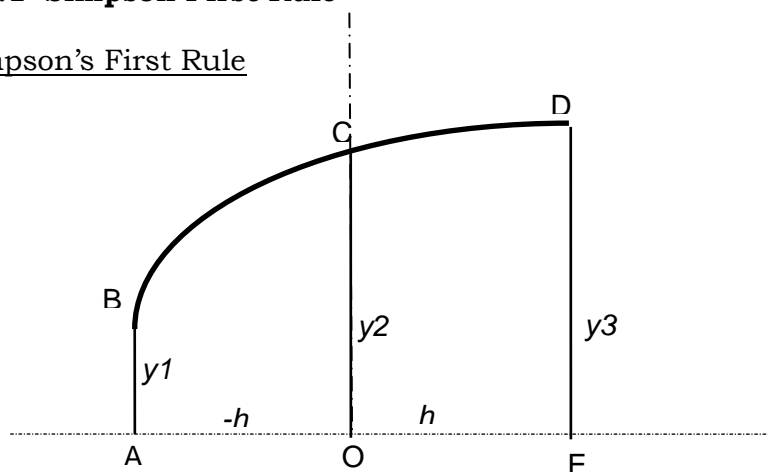


Figure 4.5 Waterline or Sectional Area Curve with Three Offsets

Assume that the offsets are points on a polynomial curve of form

$$y = a_0 + a_1x + a_2x^2 + a_3x^3$$

$$\begin{aligned} \text{Then area ABCDE} &= \int_{-h}^h y \cdot dx \\ &= 2a_0 h + \frac{2a_2 h^3}{3} \end{aligned}$$

$$\begin{aligned} \text{At } x = -h \quad y_1 &= a_0 - a_1h + a_2h^2 - a_3h^3 \\ \text{At } x = 0 \quad y_2 &= a_0 \\ \text{At } x = h \quad y_3 &= a_0 + a_1h + a_2h^2 + a_3h^3 \end{aligned}$$

Therefore,
$$a_0 = y_2 \quad a_2 = \frac{y_1 + y_3 - 2y_2}{2h^2}$$

Substituting these values into the above equation

$$\text{Area ABCD} = \frac{h}{3} [y_1 + 4y_2 + y_3]$$

First Rule is used when there is an odd number of offsets. The basic multiplier for three offsets are 1,4,1. For more stations, the multipliers are developed as follows:

Station	1	2	3	4	5	6	7
Offset	a1	a2	a3	a4	a5	a6	a7
Multiplier	1	4	1				
			1	4	1		
					1	4	1
	1	4	2	4	2	4	1

$$\text{Area} = \frac{1}{3} \times h \times \Sigma (\text{multiplier} \times \text{offset})$$

Where h = common interval

Exercise

- Use Simpson first rule to find the area of the midship section of the chine vessel in exercise 2. Explain the difference in area.

Example 1

Find the waterplane coefficient for the waterplane of a 27m LBP boat represented by the following offsets:

Station	0	1	2	3	4	5	6
Half-breadth (m)	1.1	2.7	4.0	5.1	6.1	6.9	7.7

Station	Offset	Simpson Multiplier	Product Area
0	1.1	1	
1	2.7	4	
2	4.0	2	
3	5.1	4	
4	6.1	2	
5	6.9	4	
6	7.7	1	

Area = $\frac{1}{3} \times h \times \sum \text{Product Area} = \text{_____ m}^2$
 Cw = _____

4.5.2 Simpson Second Rules

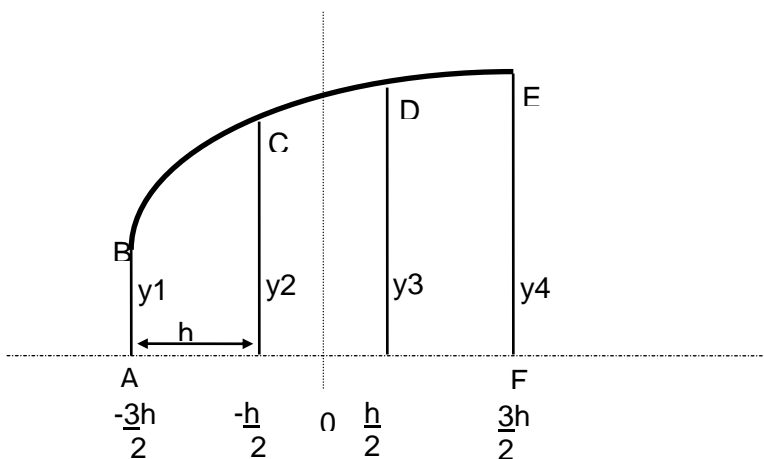


Figure 4.6 Waterline or Sectional Area Curve with Four offsets

Assume that the offsets are points on a polynomial curve of form

$$y = a_0 + a_1x + a_2x^2 + a_3x^3$$

$$\begin{aligned}
 \text{Then area ABCD} &= \int_0^{3h/2} y \cdot dx \\
 &= \left[a_0x + \frac{a_1x^2}{2} + \frac{a_2x^3}{3} + \frac{a_3x^4}{4} \right] \\
 &= 3a_0 + \frac{3}{4}a_2h^2 \tag{1}
 \end{aligned}$$

But

$$x = -\frac{3h}{2} \quad y_1 = a_0 - \frac{3a_1h}{2} + \frac{9a_2h^2}{4} - \frac{27a_3h^3}{8} \tag{2}$$

$$x = -\frac{h}{2} \quad y_2 = a_0 - \frac{a_1h}{2} + \frac{a_2h^2}{4} - \frac{a_3h^3}{8}$$

$$y_3 = a_0 - \frac{a_1h}{2} + \frac{a_2h^2}{4} - \frac{a_3h^3}{8}$$

$$x = \frac{3h}{2} \quad y_4 = a_0 - \frac{3a_1h}{2} + \frac{9a_2h^2}{4} - \frac{27a_3h^3}{8}$$

Adding (2)

and (4)

$$2a_0 - \frac{a_2h^2}{2} = y_1 + y_2$$

Adding (2) and (5)

$$2a_0 - \frac{9a_2h^2}{2} = y_1 + y_4$$

$$4a_2h^2 = (y_1 + y_4) - (y_2 + y_3)$$

$$a_2 = \frac{y_1 + y_4 - y_2 - y_3}{4h^2}$$

$$a_0 = \frac{y_2 + y_3}{2} - \frac{a_2h^2}{2}$$

$$= \frac{9y_2}{16} + \frac{9y_3}{16} - \frac{y_1}{10} - \frac{y_4}{10}$$

$$\text{Then area ABCD} = \frac{3}{8}h[y_1 + 3y_2 + 3y_3 + y_4]$$

The basic multipliers are thus 1,3,3,1 and Area = $\frac{3}{8} \times h \times \Sigma$ (multiplier x offset)

The rule can only be used when number of offsets = 3N + 1

4.5.3 Simpson Third Rule

Simpson third rule is used when we have three offsets and we require the area between two of the offsets.

Example 2

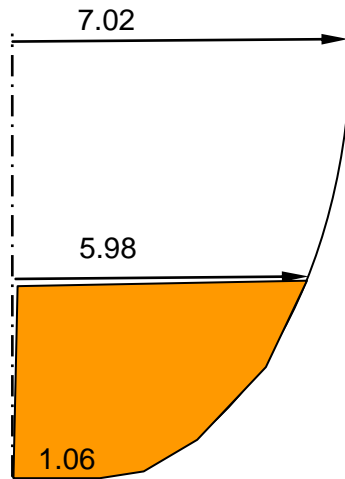


Figure 4.7 Midship Section Curve with Three offsets

A midship section curve has halfbreadth 1.06, 5.98 and 7.02 m spaced at 9.0m draught interval. Find the area between the first two draughts.

<u>½ Breadth</u>	<u>Multiplier</u>	<u>Product</u>
1.06	5	5.30
5.98	8	47.84
7.02	-1	-7.02
		<u>46.12</u>

$$\text{Area} = \frac{1}{12} \times 9 \times 46.12 \times 2 = \underline{69.18 \text{ m}^2}$$

If we require the area between two upper draughts, the calculations are as follows:

<u>½ Breadth</u>	<u>Multiplier</u>	<u>Product</u>
7.02	5	35.10
5.98	8	47.84
1.06	-1	-1.06
		<u>81.88</u>

$$\text{Area} = \frac{1}{12} \times 9 \times 81.88 \times 2 = \underline{122.82 \text{ m}^2}$$

$$\text{Total Area} = 192.0 \text{ m}^2$$

Exercise

- Find the total area under the curve using Simpson first rule and compare.

5.6 Obtaining Volume

Volumes and hence displacement of a ship at any draught can be calculated if we know either:

- i) Waterplane areas at various waterlines up to the required draught
- ii) Sectional areas up to the required draught at various stations.

Example 3

Sectional areas of a 180m LBP ship up to 5m draught in sea water at constant interval along the length are as follows. Find its volume displacement, mass displacement and prismatic coefficient.

Station	0	1	2	3	4	5	6	7	8	9	10
Area (m ²)	5	118	233	291	303	304	304	302	283	171	0

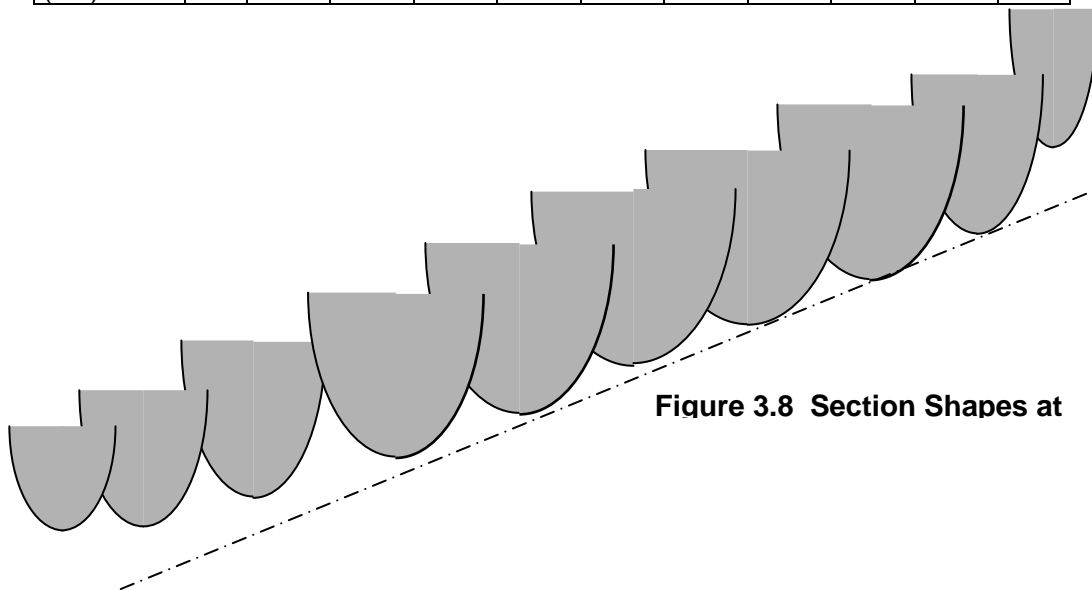


Figure 3.8 Section Shapes at

Station	Section Area	Simpson Multiplier	Product vol
0	5		
1	118		
2	233		
3	291		
4	303		
5	304		
6	304		
7	302		
8	283		
9	171		

10	0		
			$\Sigma f vol$

$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times h \times \Sigma f vol \\ &= \text{_____ m}^3 \end{aligned}$$

Similarly if we have waterplane areas, we can use Simpson rules to integrate the areas to obtain volume. In this case the common interval is the waterline spacing.

4.7 Considering Half and Quarter Stations

Rapidly changing curvature at both ends of the ship necessitates the use of half and quarter stations. To take this into consideration, Simpson Multipliers are also divided as follows:

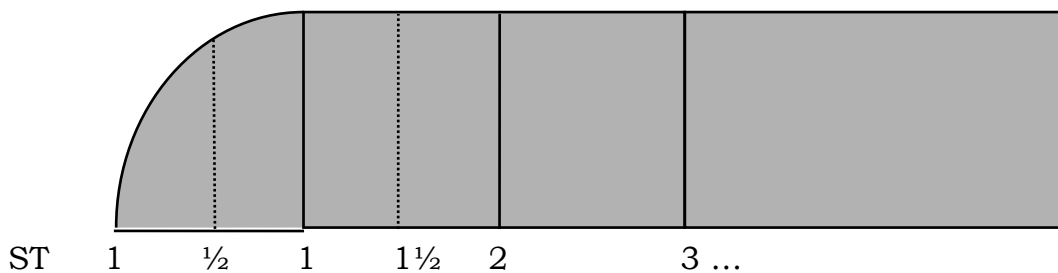


Figure 4.9 Shape with Half Stations

If we consider full stations only:

$$1 \qquad 4 \qquad 2 \qquad 4 \dots$$

If we consider half stations:

$$\begin{array}{cccccccc} \frac{1}{2} & 4 & \frac{1}{2} & & & & & \\ \frac{1}{2} & 2 & \frac{1}{2} & & & & & \\ & & \frac{1}{2} & 4 & \frac{1}{2} & & & \\ & & & & 1 & 4 & 1 & \\ \hline & \frac{1}{2} & 2 & 1 & 2 & 1 \frac{1}{2} & 4 & 1 \end{array}$$

Example 4

A waterplane for a 120 m LBP ship has the following offsets:

Station	0	1/4	1/2	3/4	1	2	3	4	5	5 1/2	6
1/2 ord	0.6	2.8	4.0	5.2	6.2	9.0	9.8	8.4	4.8	2.2	0.0

Find the waterplane area, waterplane coefficient and TPC for the waterplane.

Station	0	1/4	1/2	3/4	1	2	3	4	5	5 1/2	6
---------	---	-----	-----	-----	---	---	---	---	---	-------	---

Multiplier	$1/4$	1	$1/4$		1	4	1		$1/2$	2	$1/2$
			$1/4$	1	$1/4$		1	4	1		
	$1/4$	1	$1/2$	1	$1^{1/4}$	4	2	4	$1^{1/2}$	2	$1/2$

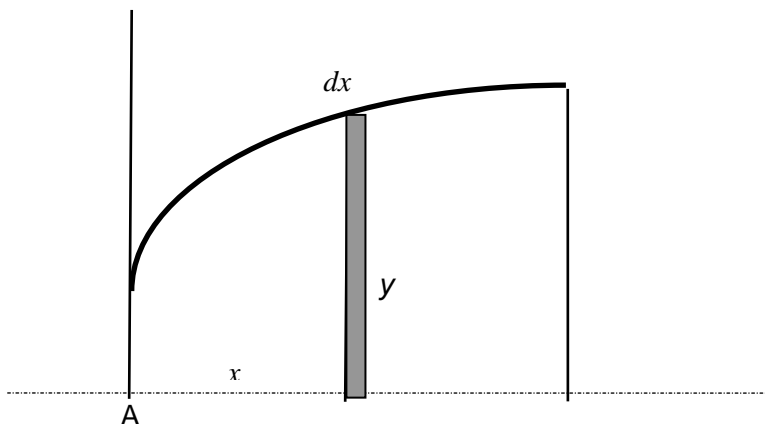
Station	1/2 ord	SM	Product Area	Lever	Product 1st moment	Product 2nd mmt
0	0.6	1/4				
1/4	2.8	1				
1/2	4.0	1/2				
3/4	5.2	1				
1	6.2	1 1/4				
2	9.0	4				
3	9.8	2				
4	8.4	4				
5	4.8	1 1/2				
5.5	2.2	2				
6	0.0	1/2				

∴ Area = $\frac{1}{3} \times h \times \sum \text{Product Area} = \underline{\hspace{2cm}} \text{ m}^2$

Cw = $\underline{\hspace{2cm}}$

TPC = $\underline{\hspace{2cm}}$

4.8 Obtaining LCF, LCB and Longitudinal Second moment of Area



$$\begin{aligned}
 \text{Area} &= \int y dx \quad \Rightarrow \quad 1/3 \times \Sigma \text{product}_A \times h \\
 \text{1st moment} &= \int y dx \cdot x \\
 &= \int x \cdot y dx \quad \Rightarrow \quad 1/3 \times \Sigma \text{product}_{1\text{st mmt}} \times h \times h \\
 \text{2st moment} &= \int y \cdot dx \cdot x^2 \\
 &= \int x^2 \cdot y dx \quad \Rightarrow \quad 1/3 \times \Sigma \text{product}_{2\text{nd mmt}} \times h \times h^2
 \end{aligned}$$

Values of x are given in multiples of h , the common interval.

If the product for area is multiplied by multiples of h , called levers, the sum of products can be used to find the first moment and hence the longitudinal position of the centroid.

$$\begin{aligned}
 \text{LCF} &= \frac{\frac{1}{3} \times h \times h \times \Sigma \text{product}_{1\text{st moment}}}{\frac{1}{3} \times h \times \Sigma \text{product}_{\text{area}}} \\
 &= \frac{\Sigma \text{product}_{1\text{st moment}} \times h}{\Sigma \text{product}_{\text{area}}}
 \end{aligned}$$

If the offsets are half-breadths, the centroid is LCF. If the offsets are sectional areas, the centroid is centre of volume i.e. LCB. The LCF is measured from the axis where levers are taken.

For second moment, Simpson's product for areas are multiplied twice with levers. Again, the second moment are taken about the axis from where levers are taken.

$$I_L = \frac{1}{3} \times h \times h \times h \times \Sigma \text{product}_{2\text{nd moment}}$$

Example 5

Find the area, LCF, second moment of area about amidships, transverse second moment of area about centreline for the waterplane of a ship LBP 180m with the following ordinates.

Stesen	AP	1/2	1	2	3	4	5	6	7	8	9	9 1/2	FP
1/2ord (m)	0	5	8	10.5	12.5	13.5	13.5	12.5	11.0	7.5	3.0	1.0	0

Station	$\frac{1}{2}$ ordinate	SM	Product Area	Lever	Product 1st mmt	Lever	Product 2nd mmt
AP	0	$\frac{1}{2}$	-	+5	-	+5	-
$\frac{1}{2}$	5.0	2	10.0	+4 $\frac{1}{2}$	+45.0	4 $\frac{1}{2}$	+202.5
1	8.0	1 $\frac{1}{2}$	12.0	+4	+48.0	+4	+192.0
2	10.5	4	42.0	+3	+126.0	+3	+378.0
3	12.5	2	25.0	+2	+50.0	+2	+100.0
4	13.5	4	54.0	+1	+54.0	+1	+54.0
5	13.5	2	27.0	0	Sum aft +323.0	0	-
6	12.5	4	50.0	-1	-50.0	-1	+50.0
7	11.0	2	22.0	-2	-44.0	-2	+88.0
8	7.5	4	30.0	-3	-90.0	-3	+270.0
9	3.0	1 $\frac{1}{2}$	4.5	-4	-18.0	-4	+72.0
9 $\frac{1}{2}$	1.0	2	2.0	-4 $\frac{1}{2}$	-9.0	-4 $\frac{1}{2}$	+40.5
FP	0	$\frac{1}{2}$	-	-5	-	-5	-
			278.5		Sum fwd -211.0		1447.0

$$\text{Waterplane Area} = \frac{1}{3} \times \frac{180}{10} \times 278.5 \times 2 = 3342.0 \text{ m}^2$$

$$\text{LCF} = \frac{\sum \text{product}_{1\text{st moment}} \times h}{\sum \text{product}_{\text{area}}}$$

$$\text{LCF} = \frac{(323 - 211) \times 180}{278.5 \times 10} = 7.24 \text{ m aft of amidships}$$

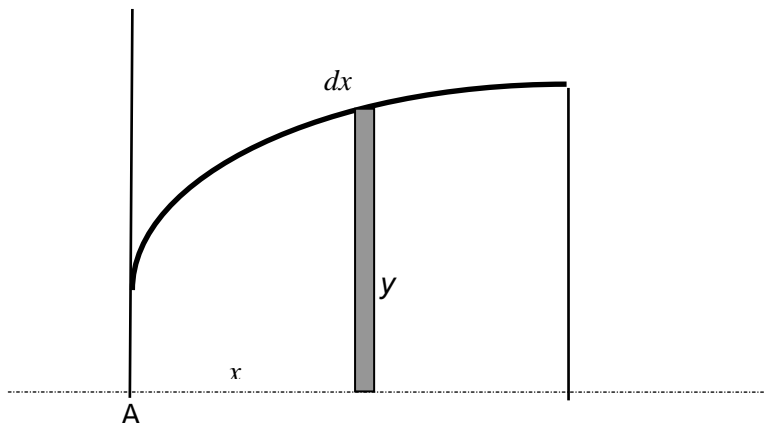
$$I_L = \frac{1}{3} \times h \times h \times h \times \sum \text{product}_{2\text{nd moment}}$$

$$I_L = 2 \times \frac{1}{3} \times 18^3 \times 1447.0 = 5,625,936 \text{ m}^4.$$

Exercise 6:

1. Repeat the Example 5 but this time,
 - Calculate LCF from AP and 2nd Moment of area about AP.
 - Check that the answers are identical.
2. Calculate the centroid of the midship section in Example 2 measured from the top-most waterline (page 8).
3. Calculate LCB of the vessel in Example 3.
4. Calculate LCF from amidship and longitudinal second moment of area about amidship of the ship in Example 4 on page 11.

4.9 Obtaining Second Moment Of Area About The Centreline



If the shaded area is a rectangle, second moment of area about the x-axis is

$$i = \frac{1}{3} dx y^3$$

for the whole area :

$$I_T = \frac{1}{3} \int y^3 dx$$

If the ordinates are cubed and Simpson multipliers are applied,

$$\begin{aligned} I_T &= \frac{1}{3} \times \frac{1}{3} \times h \times \sum \text{product}_{2\text{nd moment}} \\ &= \frac{1}{9} \times h \times \sum \text{product}_{2\text{nd moment}} \end{aligned}$$

Example

Find BM_T for a waterplane of a ship LBP = 100m with the following half breadths. At this draught the ship has a displacement of 11275 tonnes in sea water.

AP	½	1	2	3	4	5	6	7	8	9	9½	FP
0	5	8	10.5	12.5	13.5	13.5	12.5	11	7.5	3	1	0

Station	½ ordinate	(½ ordinate) ³	SM	Product for Second Moment T
AP	0	-	½	-
½	5.0	125.0	2	250.0
1	8.0	512.0	1 ½	768.0
2	10.5	1157.6	4	4630.4
3	12.5	1953.1	2	3906.2
4	13.5	2460.4	4	9841.6
5	13.5	2460.4	2	4920.8
6	12.5	1953.1	4	7812.4
7	11.0	1331.0	2	2662.0
8	7.5	421.9	4	1687.6
9	3.0	27.0	1 ½	40.5
9 ½	1.0	1.0	2	2.0
FP	0	-	½	-
				36521.5

$$2^{nd} \text{ Moment} = \frac{1}{3} \times \frac{1}{3} \times h \times \Sigma \text{product mmt} \times 2 = 81158.9 \text{ m}^4$$

about amidships

$$\text{Volume Displacement} = \frac{11275}{1.025} = 11000 \text{ m}^3$$

$$BM_T = \frac{81158.9}{11000} = 7.38\text{m}$$

4.10 Appendages

Appendages are the portion of the hull which is protruding from the main body. It may be part of underwater volume such as a skeg or keel or parts of a waterplane area which is not suitable to be integrated with the main area due to its abrupt change in area.

Areas, volumes and moment are calculated separately for the appendages and later incorporated using composite body method explained in Chapter 4 of NA1 notes.

Example

A ship length 150m, breadth 22m has the following areas at the various draft.

Draught (m)	2	4	6	8	10
Area of Waterplane(m ²)	1800	2000	2130	2250	2370

There is an appendage (between waterline 0 and 2m) with displacement 2600 tonne in sea water and Kb of 1.2m. Find the total displacement, KB and C_b of the ship at 10m draught.

Solution:

Draught (m)	Aw (m ²)	Multiplier	Product for Volume	Lever	Product for 1 st Moment
2	1800	1	1800	0	0
4	2000	4	8000	1	8000
6	2130	2	4260	2	8520
8	2250	4	9000	3	27000
10	2370	1	2370	4	9480
			25,430		53,000

$$\text{Volume Displacement} = \frac{1}{3} \times 2 \times 25430 = \underline{16960 \text{ m}^3}$$

$$\text{Mass } \Delta = 16960 \times \rho = \underline{17380 \text{ tonne}}$$

$$\text{Centre of Buoyancy} = \frac{53000 \times 2}{25430} = \underline{4.16\text{m above 2m WL.}}$$

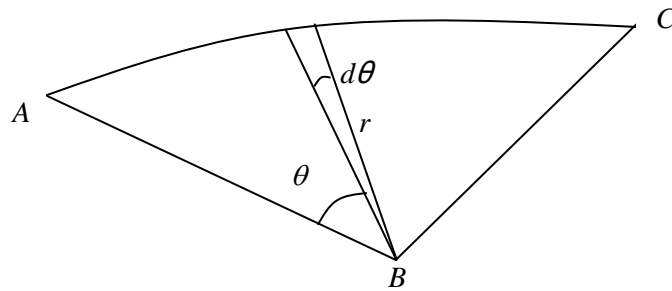
Composite Table

Portion	Displacement (tonnes)	KB	Moment
Main(2m-10m)	17380	6.16	107,000
Appendage	2600	1.20	3,120
Total	19980		110,120

$$\text{KB} = \frac{110,120}{19980} = \underline{5.51\text{m}}$$

$$\text{C}_B = \frac{19980}{150 \times 22 \times 10 \times 1.025} = \underline{0.59}$$

4.11 Simpson Rules for Radial Integration



$$\text{Strip Area} = \frac{1}{2} r^2 d\theta$$

$$\text{Total Area} = \int \frac{1}{2} r^2 d\theta$$

$$= \frac{1}{2} \int r^2 d\theta$$

In Simpson terms, if first rule is used;

$$\text{Total area} = \frac{1}{2} \times \frac{1}{3} \times h \times \sum fA$$

↓
in radians

Example

e.g. A figure is bounded by two radii at right angles to each other and a plane curve. The polar coordinates of the curve at equal interval of angle are 10,9,8,7,6,5 and 4 meters respectively. Find the area of the figure and its centroid from the 10m radius.

Angle	r	r ²	SM	Product for Area	r ³	Sin angle	r ³ xsinxSM
0	10	100	1	100			
15	9	81	4	324			
30	8	64	2	128			
45	7	49	4	196			
60	6	36	2	72			
75	5	25	4	100			
90	4	16	1	16			
				936			3157.95

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \frac{h}{3} \times fA \\ &= \frac{1}{2} \times \frac{1}{3} \times \frac{90}{6} \times \frac{\pi}{180} \times 936 \\ &= 40.6 \text{ Sq. metres} \end{aligned}$$

Centroid is measured perpendicular from one boundary

$$1^{\text{st}} \text{ moment about AB} = \frac{1}{3} \int r^3 \sin \theta \, d\theta$$

$$\begin{aligned} \therefore \text{Centroid } \bar{x} &= \frac{\text{moment}}{\text{area}} \\ &= \frac{\frac{1}{3} \times \frac{1}{3} \times h \times \sum fmmt}{\frac{1}{2} \times \frac{1}{3} \times h \times \sum fA} \\ &= \frac{2}{3} \times \frac{\sum fmmt}{\sum fA} \end{aligned}$$

$$\text{Centroid from 10m boundary} = \frac{2 \times 3157.95}{3 \times 936} = 2.25\text{m}$$

Exercise 7

Find area bounded by a plane curve and two radii 90° apart, if the lengths of the radii at equal angle intervals are 2,3,5,8, and 10 metres respectively. Also find the distance of the centroid of the figure from the 2m radius.

4.12 Tchebycheff's Rule

When y_2 is the middle ordinate and y_1 and y_3 are located $0.7071l$ to the left and right of y_2 ,

Area = $C (y_1 + y_2 + y_3)$ where $C = L/\text{number of ordinates}$ and l is $0.5L$

Ordinates are not equally spaced and their positions in the length depend on number of ordinates, n .

n	Position of ordinates from centre of length expressed as fraction of half length			
2	0.5773			
3	0	0.7071		
4	0.1876	0.7947		
5	0	0.3745	0.9325	
6	0.2666	0.4225	0.8662	
10	0.0838	0.3127	0.50	0.6873
				0.9162

Example

Find area of a 200m waterplane if the half breadth at Tchebycheff stations are as follows:

1.2, 5.0, 8.4, 10.5, 11.7, 11.8, 11.1, 9.6, 7.4, 3.8

$$C = L/10 = 200/10$$

Sum of $y =$

$$\text{Area} = 3220 \text{ m}^2$$

EXERCISES 4

Question 1

A cargo ship 120m, breadth 25m and depth 16m is floating at 8.5m draught in sea water. The area of sections at various stations are shown in the following table:

Station	AP	1	2	3	4	5	6	7	8	9	FP
As (m ²)	12.8	64.5	100.0	120.6	154.2	166.8	140.7	125.9	97.6	43.2	0.0

Calculate

- i. Mass Displacement
- ii. Longitudinal Centre of Buoyancy (LCB) from amidships.
- iii. Block Coefficient (C_B)
- iv. Midship Section Coefficient (C_M)
- v. Prismatic Coefficient (C_p)

COPYING (zero marks), UNTIDY (minus up to 1 mark)

Question 2

- a. At a draught of 4m, the waterplane of Containership *Bunga Bawang* (LBP=88m) has the following offsets.

Station	0(AP)	1	2	4	6	7	8(FP)
½ Breadth (m)	2.20	4.48	6.22	7.10	5.02	2.53	0

Calculate area of waterplane, waterplane coefficient, TPC and LCF from amidship.

- b. The waterplane areas of *Bunga Bawang* at other draughts are as follows:

Draughts	1m	2m	3m
Area (m²)	520	690	830

Between the keel and 1m waterplane, there is an appendage with volume 420 m³ and centroid 0.60m above keel.

Use all the information to calculate for the ship at draught of 4m, the total mass displacement in sea water, its block coefficient and centre of buoyancy above keel.

Question 3

The 2m waterplane of a catamaran boat LBP 20m is shown in Figure 1. The half-breadths of one hull is shown in the following table:

Station	0 (AP)	1	2	3	4	5 (amidships)
½ lebar (m)	2.20	2.18	2.16	2.14	2.12	2.10

Station	6	7	8	8.5	9	9.5	10 (FP)
½ lebar (m)	2.0	1.8	1.6	1.2	0.9	0.4	0.00
	0	0	0	0	0	0	

Calculate for the total waterplane:
 area of waterplane, TPC, LCF and second moment of area about the centreline

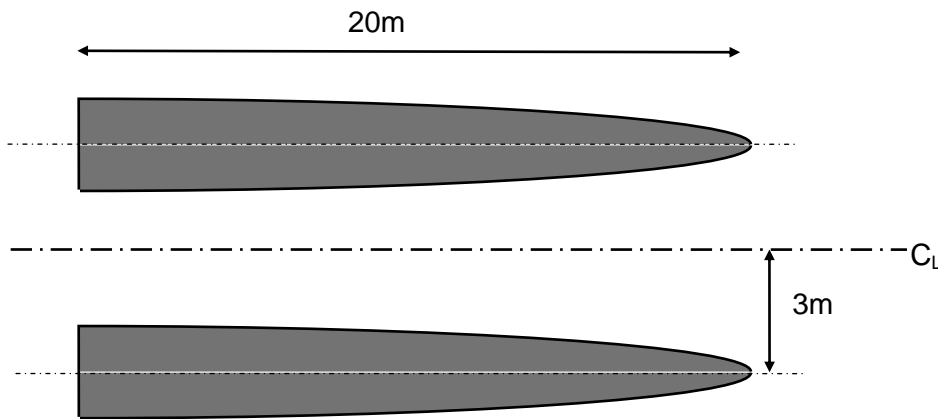


Figure 1 Catamaran Waterplane

Question 4

A ship LBP 90m, lebar 17.2 m is floating in seawater. At 5m draught, the waterplane has the following offsets.

Stn.	AP	1	2	3	4	5	6	7	8	9	FP
½ breadth (m)	0.0	5.5	8.0	8.4	8.5	8.6	8.5	8.0	7.0	4.5	0.0

The ship has the following waterplane area at other draughts:

Draught (m)	0.0	0.5	1.0	2.0	3.0	4.0
Waterplane Area(m ²)	10	500	800	1100	1200	1260

Using all the information, calculate for draught of 5.0m:

- i. Waterplane Area
- ii. Second moment of area about the centreline.
- iii. Mass displacement.
- iv. Block Coefficient C_b
- v. Height of Metacentre, KM_T

Question 5

The cross-section of a tank can be represented by a plane curve and two radii 90° apart as shown in Figure 2. The lengths of the radii at equal angle intervals are 12,14, 16,18, and 20 metres respectively. Calculate the area of the cross-section.

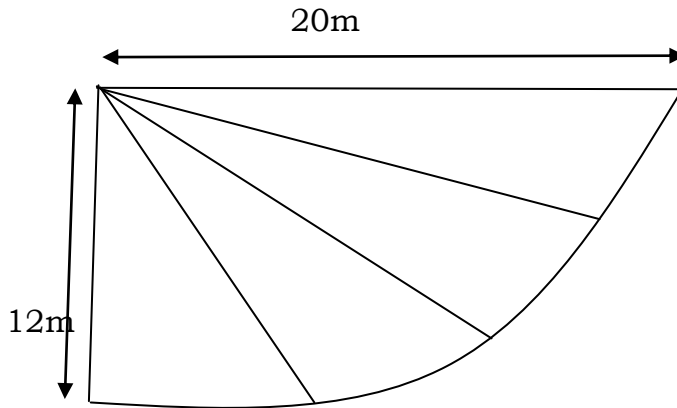


Figure 2

Question 6

a. Sebuah lengkung dinyatakan seperti berikut:

$$y = 2 + 3x + 4x^2$$

Tentukan luas di bawah lengkung yang disempadani oleh $x = 0$ to $x = 4$ dan paksi x menggunakan kaedah:

- i) Simpsons Pertama
- ii) Simpsons Kedua
- iii) Trapezoid
- iv) Pengamiran

Berikan komentar terhadap keputusan yang diperolehi.

b. Ofset bagi sebuah kapal LBP 60m adalah seperti berikut:

Stesen	0 (AP)	1	2	2 ½	3	4	5 (FP)
Separuh Lebar (m)	0.5	1.4	2.6	4.3	5.4	6.6	7.0

Kirakan :

- i) Luas Satahair
- ii) LCF dari peminggang
- iii) Momen luas kedua melintang pada garis tengah.
- iv) Momen luas kedua membujur pada pusat keapungan.

