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PhD CO-DIRECTED THESIS

by

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Fission-barriers and energy spectra of odd-mass actinide nuclei in self-consistent mean-field calculations

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To my parents, my wife and our soon-to-be born child.

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ABSTRACT

While there have been numerous microscopic calculations on fission barriers of even-mass compound nuclei, there are however, relatively few such work dedicated to odd-mass nuclei. This is due to the complications posed by the breaking of the time-reversal symmetry at the mean-field level due to the presence of an unpaired nucleon. In order to circumvent this difficulty, previous fission-barrier calculations of odd-mass nuclei have been performed by neglecting the effect of time-reversal symmetry breaking. This work aims to improve on the description of fission barriers as well as the spectroscopic properties of ground and fission-isomeric state, of some odd-mass actinide nuclei by taking the effect of time-reversal symmetry breaking into account. This has been performed within a Skyrme-Hartree-Fock-plus-BCS framework with blocking, where the BCS formalism has been adapted to accommodate this symmetry breaking. The Skyrme nucleon-nucleon effective force has been used with various sets of parameters (SIII, SkM*, SLy5*). The residual pairing interaction has been approximated by seniority forces whose neutron and proton parameters have been fitted to reproduce the odd-even mass differences of some actinide nuclei. The low-lying rotational band-head energies evaluated within the Bohr-Mottelson unified model have been determined for four well-deformed odd-nuclei (^{235}U , ^{239}Pu , ^{237}Np , ^{241}Am) yielding a good qualitative agreement to the data for odd-neutron nuclei. The agreement was significantly less good for the odd-proton nuclei, possibly due to the use of the Slater approximation for the exchange Coulomb interaction. The deformation energies of two odd-neutron nuclei (^{235}U and ^{239}Pu) have been calculated for some single-particle configurations up to a point beyond the outer fission-barrier. Axial symmetry nuclear shape has been assumed while a breaking of the left-right (or intrinsic parity) symmetry has been allowed around the outer fission-barrier. The fission-barrier heights of such odd-neutron nuclei depend significantly on the particle configurations. A special attention has been paid to the very important rotational correction to deformation energies. In particular, the correction of the moment of inertia calculated from the usual Belyaev expression was considered. Overall, a qualitative agreement with available data on fission-barrier heights for the considered odd-neutron nuclei and their even neighbours has been obtained.

RÉSUMÉ

Alors qu'il existe de nombreux calculs microscopique de barrières de fission pour des noyaux composé de masse paire, il n'y a cependant que relativement peu de tels calculs pour des noyaux de masse impaire. Ceci est dû aux complications induites par la brisure de la symétrie de reversement du sens du temps au niveau du champ moyen qui est engendrée par la présence d'un nucleon non-apparié. Pour éviter cette difficulté, des calculs existants pour des noyaux de masse impaire ont tout simplement négligé ces effets de brisure de la symétrie de reversement du sens du temps. Dans ce travail, on se donne pour but d'améliorer la description des barrières de fission, aussi bien que des propriétés spectroscopiques du niveau fondamental et de l'état isomérique de fission, pour quelques isotopes de masse impaire dans la région des actinides en prenant en compte de tels effets. Ceci a été réalisé dans le cadre du formalisme de Skyrme–Hartree–Fock plus BCS avec blocking en adaptant ce formalisme à la brisure de la symétrie considérée. L'interaction résiduelle d'appariement a été approchée par une force de séniorité dont les paramètres ont été ajustés pour reproduire les différences de masse pair-impair de quelques noyaux de la région des actinides. Les énergies des têtes de bande rotationnelle de basse énergie ont été calculées dans le cadre du modèle unifié de Bohr-Mottelson pour quatre noyaux bien déformés (^{235}U , ^{239}Pu , ^{237}Np , ^{241}Am) produisant un bon accord qualitatif avec les données pour les noyaux impairs en neutrons. L'accord significativement moins bon obtenu pour les noyaux impairs en protons pourrait résulter de l'usage de l'approximation de Slater pour l'interaction d'échange de Coulomb. Les énergies de déformation de deux noyaux impairs en neutrons (^{235}U , ^{239}Pu) ont été calculées pour quelques configurations de particule individuelle, jusqu'à après la barrières de fission externe. La symétrie axiale a été imposée tandis que la brisure de la symétrie droite-gauche (ou de parité intrinsèque) a été permise dans la région de la seconde barrière. Les hauteurs des barrières de fission pour ces noyaux impairs dépendent significativement des configurations de particule individuelle. Un accord qualitatif avec les données disponibles pour les hauteurs de barrières des noyaux impairs considérés et leurs voisins pairs a été généralement obtenu.

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