Application of matrix in electronics – Approach 1

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1 Deriving linear equations representing current through resistors

Let us take as example an electronic circuits given in Figure 1. We shall denote I as the current going through resistor R_1 while I_1 and I_2 are currents going through R_2 and R_3, respectively. The total current is

$$I = I_1 + I_2. \tag{1}$$

This is an easy circuit to solve. You will find that

$$I_1 = 1.364A \quad \text{and} \quad I_2 = 0.909A \tag{2}$$

with

$$I = 1.364 + 0.909 = 2.273A. \tag{3}$$

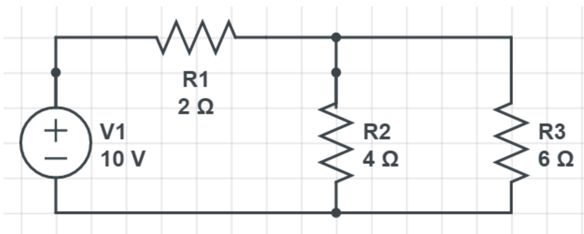

Figure 1

Now we want to solve this circuit using matrix. The purpose is to show the idea behind using matrix to solve such easy examples first, before using it for more difficult ones. Let us start.

We know from our electronics course that the sum of voltage across R_1 and R_2 is equal to 10 V. Similarly, the sum of voltage across R_1 and R_3 is also equal to 10 V – the voltage across R_2 and R_3 is the same. We can then write two equations:

$$2I + 4I_1 = 10 \tag{4}$$

$$2I + 6I_2 = 10. \tag{5}$$

Using eq. (1) we have

$$6I_1 + 2I_2 = 10 \tag{6}$$

$$2I_1 + 8I_2 = 10. \tag{7}$$

These equations can be written in matrix form

$$\underbrace{\begin{pmatrix} 6 & 2 \\ 2 & 8 \end{pmatrix}}_{M} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}. \tag{8}$$

In order to find I_1 and I_2, we just have to multiply the inverse of the matrix M with the column matrix on the right

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = M^{-1} \begin{pmatrix} 10 \\ 10 \end{pmatrix}. \tag{9}$$

2 The idea for building the matrix M

We are going to discuss the idea behind the matrix M, hoping that this would ease calculations for similar circuits. Let us recall the elements for the matrix M is:

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}. \tag{10}$$

Remember that we want to find I_1 and I_2. This is an important piece of information! The element M_{11} is then the sum of resistance that will be transversed by the currents I and I_1.[1] The sum of resistance for this Circuit 1 is 6.

Next the element M_{22} refers to the sum of resistance of Circuit 2[2] which is a total of 8 Ω.

Going to the non-diagonal elements M_{12} and M_{21}. The values that will enter these elements are the sum of resistance <u>common</u>[3] to both Circuit 1 and Circuit 2 that we considered earlier. Here the total resistance is 2 Ω.

Putting all these values into a matrix, we have

$$\begin{pmatrix} 6 & 2 \\ 2 & 8 \end{pmatrix} \tag{11}$$

[1]Take the circuit from the voltage supply to R_1 and R_2 as Circuit 1.
[2]That is the outer circuit excluding R_2.
[3]i.e. both currents goes through the same resistor(s).

3 Some exercises

Let us try to apply this idea to solve two cases. Figure 2 is exactly the same as the above except that the values for resistance have been changed, Find the current through R_2 and R_3, and the total current in the circuit.

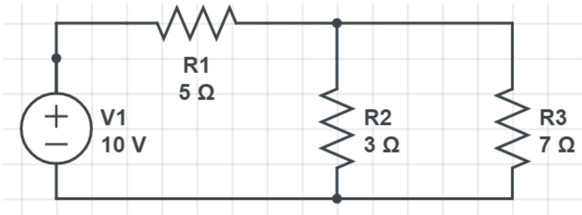

Figure 2

Next try to solve for a circuit in Figure 3 where a fourth resistor is added to the circuit. Find the current through resistor R_2 and R_3, and the total current in the circuit.

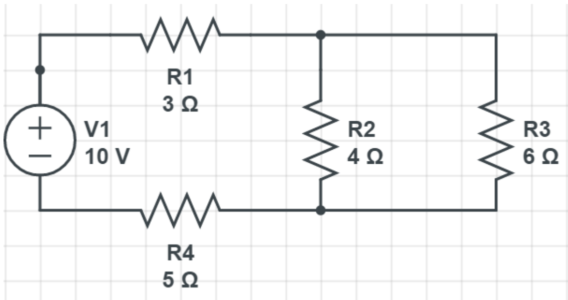

Figure 3

Answers:

$0.986A; 0.423A; 1.409A$

$0.577A; 0.385A; 0.962A$

4 Example of 3 parallel resistors

Let us now try to solve for a circuit as show in Figure 4. We will stick to the notation of previous example where I_1, I_2 and I_3 are currents going through resistor R_2, R_3 and R_4.

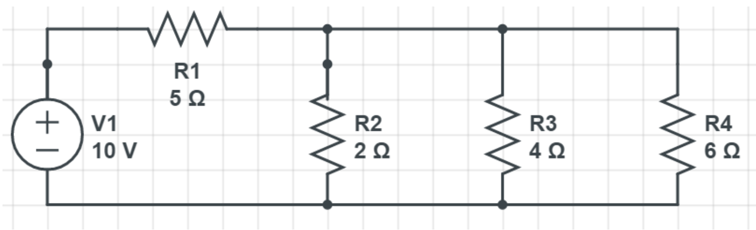

Figure 4

To solve for I_1, I_2 and I_3, we write in matrix form

$$\begin{pmatrix} 7 & 5 & 5 \\ 5 & 9 & 5 \\ 5 & 5 & 11 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \\ 10 \end{pmatrix}. \tag{12}$$

Here the values of elements M_{13} and M_{31} are the same as M_{12} and M_{21} since the there is only one resistor common to both Circuit 2[4] and Circuit 3[5]. Solving the matrix, one obtains:

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 0.896 \\ 0.448 \\ 0.299 \end{pmatrix}. \tag{13}$$

The total current in the circuit is

$$0.896 + 0.448 + 0.299 = 1.643A. \tag{14}$$

[4]Circuit with the voltage supply, R_1 and R_3 only.
[5]Circuit with the voltage supply, R_1 and R_4 only.

5 More exercises

Consider now Figure 5 where a fifth resistor has been addded. Find the current through the resistors R_2, R_3 and R_4, and the total current in the circuit.

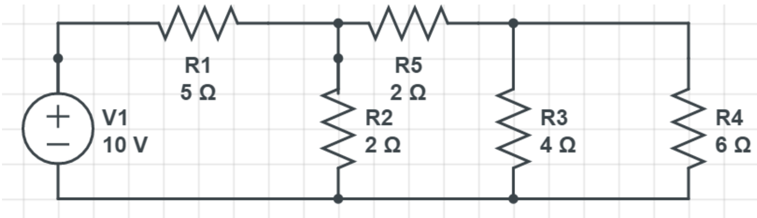

Figure 5

Figure 6 shows a circuit with two voltage supplies. Find the current through each resistor.

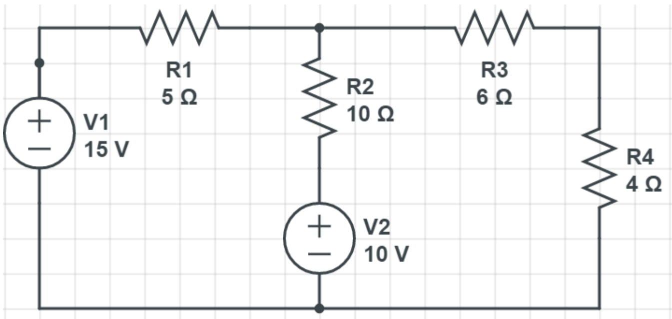

Figure 6

Note the positive and negative terminals of the voltage supplies – is it $V_1 - V_2$ or $V_1 + V_2$?