Application of matrix in electronics – Approach 2

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In a previous write-up, I have introduced to you one way to solve electronics circuits by using a matrix approach. We will now consider another matrix approach to solve electronics circuits. We are still going to build the matrix M but through a slightly different perspective.

1 Example of two parallel resistors

Let us consider the first example in Figure 1. As we have done before, we are going to find the current through each resistor.

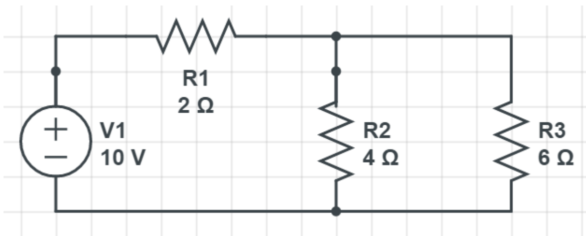

Figure 1

To build the matrix M, we will imagine the left loop (consisting of the voltage supply, resistor R_1 and R_2) as Loop 1 with current I_1 and the right loop (consisting of resistor R_2 and R_3 only) as Loop 2 with current I_2. The currents I_1 and I_2 are sketched in Figure 2.

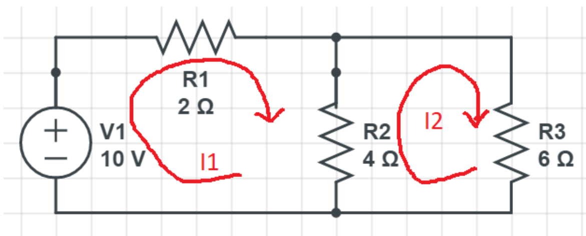

Figure 2

In matrix M the element M_{11} is the sum of resistance in Loop 1. The element M_{22} is the sum of resistance in Loop 2. This gives us the matrix M

$$\begin{pmatrix} 6 & M_{12} \\ M_{21} & 10 \end{pmatrix} \tag{1}$$

For the non-diagonal elements, the values are the negative of the sum of resistances shared by both Loop 1 and Loop 2. In this case, there is only one resistor i.e. R_2. Therefore the matrix M is now

$$\begin{pmatrix} 6 & -4 \\ -4 & 10 \end{pmatrix} \tag{2}$$

Let us now look at the column matrix on the right-hand side which refers to the total voltage of Loop 1 and Loop 2. The total voltage in Loop 1 is 10 V, while it is 0 in Loop 2. Putting all these together, we have the matrix equation to solve:

$$\begin{pmatrix} 6 & -4 \\ -4 & 10 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}. \tag{3}$$

We can solve this immediately by taking the inverse of the square matrix and multiply it with the column matrix on the right-hand side. We get the answers:

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 2.2727 \\ 0.9091 \end{pmatrix} \tag{4}$$

Here $I_1 = 2.2727A$ refers to the total current in the circuit, and $I_2 = 0.9091A$ is the amount of current through R_3. The amount of current through R_2 is

$$I_1 - I_2 = 2.2727 - 0.9091 = 1.3636A. \tag{5}$$

Compare this to the earlier approach which we took to solve the circuit here.

2 Using determinants to solve for currents

Instead of finding the inverse of the matrix M, we can also solve for the current I_1 and I_2 using determinants.

From eq. (3) we will find three sets of determinants.

- That which would be the denominator:

$$\begin{vmatrix} 6 & -4 \\ -4 & 10 \end{vmatrix} = 44 \tag{6}$$

- Which will give us the current I_1 later on:

$$\begin{vmatrix} 10 & -4 \\ 0 & 10 \end{vmatrix} = 100 \tag{7}$$

- And the last which will give us I_2 later on:

$$\begin{vmatrix} 6 & 10 \\ -4 & 0 \end{vmatrix} = 40 \tag{8}$$

Notice how the matrix in equations (7) and (8) are arranged from equatio (3). Now to get the currents I_1 and I_2 we divide these two values with a denominator given by equation (6)

$$I_1 = \frac{100}{44} = 2.2727A \tag{9}$$

$$I_2 = \frac{40}{44} = 0.909A. \tag{10}$$

3 Example with 3×3 matrix

Let us now consider a circuit given in Figure 3.

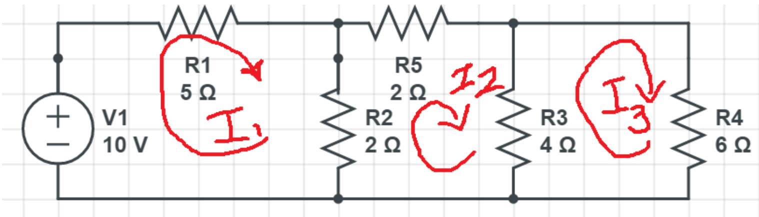

Figure 3

The matrix equation to be solved is

$$\begin{pmatrix} 7 & -2 & 0 \\ -2 & 8 & -4 \\ 0 & -4 & 10 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}. \tag{11}$$

We have four sets of determinants in this case:

- The denominator:

$$\begin{pmatrix} 7 & -2 & 0 \\ -2 & 8 & -4 \\ 0 & -4 & 10 \end{pmatrix} = 408 \tag{12}$$

- The determinant to find I_1

$$\begin{pmatrix} 10 & -2 & 0 \\ 0 & 8 & -4 \\ 0 & -4 & 10 \end{pmatrix} = 640 \tag{13}$$

- The determinant to find I_2

$$\begin{pmatrix} 7 & 10 & 0 \\ -2 & 0 & -4 \\ 0 & 0 & 10 \end{pmatrix} = 200 \tag{14}$$

- The determinant to find I_3

$$\begin{pmatrix} 7 & -2 & 10 \\ -2 & 8 & 0 \\ 0 & -4 & 0 \end{pmatrix} = 80 \tag{15}$$

Dividing the determinants with the denominator (12), we get $I_1 = 1.569A$, $I_2 = 0.490A$

and $I_3 = 0.196A$. The current going through R_2 is

$$1.569 - 0.490 = 1.079A \tag{16}$$

and the current through R_3 is

$$0.490 - 0.196 = 0.294A. \tag{17}$$

The current through R_4 is simply $0.196A$.

4 Exercises

Find the current through the resistors in the following circuits.

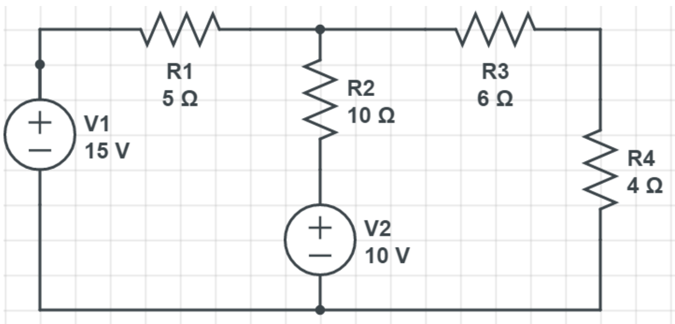

Figure 4

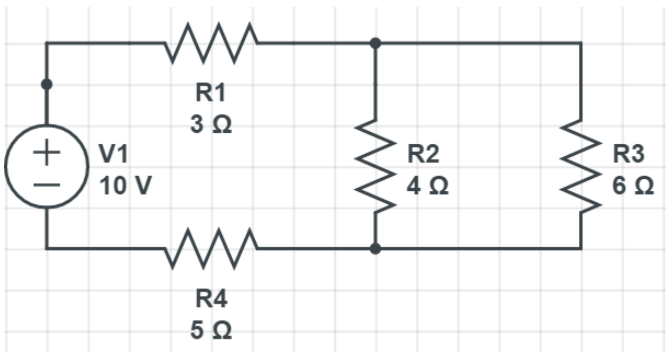

Figure 5