

[Part 3]
Integrity, Authenticity, and Key
Management

Chapter 15

Key Management

Forouzan, B.A. Cryptography and Network Security (International Edition). United States: McGraw Hill, 2008.

Chapter 15 Objectives

- To explain the need for a *Key-Distribution Center* (KDC).
- To show how a KDC can create a session key between two parties.
- To show how two parties can use a symmetric-key agreement protocol to create a session between themselves without using the services of a KDC.

15.2

| | |
|---|----------|
| Chapter 15 | Contents |
| 15.1 Introduction | |
| 15.2 Symmetric-Key Distribution | |
| 15.3 Symmetric-Key Agreement: <i>Diffie-Hellman</i> | |
| 15.4 Summary | |

15.3

| | |
|------------|-------------------|
| Chapter 15 | 15.1 Introduction |
|------------|-------------------|

- In previous chapters, we have discussed symmetric-key and asymmetric-key cryptography.
- However, we have not discussed yet on how *secret key* in symmetric-key cryptography, and *public key* in asymmetric-key cryptography are distributed and maintained.

15.4

Chapter 15 15.1 Introduction

- This chapter will touches the *secret key* in symmetric-key cryptography only.
- The discussion will be on:
 - the distribution of symmetric keys using a trusted third party.
 - How two parties can establish a symmetric key between themselves without using a trusted third party.

15.5

Chapter 15 15.1 Introduction

Key Management

- Key management is the way how we manage the cryptographic keys in a cryptosystem.
- This may includes the dealing with the key generation, key exchange, key storage, key use, and key replacement.
- Successful key management is critical to the security of a cryptosystem.

https://en.wikipedia.org/wiki/Key_management

15.6

| | |
|---|----------|
| Chapter 15 | Contents |
| 15.1 Introduction | |
| 15.2 Symmetric-Key Distribution | |
| 15.3 Symmetric-Key Agreement: <i>Diffie-Hellman</i> | |
| 15.4 Summary | |

15.7

| | |
|------------|---------------------------------|
| Chapter 15 | 15.2 Symmetric-Key Distribution |
|------------|---------------------------------|

- Symmetric-key cryptography is more efficient than asymmetric-key cryptography for encrypting large messages.
- Symmetric-key cryptography, however, needs a shared secret key between two parties.
 - For n people, we need:
 $n(n-1)/2$ shared secrets.
- **Problems:**
 - The **number of keys**;
 - The **distribution of keys** is another **problem**.
- **Solution:** Use a trusted third party.

15.8

Chapter 15 15.2 Symmetric-Key Distribution
Key-Distribution Center (KDC)

- KDC is a practical solution that use a trusted third party.
- To reduce the number of keys, each person establishes a shared secret key with the KDC.

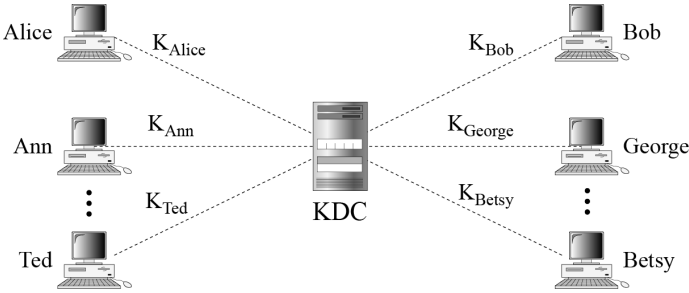


Figure 15.1: Key-distribution center (KDC)

15.9

Chapter 15 15.2 Symmetric-Key Distribution

- (Figure 15.1)
- A secret key is established between the KDC and each members.

- Alice can sends a confidential message to Bob with the following process:
 - ① Alice sends a request to the KDC stating that she needs a session (temporary) secret key with Bob.
 - ② The KDC informs Bob about Alice's request.
 - ③ If Bob agrees, a session key is created between them.

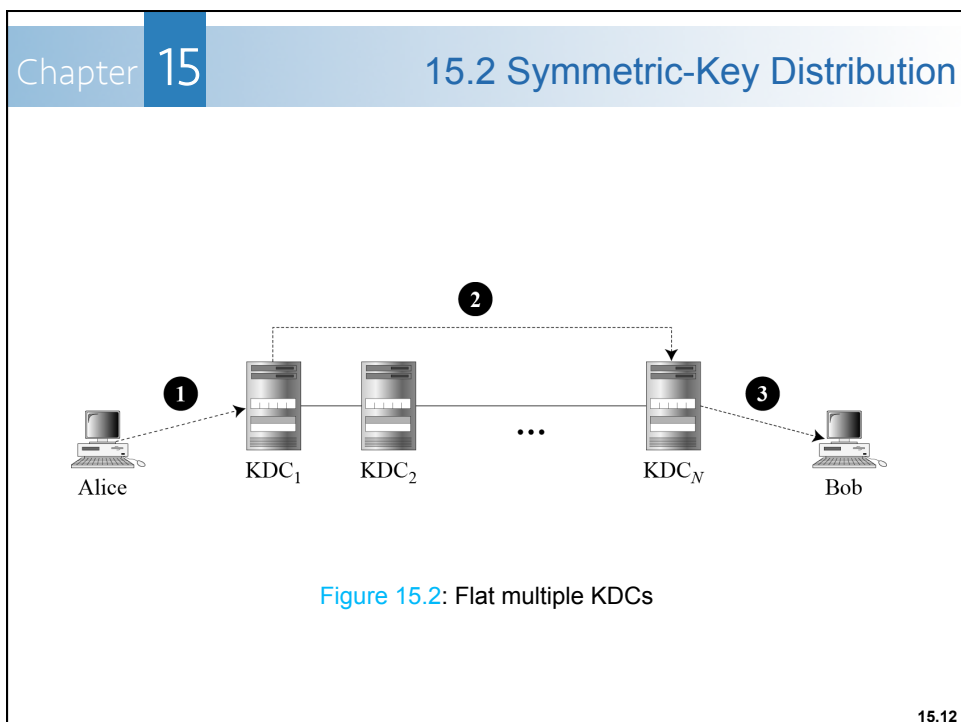
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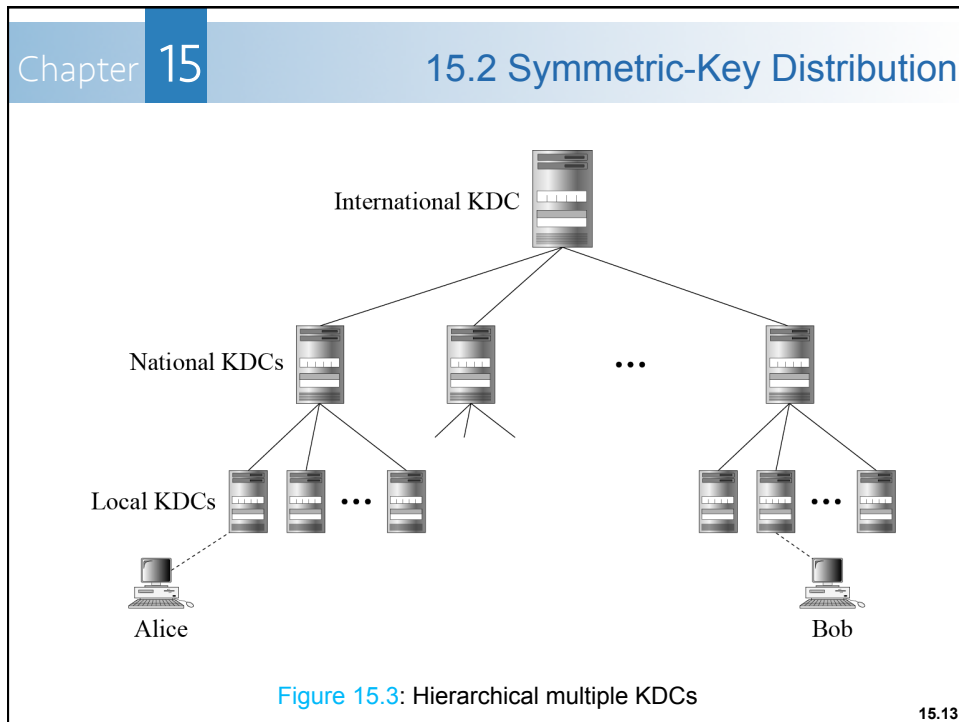
Chapter 15 15.2 Symmetric-Key Distribution

- When the number of people using a KDC increases, the system becomes **unmanageable** and a **bottleneck** can result.
- **Solution:** Multiple KDCs.

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graph TD; A[Types of KDC] --> B[Flat Multiple KDCs]; A --> C[Hierarchical Multiple KDCs];
```

15.11





Chapter 15 15.2 Symmetric-Key Distribution

Session Keys:

- A KDC creates a secret key for each member.
- This secret key can be used only between the member and the KDC, not between two members.

Note

A session symmetric key between two parties is used only once.

15.14

| | |
|-------------------------------------|----------|
| Chapter 15 | Contents |
| 15.1 Introduction | |
| 15.2 Symmetric-Key Distribution | |
| 15.3 Symmetric-Key Agreement | |
| • <i>Diffie-Hellman</i> | |
| • Analysis of <i>Diffie-Hellman</i> | |
| • Security of <i>Diffie-Hellman</i> | |
| 15.4 Summary | |

15.15

| | |
|------------|------------------------------|
| Chapter 15 | 15.3 Symmetric-Key Agreement |
|------------|------------------------------|

- Sender and receiver can create a session key between themselves without using a KDC.
- This method of session-key creation is referred to as the *symmetric-key agreement*.
- There are several ways to accomplish this.
- Two common methods are *Diffie-Hellman* and station-to-station.
- However, we shall discuss the *Diffie-Hellman* only.

15.16

Chapter 15 15.3 Symmetric-Key Agreement

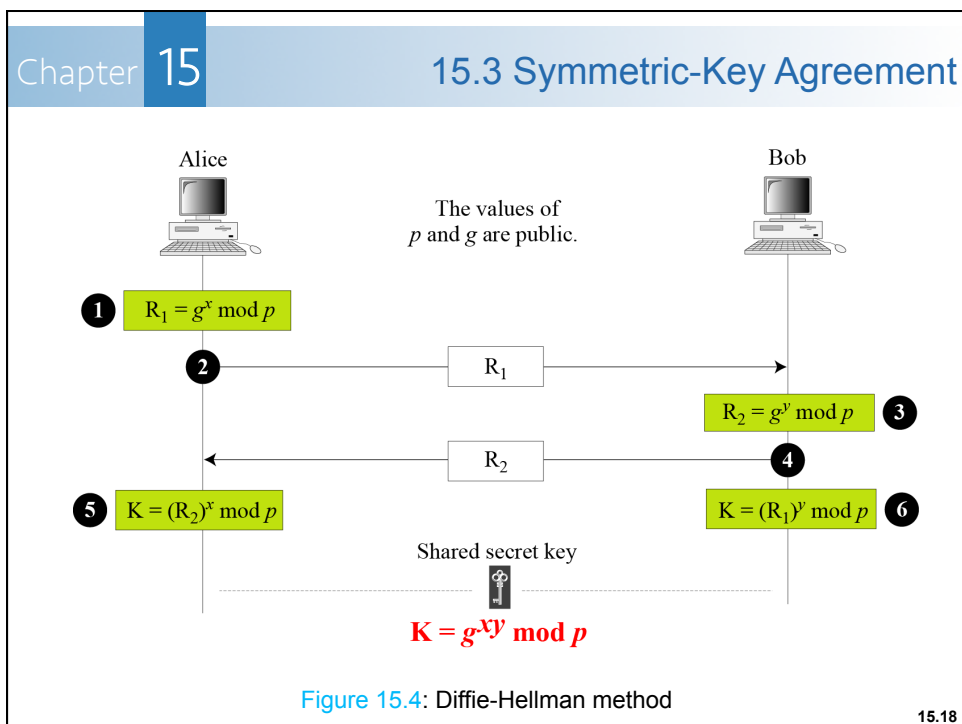
Diffie-Hellman (DH)

- In DH protocol, two parties create a symmetric session key without the need of a KDC.
- Before establishing a symmetric-key, the two parties need to choose two numbers :

| | |
|-----|---|
| p | A large prime number on the order of 300 decimal digits (1024 bits) |
| g | A generator of order $p - 1$ in the group $\langle Z_p^*, \times \rangle$ |

- These p and g do not need to be confidential.
- They can be sent through the Internet; can be public.

15.17



Chapter 15 15.3 Symmetric-Key Agreement

- **Figure 15.4** shows the procedure.
- The steps are as follows:
 - ① Alice chooses a large random number x such that $0 \leq x \leq (p - 1)$ and calculates $R_1 = g^x \text{ mod } p$.
 - ② Bob chooses another large random number y such that $0 \leq y \leq (p - 1)$ and calculates $R_2 = g^y \text{ mod } p$.
 - ③ Alice sends R_1 to Bob (but not sends the value of x).
 - ④ Bob sends R_2 to Alice (but not sends the value of y).
 - ⑤ Alice calculate $K = (R_2)^x \text{ mod } p$.
 - ⑥ Bob calculate $K = (R_1)^y \text{ mod } p$.

K is the symmetric key for the session.

Chapter 15 15.3 Symmetric-Key Agreement

- Bob has calculated K as:

$$R_1 = g^x \text{ mod } p$$

$$K = (R_1)^y \text{ mod } p = (g^x \text{ mod } p)^y \text{ mod } p = g^{xy} \text{ mod } p$$
- Alice has calculated K as:

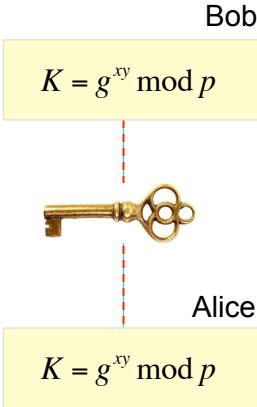
$$R_2 = g^y \text{ mod } p$$

$$K = (R_2)^x \text{ mod } p = (g^y \text{ mod } p)^x \text{ mod } p = g^{xy} \text{ mod } p$$

15.20

Chapter 15 15.3 Symmetric-Key Agreement

- Both have reached the same value without Bob knowing the value of x and without Alice knowing the value of y .



Bob

$$K = g^{xy} \bmod p$$

Alice

$$K = g^{xy} \bmod p$$

15.21

Chapter 15 15.3 Symmetric-Key Agreement

Example 15.1: Let us give a trivial example to make the procedure clear.

Our example uses small numbers, but note that in a real situation, the numbers are very large. Assume that $g = 7$ and $p = 23$.

The steps are as follows:

- ① Alice chooses $x = 3$ and calculates $R_1 = 7^3 \bmod 23 = 21$.
- ② Bob chooses $y = 6$ and calculates $R_2 = 7^6 \bmod 23 = 4$.
- ③ Alice sends the number 21 to Bob.
- ④ Bob sends the number 4 to Alice.
- ⑤ Alice calculates the symmetric key $K = 4^3 \bmod 23 = 18$.
- ⑥ Bob calculates the symmetric key $K = 21^6 \bmod 23 = 18$.

The value of K is the same for both Alice and Bob;
 $g^{xy} \bmod p = 7^{18} \bmod 23 = 18$.

Chapter 15 15.3 Symmetric-Key Agreement

Exercise 15.1: Given $p = 97$ and $g = 5$. Assume that Alice and Bob choose the private key as $x = 36$ and $y = 58$ respectively.

- Calculate Alice public key, R_A .
- Calculate Bob public key, R_B .
- Calculate the symmetric key K_A value for Alice.
- Calculate the symmetric key K_B value for Bob.
- Do they reach the same value of symmetric key?

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Chapter 15 15.3 Symmetric-Key Agreement

Exercise 15.2: Alice and Bob who wish to swap keys agree on $p = 353$ and $g = 3$. Assume that Alice and Bob choose the private key as $x = 97$ and $y = 233$ respectively.


- Calculate Alice public key, R_A .
- Calculate Bob public key, R_B .
- Calculate the symmetric key K_A value for Alice.
- Calculate the symmetric key K_B value for Bob.

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Chapter 15
15.3 Symmetric-Key Agreement

Exercise 15.3: Choose a partner in the class. Assume the prime $p = 97$ and $g = 5$.

- Each person select a random secret key that must less than p .
- Compute your public key.
- Tell your public key to your partner.
- Compute your shared session key.
- Check with your partner whether the keys are the same.



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<https://kellysico.files.wordpress.com/2014/06/two-people-speech-bubbles1.jpg>

15.25

Chapter 15
15.3 Symmetric-Key Agreement

Example 15.2: Let us give a more realistic example.

We used a program to create a random integer of 512 bits (the ideal is 1024 bits).

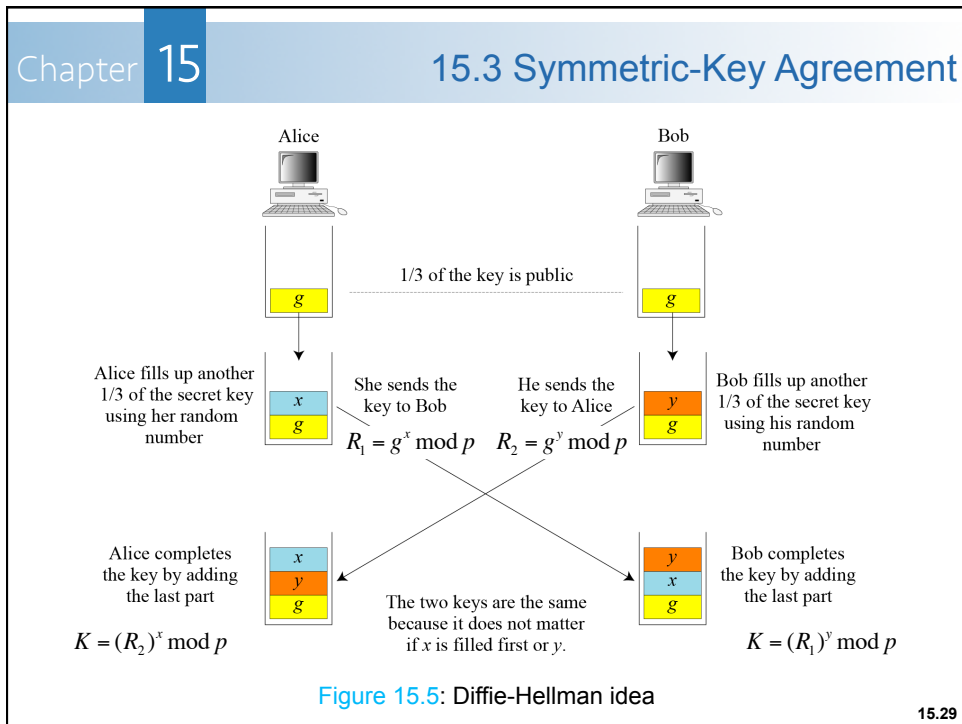
The integer p is a 159-digit number.

We also choose g , x , and y as shown below:

| | |
|-----|--|
| p | 764624298563493572182493765955030507476338096726949748923573772860925 235666660755423637423309661180033338106194730130950414738700999178043 6548785807987581 |
| g | 2 |
| x | 557 |
| y | 273 |

| Chapter 15 | 15.3 Symmetric-Key Agreement |
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| (Continued) | |
| The following shows the values of R_1 , R_2 , and K . | |
| R₁ | 844920284205665505216172947491035094143433698520012660862863631067673 619959280828586700802131859290945140217500319973312945836083821943065 966020157955354 |
| R₂ | 435262838709200379470747114895581627636389116262115557975123379218566 310011435718208390040181876486841753831165342691630263421106721508589 6255201288594143 |
| K | 155638000664522290596225827523270765273218046944423678520320400146406 500887936651204257426776608327911017153038674561252213151610976584200 1204086433617740 |

| Chapter 15 | 15.3 Symmetric-Key Agreement |
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| <p><u>Analysis of Diffie-Hellman:</u></p> <ul style="list-style-type: none"> • The DH concept is simple but elegant. • The secret key between Alice and Bob is made of three parts: g, x, and y. • However, 1/3 of the key is public: g. • The other 2/3 of the key must be added by Alice and Bob. | |
| Note | <p>Although the key in Alice's hand (g, y, and x) and the key in Bob's hand (g, x, and y), these two keys are the same because $g^{xy} = g^{yx}$</p> <p>Although the two keys are the same, Alice cannot find the value of y used by Bob because the calculation is done in modulo p.</p> |



Chapter 15 15.3 Symmetric-Key Agreement

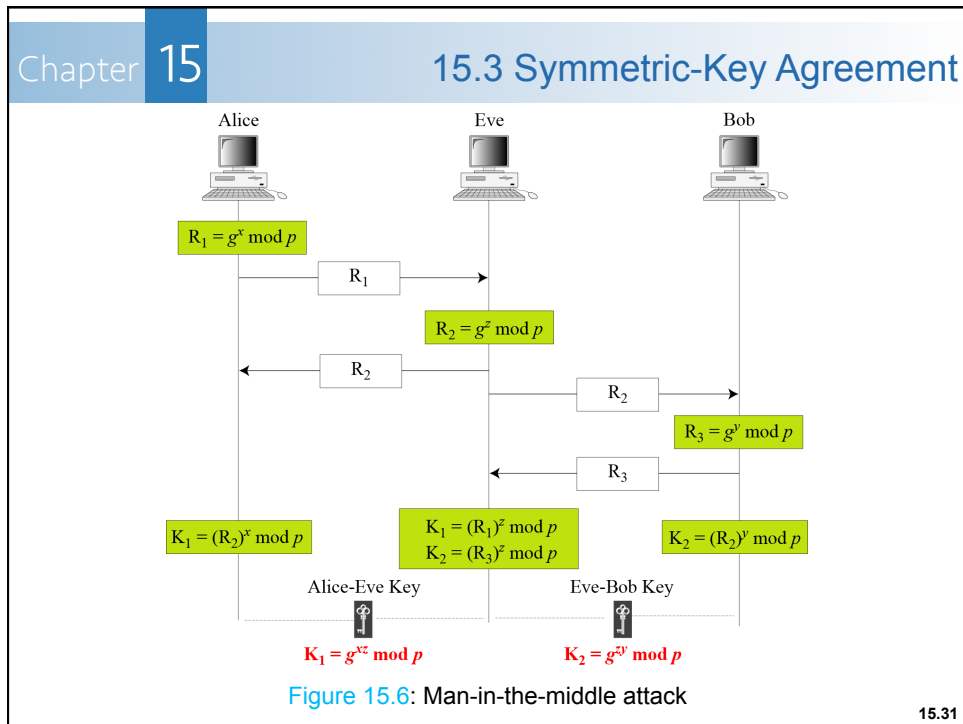
Security of Diffie-Hellman:

- The DH key exchange is susceptible to two attacks:

DH Attacks

- Discrete Logarithm Attack
- Man-in-the-Middle Attack

- Eve can intercept R_1 and R_2 .
- If the x and y found from R_1 and R_2 , the symmetric key K can be calculated.
- Eve does not have to find the value of x and y .
- Eve can fool Alice and Bob by creating two keys; one between herself and Alice, one between herself and Bob. (Figure 15.6)



Chapter 15 Contents

- 15.1 Introduction
- 15.2 Symmetric-Key Distribution
- 15.3 Symmetric-Key Agreement: *Diffie-Hellman*
- 15.4 Summary

15.32

Chapter 15 15.4 Summary

- Symmetric-key cryptography needs a shared secret key between two parties.
 - N people need $N(N-1)/2$ keys.
 - The problem are the number of keys and the distribution of the keys.
- A practical solution is the use of a trusted third party, referred to as a *Key-Distribution Center* (KDC).
 - Create a session key temporarily between Alice and Bob.
 - Their keys are used to authenticate themselves to the center.

15.33

Chapter 15 15.4 Summary

- Alice and Bob can create a session key between themselves without using KDC referred to as the *Symmetric-Key Agreement*.
 - One method discussed: *Diffie-Hellman* (DH).
 - However, DH is susceptible to the man-in-the-middle attack.

15.34

Chapter 15 Exercises

Exercise 15.4:

- What is the main duties of a KDC?
- Define a session key and show how a KDC can create a session key between Alice and Bob.
- Define the *Diffie-Hellman* protocol and its purpose.
- Define man-in-the-middle attack.

Forouzan, B.A. Cryptography and Network Security (International Edition), Singapore: McGraw-Hill, 2008. (page 463)

15.35

Chapter 15 Exercises

Exercise 15.5:

In the *Diffie-Hellman* protocol, $g = 7$, $p = 23$, $x = 3$, and $y = 5$.

- What is the value of R_1 and R_2 ?
- What is the value of the symmetric key?

Forouzan, B.A. Cryptography and Network Security (International Edition), Singapore: McGraw-Hill, 2008. (page 463)

Chapter 15Exercises

Exercise 15.6: In the *Diffie-Hellman* protocol, what happen if x and y have the same value, that is, Alice and Bob have accidentally chosen the same number? Are the R_1 and R_2 the same? Do the session keys calculated by Alice and Bob have the same value?

Use an example to prove your claims.

Forouzan, B.A. *Cryptography and Network Security (International Edition)*, Singapore: McGraw-Hill, 2008. (page 464)