Continuous Random Variables

Continuous Probability Distributions

- A continuous random variable may assume any numerical value in one or more intervals
- Use a continuous probability distribution to assign probabilities to intervals of values

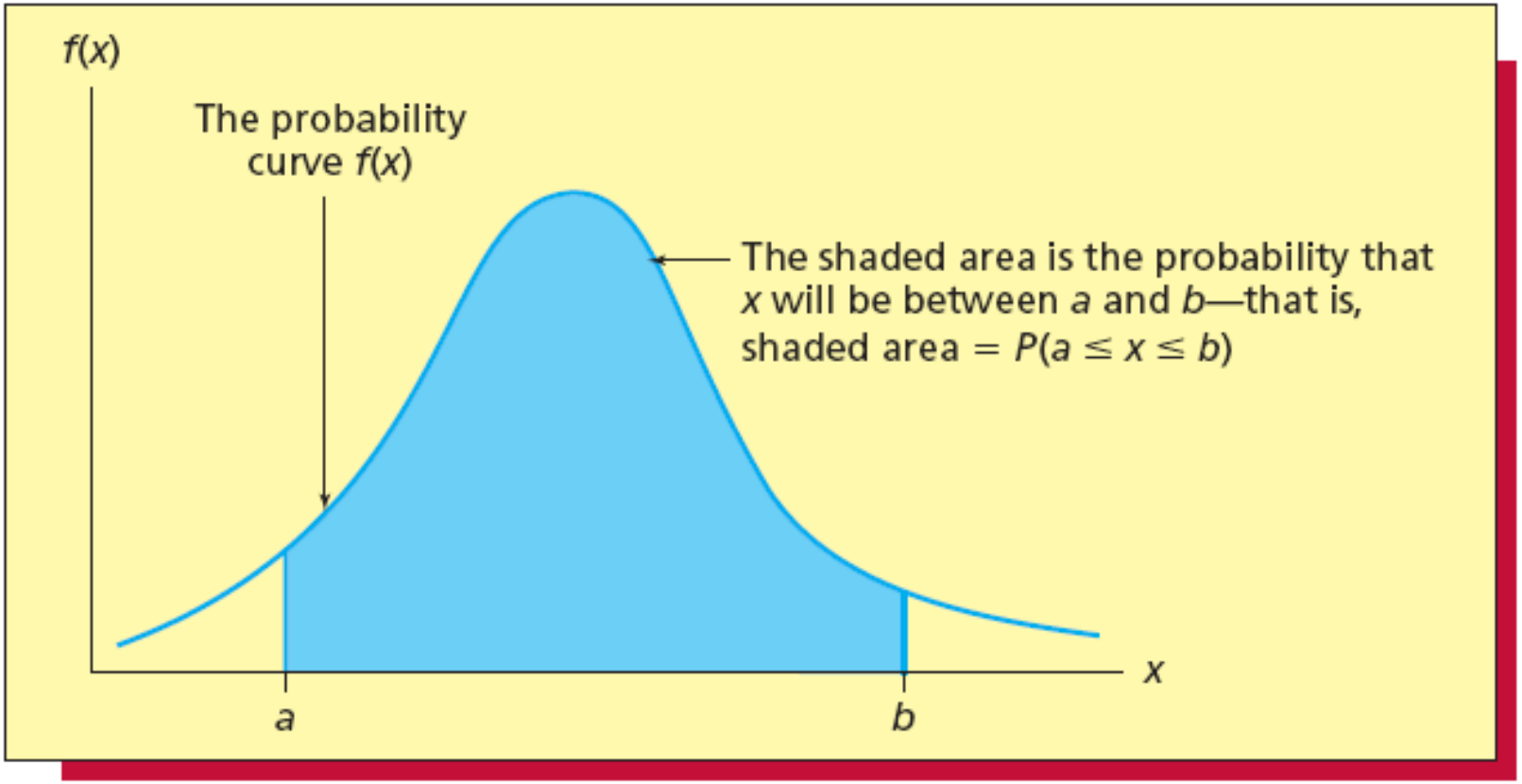
Continuous Probability Distributions Continued

- The curve f(x) is the continuous probability distribution of the continuous random variable x if the probability that x will be in a specified interval of numbers is the area under the curve f(x) corresponding to the interval
- Other names for a continuous probability distribution:
  - Probability curve
  - Probability density function

Properties of Continuous Probability Distributions

- Properties of f(x): f(x) is a continuous function such that
  1. $f(x) \geq 0$ for all x
  2. The total area under the curve of f(x) is equal to 1
- Essential point: An area under a continuous probability distribution is a probability

Area and Probability



The Uniform Distribution

$$\mathrm{f}(x)=\begin{cases}\dfrac{1}{d-c} & \text{for } c \le x \le d \\ 0 & \text{otherwise}\end{cases}$$

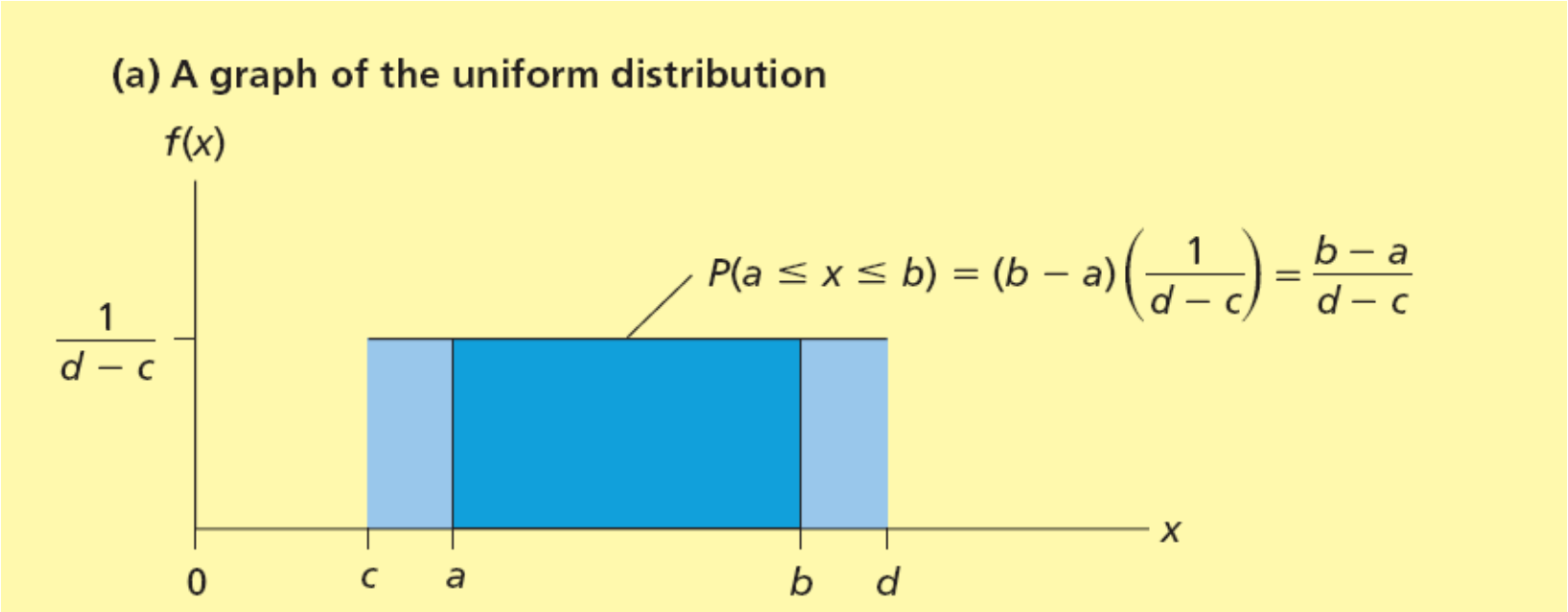
$$P(a \le x \le b) = \frac{b-a}{d-c}$$

The Uniform Distribution Mean and Standard Deviation

$$\mu_X = \frac{c+d}{2}$$

$$\sigma_X = \frac{d-c}{\sqrt{12}}$$

The Uniform Probability Curve



The Normal Probability Distribution

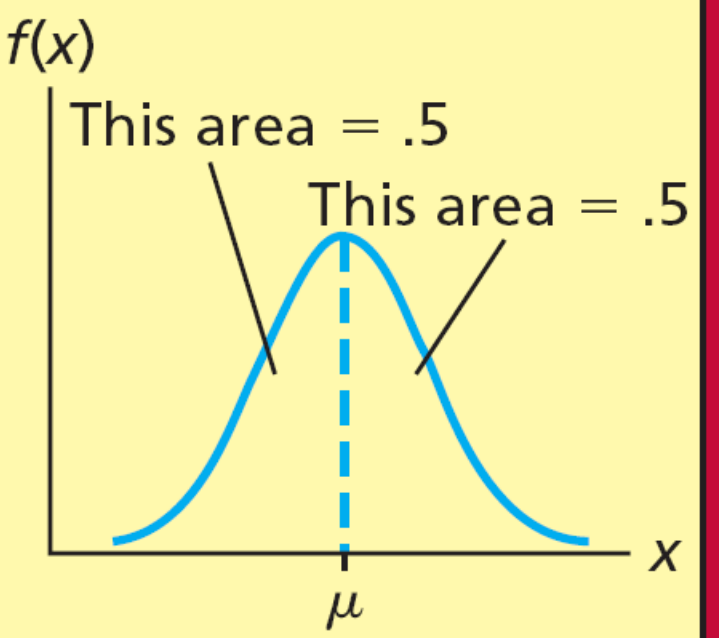
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

π = 3.14159

e = 2.71828

The Normal Probability Distribution Continued

The normal curve is symmetrical around μ, and the total area under the curve equals 1.



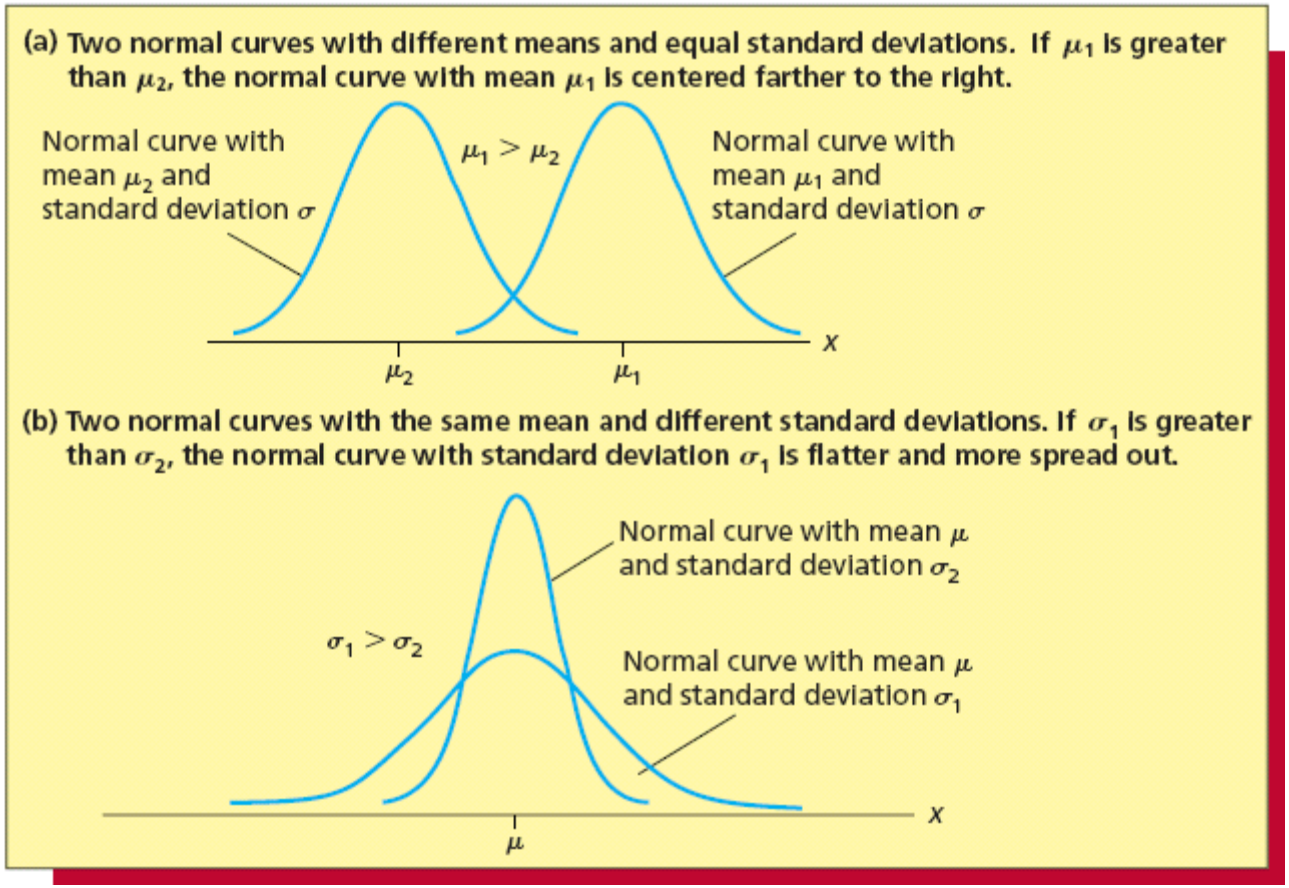
Properties of the Normal Distribution

1. There are an infinite number of normal curves
   - The shape of any individual normal curve depends on its specific mean and standard deviation
2. The highest point is over the mean
   - Also the median and mode

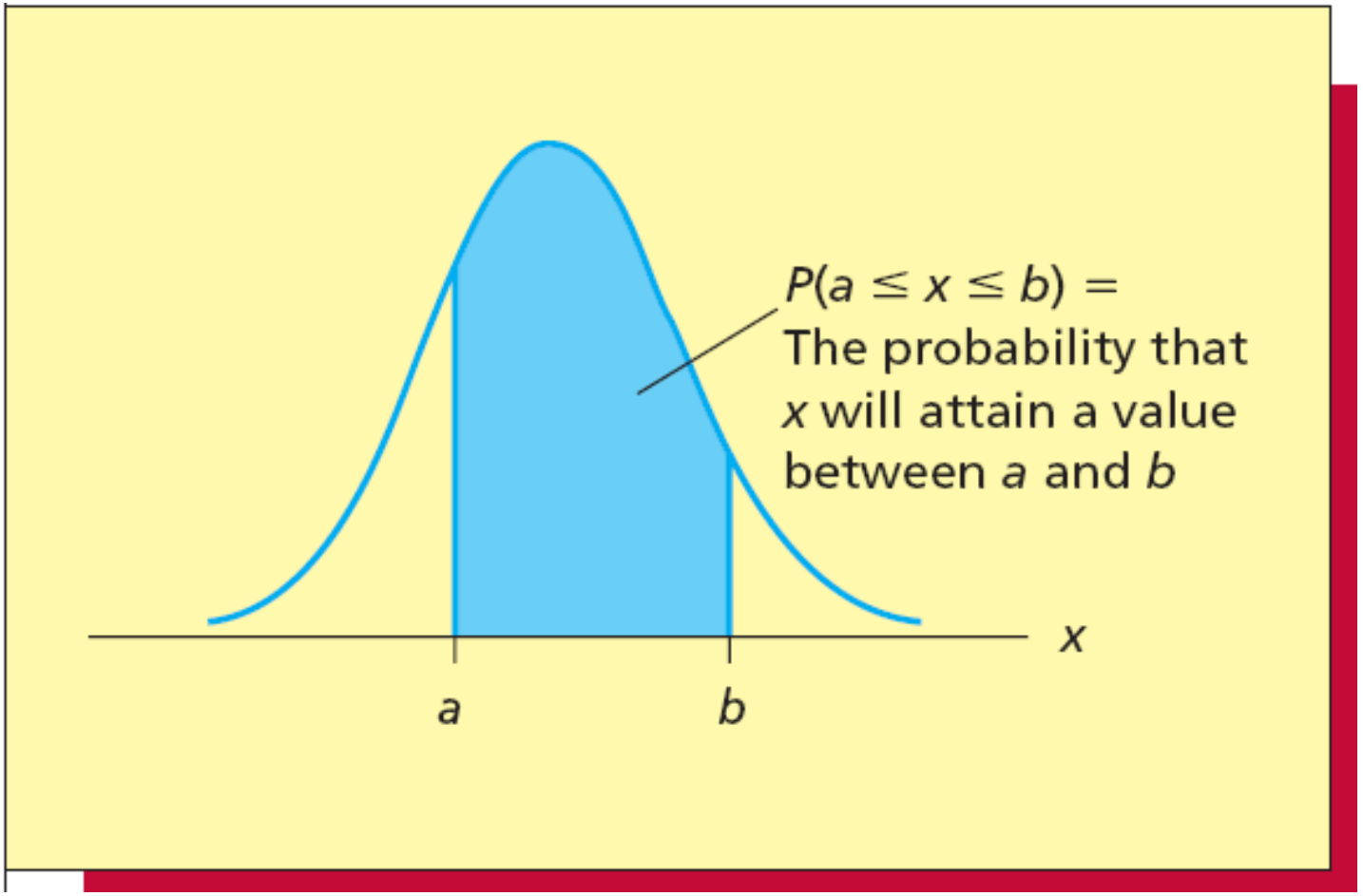
Properties of the Normal Distribution Continued

3. The curve is symmetrical about its mean
   - The left and right halves of the curve are mirror images of each other
4. The tails of the normal extend to infinity in both directions
   - The tails get closer to the horizontal axis but never touch it
5. The area under the normal curve to the right of the mean equals the area under the normal to the left of the mean
   - The area under each half is 0.5

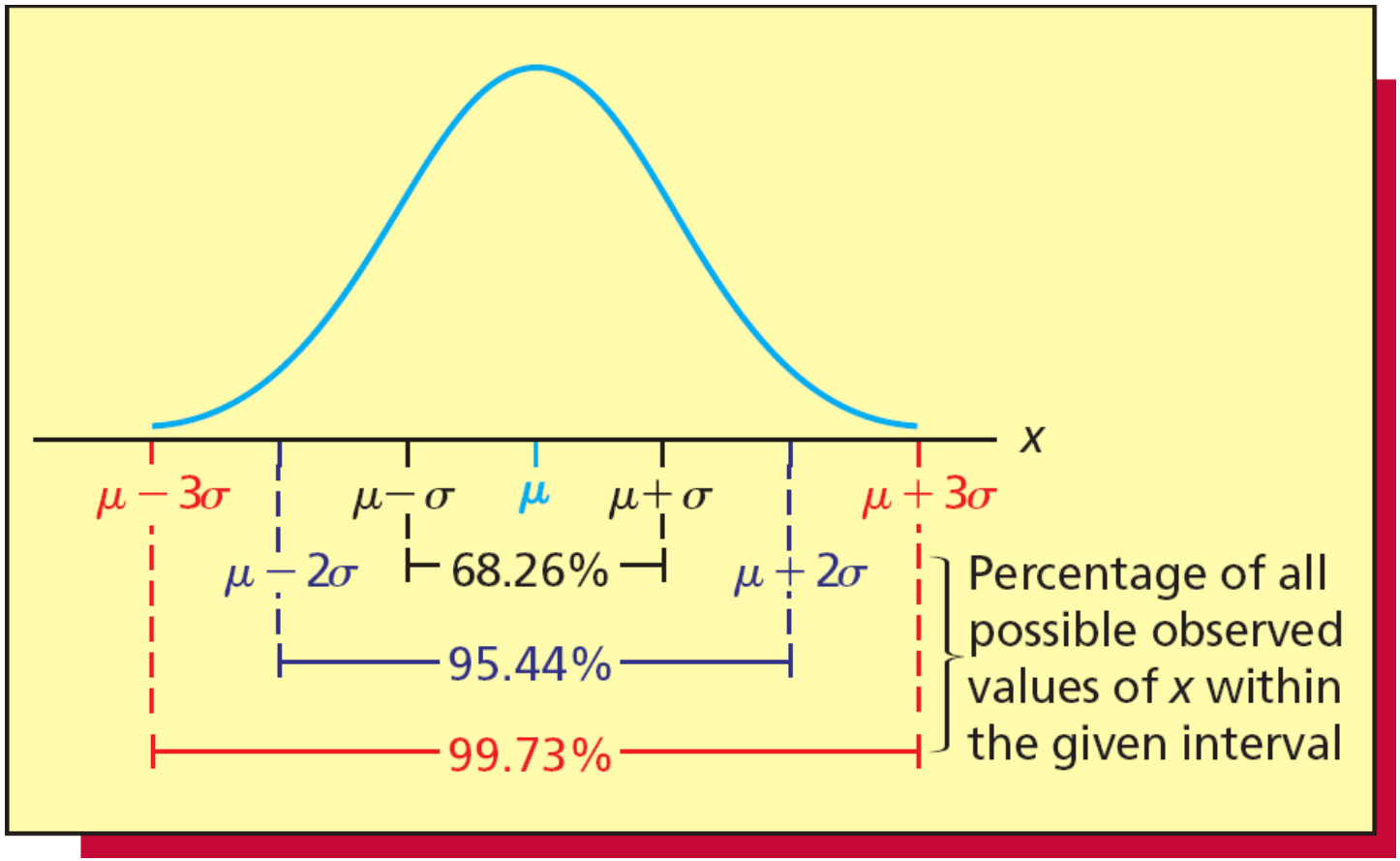
The Position and Shape of the Normal Curve



Normal Probabilities

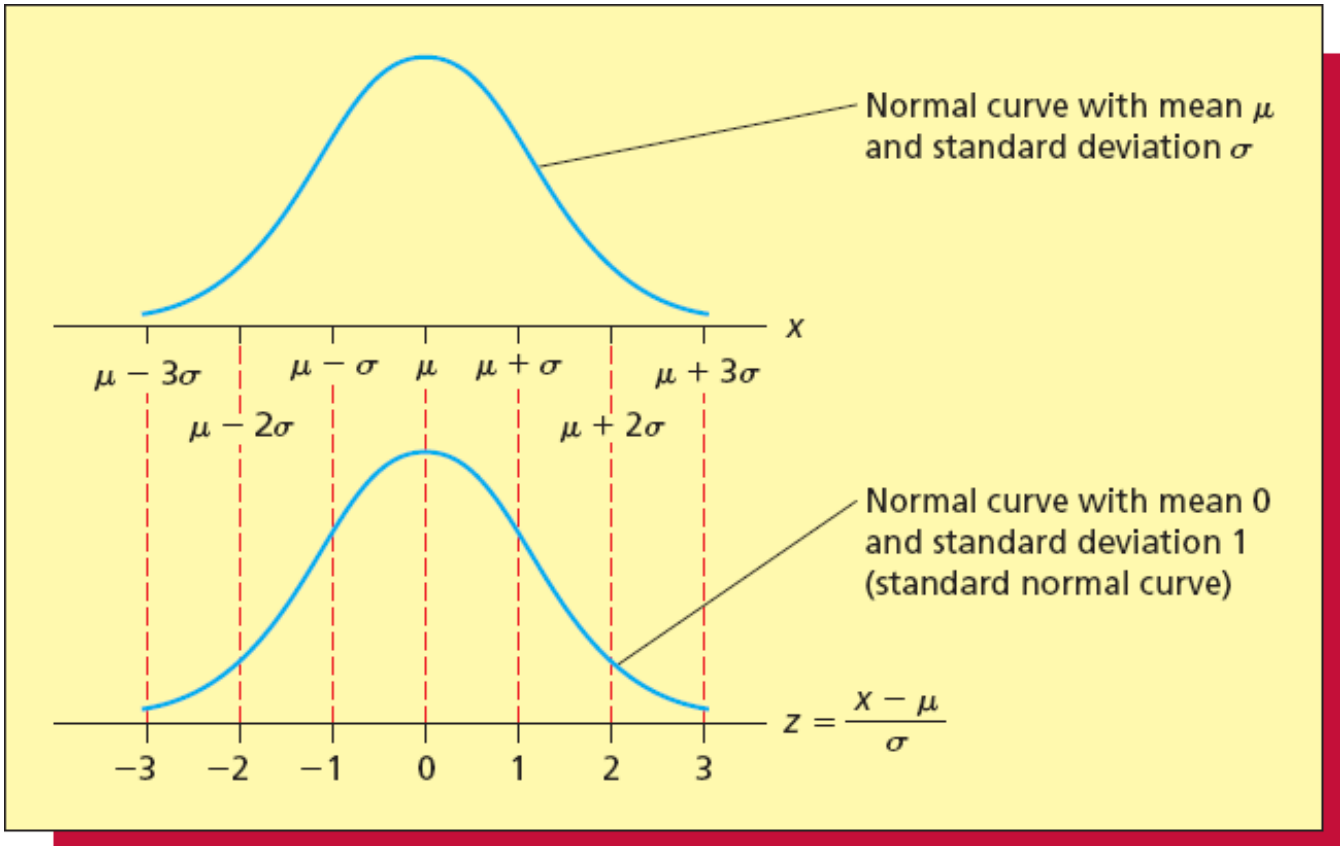


Three Important Percentages



Finding Normal Curve Areas

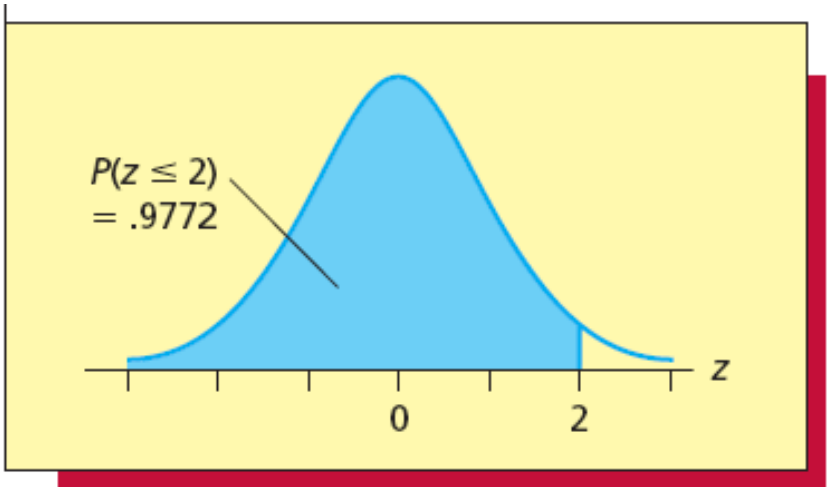
$$z = \frac{x - \mu}{\sigma}$$

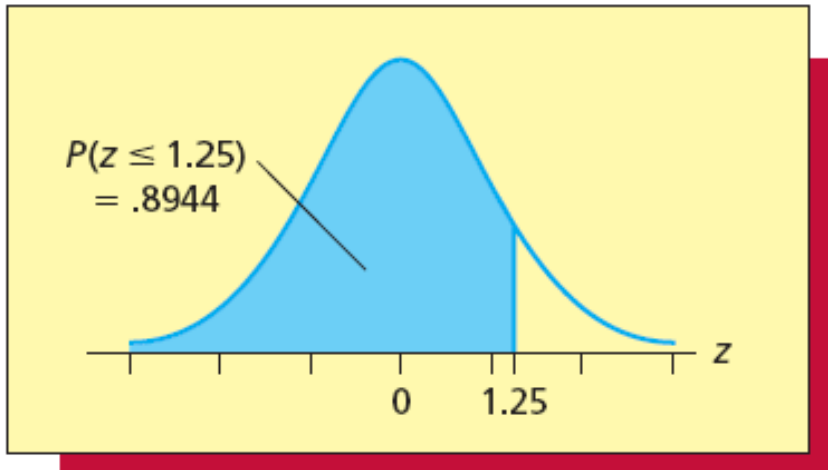


The Cumulative Normal Table

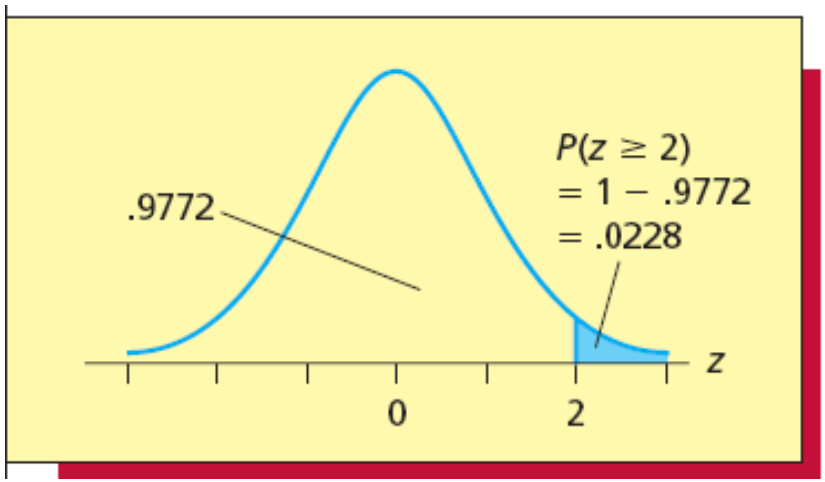
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003
−3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
−3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
−3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
−3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
−3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
−3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
−3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
−3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
−3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00103	0.00100
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183

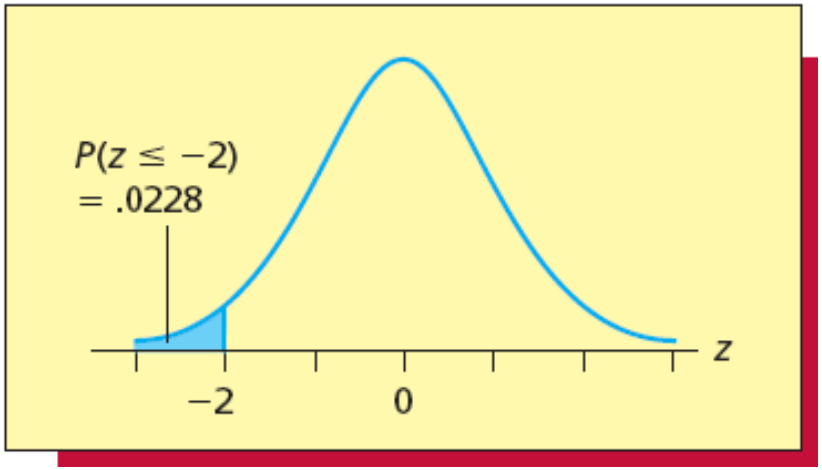
Examples



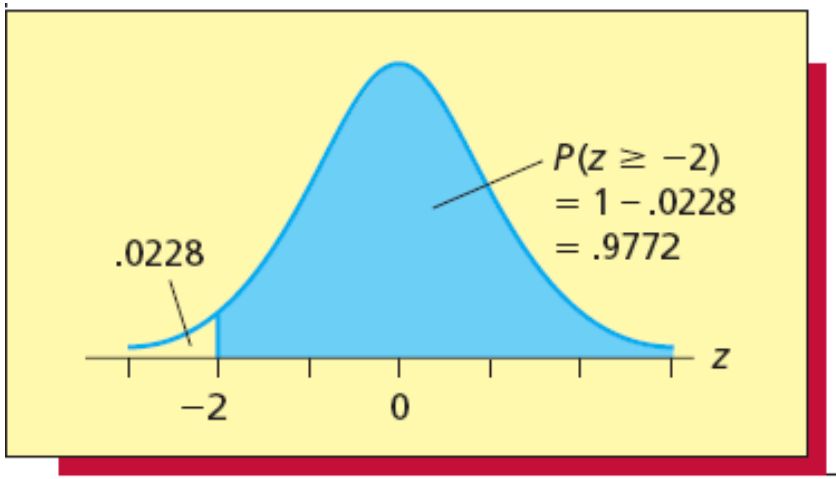


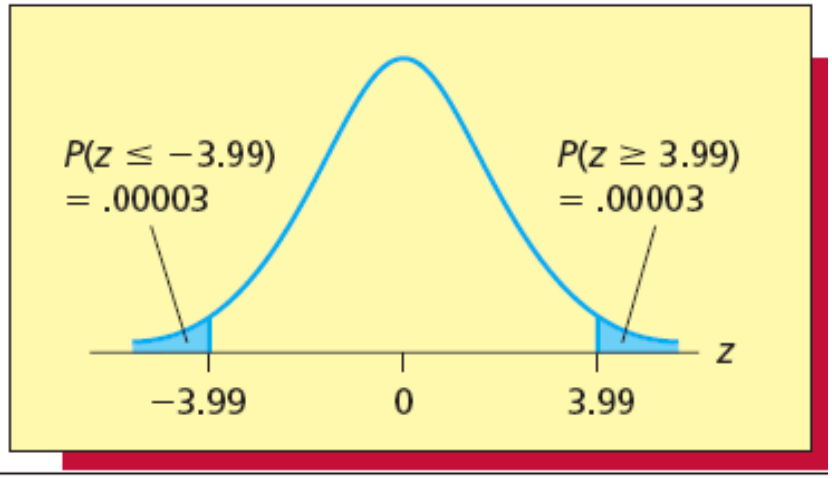
Examples Continued





Examples Continued



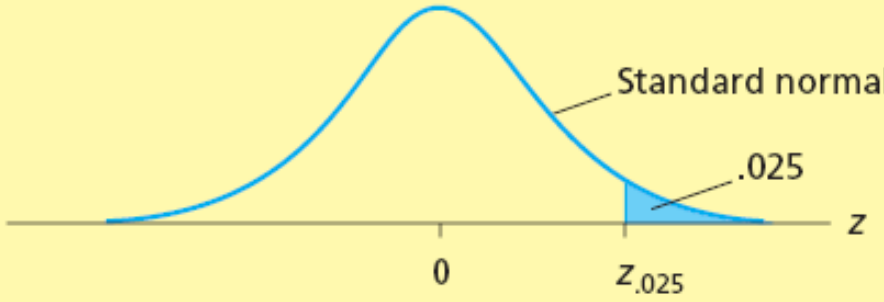


Finding Normal Probabilities

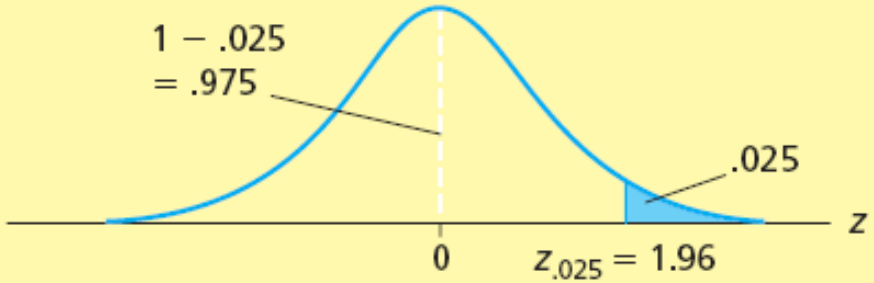
1. Formulate the problem in terms of x values
2. Calculate the corresponding z values, and restate the problem in terms of these z values
3. Find the required areas under the standard normal curve by using the table

Note: It is always useful to draw a picture showing the required areas before using the normal table

Finding a Point on the Horizontal Axis Under a Normal Curve

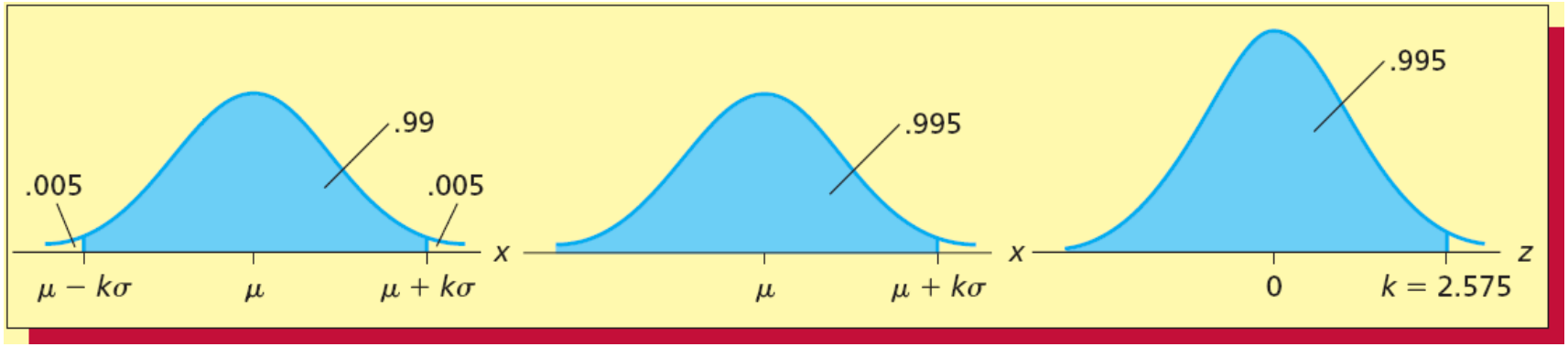


(a) $z_{.025}$ is the point on the horizontal axis under the standard normal curve that gives a right-hand tail area equal to .025

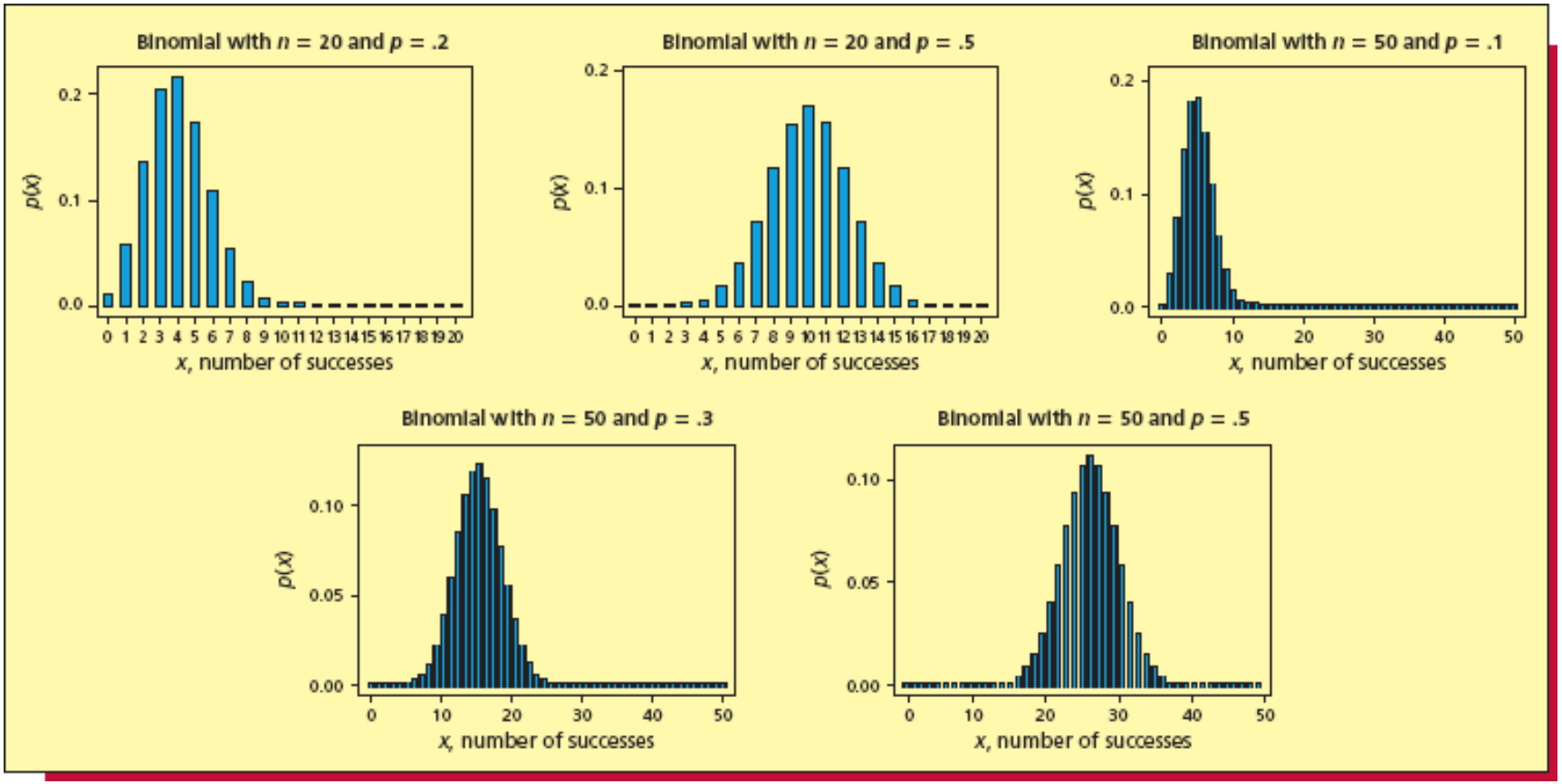


(b) Finding $z_{.025}$

Finding a Tolerance Interval



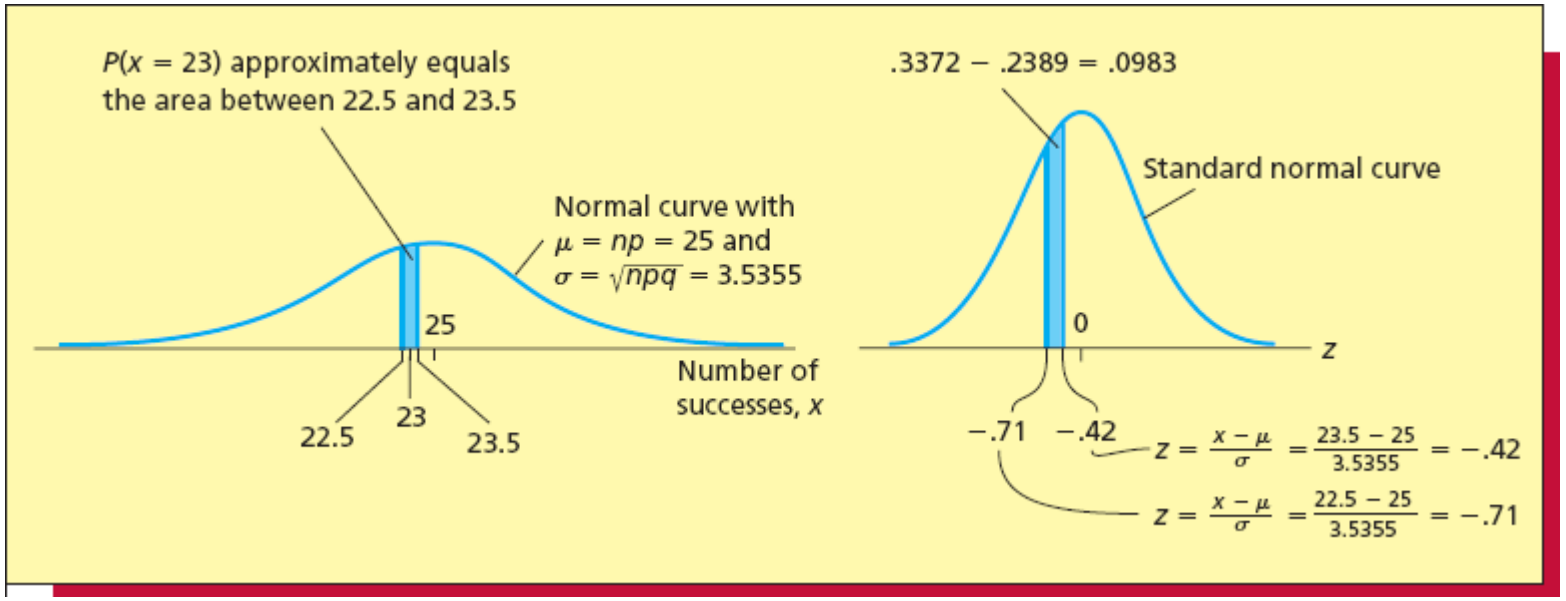
Approximating the Binomial Distribution by Using the Normal Distribution (Optional)



Normal Approximation to the Binomial Continued

- Suppose x is a binomial random variable
  - n is the number of trials
  - Each having a probability of success p
- If $np \geq 5$ and $nq \geq 5$, then x is approximately normal with a mean of np and a standard deviation of the square root of npq

Example


P(x = 23) approximately equals the area between 22.5 and 23.5
Normal curve with μ = np = 25 and σ = √npq = 3.5355
25
22.5
23
23.5
Number of successes, x
.3372 − .2389 = .0983
Standard normal curve
0
z
−.71
−.42
z = (x − μ)/σ = (23.5 − 25)/3.5355 = −.42
z = (x − μ)/σ = (22.5 − 25)/3.5355 = −.71